

# Depletion forces on soft colloids

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# Outline

Introduction

Star polymer solutions with no depletion

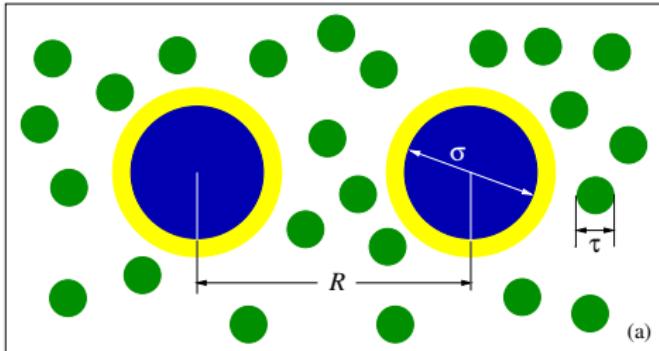
Linear polymers as depletants of soft colloids

Hard colloids as depletants of soft colloids

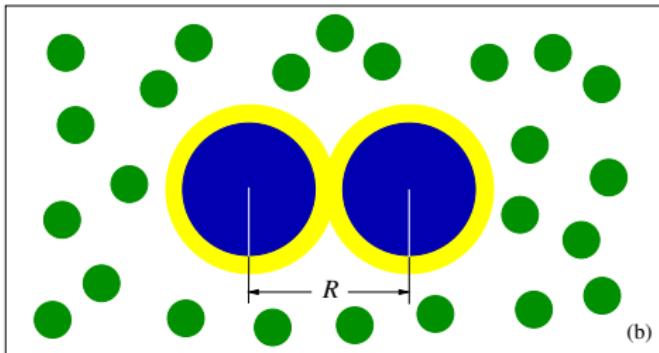
Conclusions

# Introduction

- ▶ The classical Asakura-Oosawa depletion picture



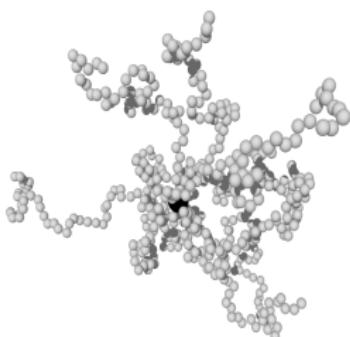
(a)



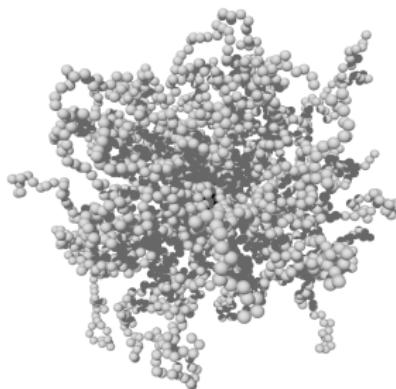
(b)

# Star polymers

- ▶ Chemically anchor  $f$  polymers on a common center
- ▶  $f$  is called *functionality* of the star
- ▶ Below:  $f = 10$  (left),  $f = 50$  (right)

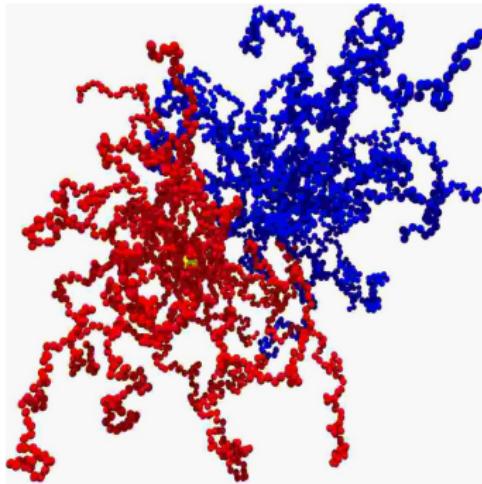


(a)



(b)

# One-component star polymer solutions



- ▶ Center-to-center effective interaction<sup>1</sup>

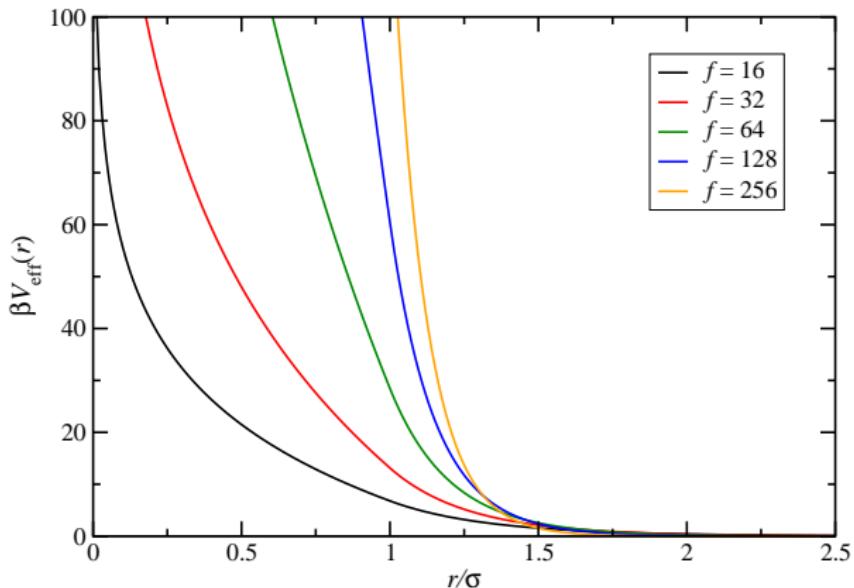
$$\beta V(r) = \frac{5}{18} f^{3/2} \begin{cases} -\ln\left(\frac{r}{\sigma}\right) + \frac{1}{1+\sigma\kappa} & \text{for } r \leq \sigma \\ \frac{1}{1+\sigma\kappa} \frac{\sigma}{r} \exp[-\kappa(r-\sigma)] & \text{for } r > \sigma, \end{cases}$$

where  $\kappa = \sqrt{f}/(2\sigma)$  and  $\sigma \cong 1.3 R_g$ .

<sup>1</sup>CNL *et al.*, Phys. Rev. Lett. **80**, 4450 (1998)

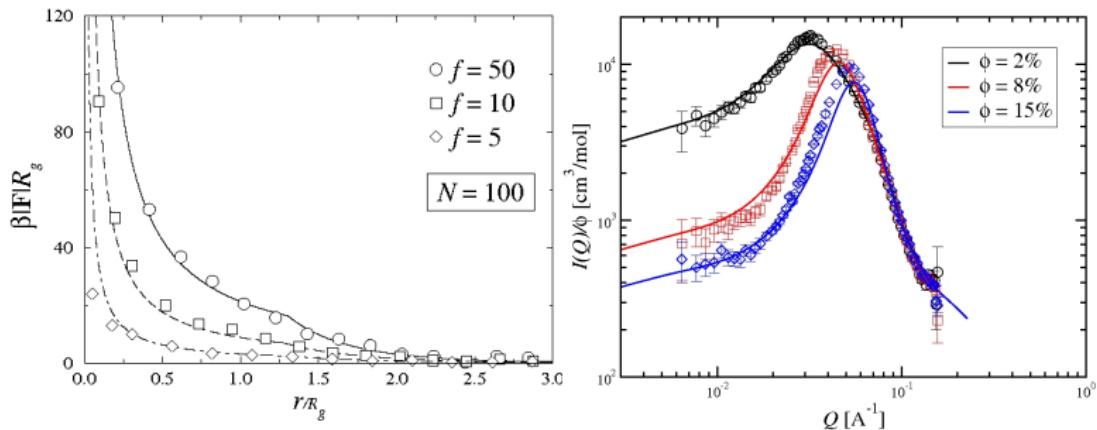
# Effective interaction

- Dependence on  $f$ : Bridging between polymer and hard-sphere behavior



# Effective interaction

- ▶ Comparison with simulations<sup>2</sup> and SANS<sup>3</sup>

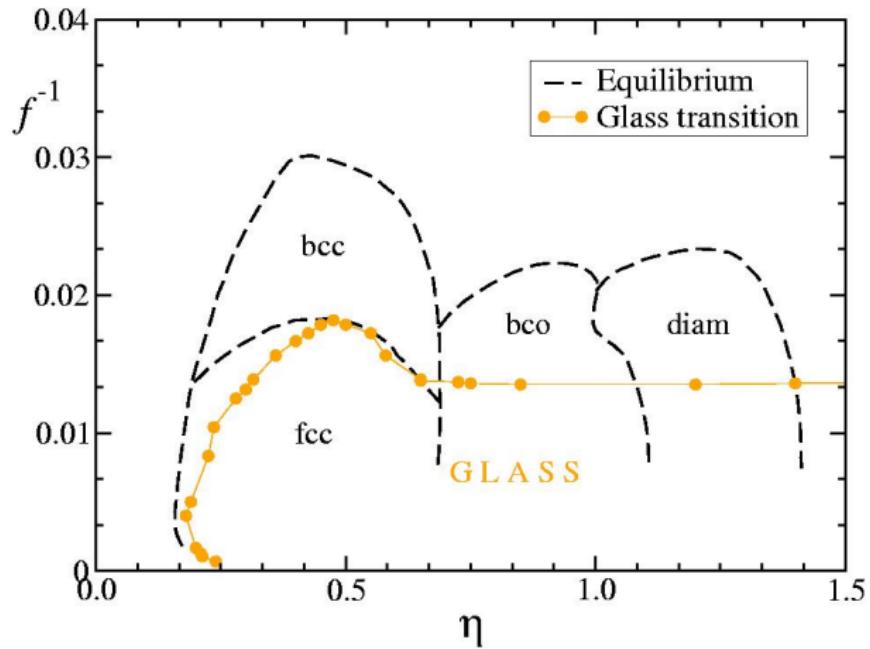


<sup>2</sup>A. Jusufi *et al.*, Macromolecules **32**, 4470 (1999).

<sup>3</sup>CNL *et al.*, Phys. Rev. Lett. **80**, 4450 (1998).

# Phase behavior

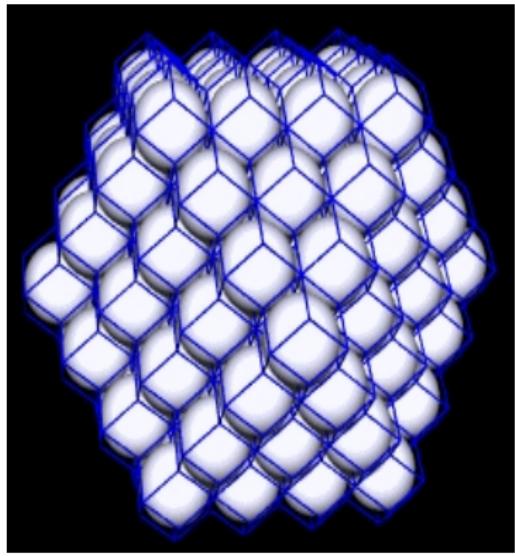
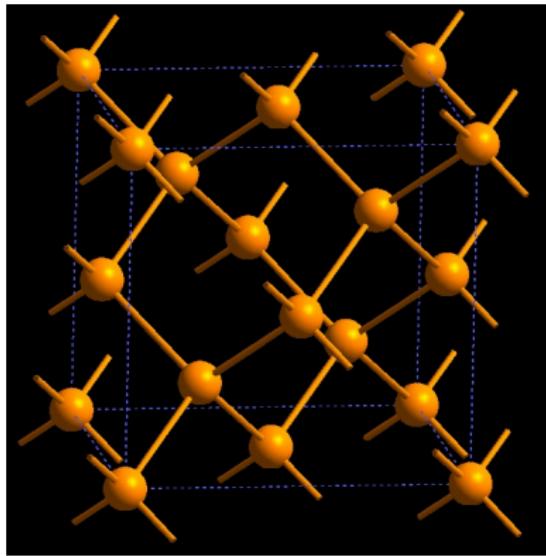
- ▶ Equilibrium<sup>4</sup> and glass<sup>5</sup> phase diagrams



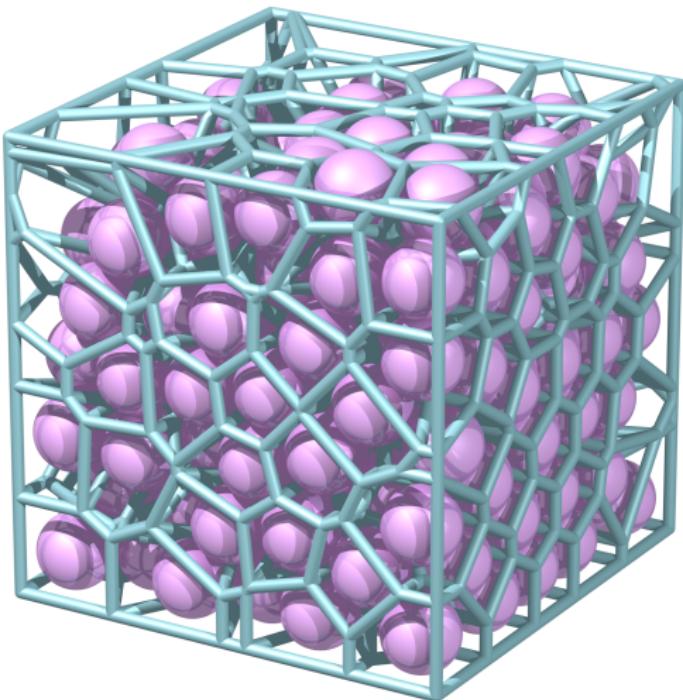
<sup>4</sup>M. Watzlawek *et al.*, Phys. Rev. Lett. **82**, 5289 (1999)

<sup>5</sup>G. Foffi *et al.*, Phys. Rev. Lett. **90**, 238301 (2003)

# Crystals ...

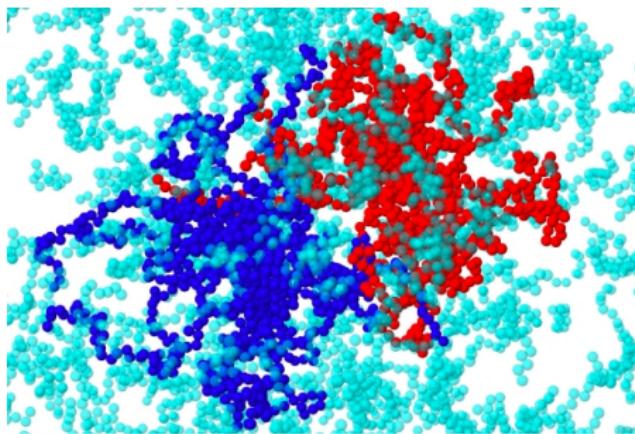


... and glasses



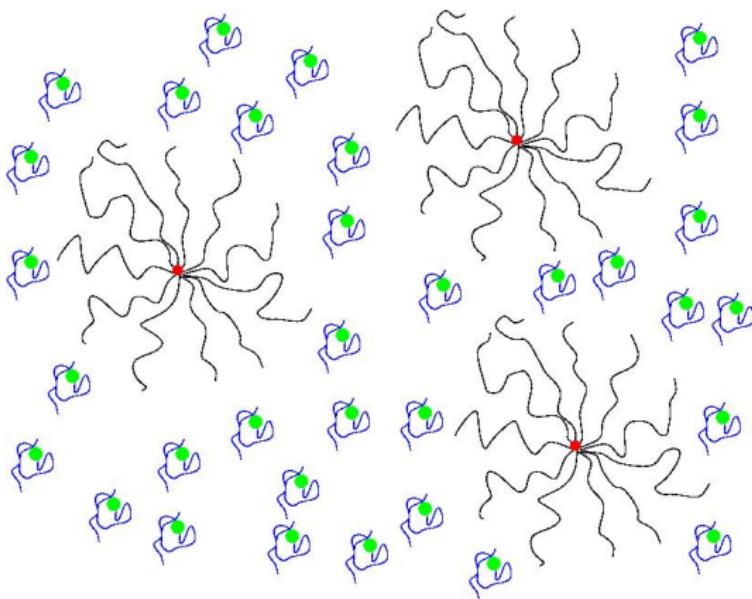
# Star-chain mixtures

- ▶ Add some homopolymer chains
- ▶ Why? Sheer curiosity!



# Star-chain mixtures

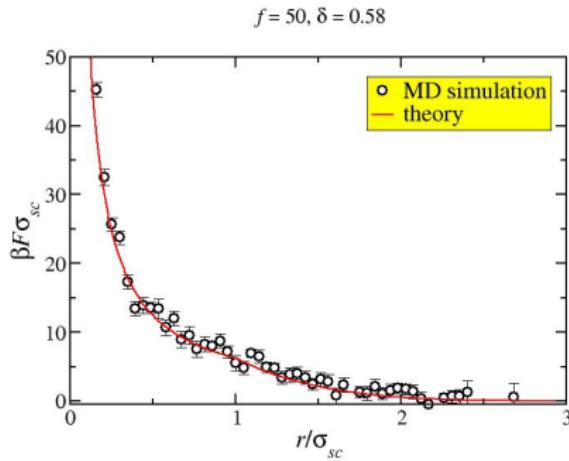
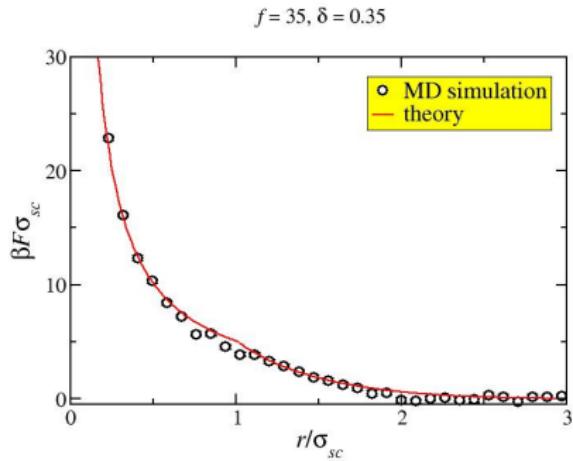
- ▶ Star centers and chain central monomers as effective coordinates



# Star-chain mixtures

Cross interaction  $V_{12}(r)$ : MD and theory:<sup>6</sup>

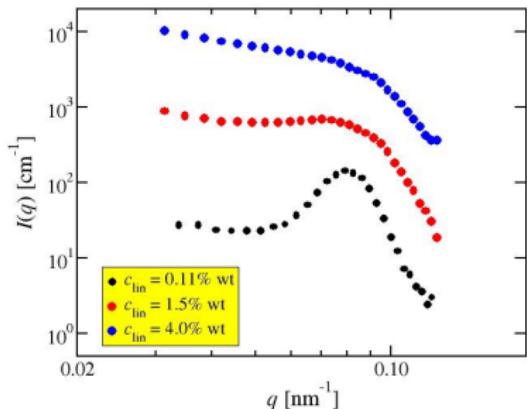
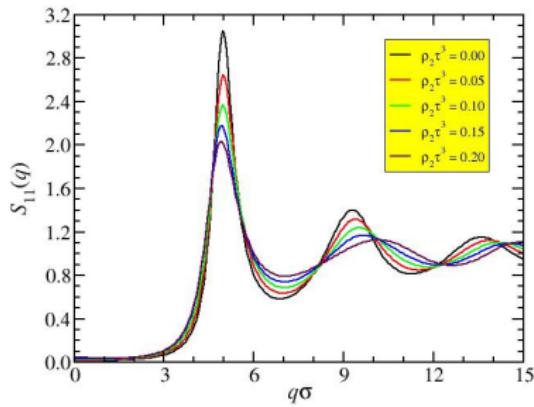
- ▶ For  $r \leq \sigma_{sc}$ , scaling form,  $V_{12}(r) \sim -\ln(r/\sigma_{sc})$ .
- ▶ For  $r > \sigma_{sc}$ , apply a Flory theory for overlapping density profiles.



<sup>6</sup>C. Mayer, CNL, Macromolecules **40**, 1196 (2007).

# Pair structure: theory and experiment

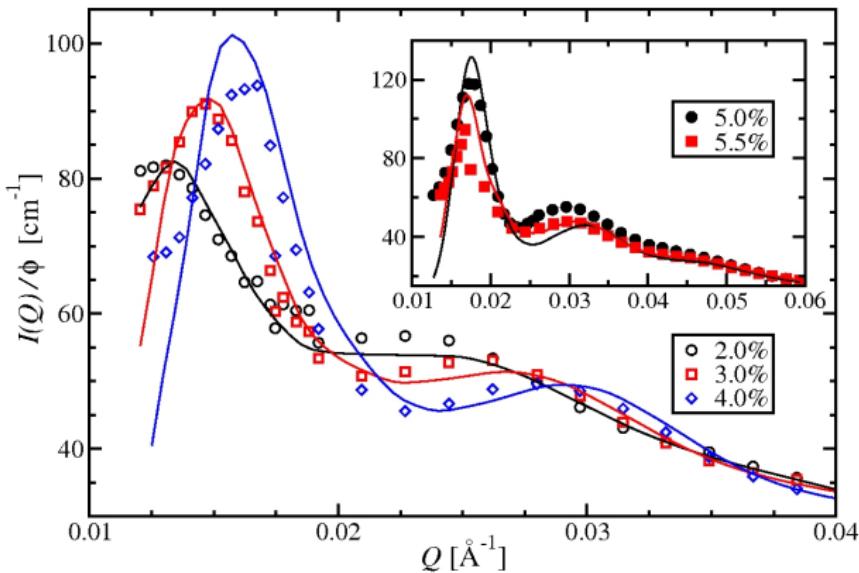
Here:  $f = 263$  and  $\delta = 0.3$   
Star solution at the overlap concentration  $c_{\text{star}}^*$



- The added chains cause a **loss of structure** of the stars.

# Pair structure: theory and experiment

Here:  $f = 91$  and  $\delta = 0.3$ , increasing concentration<sup>7</sup>



- ▶ Quantitative agreement **without free parameters** in a broad density range.

<sup>7</sup>B. Lonetti, M. Camargo, CNL et al., Phys. Rev. Lett. **106**, 228301 (2011).

# Chain-mediated depletion forces between stars

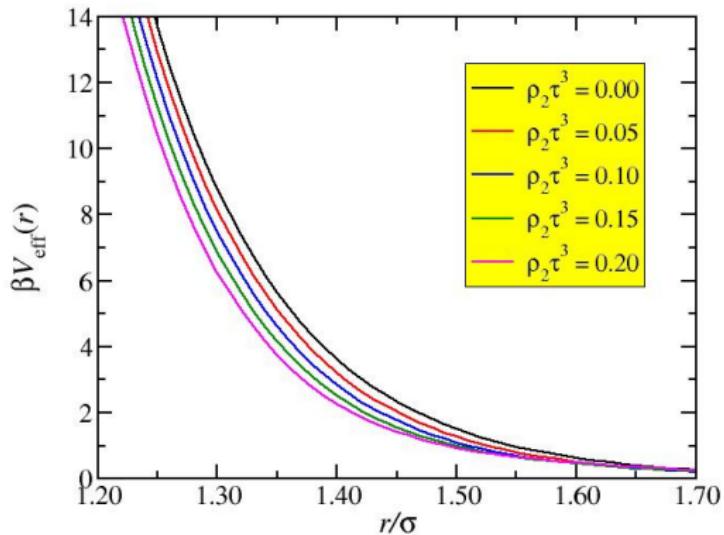
- ▶ Consider  $N_1 = 2$  stars in a sea of linear chains, density  $\rho_2$ ;
- ▶ Volume  $\Omega \rightarrow \infty$ , hence  $\rho_1 \rightarrow 0$ ;
- ▶ Star-star distance is  $r$ , direct **and** chain-mediated force is  $\mathbf{F}_{\text{eff}}(r; \rho_2) = -\nabla V_{\text{eff}}(r; \rho_2)$ ;
- ▶ Obtain as

$$V_{\text{eff}}(r; \rho_2) = - \lim_{\rho_1 \rightarrow 0} [\ln g_{11}(r; \rho_1, \rho_2)];$$

- ▶ Readily available from liquid structure.

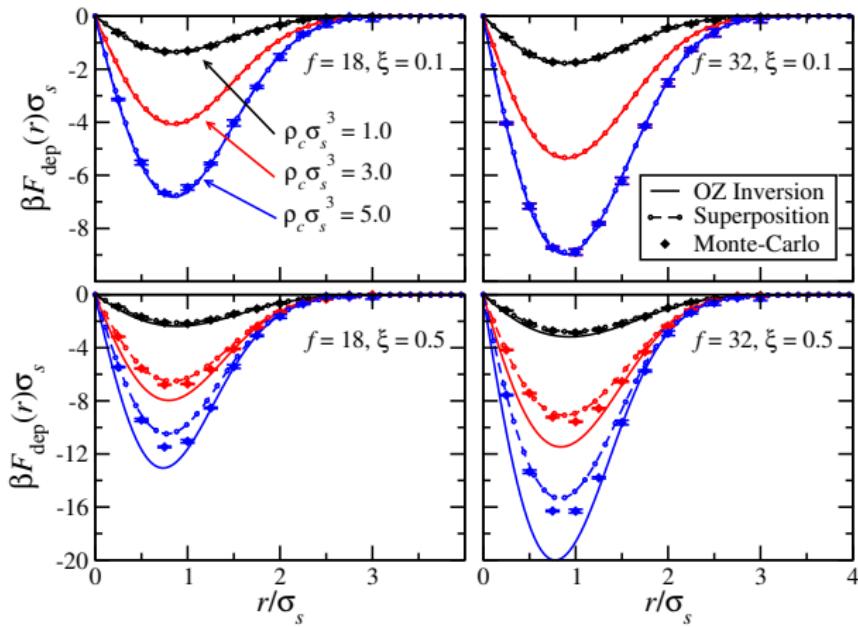
# Chain-mediated depletion forces between stars

- Here:  $f = 263$ ,  $\delta = 0.3$



# Chain-mediated depletion forces between stars<sup>8</sup>

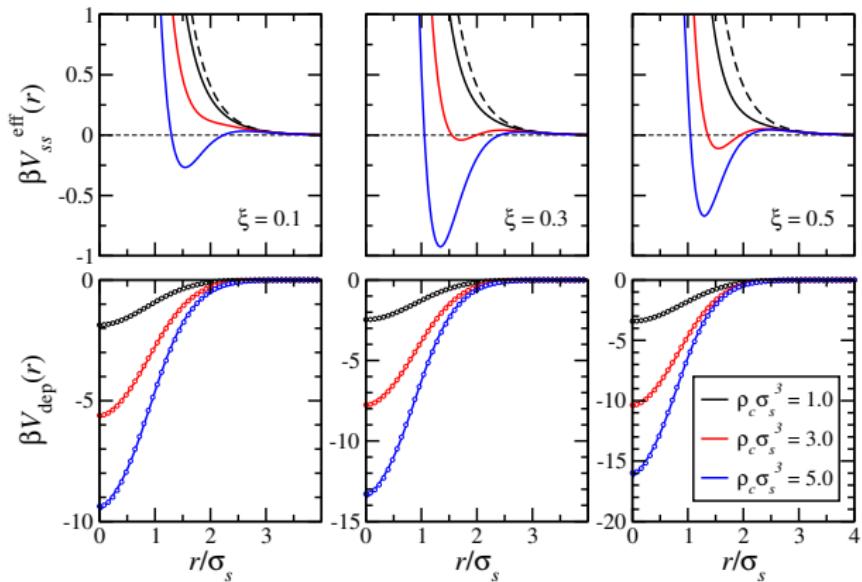
- Here:  $f = 18, 32$ , varying  $\xi \equiv \delta$  and chain density



<sup>8</sup>M. Camargo and CNL, Phys. Rev. Lett. **104**, 078301 (2010).

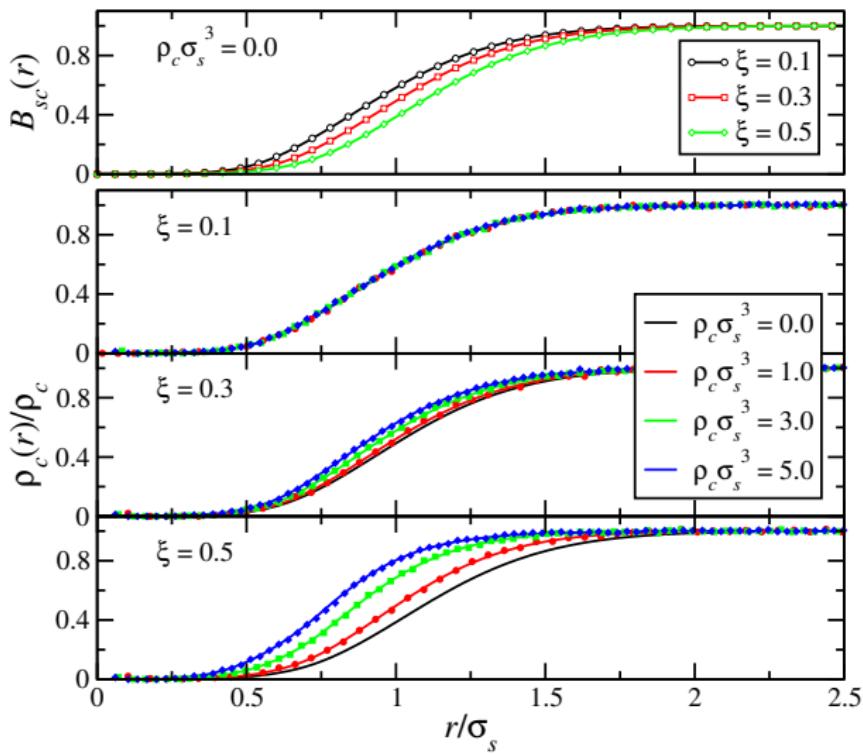
# Chain-mediated depletion forces between stars

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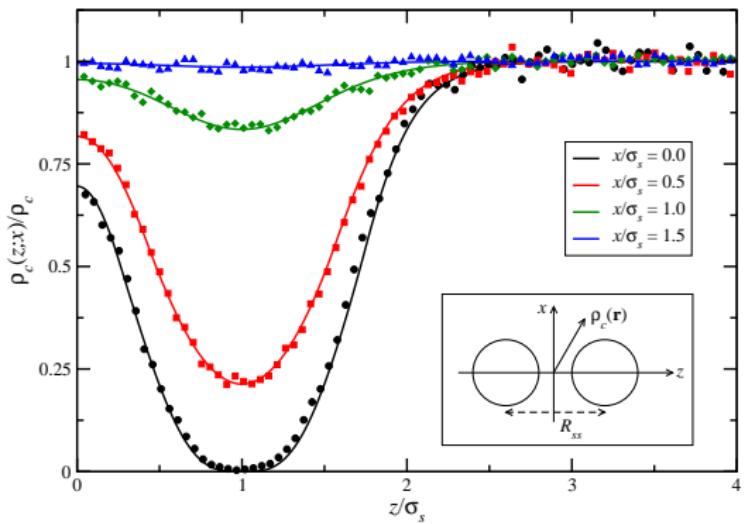
# Polymer density profiles around a star

- Here:  $f = 32$ , varying  $\xi \equiv \delta$  and chain density



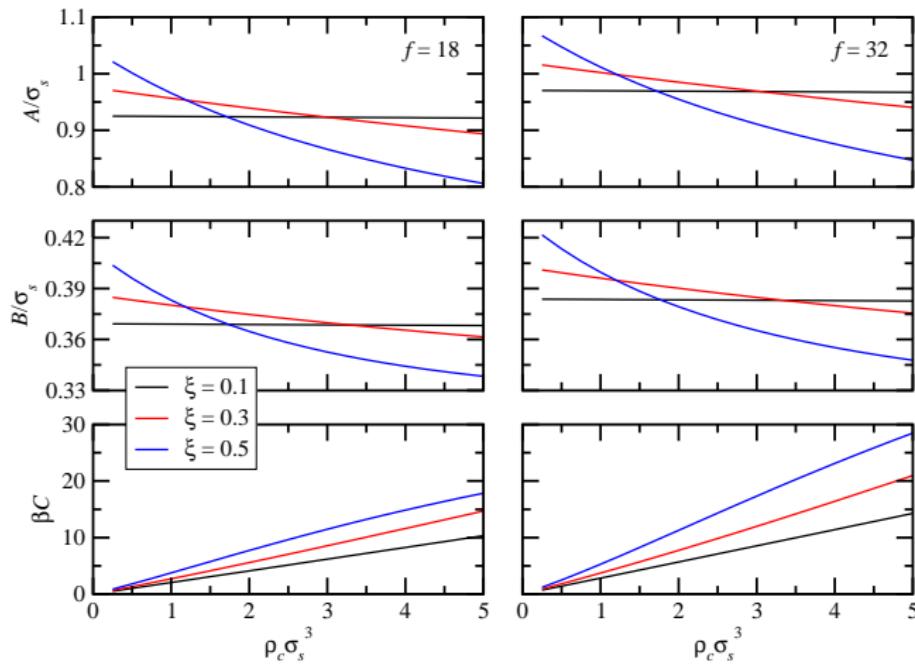
# Polymer density profiles around two interacting stars

- Here:  $f = 18$ , varying  $\xi = 0.5$  and  $\rho_c \sigma_s^3 = 4.0$



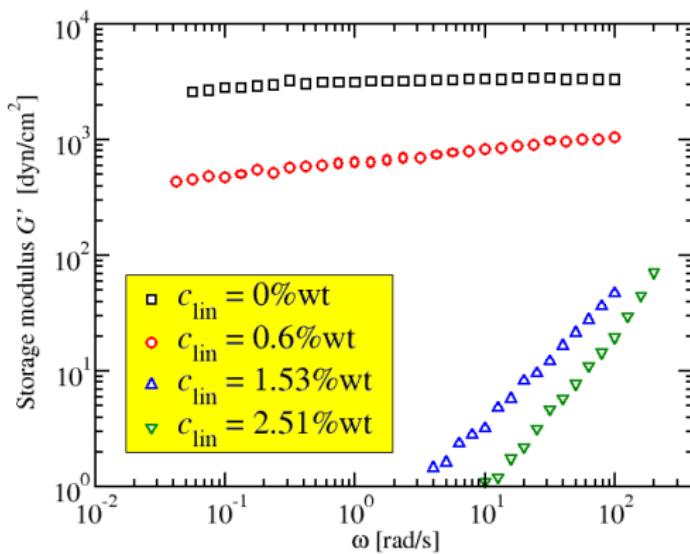
# Chain-mediated depletion forces between stars

- ▶ Fit to  $\beta V_{\text{dep}}(r) = -\frac{C}{\exp[(r-A)/B]+1}$



# Star-chain mixtures: glass melting

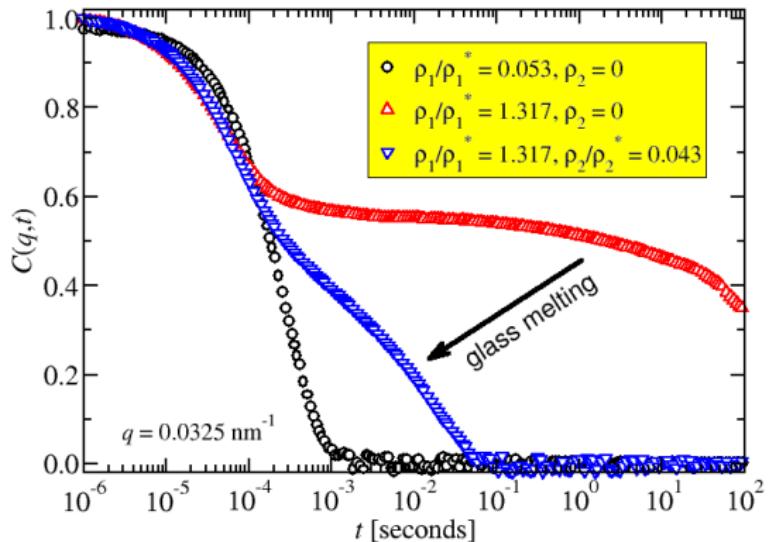
- ▶  $f = 263$ ,  $\delta = 0.3$ , rheology<sup>9</sup>



<sup>9</sup>E. Stiakakis *et al.*, Phys. Rev. Lett. **89**, 208302 (2002).

# Star-chain mixtures: glass melting

- ▶  $f = 122$ ,  $\delta = 0.44$ , Dynamic Light Scattering<sup>10</sup>



<sup>10</sup>E. Stiakakis *et al.*, Europhys. Lett. **72**, 664 (2005).

# Mode Coupling Theory

- ▶ Focus **only** on stars, chains implicit.
- ▶ Consider  $\hat{\rho}(\mathbf{q}, t) = \sum_{\ell} \exp[-i\mathbf{q} \cdot \mathbf{r}_{\ell}]$ .
- ▶ Non-ergodicity factor  $f(q)$  of the stars<sup>11</sup>

$$f(q) = \lim_{t \rightarrow \infty} \langle \hat{\rho}^*(\mathbf{q}, 0) \hat{\rho}(\mathbf{q}, t) \rangle / \langle \hat{\rho}^*(\mathbf{q}, 0) \hat{\rho}(\mathbf{q}, 0) \rangle.$$

- ▶ MCT:

$$\frac{f(q)}{1 - f(q)} = \frac{1}{2(2\pi)^3} \int d\mathbf{k} V^{(2)}(q, k, |\mathbf{q} - \mathbf{k}|) f(k) f(|\mathbf{q} - \mathbf{k}|),$$

- ▶ where

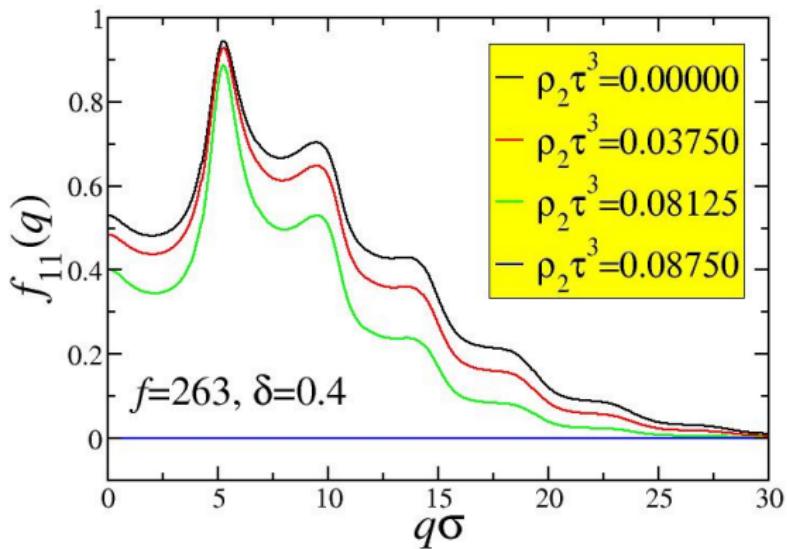
$$\begin{aligned} V^{(2)}(q, k, |\mathbf{q} - \mathbf{k}|) &= \frac{\rho}{q^2} S(q) S(k) S(|\mathbf{q} - \mathbf{k}|) \\ &\times \left( \frac{\mathbf{q}}{q} [\mathbf{k} c(k) + (\mathbf{q} - \mathbf{k}) c(|\mathbf{q} - \mathbf{k}|)] \right)^2. \end{aligned}$$

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<sup>11</sup>C. Mayer, CNL, Macromolecules **40**, 1196 (2007).

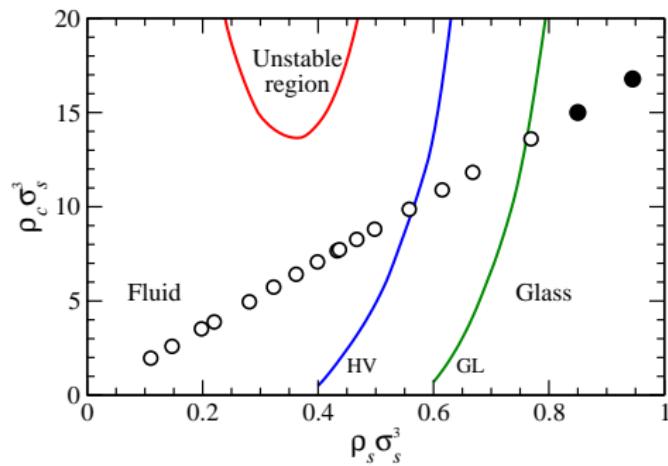
# Star-linear mixtures: glass melting

- MCT-results



# Star-linear mixtures: glass melting

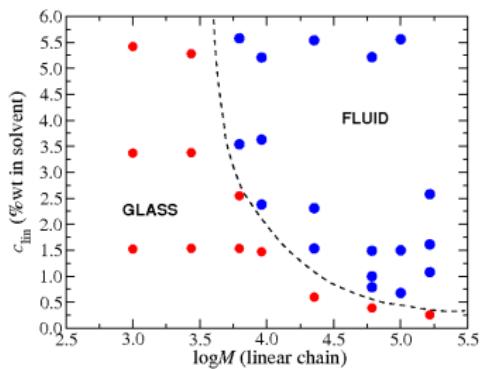
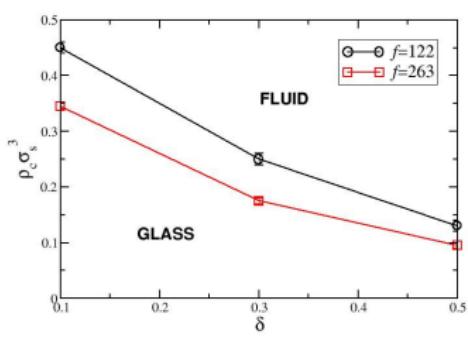
- ▶ Shifting of glass line<sup>12</sup>



<sup>12</sup>B. Lonetti, M. Camargo, CNL et al., Phys. Rev. Lett. **106**, 228301 (2011).

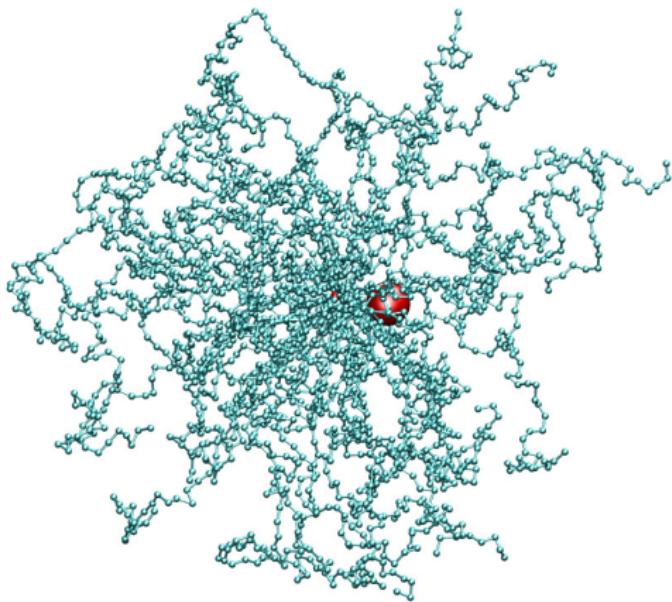
# Star-linear mixtures: glass melting

- ▶ Comparison with experiment<sup>13</sup>



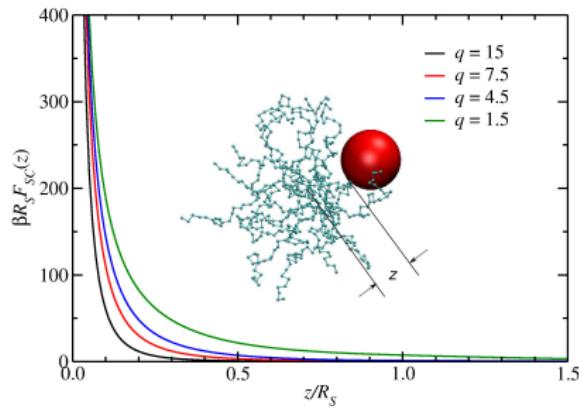
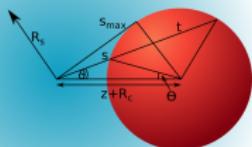
<sup>13</sup>E. Stiakakis *et al.*, Phys. Rev. Lett. **89**, 208302 (2002).

Turn it around: big stars, small colloids



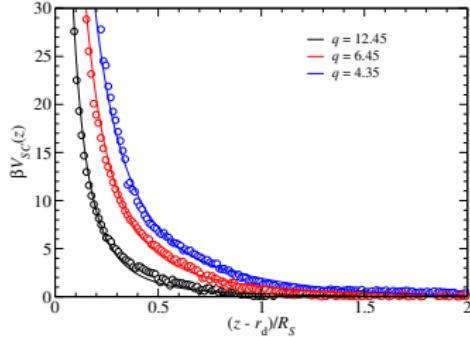
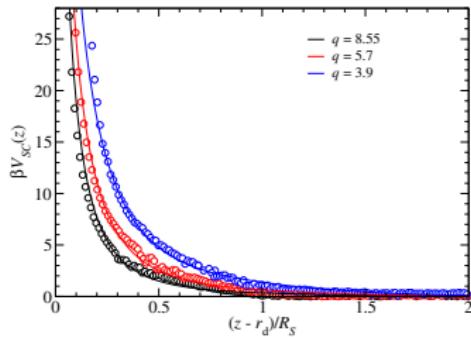
# Turn it around: big stars, small colloids

- ▶ Coarse-graining: calculate  $V_{sc}(r)$



# Turn it around: big stars, small colloids

- ▶  $V_{sc}(r)$ : left,  $f = 30$ ; right,  $f = 50$ . Compare with simulations<sup>14</sup>

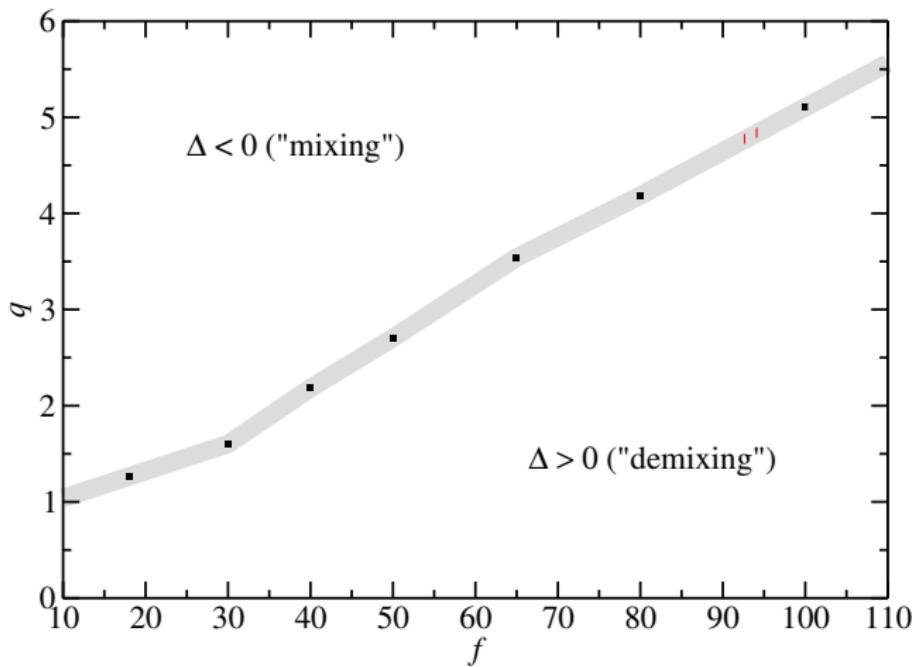


- ▶ NB,  $V_{sc}(z) \sim -\ln z$  as  $z \rightarrow 0$ .

<sup>14</sup>D. Marzi, CNL, B. Capone, J. Chem. Phys. **137**, 014902 (2012).

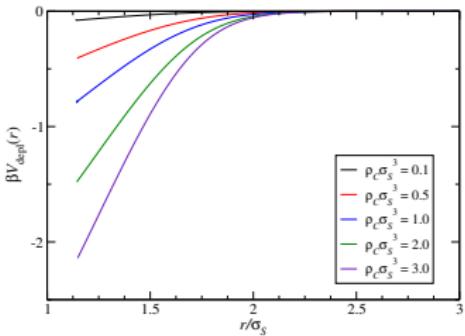
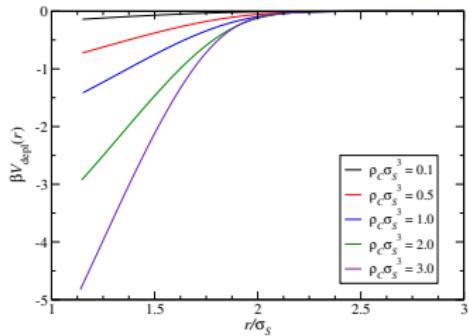
# Turn it around: big stars, small colloids

- ▶ A rough-and-ready prediction on phase behavior



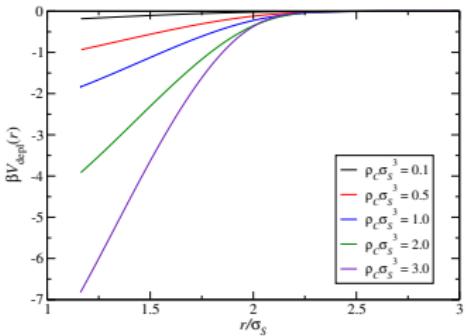
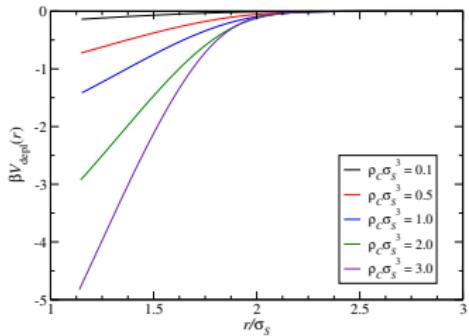
# Depletion potentials on the stars due to colloids

- ▶  $V_{depl}(r)$ : left,  $f = 200, q = 3$ ; right,  $f = 200, q = 4$ .



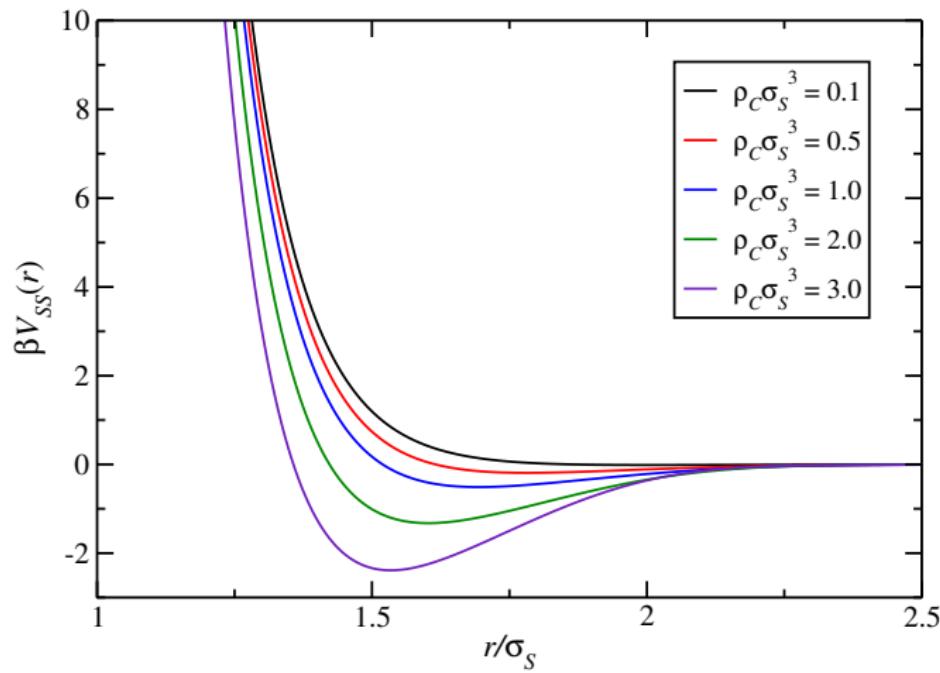
# Depletion potentials on the stars due to colloids

- ▶  $V_{depl}(r)$ : left,  $f = 200, q = 3$ ; right,  $f = 300, q = 3$ .



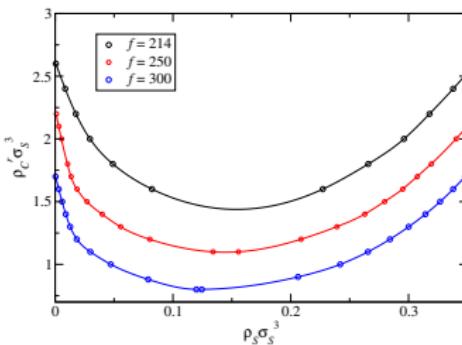
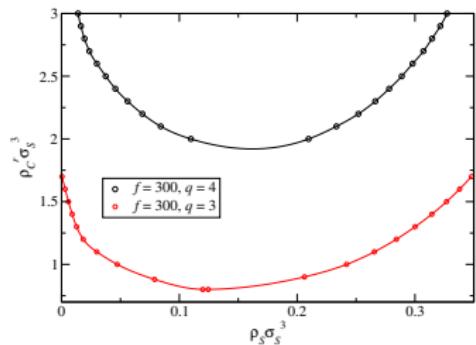
# Total effective star-star potential

►  $f = 300, q = 3$



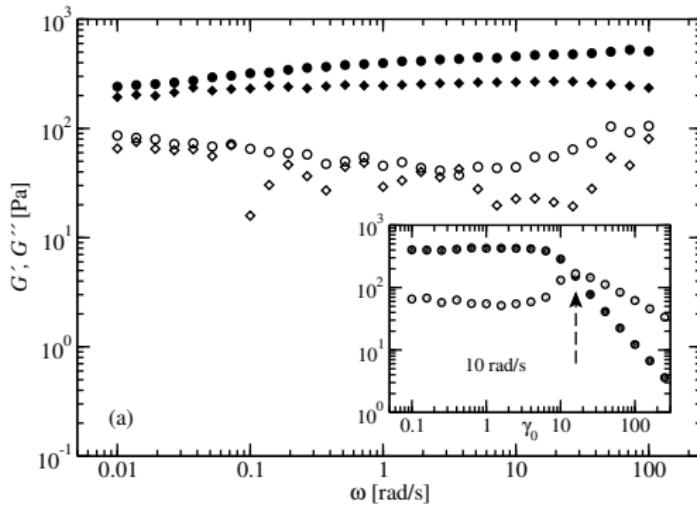
# Phase behavior of star-polymer/colloid mixtures

- ▶ Demixing binodals



# Glassy behavior – influence of colloids<sup>15</sup>

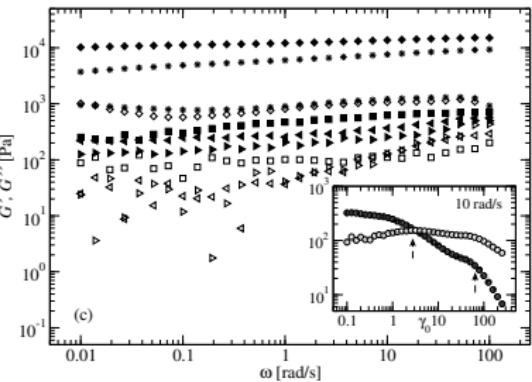
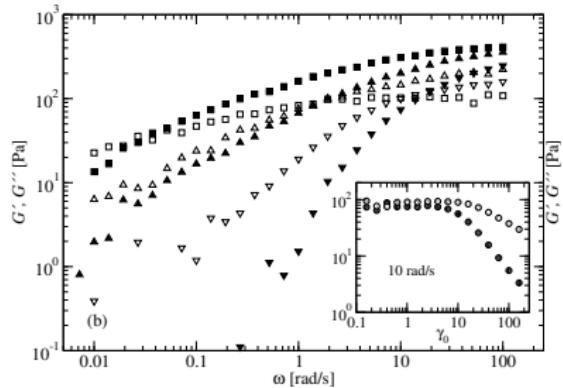
- ▶ The stars: 1,4 PB,  $f_s = 214$ ,  $M_a = 67\,000$  g/mol.
- ▶ The colloids: 1,4 PB,  $f_c = 1109$ ,  $M_a = 1270$  g/mol.
- ▶ Oscillatory shear rheology: measure  $G'(\omega)$  and  $G''(\omega)$ .



<sup>15</sup>D. Truzzolillo *et al.*, Phys. Rev. Lett. **111**, 208301 (2013).

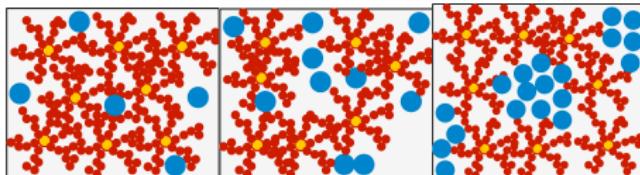
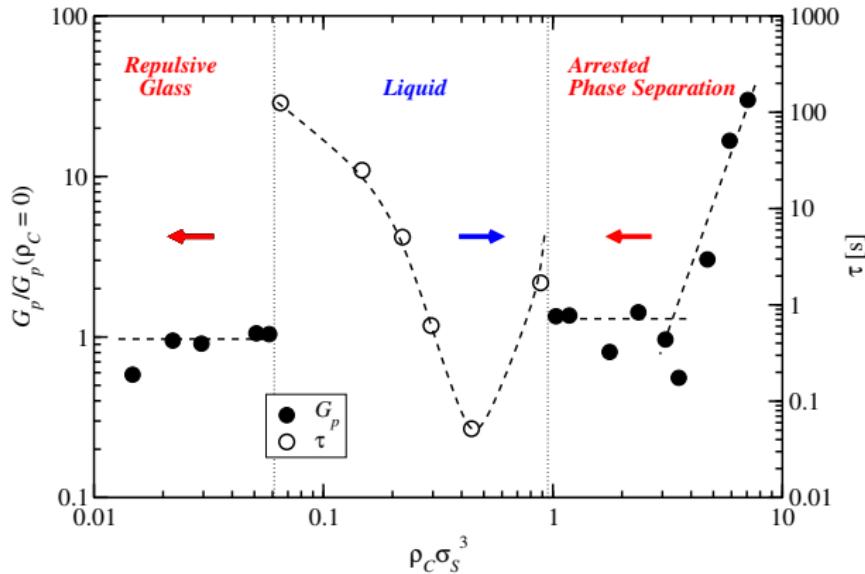
# Glassy behavior

## ► Rheology



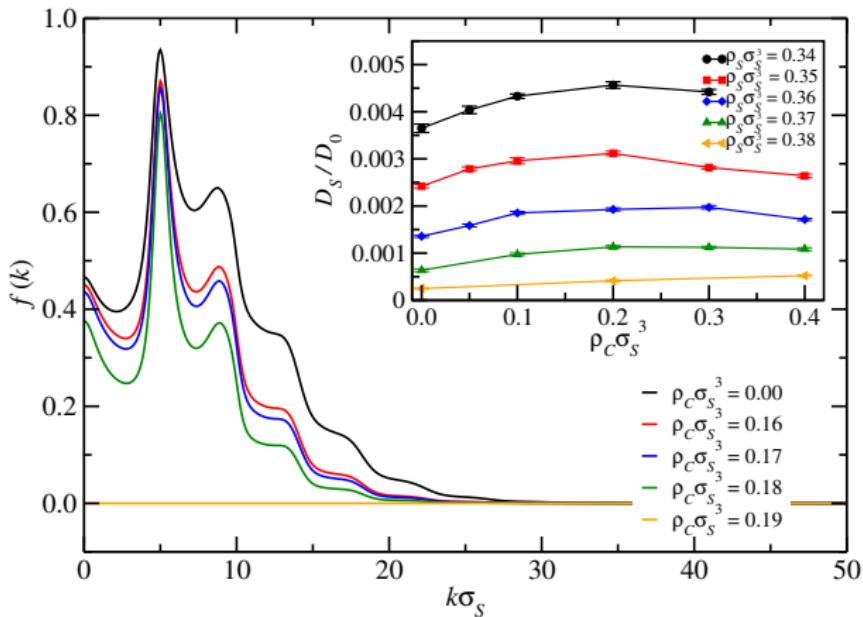
# Glassy behavior

## ► Rheology – Summary



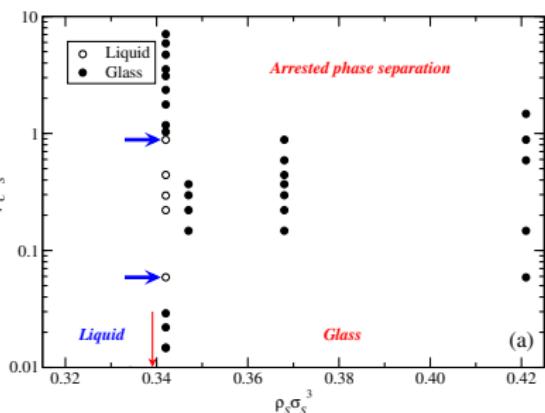
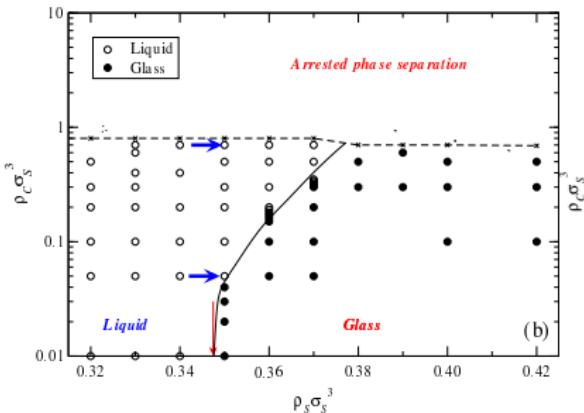
# Glassy behavior

- Mode-Coupling-Theory and simulations



# Glassy behavior

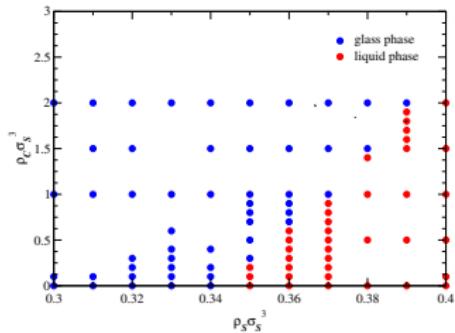
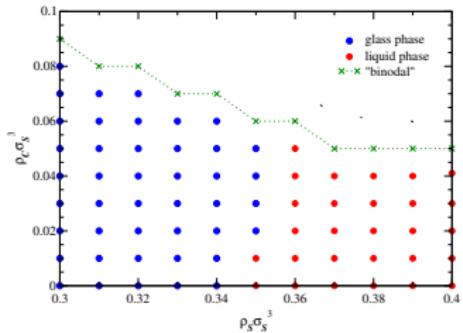
- ▶ Comparison experiment-theory



- ▶ Re-entrant glass due to arrested phase separation

# Glassy behavior

- ▶ Interplay between glass melting and demixing
- ▶ Left:  $f = 214$ ,  $q = 2$ ; right:  $f = 214$ ,  $q = 4$ .



# Conclusions

- ▶ Soft colloids hide many surprises
- ▶ Depletion potentials have novel, unexpected properties
- ▶ Multitude of arrested states in concentrated soft colloids
- ▶ Depletion dictates phase- and glassy-behavior

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- ▶ Multitude of arrested states in concentrated soft colloids
- ▶ Depletion dictates phase- and glassy-behavior
- ▶ Asakura-Oosawa theory has set a paradigm ...

# Conclusions

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- ▶ Depletion potentials have novel, unexpected properties
- ▶ Multitude of arrested states in concentrated soft colloids
- ▶ Depletion dictates phase- and glassy-behavior
- ▶ Asakura-Oosawa theory has set a paradigm ...
- ▶ ... but it can be quantitatively misleading!

# Acknowledgments

- ▶ **Dimitris Vlassopoulos** (FORTH, Greece)
- ▶ **Dieter Richter** (FZ Jülich)
- ▶ **Emanuela Zaccarelli** (University of Rome *La Sapienza*)
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- ▶ **Daniela Marzi** (University of Vienna)
- ▶ **Barbara Capone** (University of Vienna)

