

# Self-assembly of virus capsids

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The Netherlands

TU/e

(1) Spherical virus capsids: HBV

See also: 'Physical Biology of the Cell' Rob Phillips et al  
T&F 2009; 2<sup>nd</sup> ed 2012.

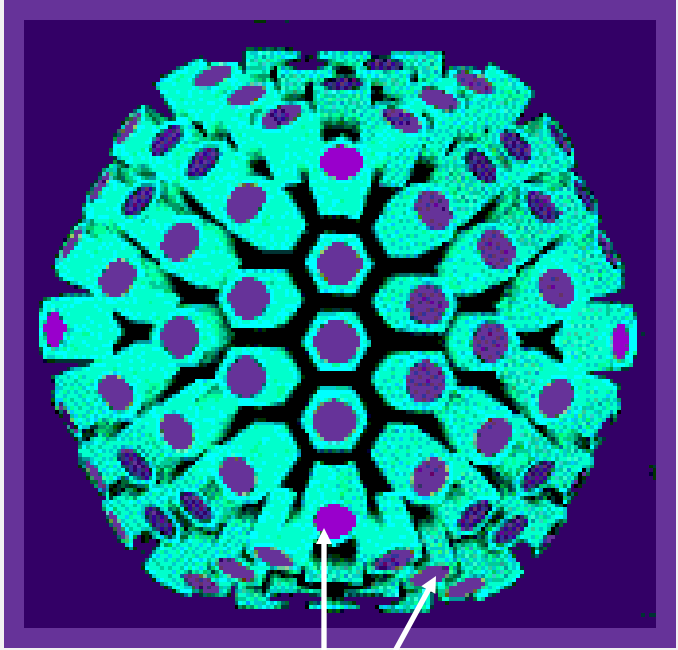
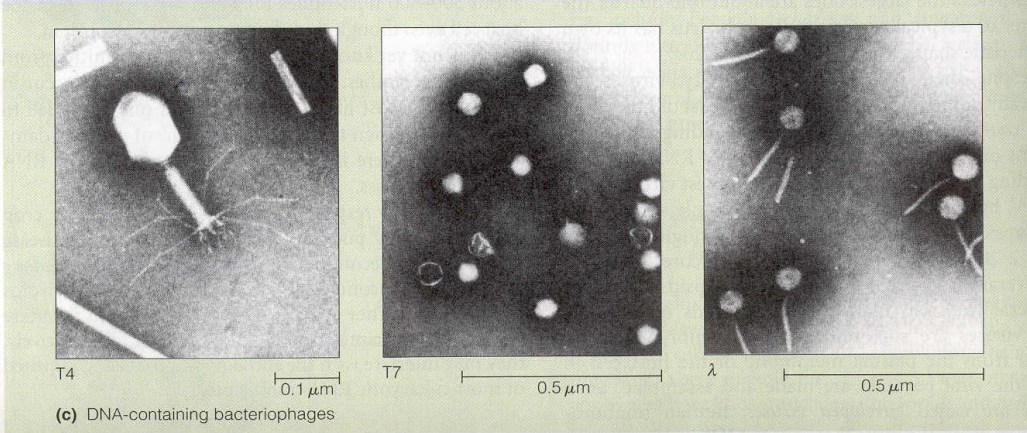
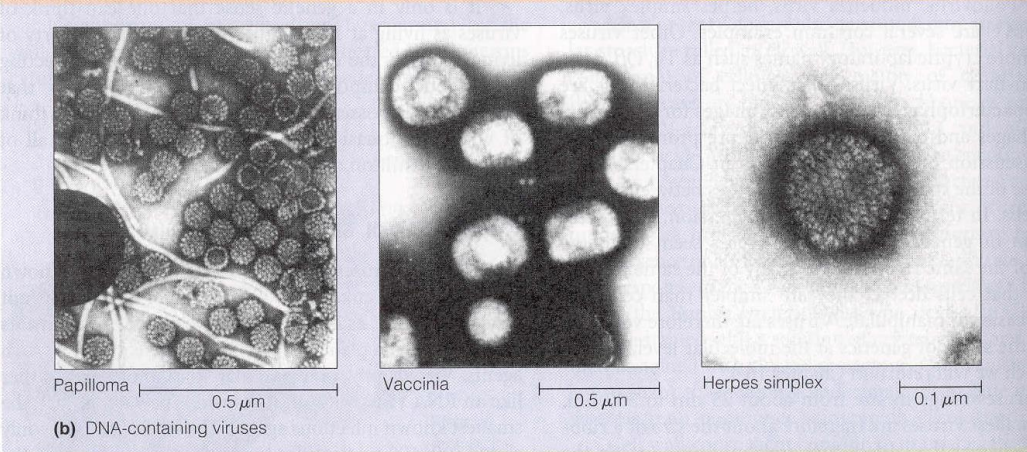
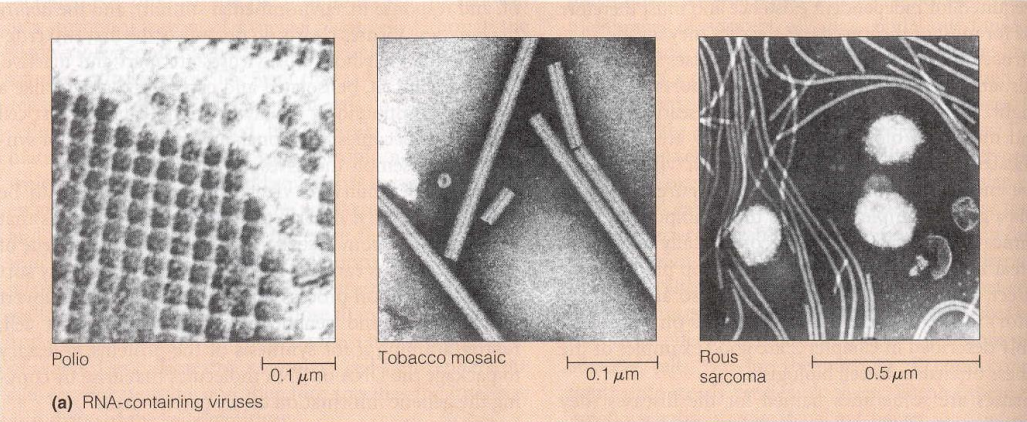
(2) (Briefly) cylindrical viruses: TMV disk-helix –  
comparing the numbers

(3) A virus shell – like structure made of colloids

# Viruses are (often) assemblies of protein subunits around RNA or DNA

(Bad news wrapped in protein)

Spherical virus **capsid**



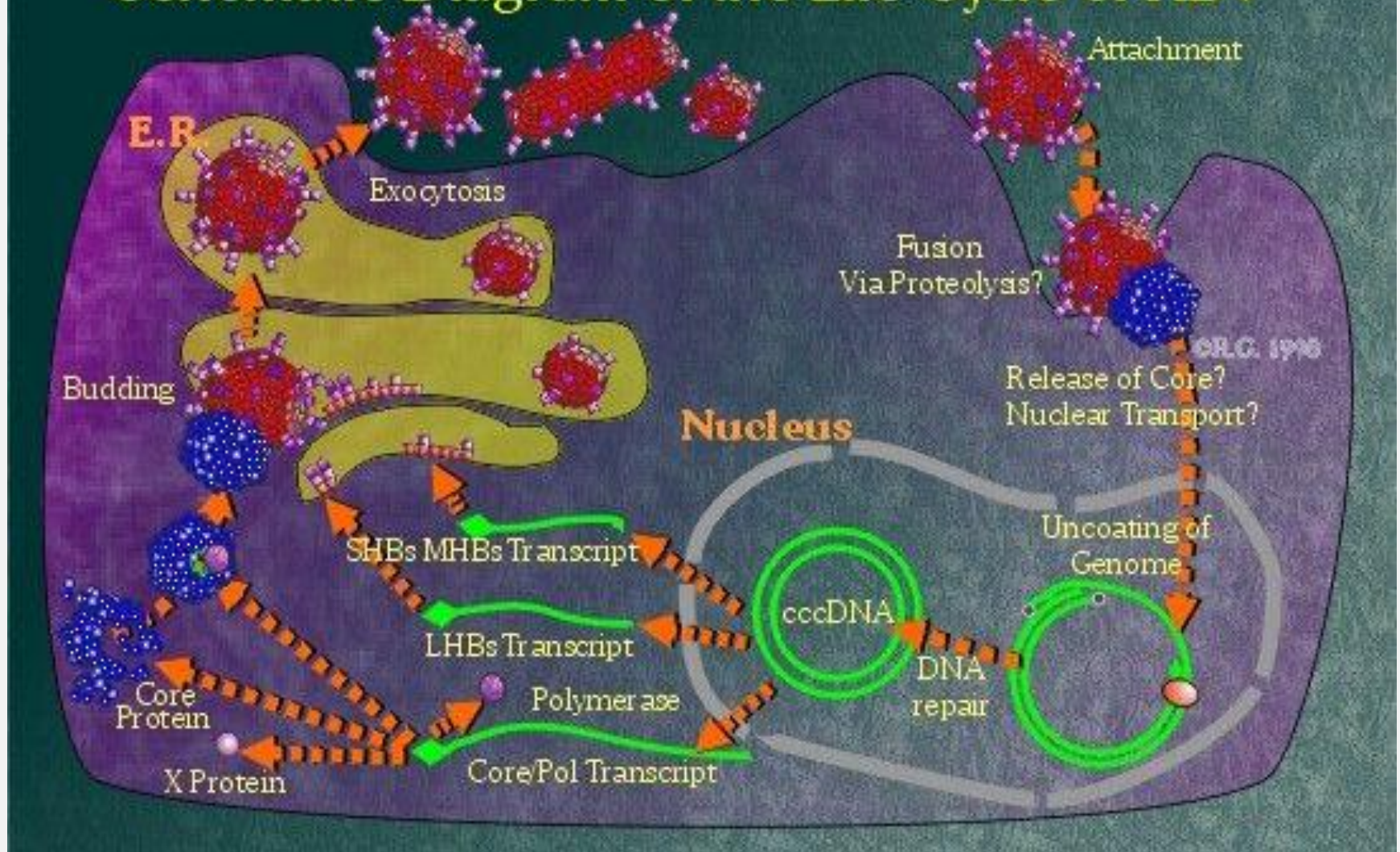
Coat protein subunits  
**capsomeres**

# Quiz

- Why subunits?

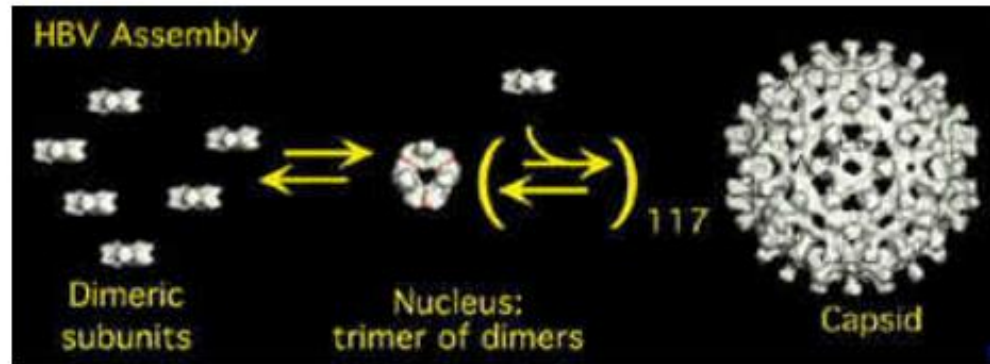


# Schematic Diagram of the Life Cycle of HBV

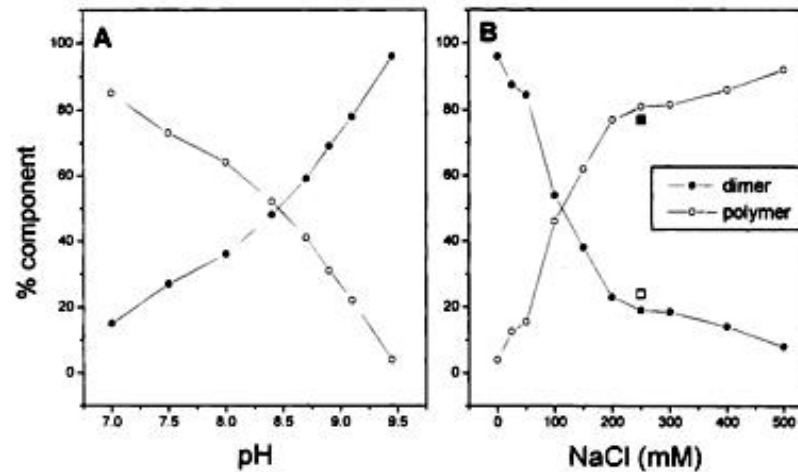


Current “paradigm”: monomers switch between 2 conformations: assembly-active and assembly-inactive state

## In vitro self-assembly of hepatitis B virus capsids



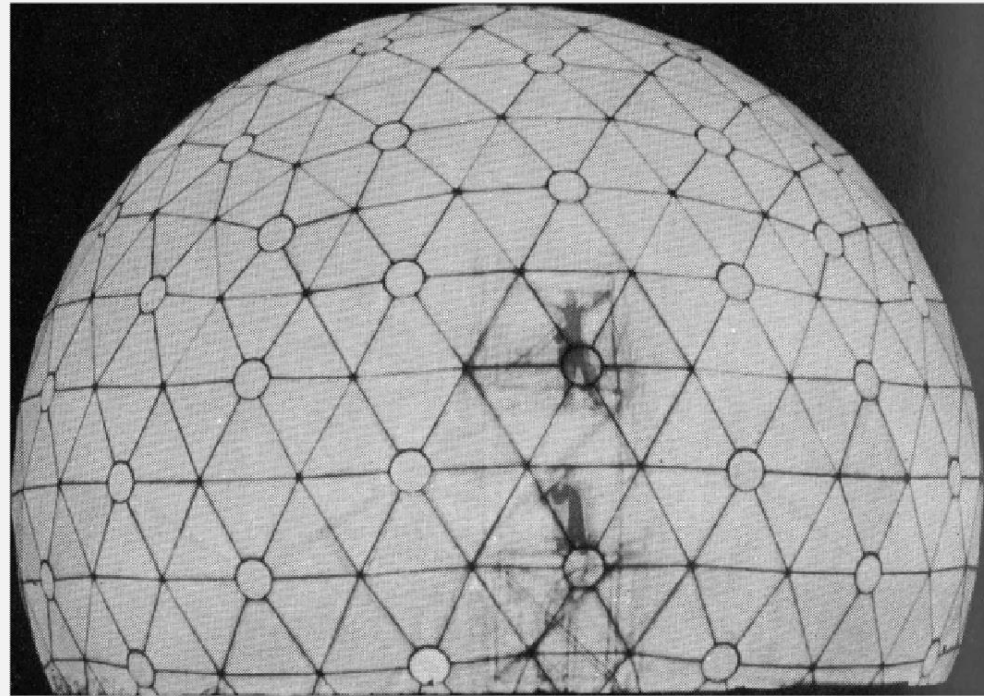
Association  
=  
Reversible!



Wingfield et al. *Biochem* 34 (1995) 4919.



## SHAPE of virus capsids [Caspar & Klug, 1962]:

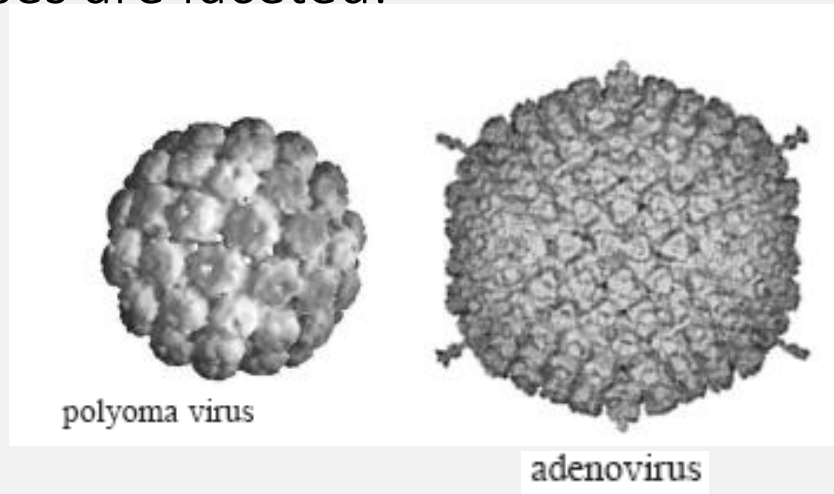


Spherical viruses are structured as miniature geodesic domes (quasi-equivalent triangular subunits arranged into groups of five and six)

“**Magic number**”  $S=60T=60(h^2+hk+k^2)$  – dodecahedron  $T=1$   
 - buckyball  $T=3$

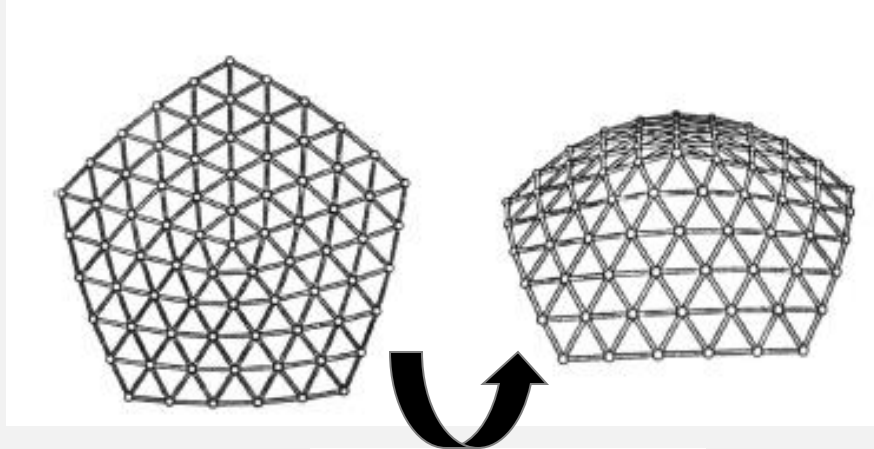
(triangle) vertices      ‘triangulation number’

Large viruses are faceted:



(not to scale)

Caused by elastic energy  $\sim R^2$  [Lidmar Mirny Nelson PRE 2003]



Buckling

Foppl-von Kármán number

$$YR^2/\kappa \geq 154.$$

Young modulus

Bending elastic modulus



Pick your target















Conformational change necessary for assembly regulation?

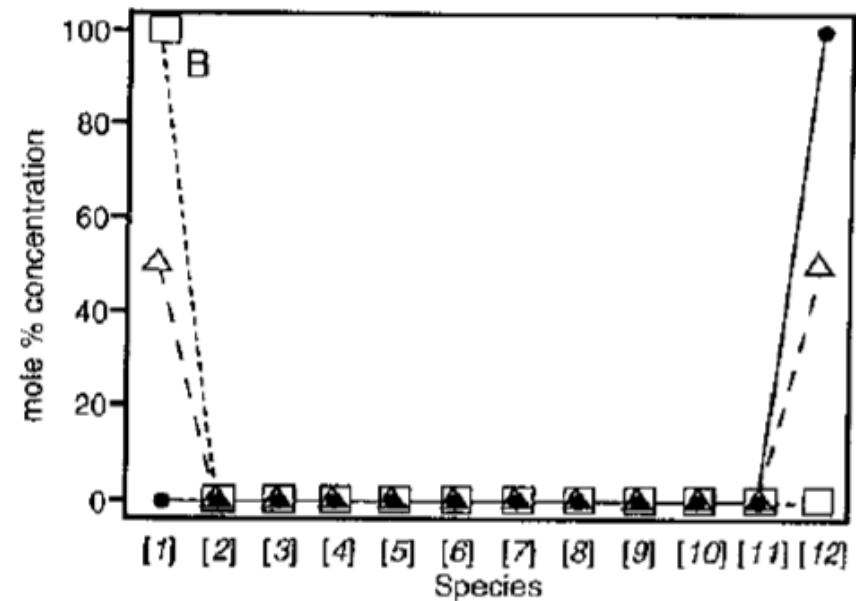
Can we get away with 'simple' physics / phys chem?

Choose level of description

# Thermodynamic reason for dominant occurrence of **closed shells** of coat proteins— example: dodecahedron (T=1)

q		# bonds/ monomer
1		0
2		1/2
3		3/3
4		5/4
5		7/5
6		10/6
7		12/7
8		15/8
9		18/9
10		21/10
11		25/11
12		30/12

Maximizing # bonds under geometrical constraints leads to either monomers OR 12-mers being present



[A. Zlotnick, J.Mol.Biol. (1994)]

Boltzmann weight  $\propto \exp[- \text{\#bonds} \times \text{bond energy}]$

# Analog: micelles (archetypical self-assembly)

## LIGHT SCATTERING IN SOAP SOLUTIONS

BY P. DEBYE\*

*Baker Laboratory, Cornell University, Ithaca, New York*

[Ann. NY Acad. Sci. **51**, 575, (1949)]

Consider the following idealized reaction between fatty ions  $A$  and micelles  $A_n$ , where  $n$  is the number of fatty ions per micelle:



If we let  $c_n$  be the concentration of micelles,  $c_1$  the concentration of unaggregated paraffin chains, and  $c$  the total concentration of fatty ions the following relationships hold:

$$\frac{c_1^n}{c_n} = K \quad (10)$$

$$c = c_1 + nc_n \quad (11)$$

$K$  is the equilibrium constant. It has been assumed that, for this simple treatment, activity coefficients are equal to unity.

The equilibrium constant  $K$  has the dimension of a concentration to the power  $(n - 1)$ . We write  $K = c_0^{n-1}$  and express our concentrations as multiples of  $c_0$ . For the relative concentrations

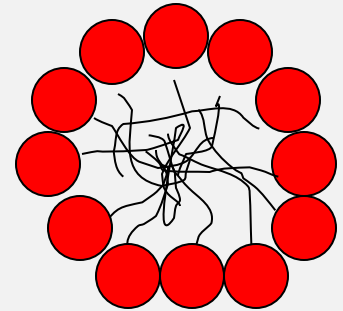
$$\gamma_1 = \frac{c_1}{c_0}, \quad \gamma_n = \frac{cn}{c_0}, \quad \gamma = \frac{c}{c_0}, \quad (12)$$

the relations

$$\gamma_1^n = \gamma_n, \quad \gamma_1 + n\gamma_n = \gamma \quad (13)$$

hold.

For very large values of  $n$ , it turns out that the relative concentration  $\gamma_1$  of the monomer is equal to  $\gamma$  for  $\gamma < 1$ . From  $\gamma = 1$  on the concentration  $\gamma_1$  remains constant. The relative concentration of the polymeric particle, on the other hand, is 0 from  $\gamma = 0$  to  $\gamma = 1$  and equal to  $\gamma - 1$  from there on. It is seen that  $\gamma = 1$  corresponds to a critical point and we shall have to identify  $c_0$  with the critical concentration.





# Modeling self-assembly of protein subunits into capsids.

## Strategy:

1. “Guess” potential of mean force between coat protein subunits
2. Use ‘all or nothing’ approach (monomers or fully assembled virus capsids);
3. Predict behavior of capsid/ monomer ratio with  $T$ , ionic strength, pH, (...); compare with experiments.

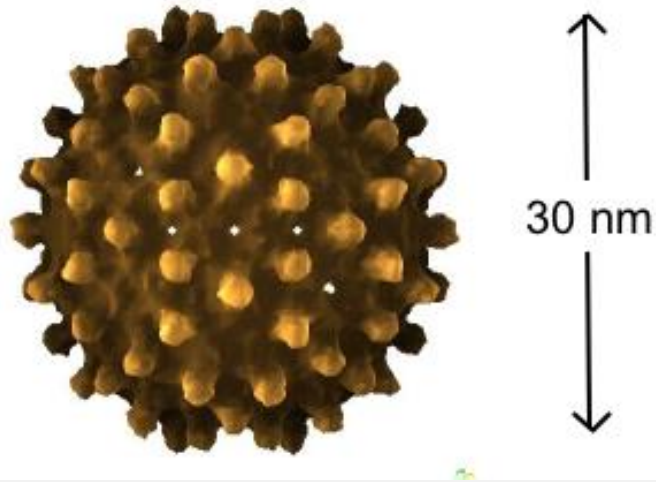
Focus on Hepatitis B virus capsids

– results expected to be more general

See: [WKK & Paul van der Schoot, Bioph. J. 2004, 2006]

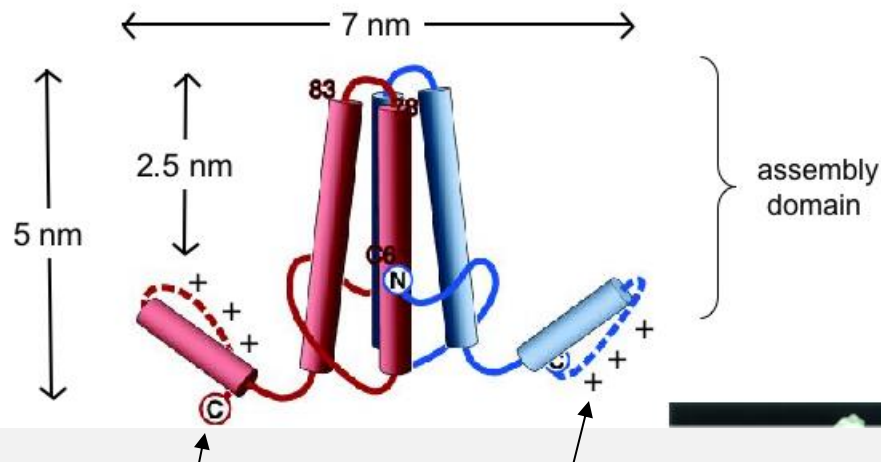
Potential applications e.g., artificial drug / gene delivery,  
controlled release,...

## Structure of the hepatitis B virus capsid



$T=4$

## Hepatitis B virus 22 kDa core protein dimer

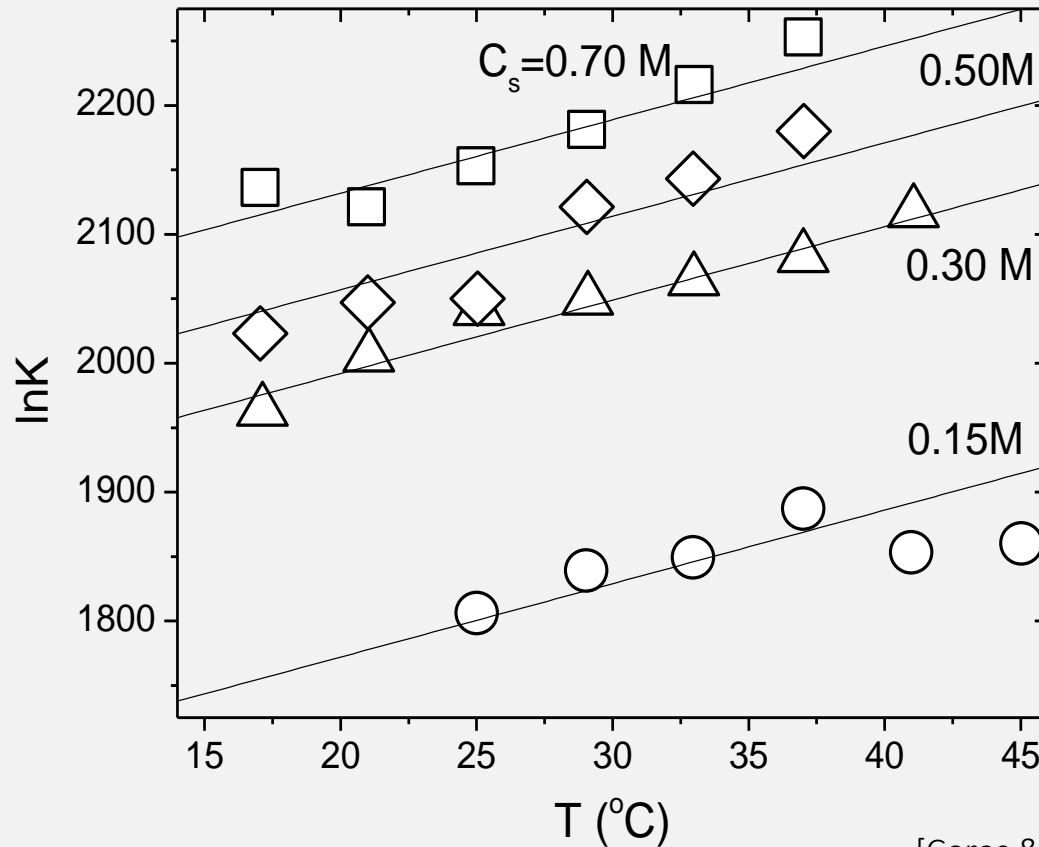
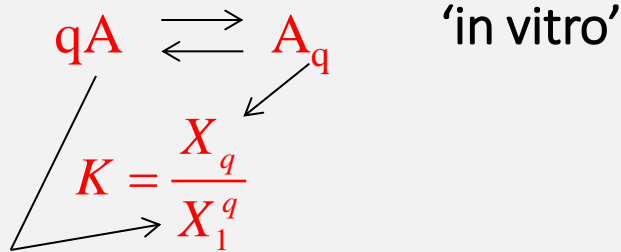


RNA-binding protamine domain  
(absent in experiments shown here)

# What's driving virus formation?

Virus in equilibrium with capsomers:

Equilibrium constant

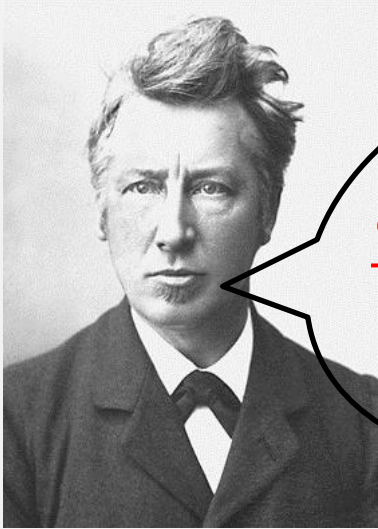


More virus:

- Higher T
- Higher ionic strength

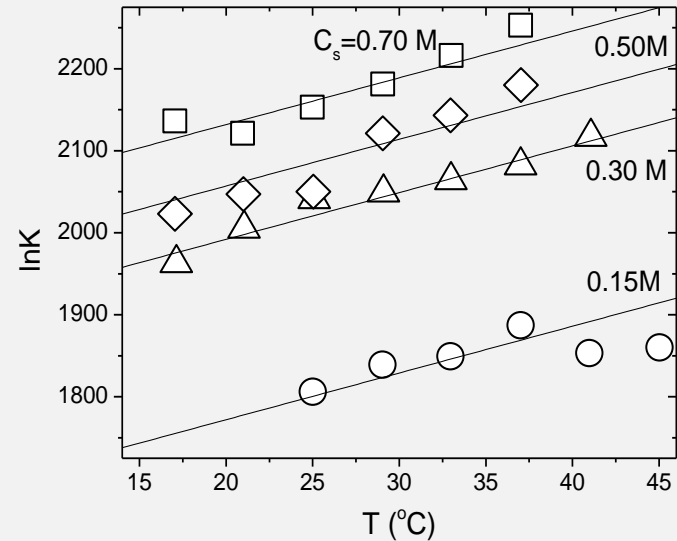


# Thermodynamics



$$\frac{d \ln K}{dT} = \frac{\Delta H^0}{kT^2}$$

Van 't Hoff law



➔  $\Delta H^0 > 0$  Virus formation is ENDOTHERMAL

$$\Delta G^0 = -kT \ln K = \Delta H^0 - T\Delta S^0$$

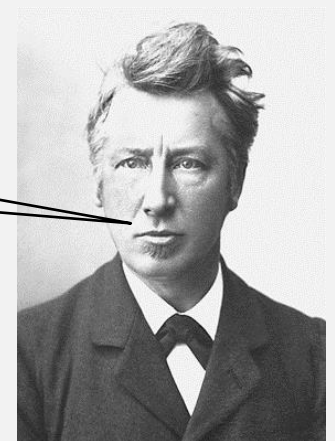
< 0

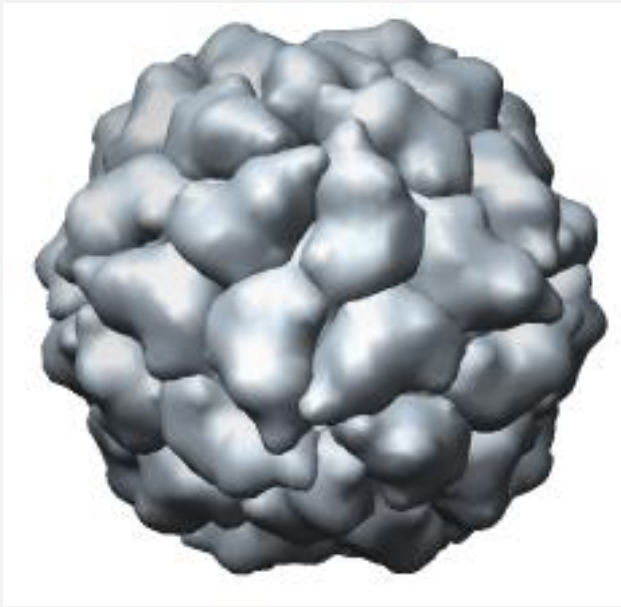
> 0



> 0

Entropy INCREASES upon virus formation!

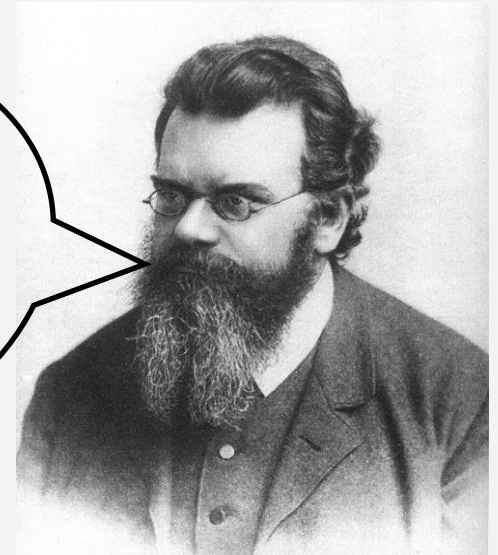




... has higher entropy than its 'free' subunits

More 'ordered' yet higher entropy

$$S = k \ln W$$



L. Boltzmann

# Realizations / complexions

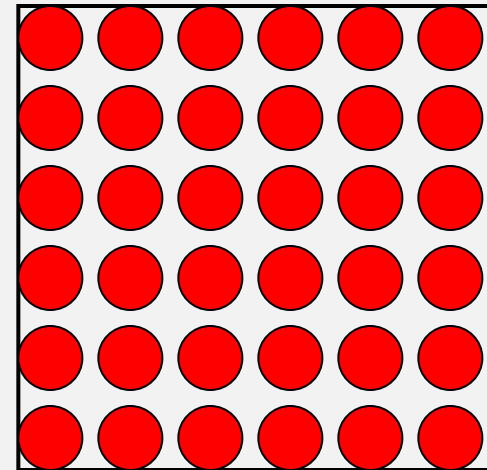
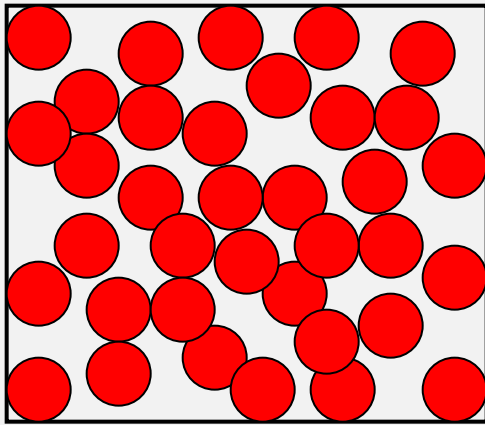
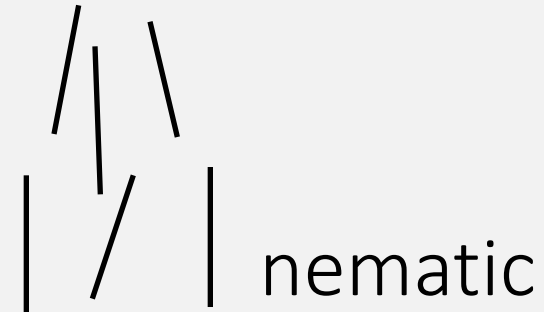
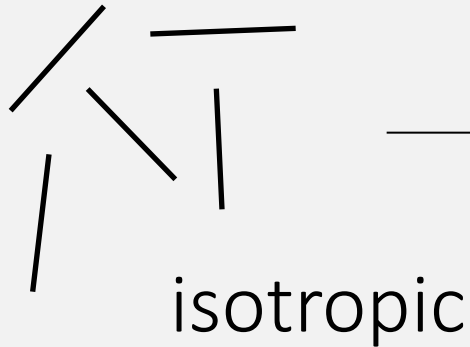
-Positions / momenta

-Distribution over energy levels

How does a virus get more entropy??

# Entropy-increase without increase of 'disorder'

Onsager 1943

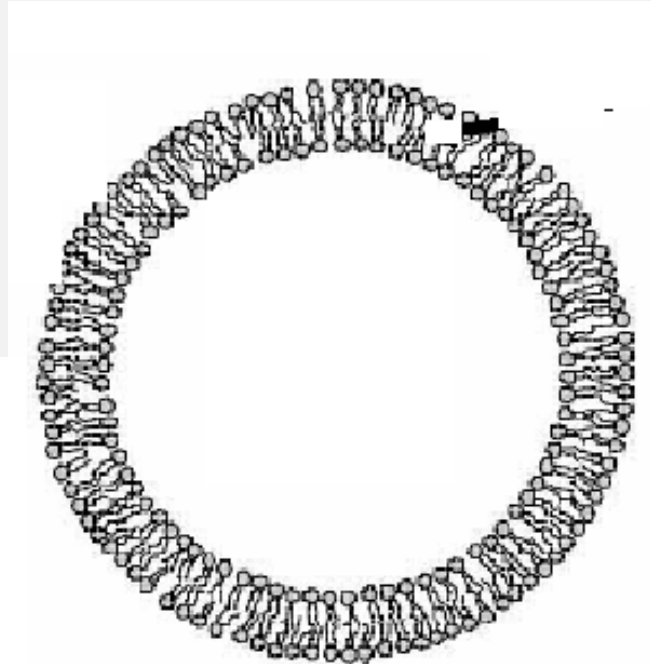
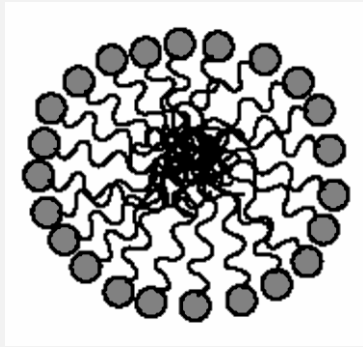


Configurational entropy high  
Translational entropy low

Configurational entropy low(er)  
Translational entropy high(er)  
 $\sim \text{Log}(\text{available volume})$

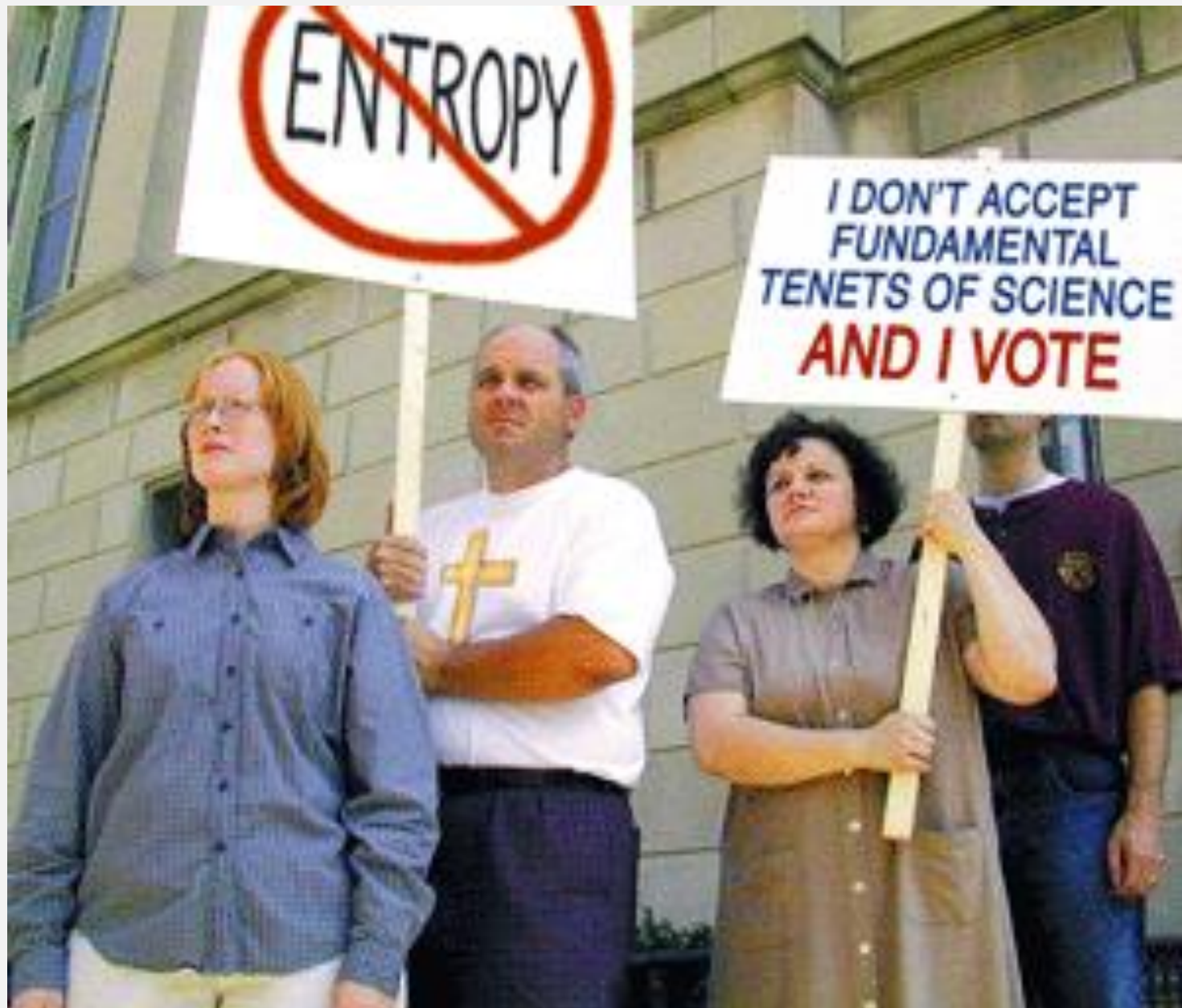


(dense) objects with larger entropy  
than their - dilute - building blocks



RNA nucleotides





‘The Onion’

# 1. Interactions between coat protein subunits: increasing attraction with temperature, ionic strength

Quiz: what interactions are expected to be important?

## Attractive

- Van der Waals  
(incl. specific ion effects)
- Hydrogen bonds
- Hydrophobic interactions
- Ion bonds

## Repulsive

- Steric
- (screened) Coulomb

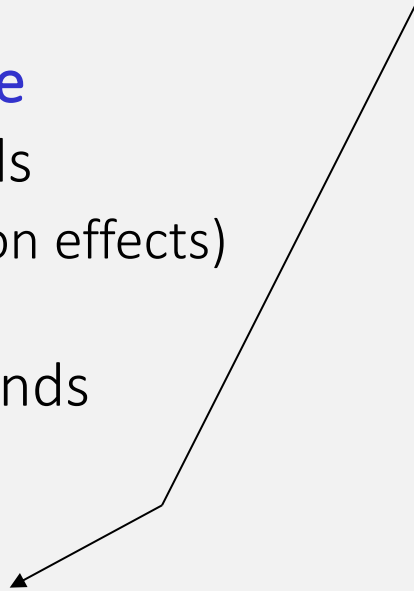
# 1. Interactions between coat protein subunits: increasing attraction with temperature, ionic strength

## Attractive

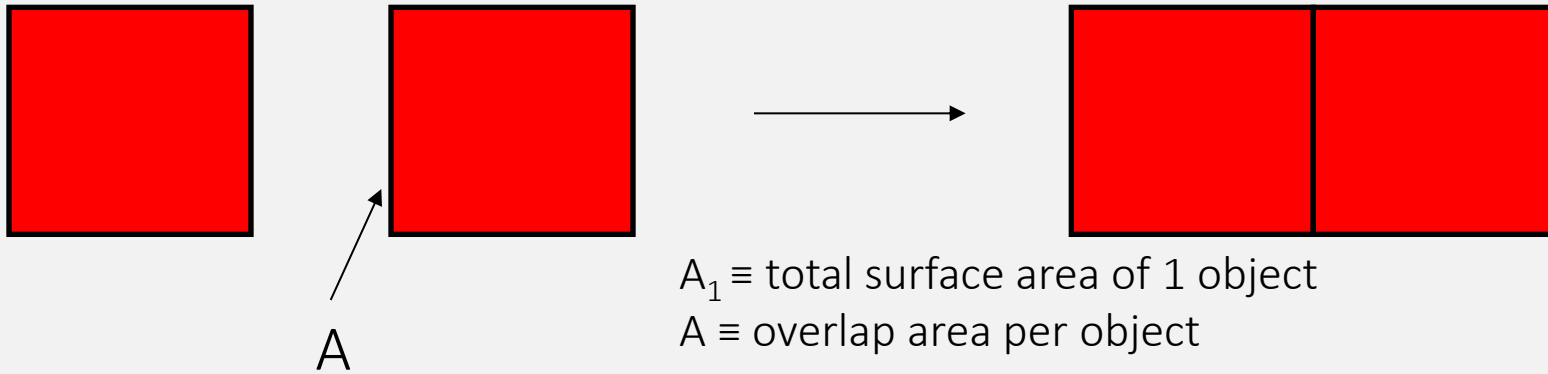
- Van der Waals  
(incl. specific ion effects)
- Hydrogen bonds
- Hydrophobic interactions
- Ion bonds

## Repulsive

- Steric
- (screened) Coulomb



# Hydrophobic interactions (macroscopic view)



surface free energy

$$F_1 = 2\gamma A_1$$

$$F_2 = 2\gamma(A_1 - A)$$

Gain upon sticking:  $G_H = F_2 - F_1 = -2\gamma A$

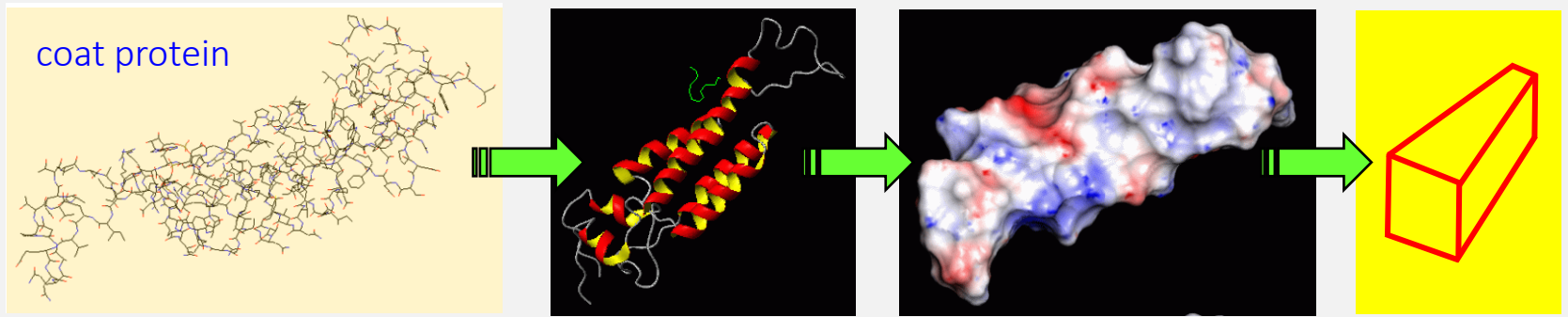
Temperature dependence:

$$\gamma(T) = \gamma(T_0) + \left( \frac{\partial \gamma}{\partial T} \right)_{T=T_0} (T - T_0) + \dots \approx \gamma(T_0) - s(T_0)(T - T_0)$$

$s \equiv$  surface excess entropy (here  $< 0$  : stronger attraction with T!).



# Coarse – graining: average out details



area  $A$   
 interfacial tension  $\gamma$   
 charge density  $\sigma$

Potential of mean force between/per subunits:  $G = G_H + G_C$   
 (objects in a “dressed vacuum”)

**Sticky part:** hydrophobic interactions

$$G_H \approx -2\gamma A e^{-x/D}$$

**Soft part:** screened Coulomb

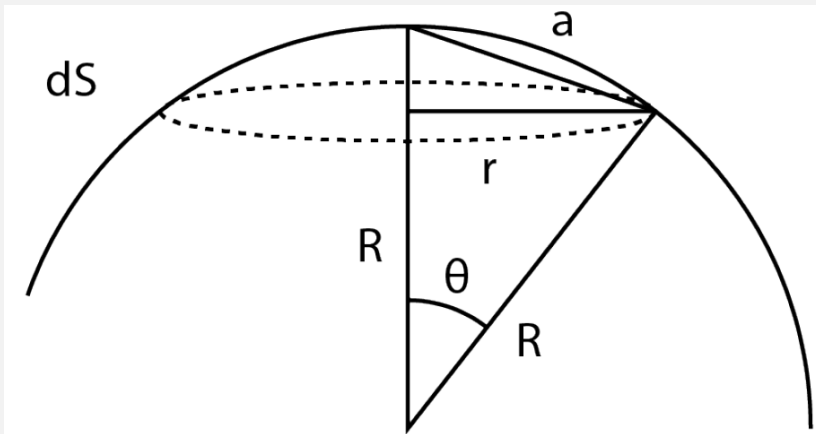
$$G_C \approx k_B T A \sigma^2 \lambda_B \kappa^{-1}$$

$\kappa^{-1} \approx 0.3 / \sqrt{c_{salt}}$

Charge density

Debye length

Bjerrum length



1 charge with all other charges:

$$G_c = \frac{kTq}{2} \int_S \rho \frac{z^2 \lambda_B}{a} e^{-\kappa a} dS,$$

Screened-Coulomb  
potential

$$\rho = \frac{q}{4\pi R^2}$$

$$\lambda_B = \frac{e^2}{4\pi\epsilon\epsilon_0 kT}$$

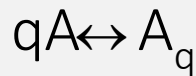
$$\kappa^{-1} \approx 0.3 / \sqrt{c_{salt}}$$

(partially) integrate etc – see handout.

Result:

$$G_c \approx AkT\sigma^2 \lambda_B \kappa^{-1}$$

## 2. Equilibrium between (q) monomers & fully assembled virus capsids



TD equilibrium:  $q\mu_1 = \mu_q$

$$\mu_i = \mu_i^0 + kT \ln x_i$$

Equilibrium constant  $K = \frac{X_q}{X_1^q} = \exp[-(\mu_q^0 - q\mu_1^0) / kT] \approx \exp[-G(q) / kT]$

Expand  $\ln K$  (*via*  $G$ ) to linear order in  $T$ ; result:

$$\ln K \approx \ln K(T_0) - \frac{qA_H h}{k_B T_0^2} (T - T_0) \quad h(T_0) = \gamma(T_0) + T_0 s(T_0)$$

Excess surface enthalpy ( $< 0$ )

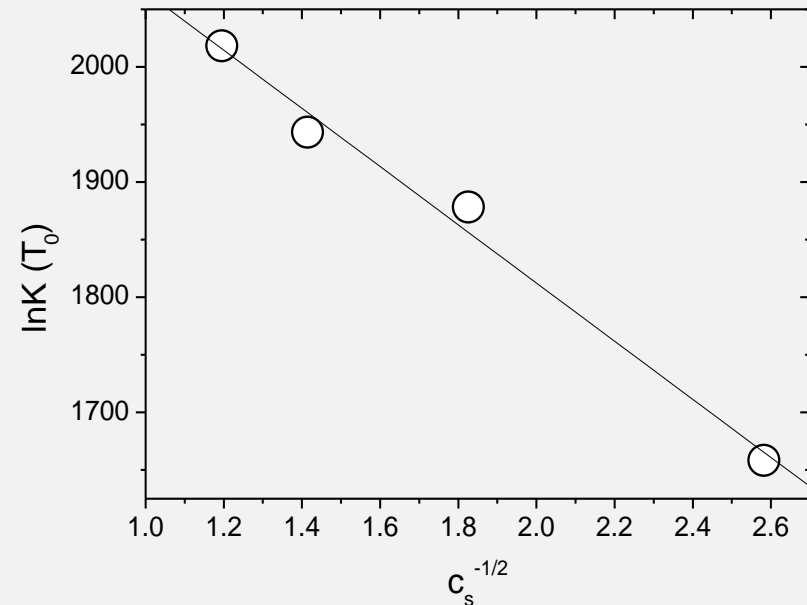
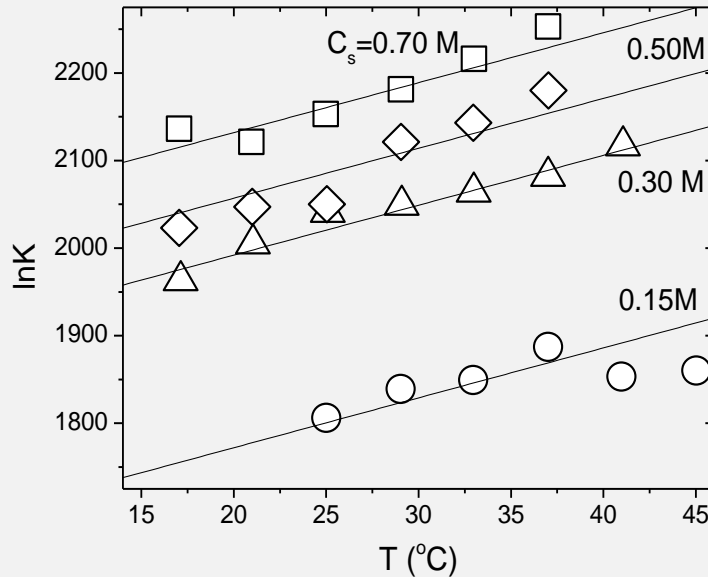
$$\ln K(T_0) = q \left( \frac{A_H \gamma}{k_B T_0} - A_C \sigma^2 \lambda_B \kappa^{-1} \right)$$

Calculate equilibrium constant

$$K = \frac{X_q}{X_1^q} \approx \exp[-G(q)/kT]$$

Predict: (1)  $\ln K \propto T$

(2)  $\ln K \propto c_s^{-1/2}$



Large  $K$  – mainly capsids:

large  $\gamma$

small  $\sigma$  – regulation by pH!

small  $\kappa^{-1} \equiv$  high ionic strength

Viruses regulate assembly – disassembly by **physical** conditions

From fits (& estimate of  $A_H$ ,  $A_C$ ):

$\gamma(T_0) \approx 5.5 \text{ mN/m}$  (comparable to water-hexanol- [Villers & Platten JPC 1988] )

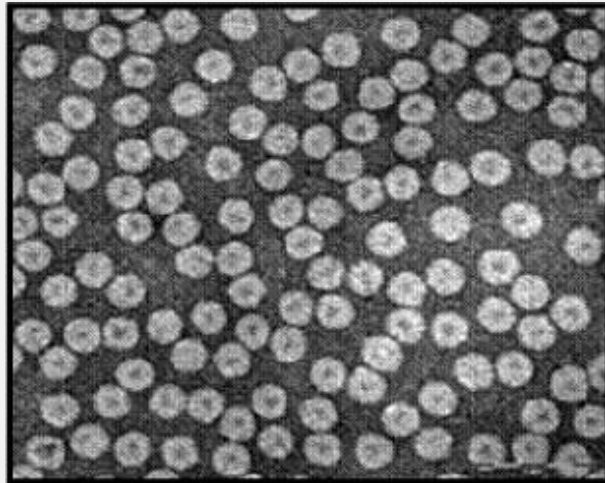
$s(T_0) \approx -0.03 \text{ mN/mK}$  (consistent with data on hydrophobic surfaces –  
[Claesson et al. J.Chem.Soc.Faraday Trans (1986)] )

$\sigma \approx 0.7e/\text{nm}^2 \equiv 7\text{-}8 \text{ charges/ subunit}$  (From titrations, estimate  $\approx 10$  charges)

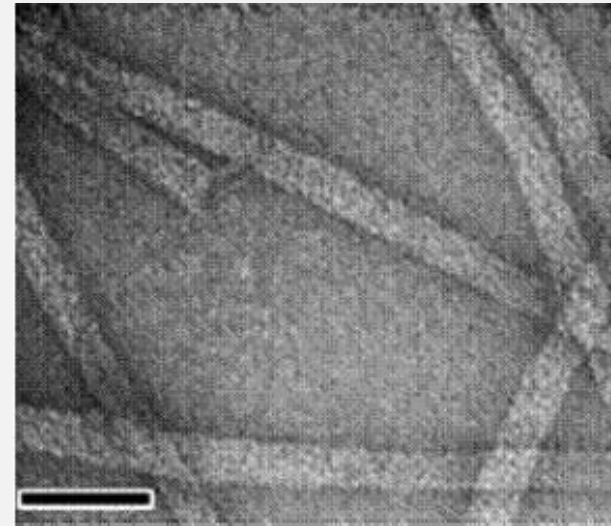
$h(T_0) \approx -3.8 \text{ mN/m} \equiv -13kT/ \text{ subunit}$  (comparable to figure for surfactant micelles)



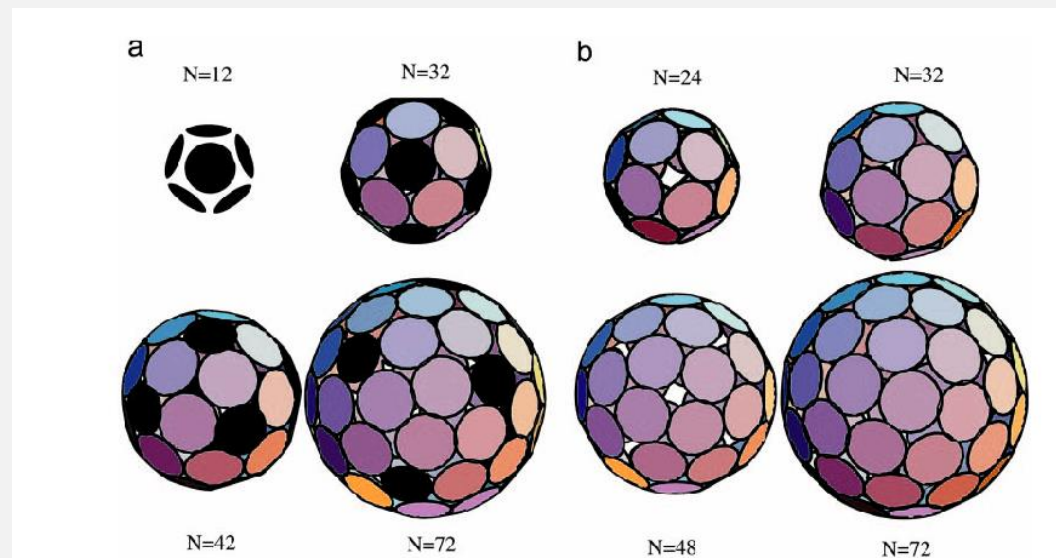
Challenge: predict 'cargo' (ccmv) influence, ...



different template



...Relative  
stability of  
T-number



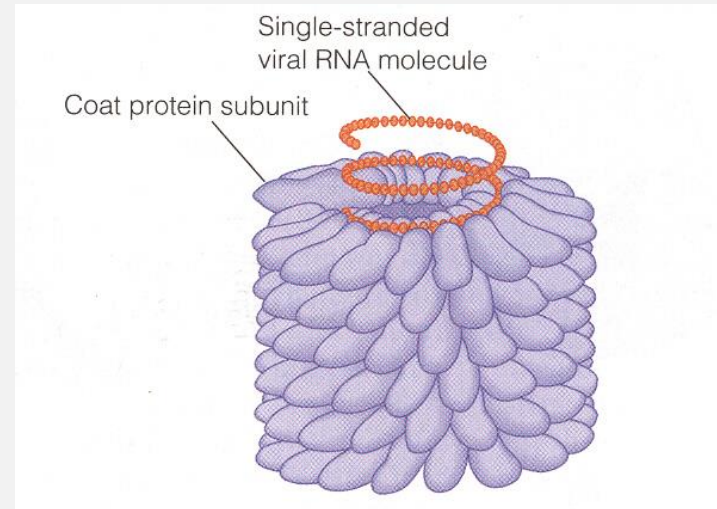
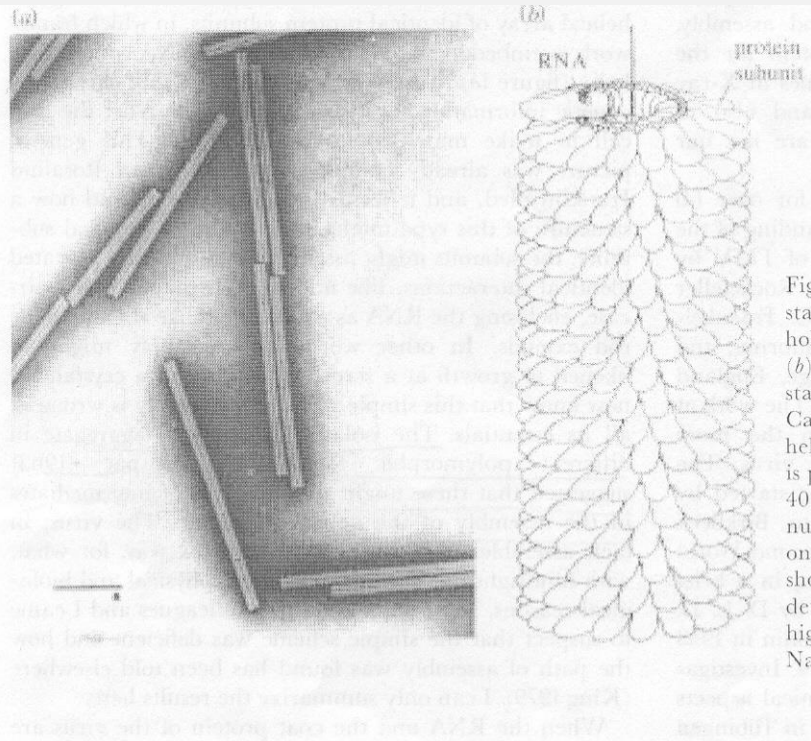
Requires fundamental insight in building block **geometry**

# Something completely different: Tobacco Mosaic Virus (TMV)

Helix length  $\sim 300$  nm

Diameter  $\sim 18$  nm

# of subunits = 2130

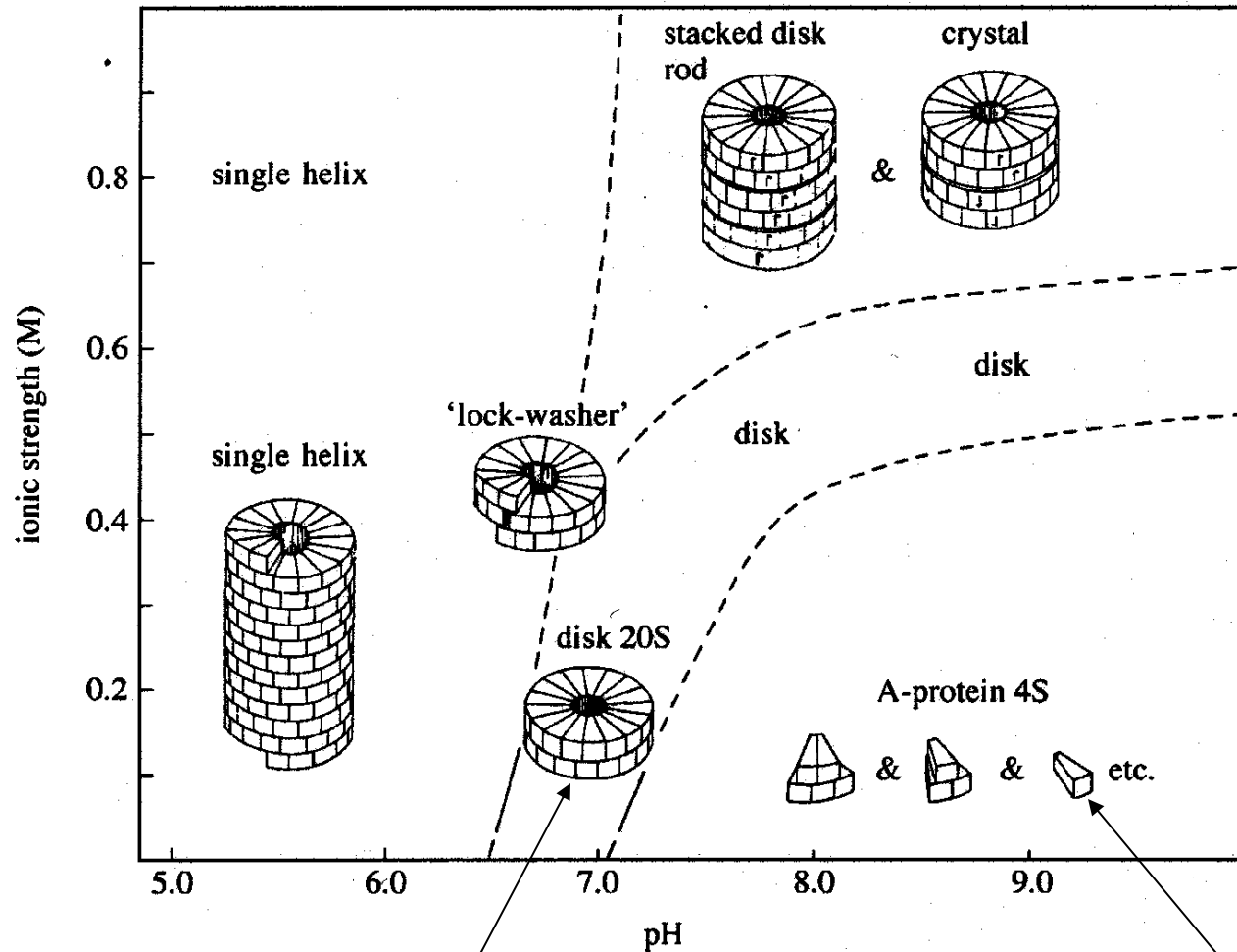


First observation of reversible virus formation: Fraenkel-Conrat & Williams, PNAS **41**, 690, (1955);

# NO RNA

TMV stability diagram [Fraenkel-Conrat & Williams, PNAS **41**, 690, (1955);  
Klug, Phil. Trans. R. Soc. B**354**, 531, (1999)]

Total conc. subunit =  $3 \times 10^{-4} \text{ M}$



Double disk of (total) 34 subunits

Taper shaped monomer;  
largest dim. ~8nm;  
smallest dim. ~2.5 nm

## Approach:

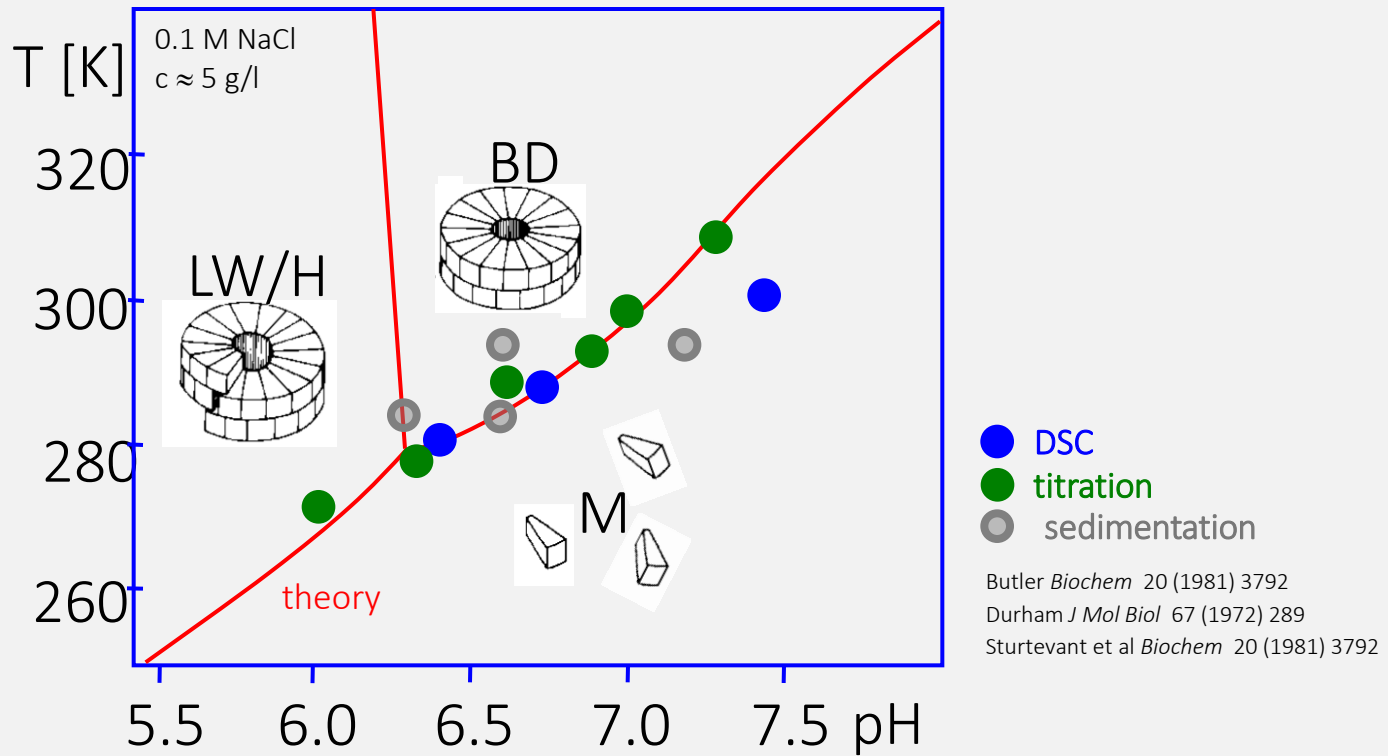
Same as HBV – assume potential of mean force:

$$G = G_H + G_C \quad (\text{bound})$$

$$G = 0 \quad (\text{unbound})$$

Additional ingredient: ‘Caspar pairs’

# Temperature and pH dependence





## Comparing HBV & TMV

	Attr. Energy $\gamma A/kT$	Excess entropy $A_s/(kT/K)$
TMV Disks	-17.3	-0.121
TMV Helices	-16.3	-0.115
HBV	-19.3	-0.107

Different virus, (almost) equal numbers....

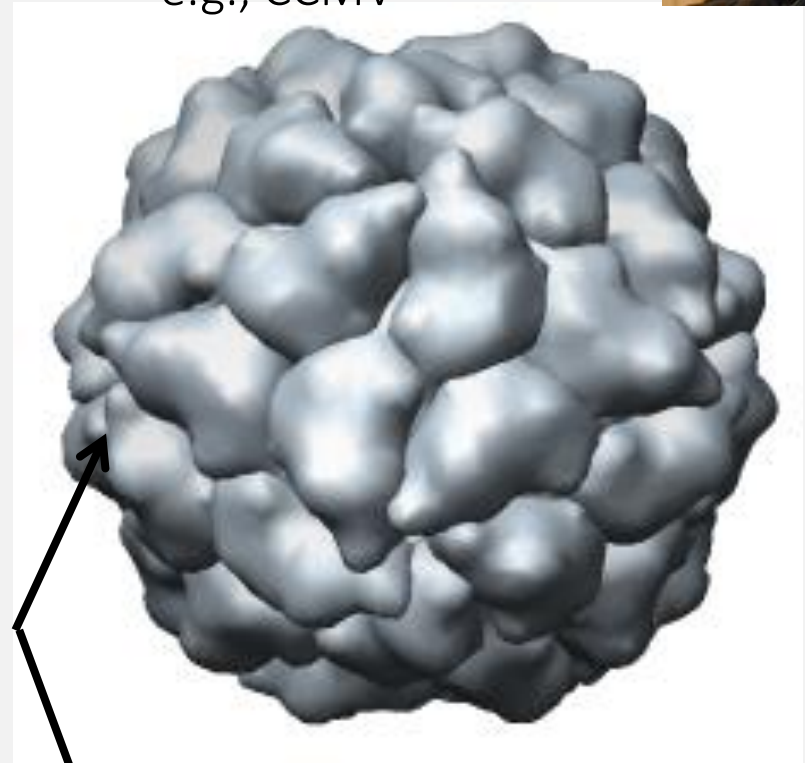
# Colloids as model systems for virus capsids

with Chris Evers



e.g., CCMV

Sorry this story is under review so no pic – it looks like the structure on the right)



a capsomer

5-fold



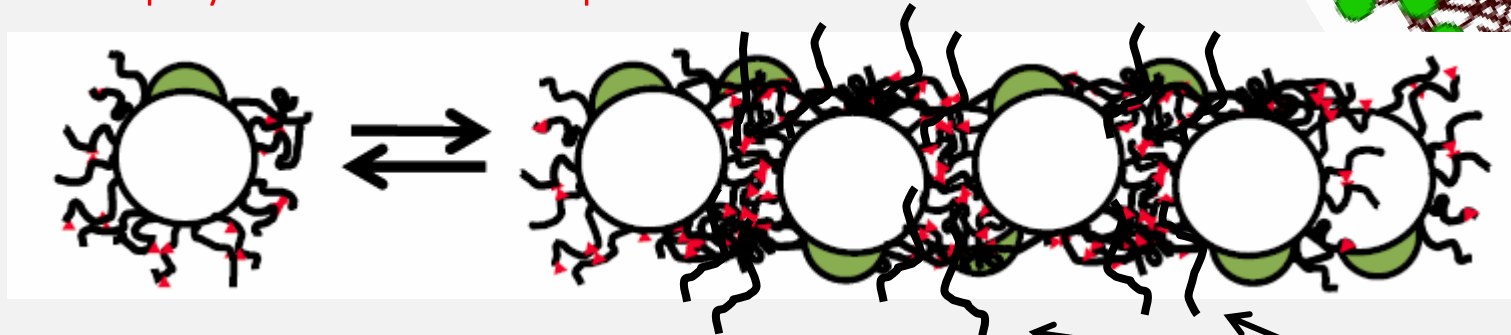
~ 30 nm

Capsid proteins are soft, asymmetric, deformable objects

-> asymmetry

-> charge + mild 'hydrophobicity' required

-> charged (PAA) floppy brush with hydrophobic bits..... **physicist view of a protein!**

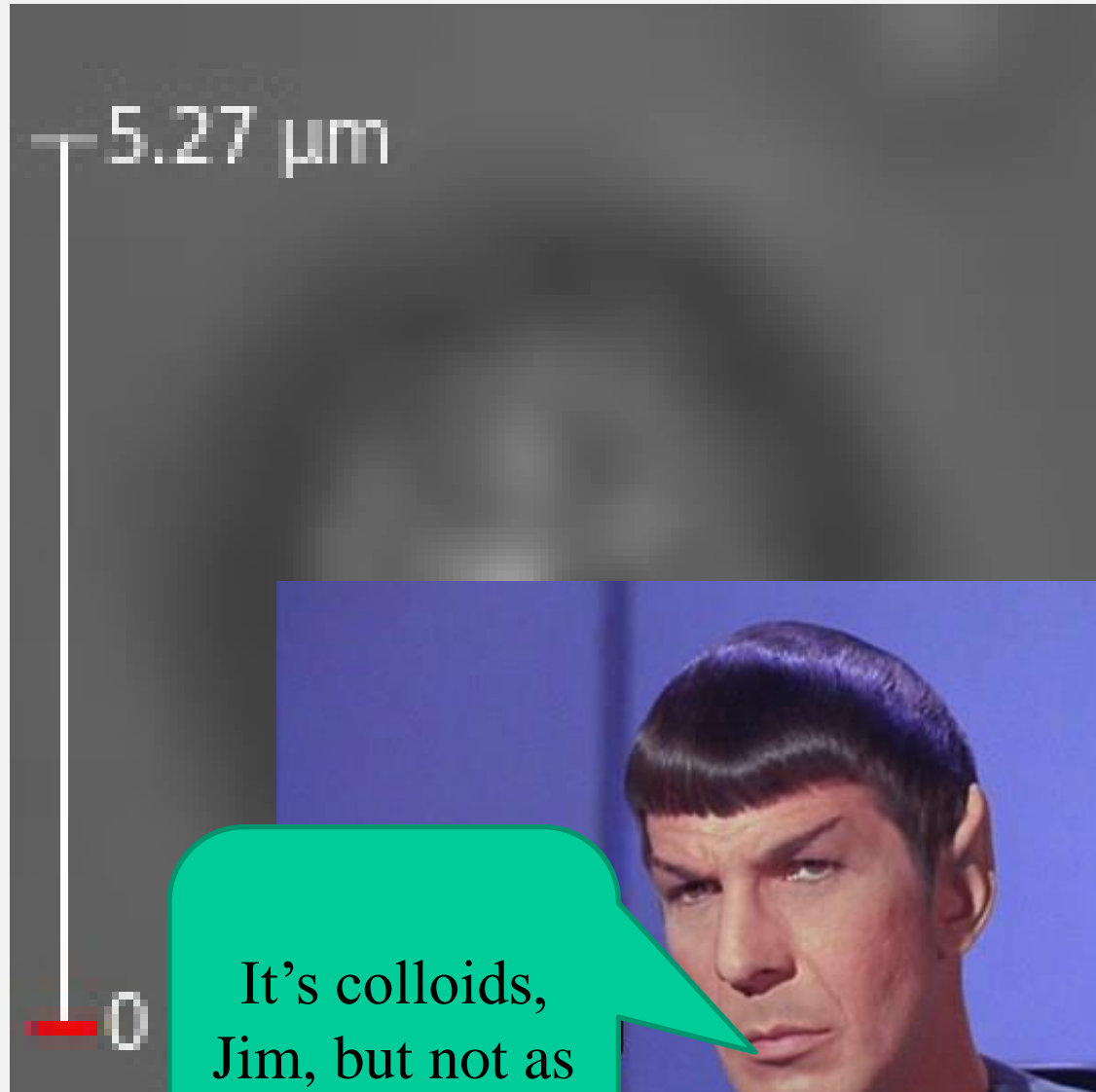
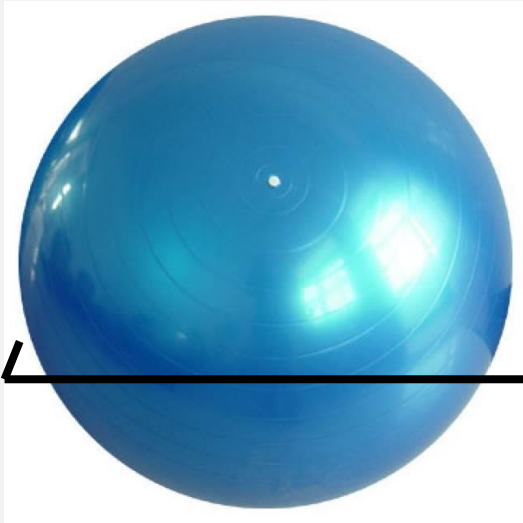


Redistribution of hydrophobic & hydrophilic parts upon approach:

(repulsive) PAA chains stick out (of plane)

(as a consequence of)

Attractive bits condense in plane

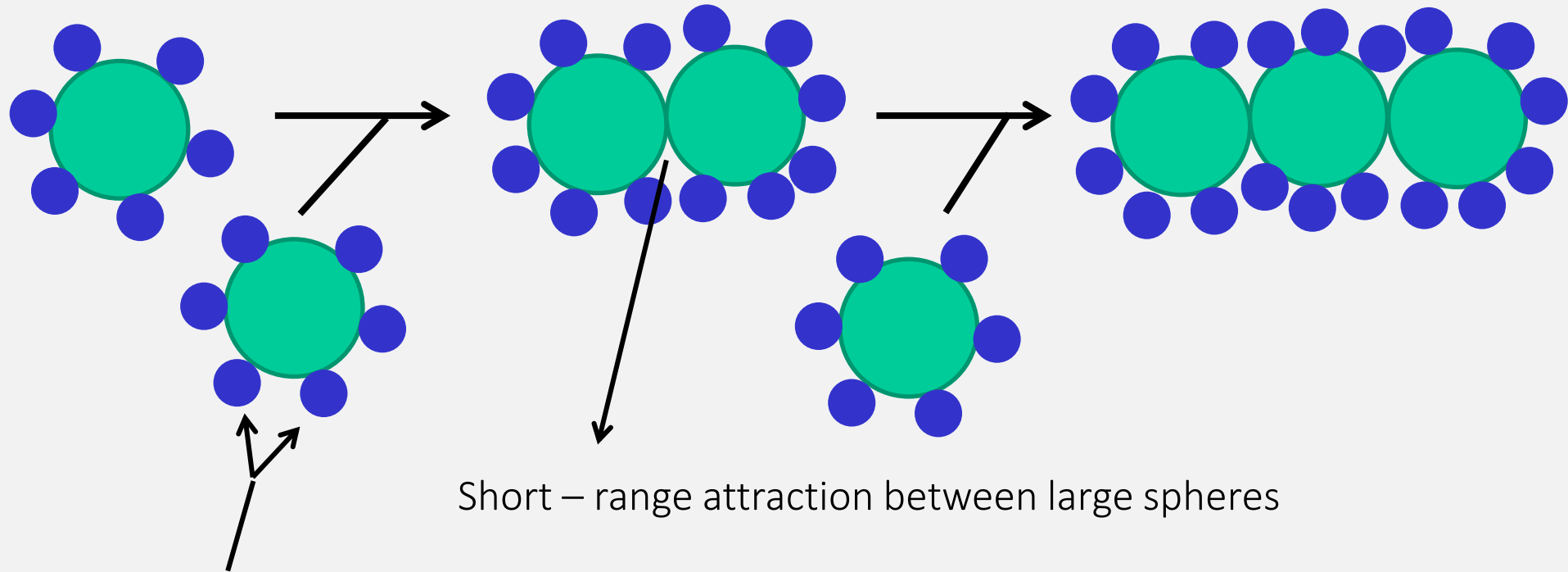


It's colloids,  
Jim, but not as  
we know it ..

RIP, Leonard Nimoy

# Toy model for ‘floppyness’ ---in computro---

– with Jurriaan Luiken & Peter Bolhuis (UvA)



Short – range attraction between large spheres

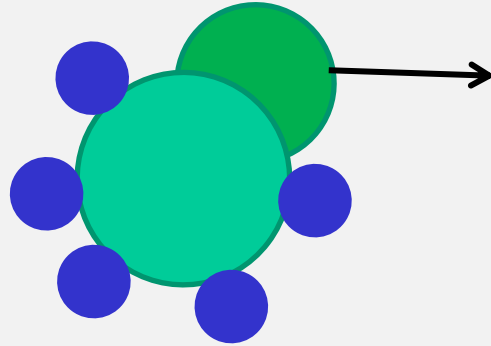
Fixed number of small ‘satellite’ spheres that move over surface of large sphere;

- Repulsion (excluded volume) between, small – large sphere.
- Small sphere interactions: (1) overlap (AO) ; (2) excluded volume

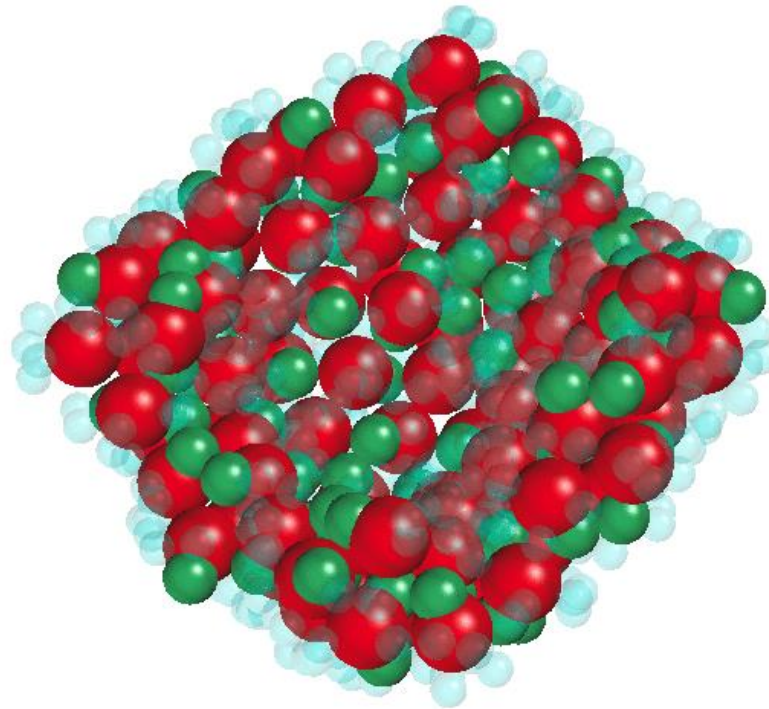
-> under appropriate conditions leads to stable FLAT sheets



Break symmetry:



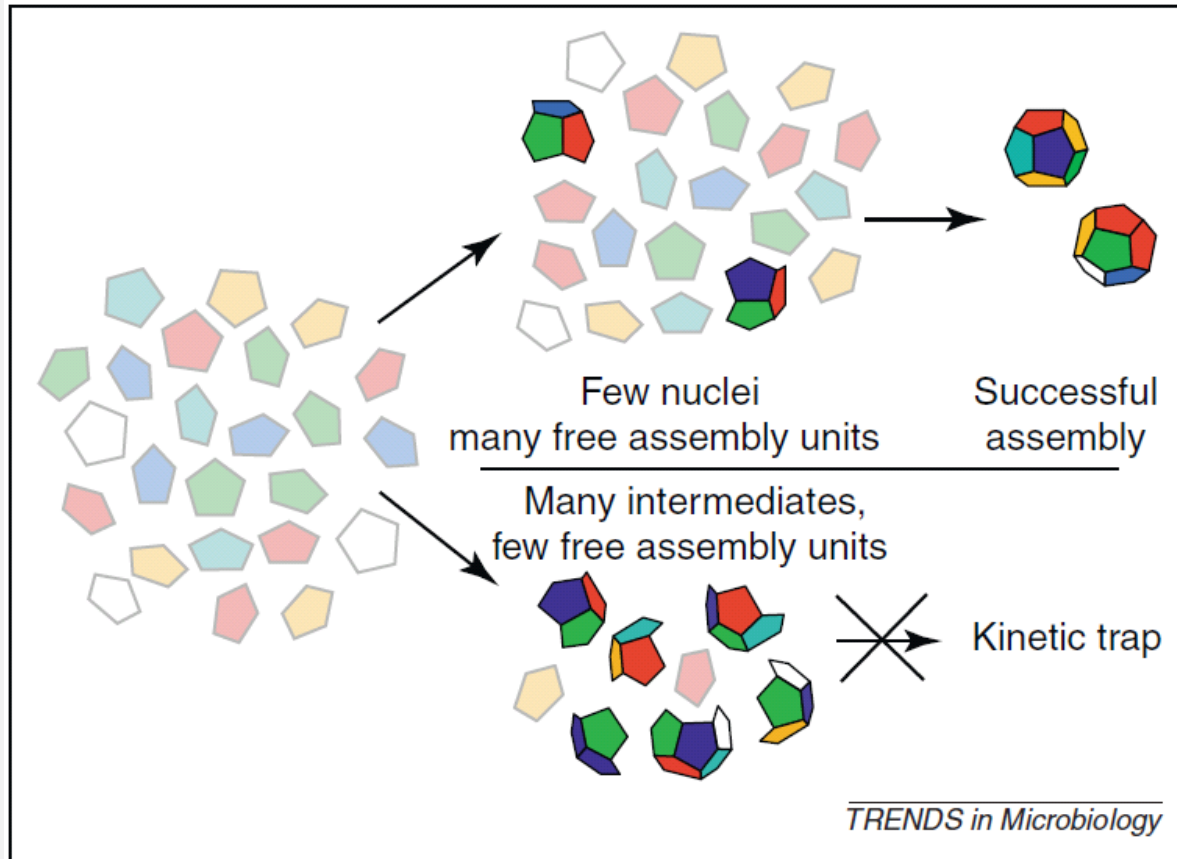
- Immobile (on particle surface)
- Excluded volume interactions (with all other objects)



-> curved sheet – globally as in colloidal shells

In progress:

Under what conditions are long-lived intermediate states expected?



[A. Zlotnick 2012]

- Take capsomer escape time as a measure for equilibration time
- Typical formation times of complete capsids  $\sim 10$  s – several minutes
- -> Non-ergodic crossover time  $\sim$  seconds

# Summary

- Hydrophobic attraction & Coulomb repulsion compete in virus capsid assembly: virus capsids are assemblies of soft, sticky objects –  
NO need to invoke conformational changes;
- Ionic strength, pH dependence by  
screened Coulomb interaction
- Typical interaction energy  $G \approx - (10 - 20) \text{ kT / monomer}$   
( $\sim - (25 - 50) \text{ kJ/ mole}$ )
- Virus –like shells form using ‘colloidal bond hybridization’ –  
new theory required.