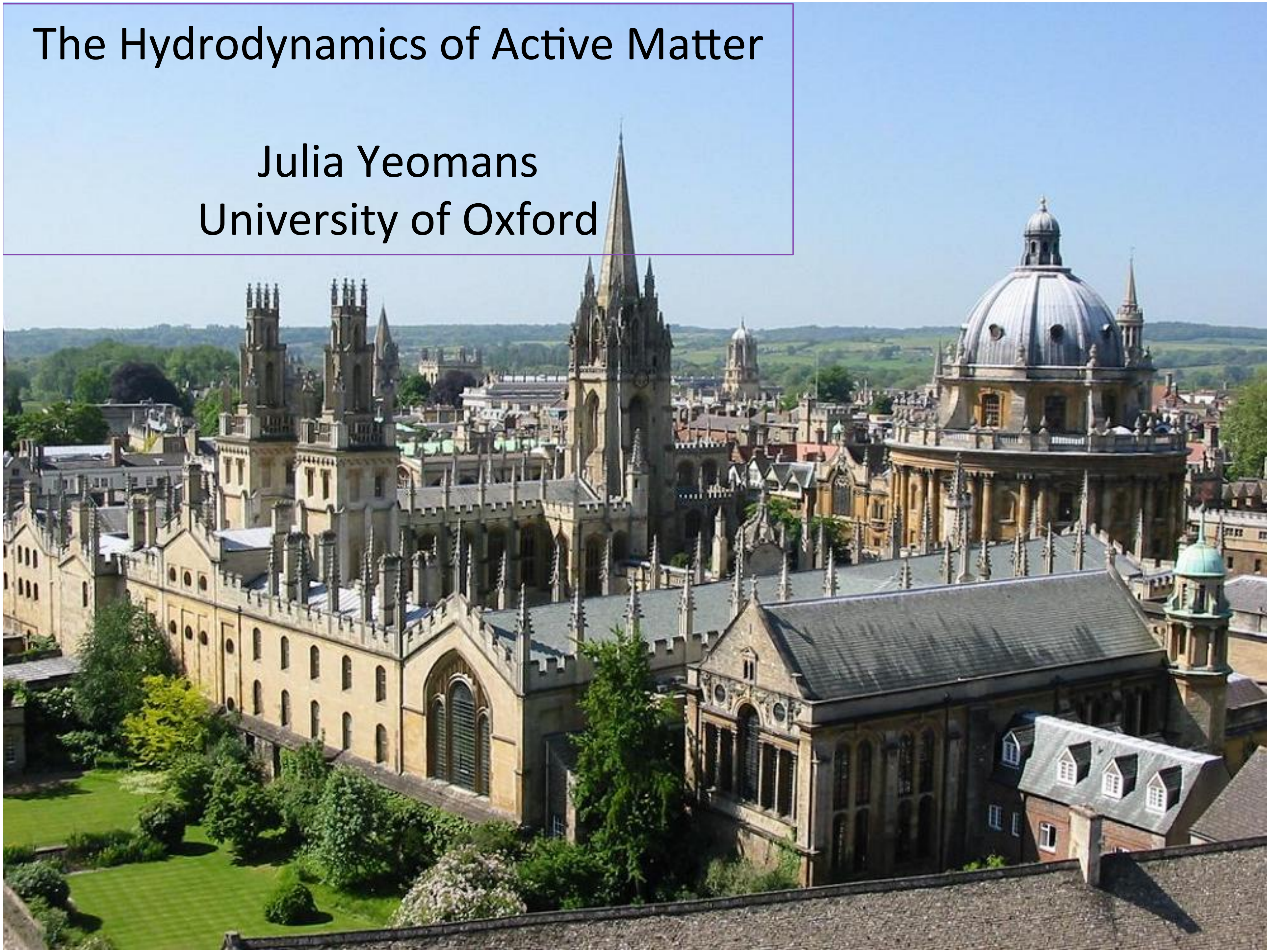
The Hydrodynamics of Active Matter

Julia Yeomans
University of Oxford



Lecture 1: The mathematics and physics of bacterial swimming

1. Low Re and the Stokes equations
2. The Scallop theorem
3. Dipolar flow fields
4. Multipole expansion for the Stokes equations

one active particle

Lecture 2: Applications

1. Swimming in Poiseuille flow and …taxis

2. Stirring by microswimmers

Lecture 3: Continuum models of dense active matter

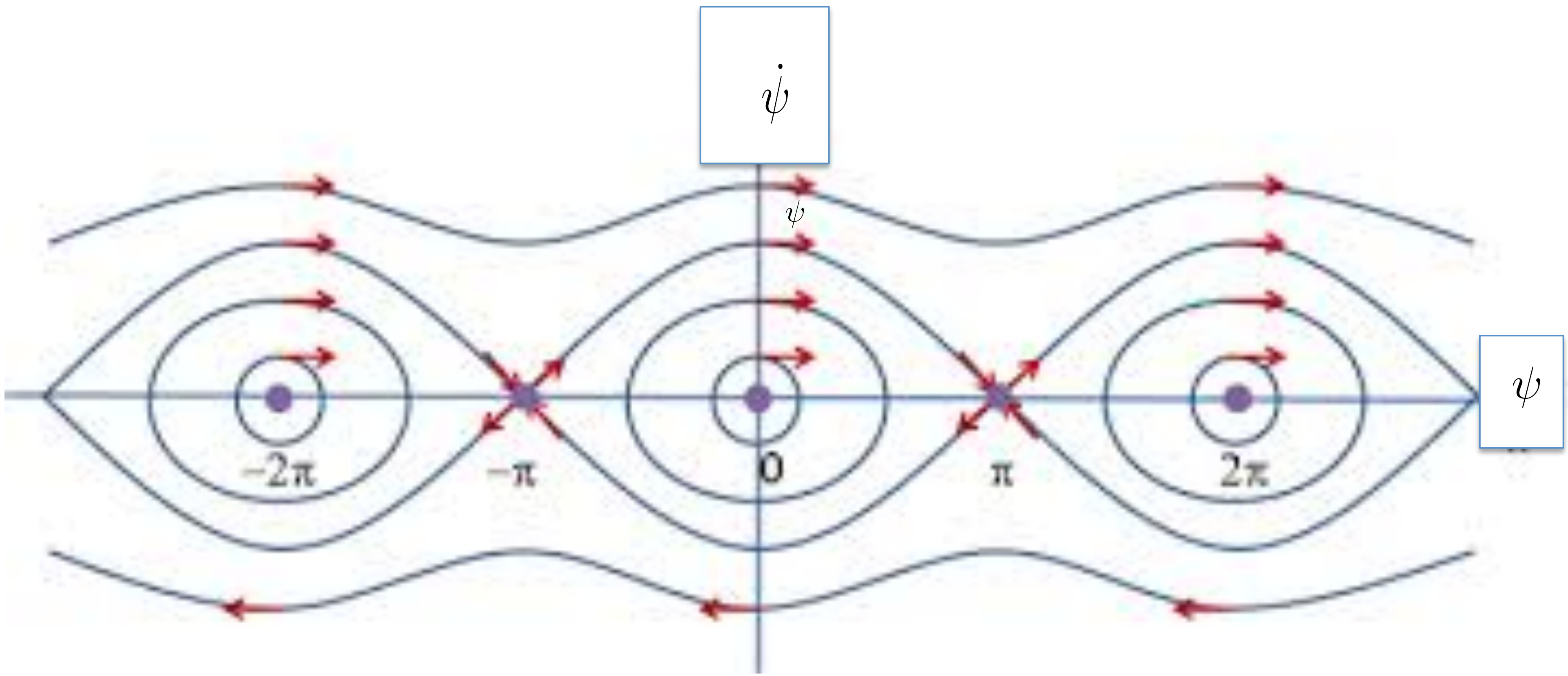
lots of active particles

Lecture 4: Active turbulence and lyotropic active nematics

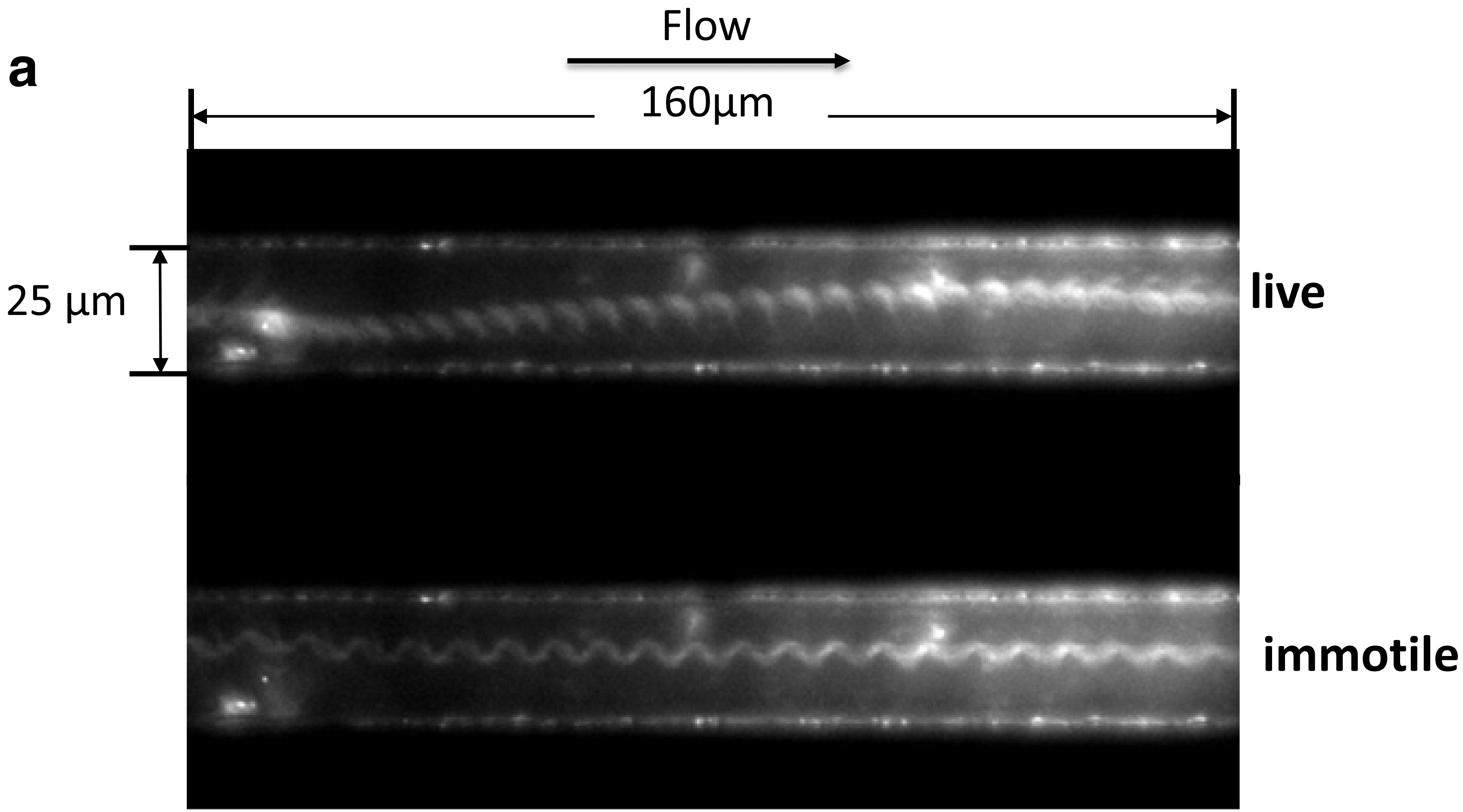
Phase portrait of a pendulum

$$\ddot{\psi} + \sin\psi = 0.$$

$$\dot{x} = -\sin\psi, \qquad \dot{\psi} = x$$



a



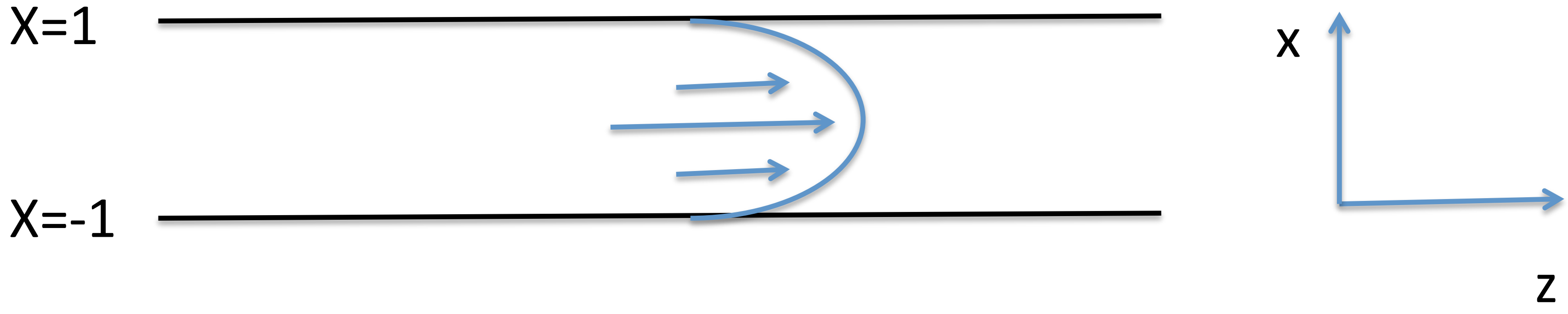
Flow velocity 1.6mm/s @ 300Hz

b

Uppaluri Biophys J (2012)

Typannosome

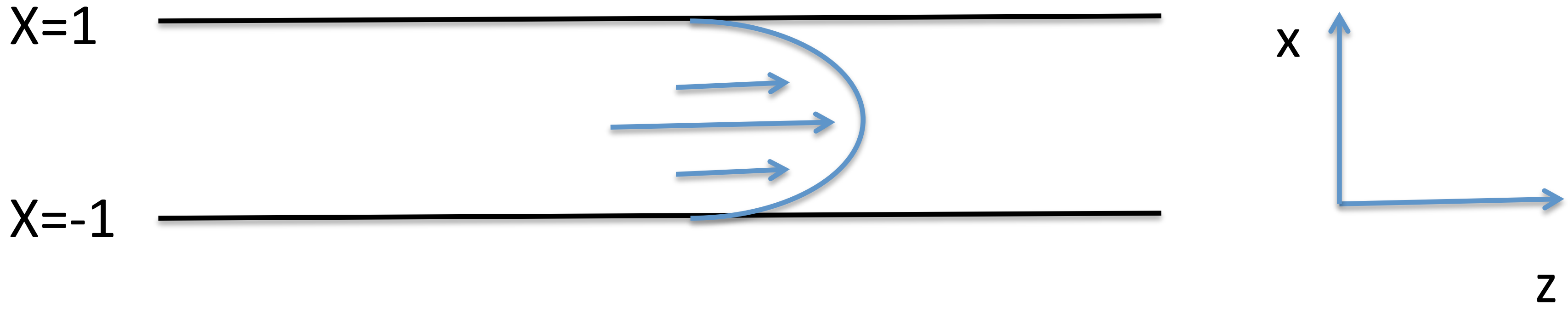
Poiseuille flow and notation



$$\mathbf{v}_f = v_f(1 - x^2)\hat{\mathbf{z}}$$

Zottl and Stark, Phys Rev Lett 108 (2012)

Poiseuille flow and notation



$$\mathbf{v}_f = v_f(1 - x^2)\hat{\mathbf{z}}$$

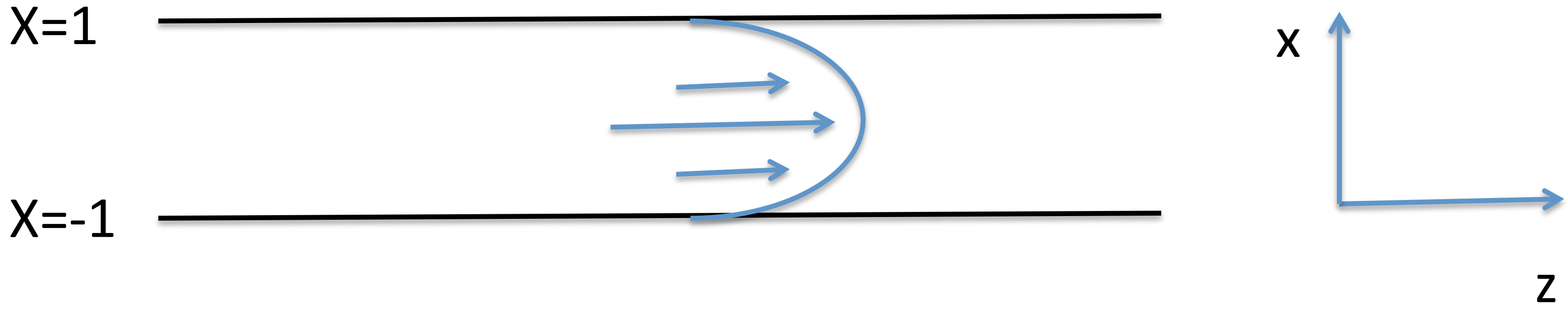
vorticity

$$\mathbf{\Omega}_f = \nabla \wedge \mathbf{v}_f$$

$$= (0, 2xv_f, 0)$$

Zottl and Stark, Phys Rev Lett 108 (2012)

Poiseuille flow and notation



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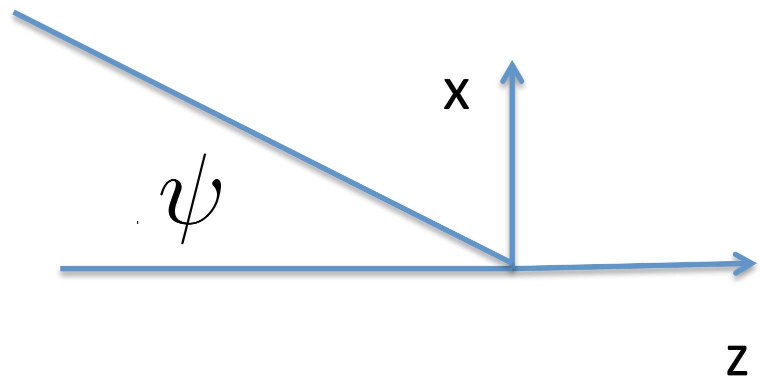
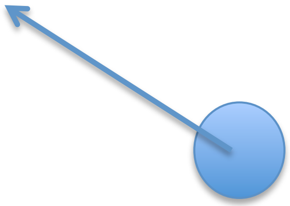


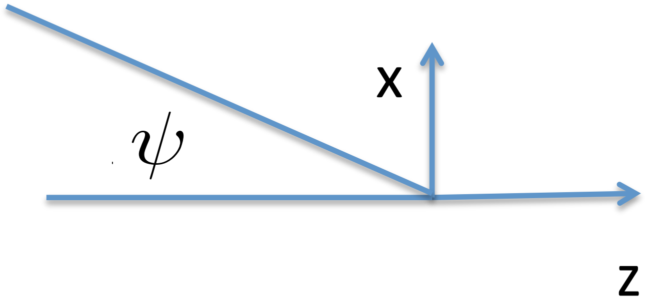
Zottl and Stark, Phys Rev Lett 108 (2012)

Swimmer model



$$\hat{\mathbf{e}} = -\sin\psi\hat{\mathbf{x}} - \cos\psi\hat{\mathbf{z}},$$

speed v_0



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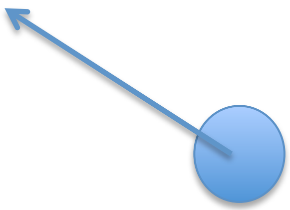

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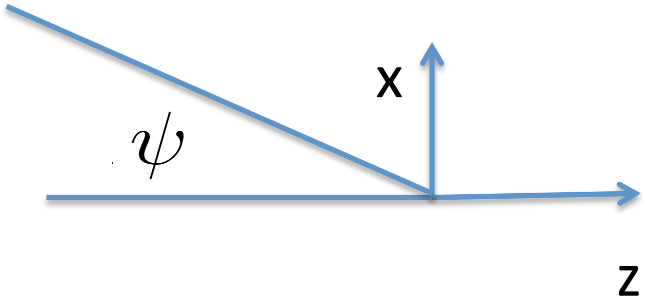
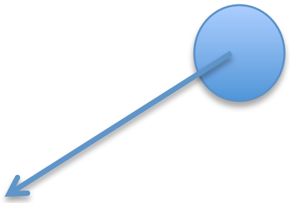


$$\frac{d}{dt}\mathbf{r} = v_0\hat{\mathbf{e}} + \mathbf{v}_f, \qquad \frac{d}{dt}\hat{\mathbf{e}} = \frac{1}{2}\boldsymbol{\Omega}_f \wedge \hat{\mathbf{e}}.$$

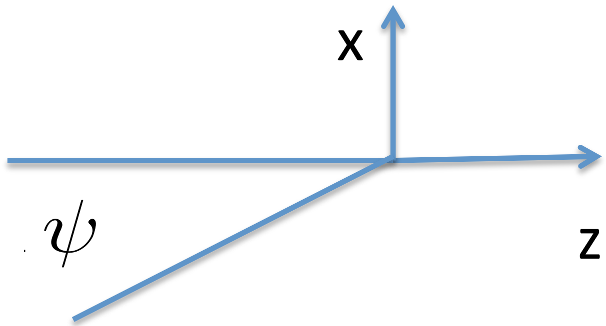
Swimmer model

direction

$$\hat{\mathbf{e}} = -\sin\psi\,\hat{\mathbf{x}} - \cos\psi\,\hat{\mathbf{z}},$$



speed v_0



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1

NB scale time with v_0

2

$$\hat{\mathbf{e}} = -\sin\psi\hat{\mathbf{x}} - \cos\psi\hat{\mathbf{z}},$$

$$\mathbf{v}_f = v_f(1 - x^2)\hat{\mathbf{z}}$$

1

$$\frac{d}{dt}\mathbf{r} = \hat{\mathbf{e}} + \frac{\mathbf{v}_f}{v_0}$$

x-component

$$\frac{d}{dt}x = -\sin\psi$$

z-component

$$\frac{d}{dt}z = -\cos\psi + \frac{v_f}{v_0}(1 - x^2)$$

$$\mathbf{\Omega}_f = \nabla \wedge \mathbf{v}_f \quad = (0, 2xv_f, 0)$$

$$\hat{\mathbf{e}} = -\sin\psi\hat{\mathbf{x}} - \cos\psi\hat{\mathbf{z}},$$

2

$$\frac{d}{dt}\hat{\mathbf{e}} = \frac{1}{2v_0}\mathbf{\Omega}_f \wedge \hat{\mathbf{e}}.$$

x-component

$$-\cos\psi\dot{\psi} = -\frac{xv_f\cos\psi}{v_0}$$

z-component

$$+\sin\psi\dot{\psi} = \frac{xv_f\sin\psi}{v_0}$$

$$\frac{d\psi}{dt} = \frac{v_f}{v_0} x$$

$$\frac{d}{dt} x = -\sin\psi$$

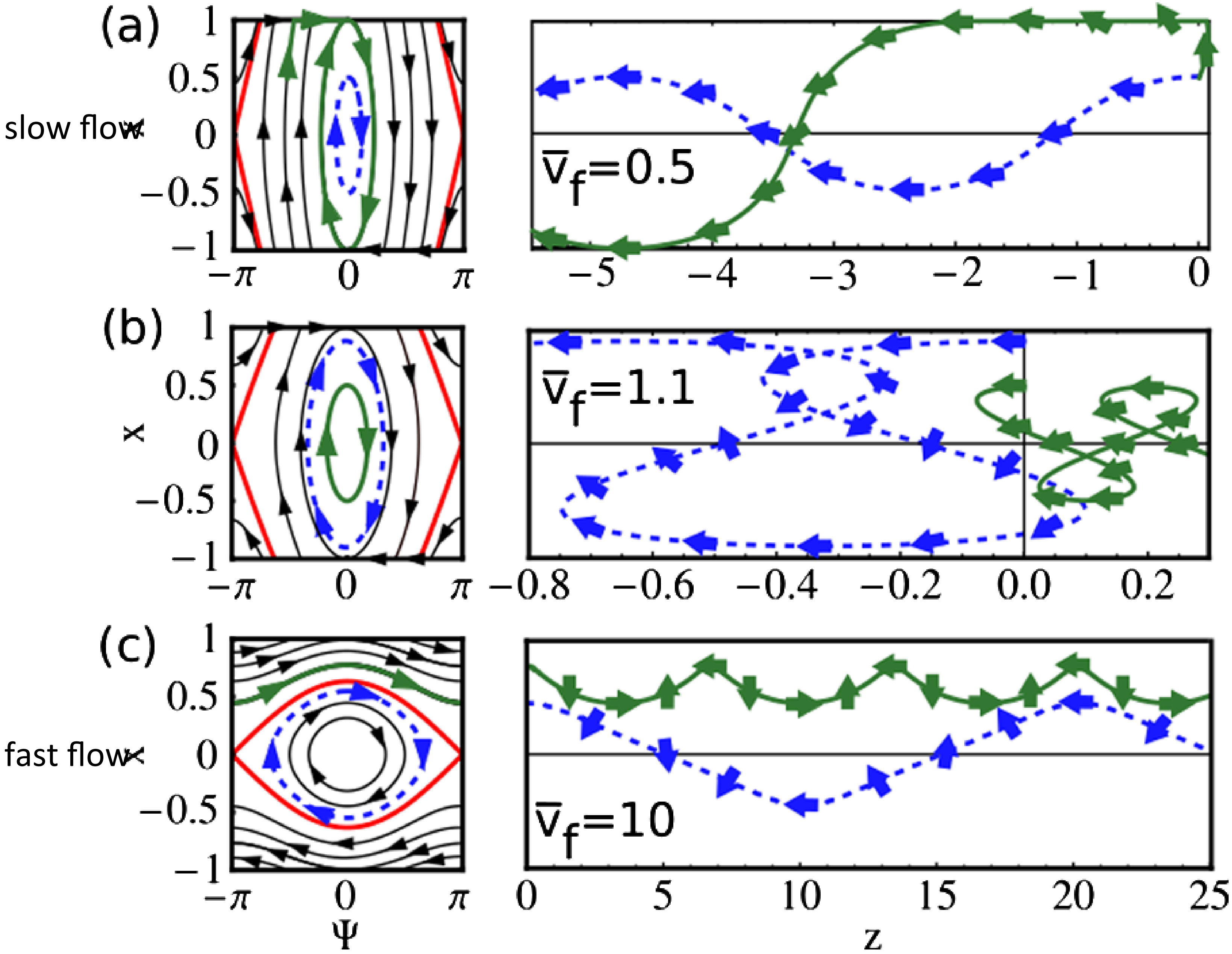
$$\frac{d}{dt} z = -\cos\psi + \frac{v_f}{v_0}(1 - x^2)$$

$$\frac{d\psi}{dt} = \frac{v_f}{v_0} x$$

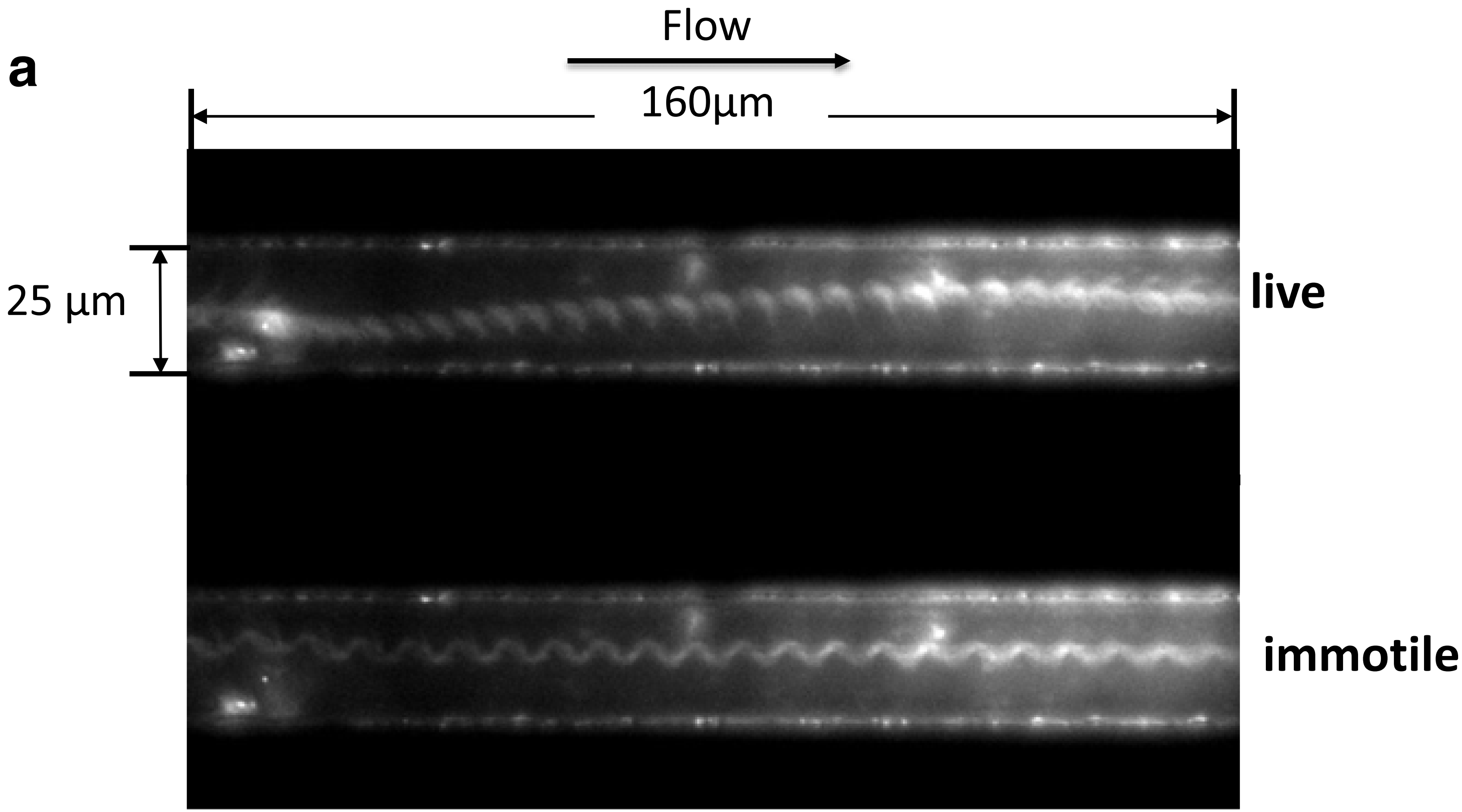
$$\frac{d}{dt} x = -\sin\psi$$

$$\ddot{\psi} + \frac{v_f}{v_0} \sin\psi = 0.$$

$$\frac{d}{dt} z = -\cos\psi + \frac{v_f}{v_0}(1 - x^2)$$



(a)
slow flow
$\bar{v}_f$=0.5
(b)
$\bar{v}_f$=1.1
(c)
fast flow
$\bar{v}_f$=10
x
Ψ
z



Flow velocity 1.6mm/s @ 300Hz

b

Typannosome

Uppaluri Biophys J (2012)

what is missing?

3D
interactions with walls
swimmer size and shape
fluctuations

…taxis

alignment with a given direction

gravitaxis – like to swim upwards

chemotaxis – follow a chemical gradient

magnetotaxis – follow a magnetic field

rheotaxis – shear + gravity

phototaxis – follow light

Why can't you trust an atom?

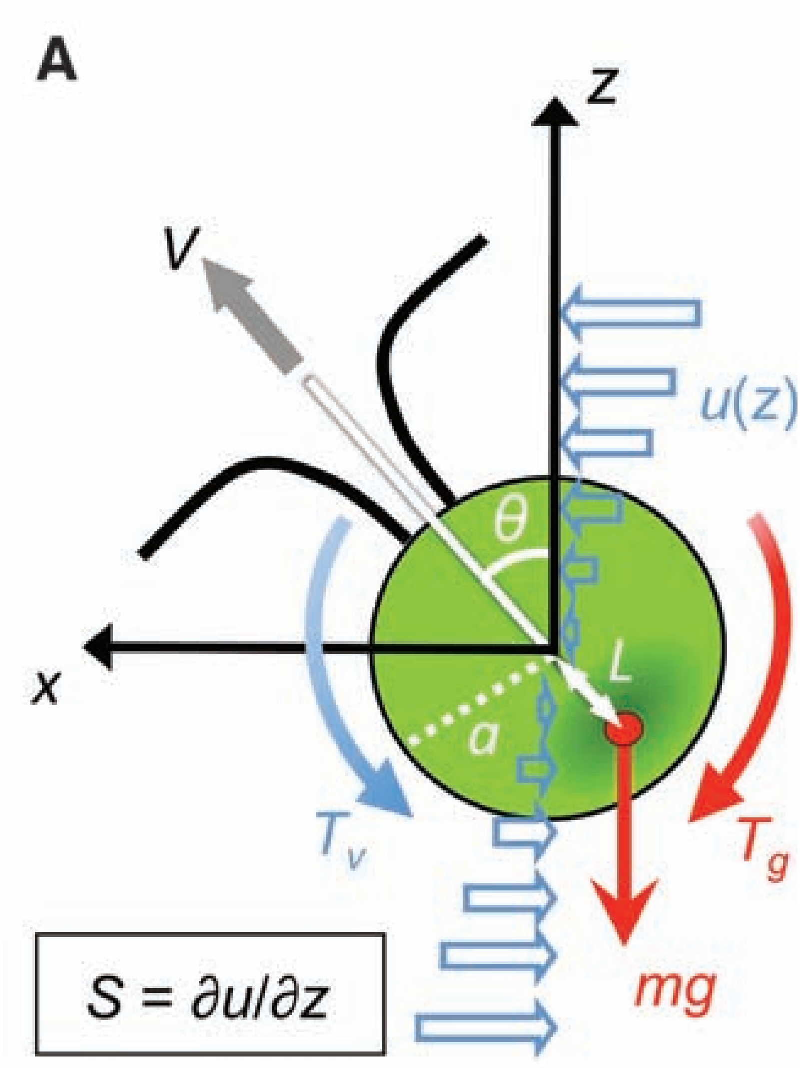
They make up everything.

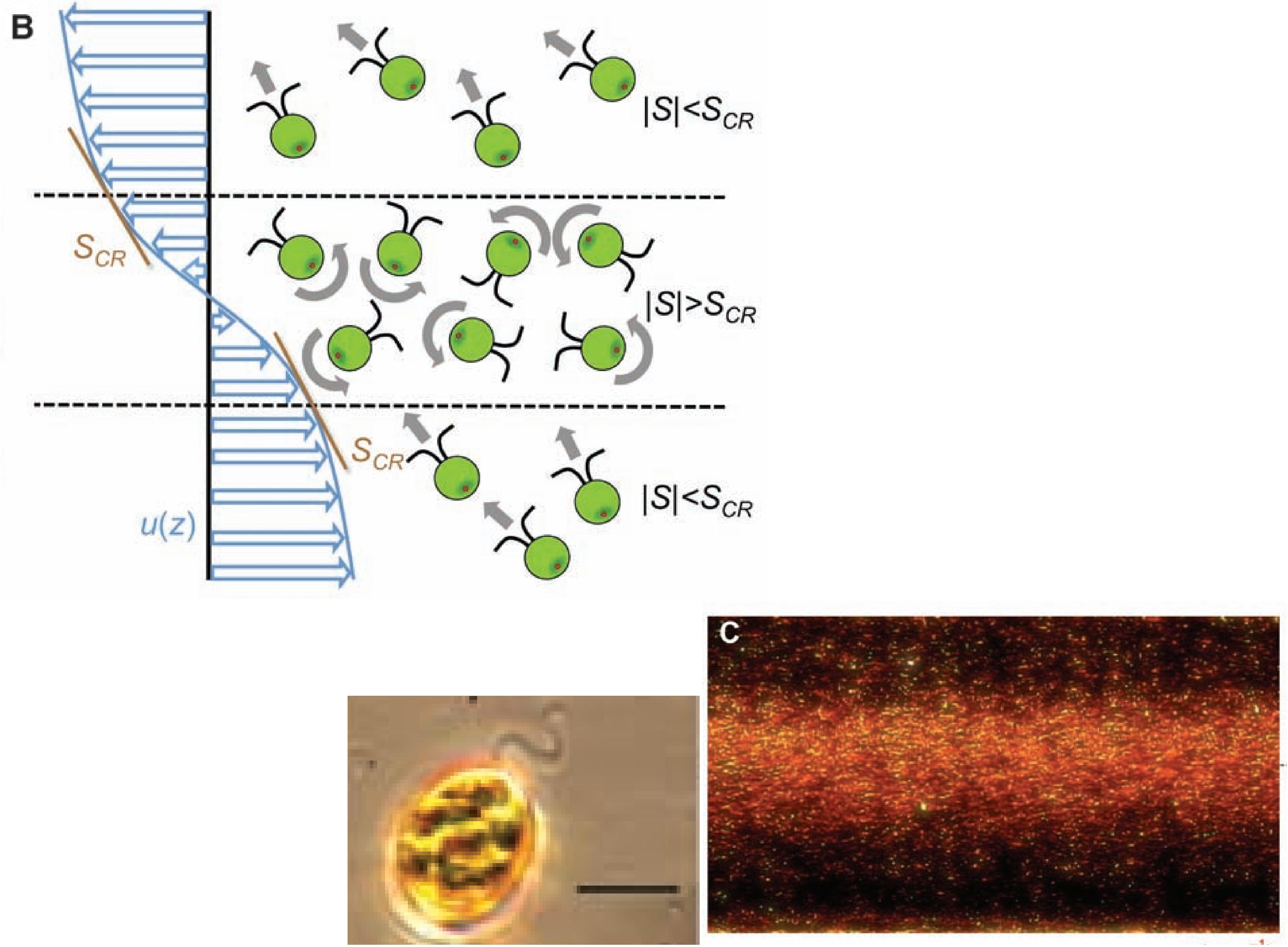
...taxis

$$\frac{d}{dt}\hat{\mathbf{e}} = \frac{1}{2}\{\boldsymbol{\Omega}_f \wedge \hat{\mathbf{e}} + \frac{1}{\beta}(\hat{\mathbf{z}} - (\hat{\mathbf{z}} \cdot \hat{\mathbf{e}})\hat{\mathbf{e}})\}$$

$$\sin\theta = \beta\Omega$$

Durham, Kessler, Stocker
Science 323 (2009)




B
|S|<S_CR
S_CR
|S|>S_CR
S_CR
|S|<S_CR
u(z)
C

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lots of active particles

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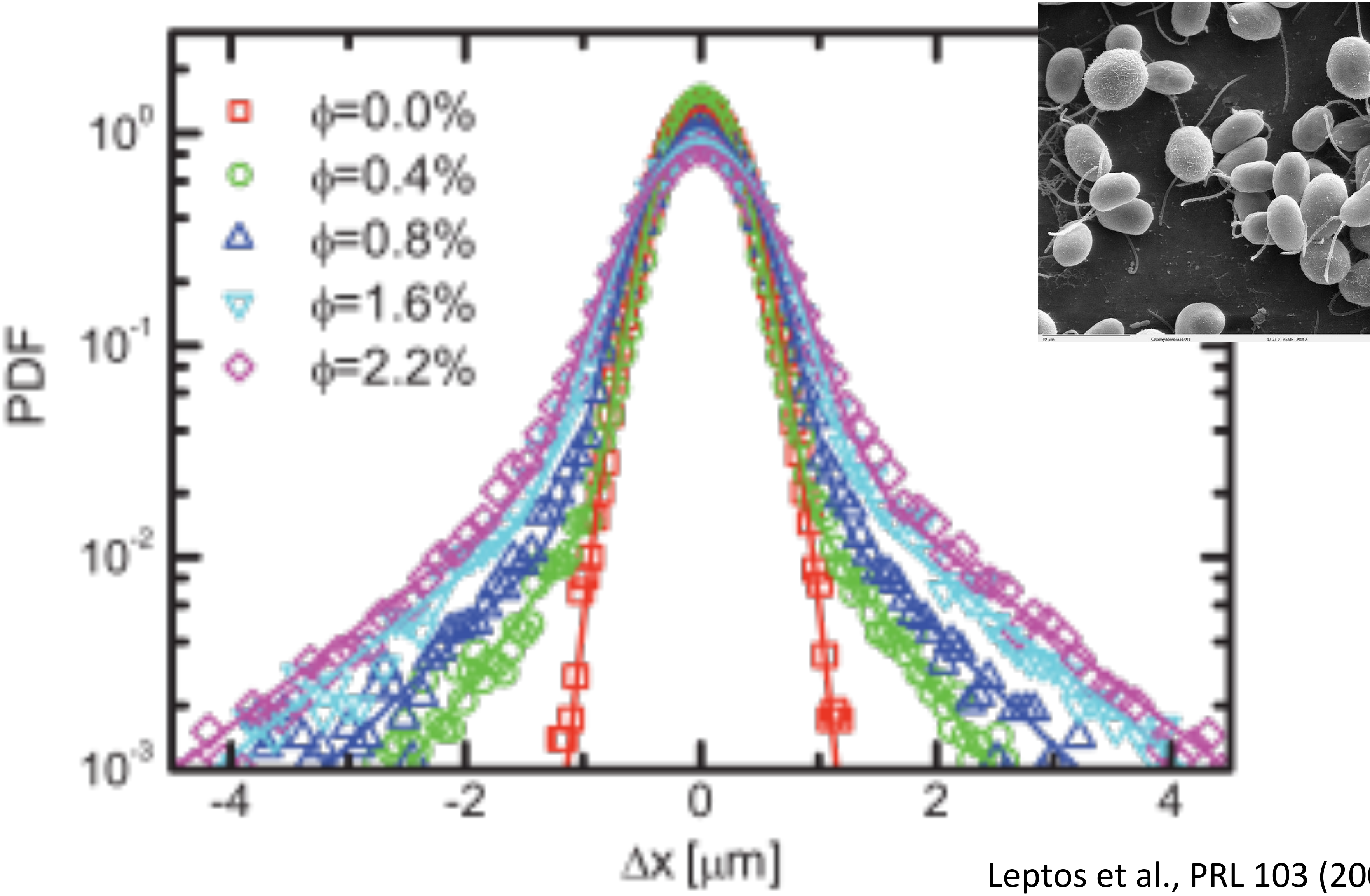
Stirring by microswimmers

Do microswimmers stir the fluid they swim in?

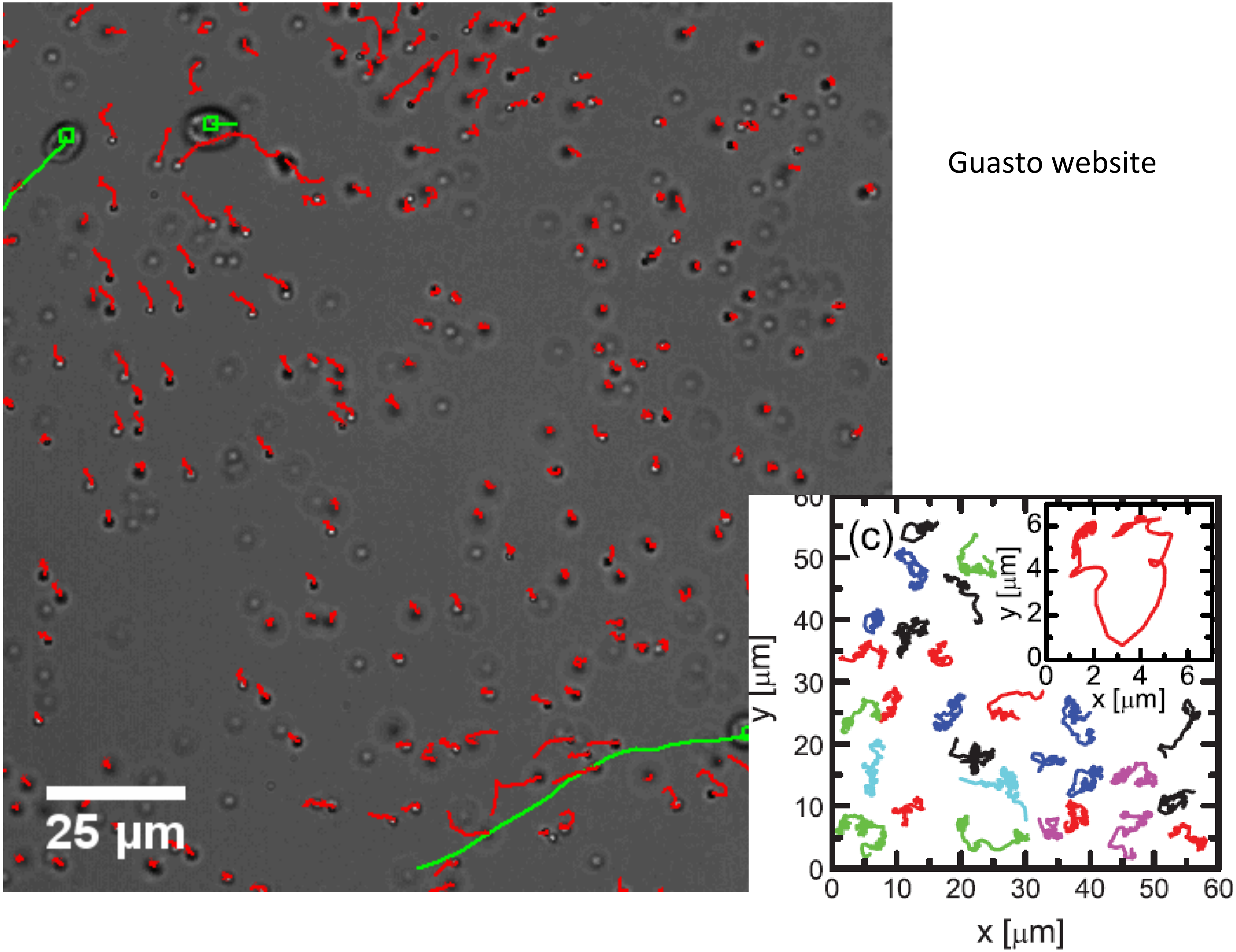
Why do microswimmers stir the fluid they swim in?

How do microswimmers stir the fluid they swim in?

Swimmers enhance diffusion



Leptos et al., PRL 103 (2009)



Guasto website

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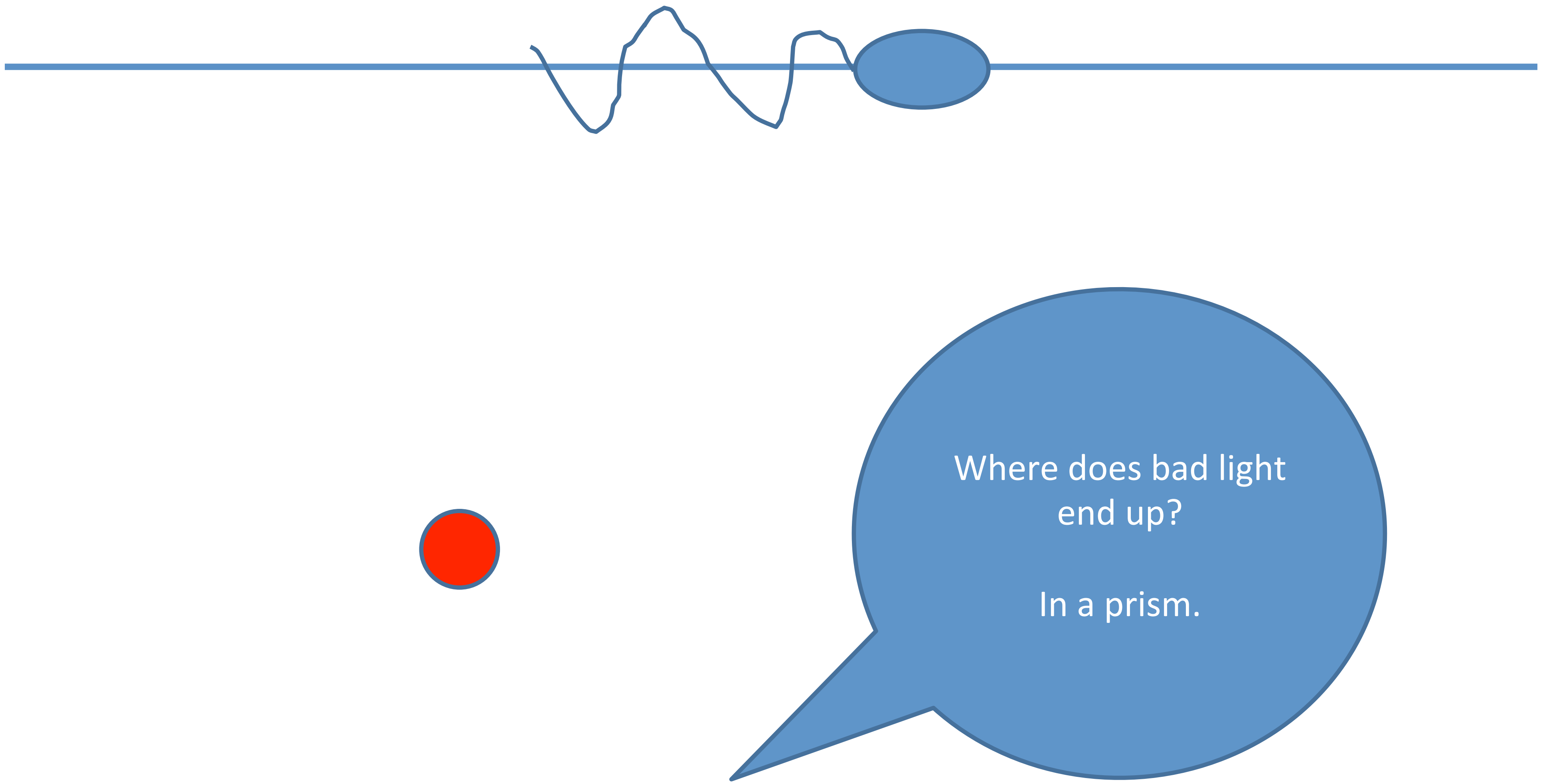
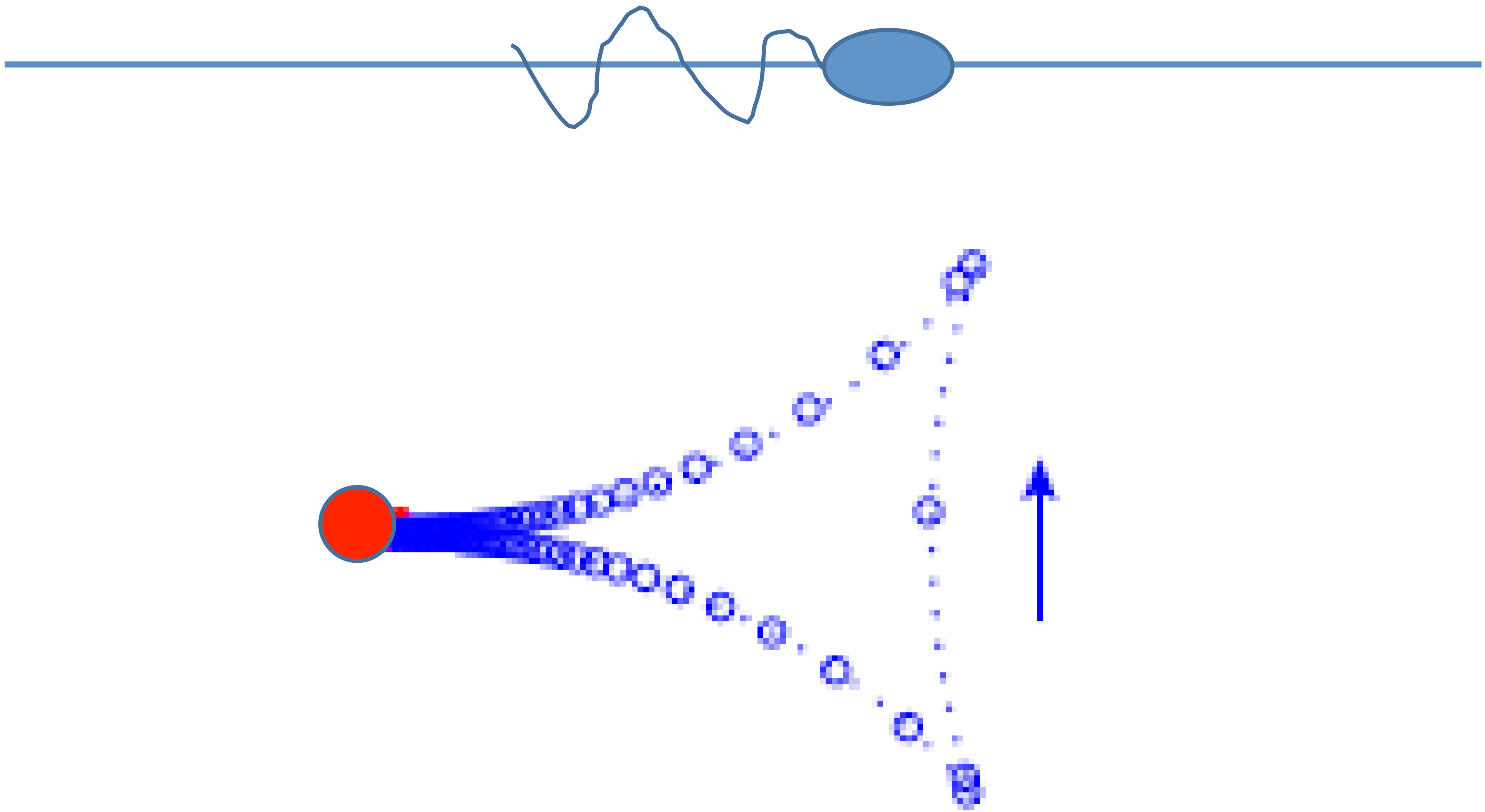
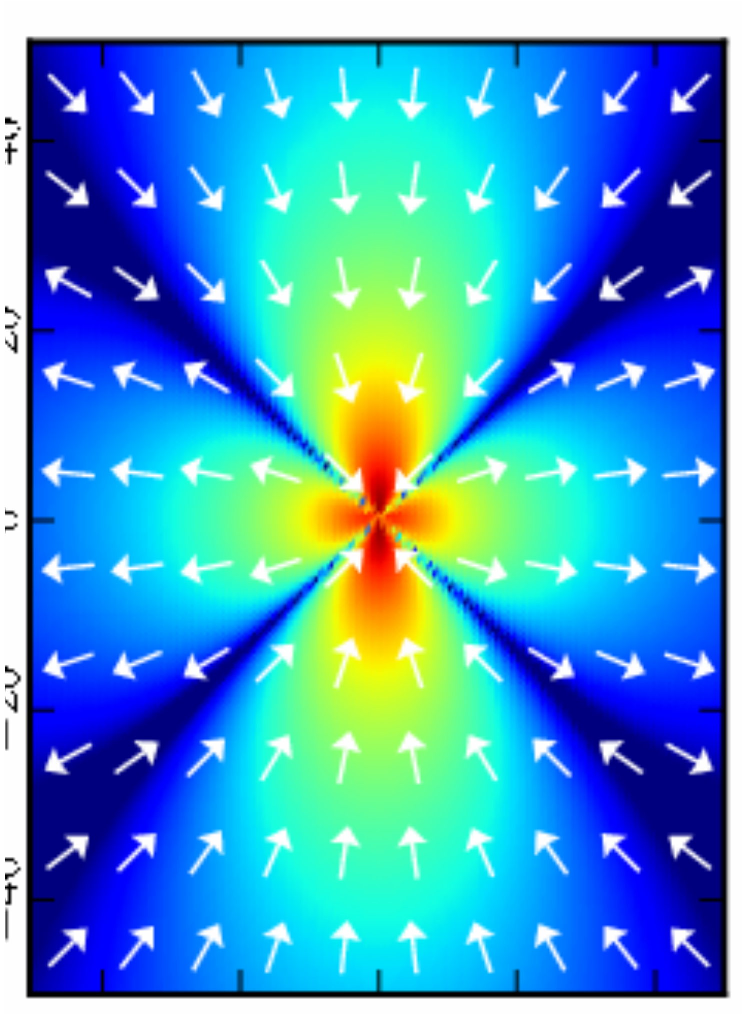


Fluid transport by individual microswimmers,
D.O. Pushkin, H. Shum and J. M. Yeomans,
Journal of Fluid Mechanics **726** 5 (2013).

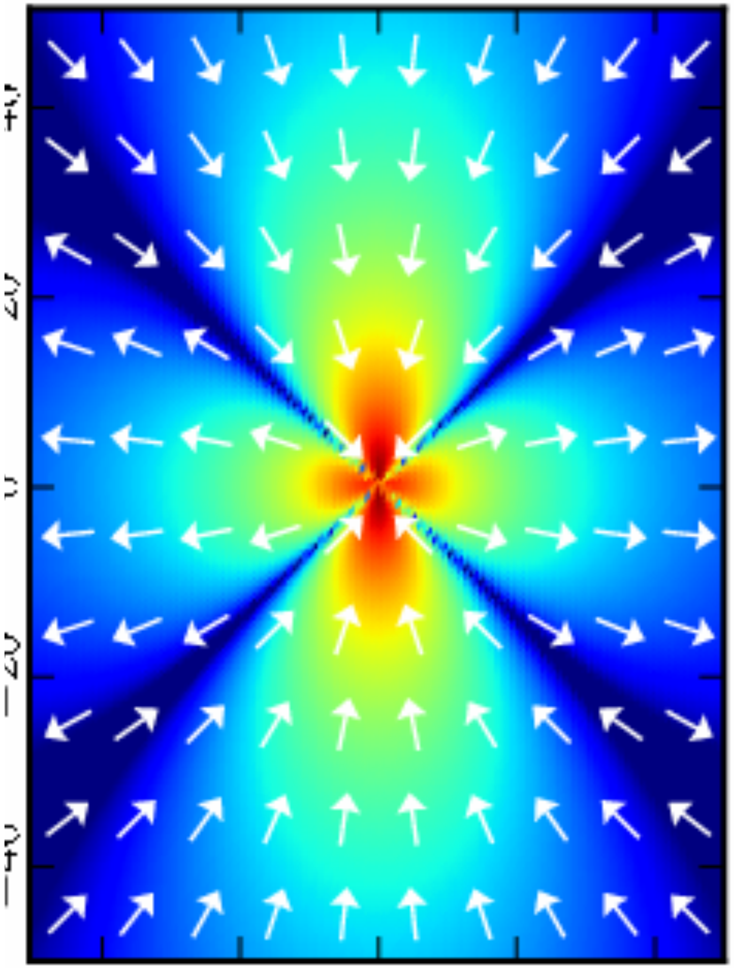


Multipole flow fields



Dipole flow field

Multipole flow fields



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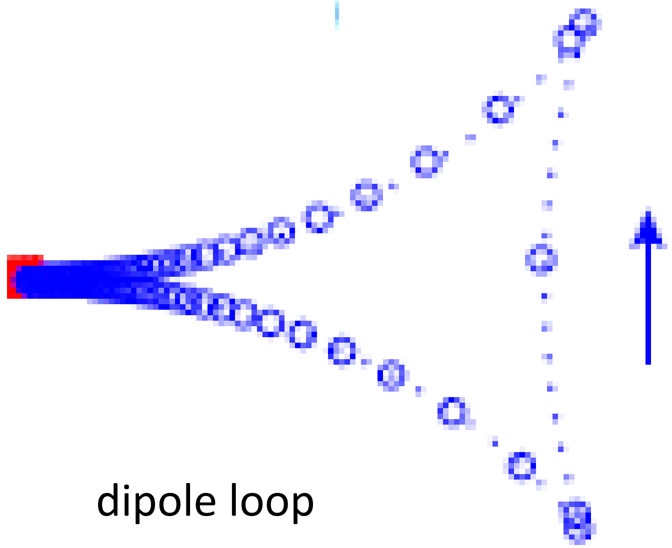
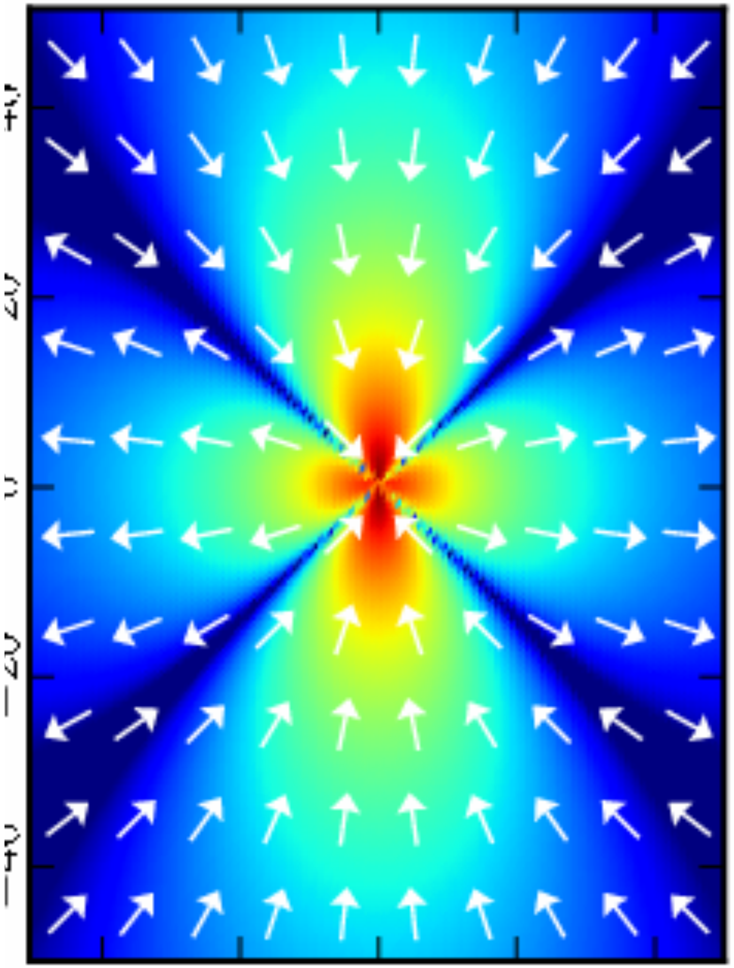


dipole loop

Multipole flow fields



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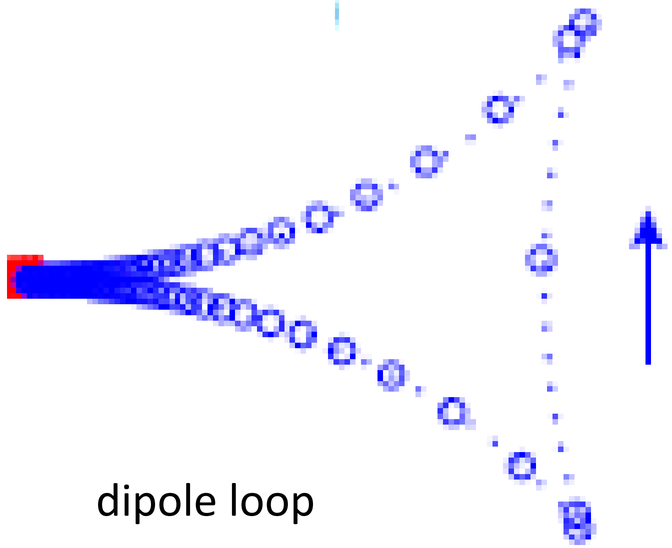

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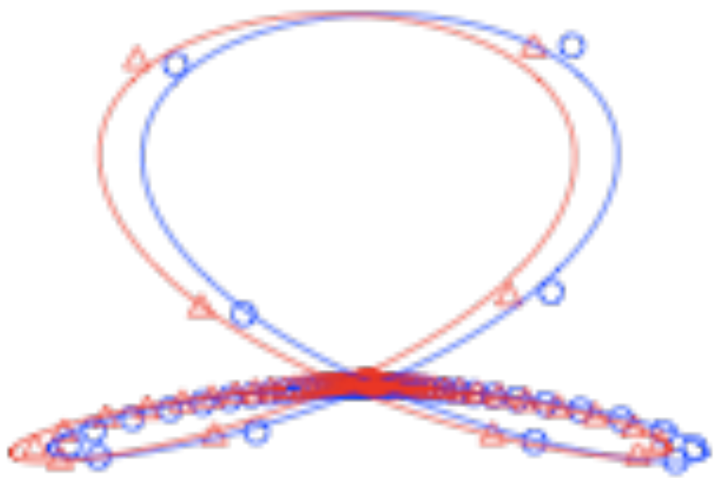


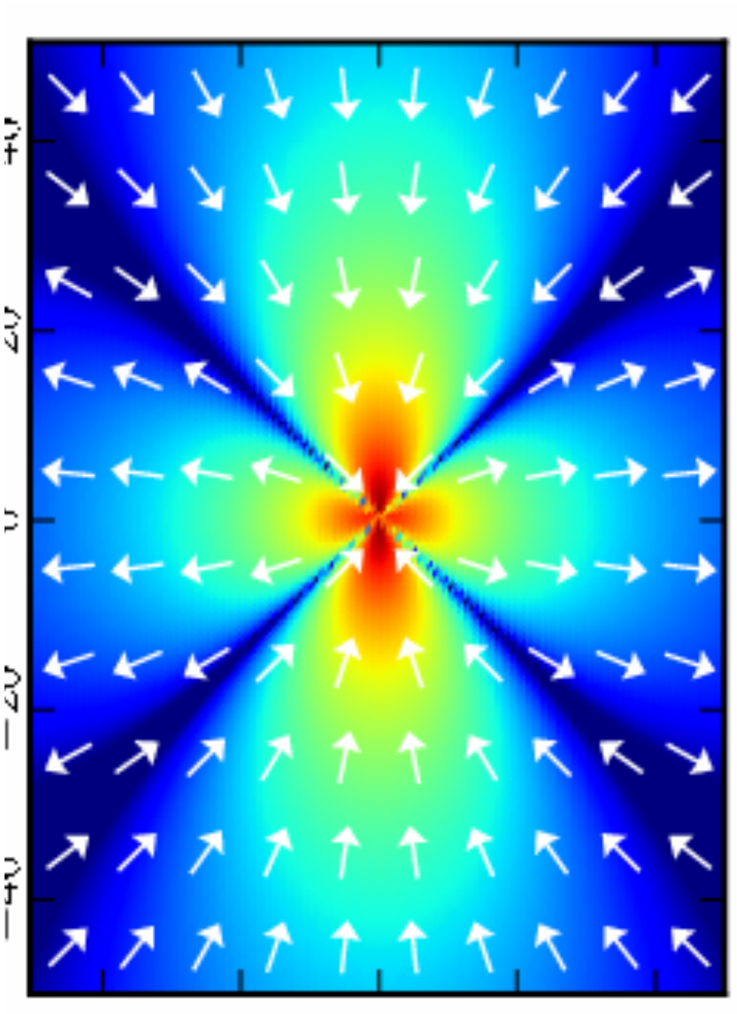
quadrupole loop

?? enhanced diffusion and loops ??

Entrainment

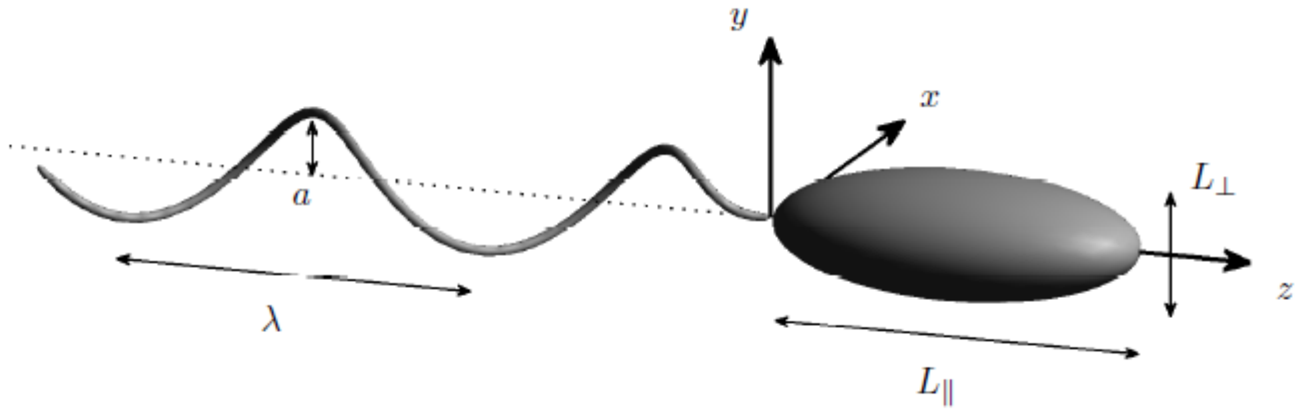
Swimmer re-orientations

Entrainment



Dipole flow field

Rhodobacter sphaeroides



Boundary element simulations

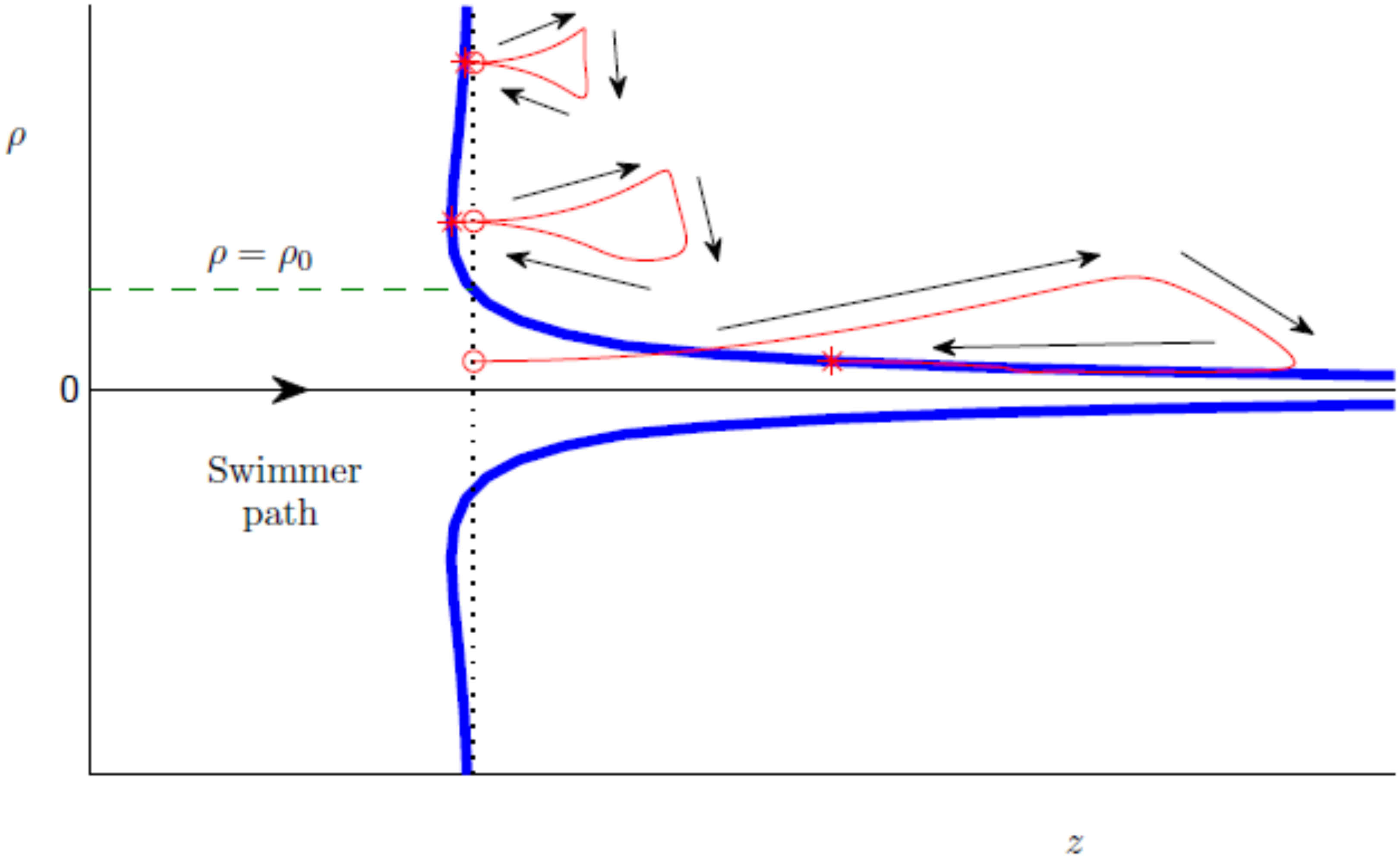
Solve Stokes equations, no slip on swimmer surface, swimmer force and torque free

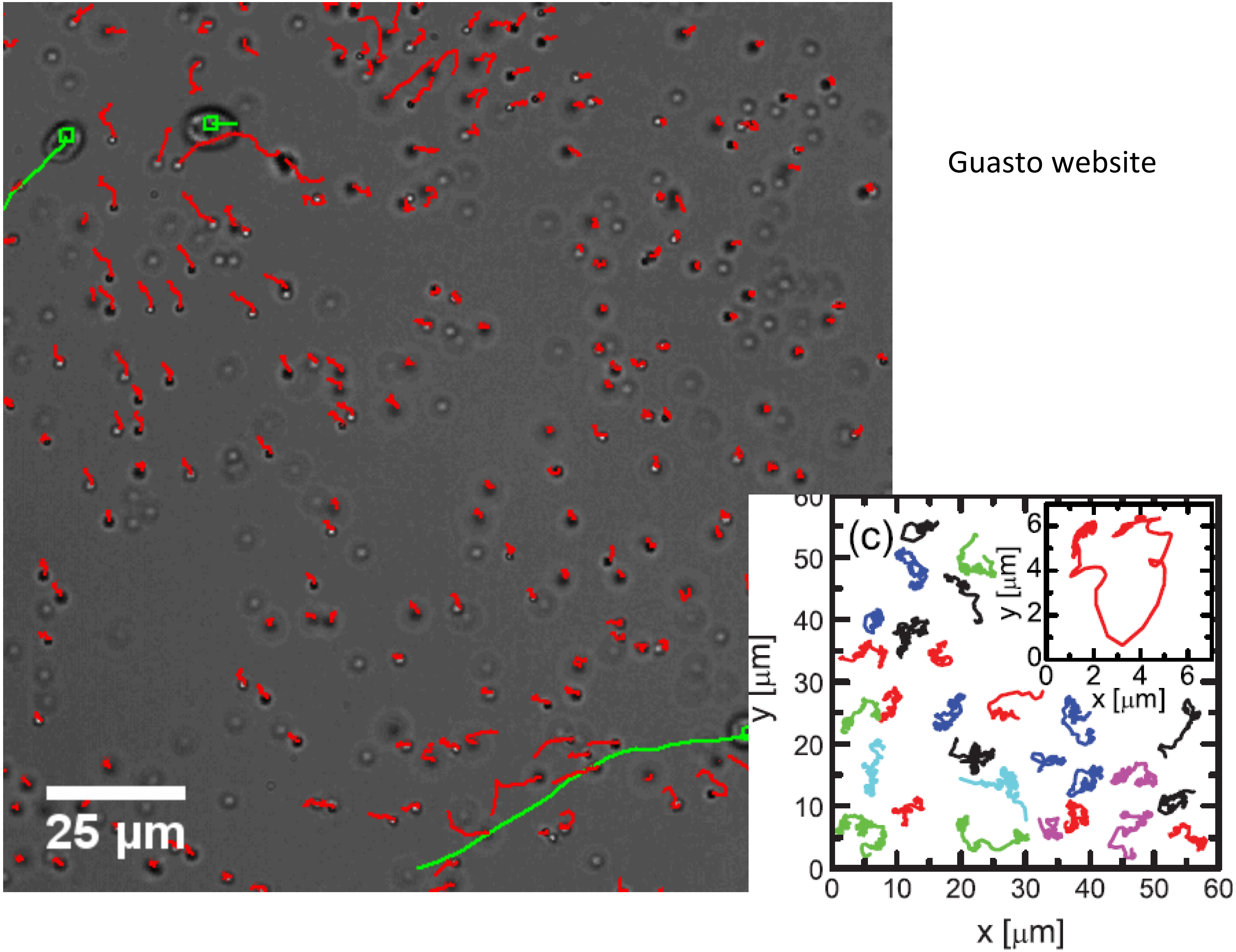
Swimmer radius 1; swimmer velocity 1; ~ 10 rotations of tail to advance one body length

Net tracer displacement along z – deviations from the z-direction very small

Swimmer moves from z= -1000 to z= +1000, and extrapolate to infinite swimmer path

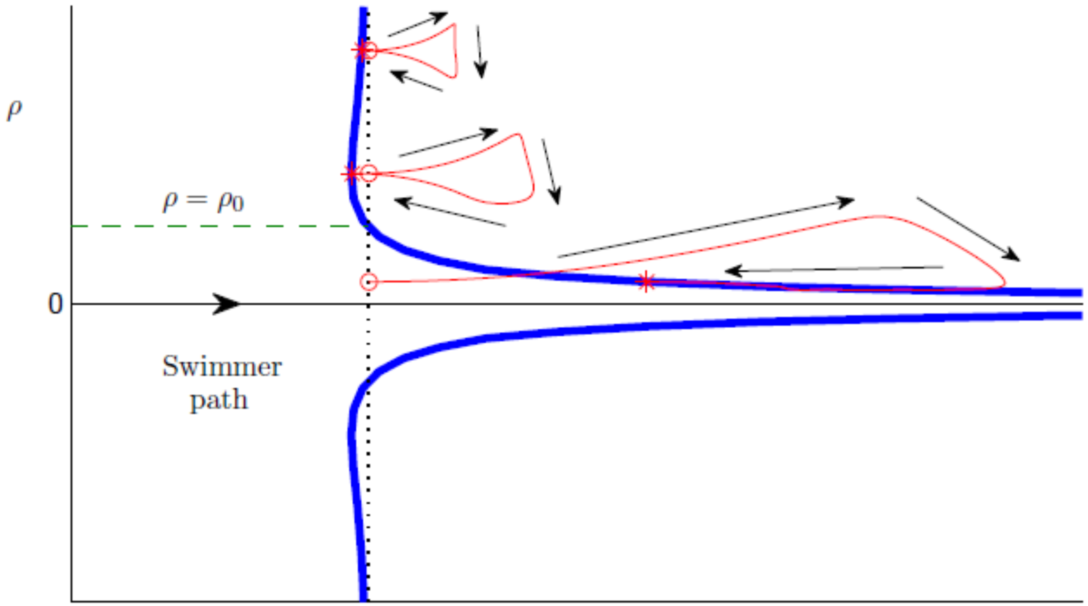
Darwin drift





Guasto website

Darwin drift



Darwin
Benjamin
Eames
Belcher
Hunt
Gobby
Dalziel
Leshansky
Pismen

Total fluid volume moved by swimmer

Darwin drift:

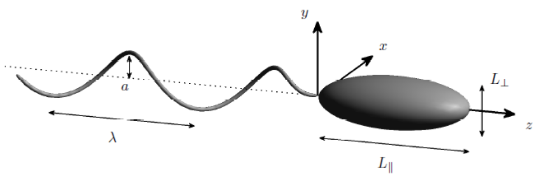
$$v_D = \frac{4\pi Q_\perp}{V} - v_*,$$
$$v_* = v_s + v_{wake}$$

$$Q_\perp = -\frac{1}{2}\int_S f_z \rho^2 dS$$

Comparison of analytic and numerical results for the Darwin drift

TABLE 1. Base parameters: $L_{\|}/L_{\perp} = 2$, $\lambda = 2$, $L = 10$, $a = \lambda/2\pi$.

Shape	$Q_{\perp}/V$	v_D (from equation)	v_D (from simulations)
Base	-0.15	-6.10	-6.11
$L_{\|}/L_{\perp} = 0.5$	-0.68	-12.74	-12.78
$L_{\|}/L_{\perp} = 3.5$	-0.08	-5.24	-5.33
$L = 5$	-0.17	-6.36	-6.30
$L = 15$	-0.14	-5.98	-6.03
$\lambda = 0.5$	-0.20	-6.76	-6.78
$\lambda = 3.5$	-0.04	-4.71	-4.68
$\lambda = 8, L = 20$	0.58	3.04	3.07



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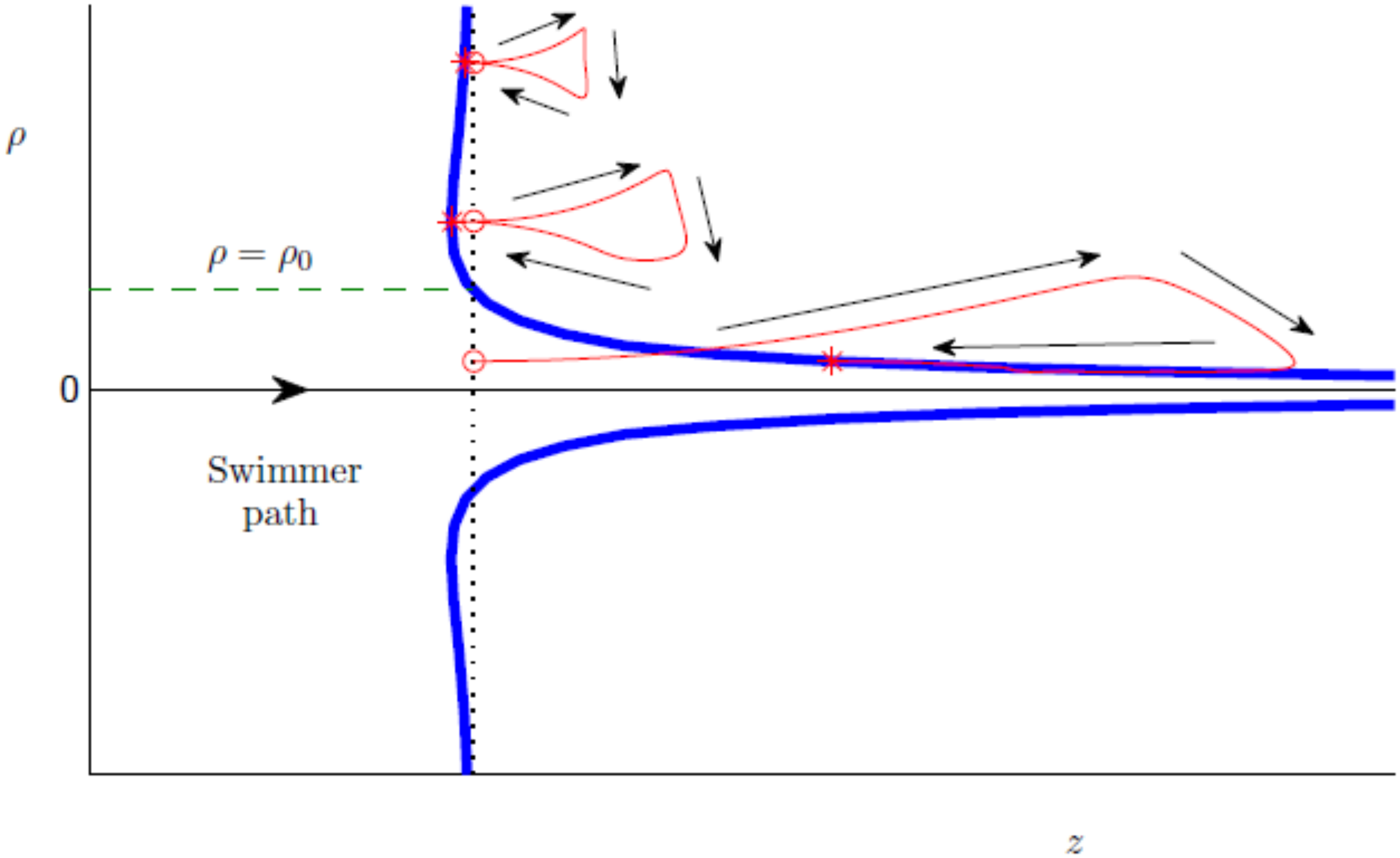

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Entrainment

Tracer moves in loops far from swimmer
Entrainment close to the swimmer

Volume of fluid moved by the swimmer:

Darwin drift:

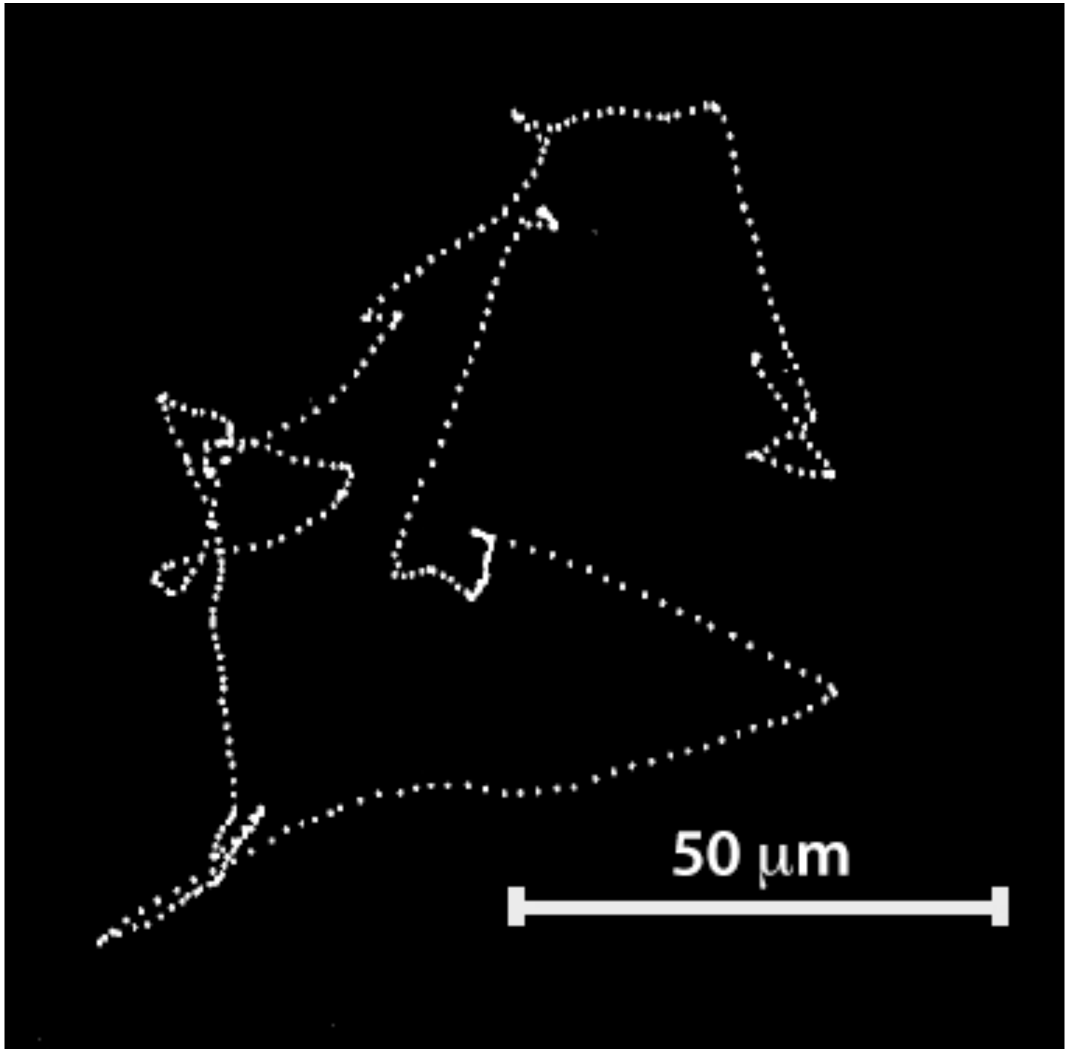
$$v_D = \frac{4\pi Q_\perp}{V} - v_*,$$
$$v_* = v_s + v_{wake}$$

$$D_{entr} \approx \frac{1}{6} n\, V\, a\, \frac{4\pi}{3} a^3$$

?? enhanced diffusion and loops ??

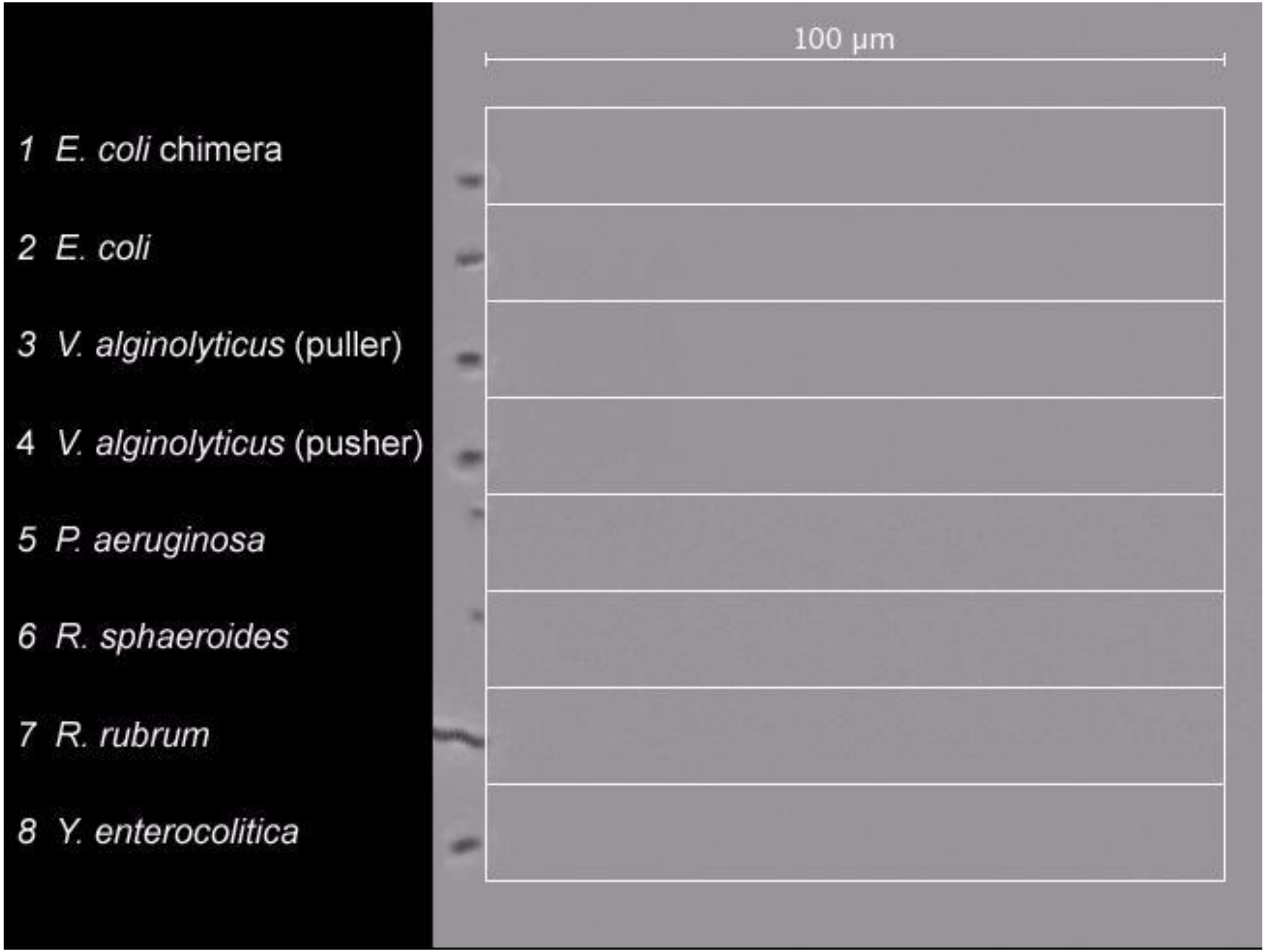
Entrainment

Swimmer re-orientations



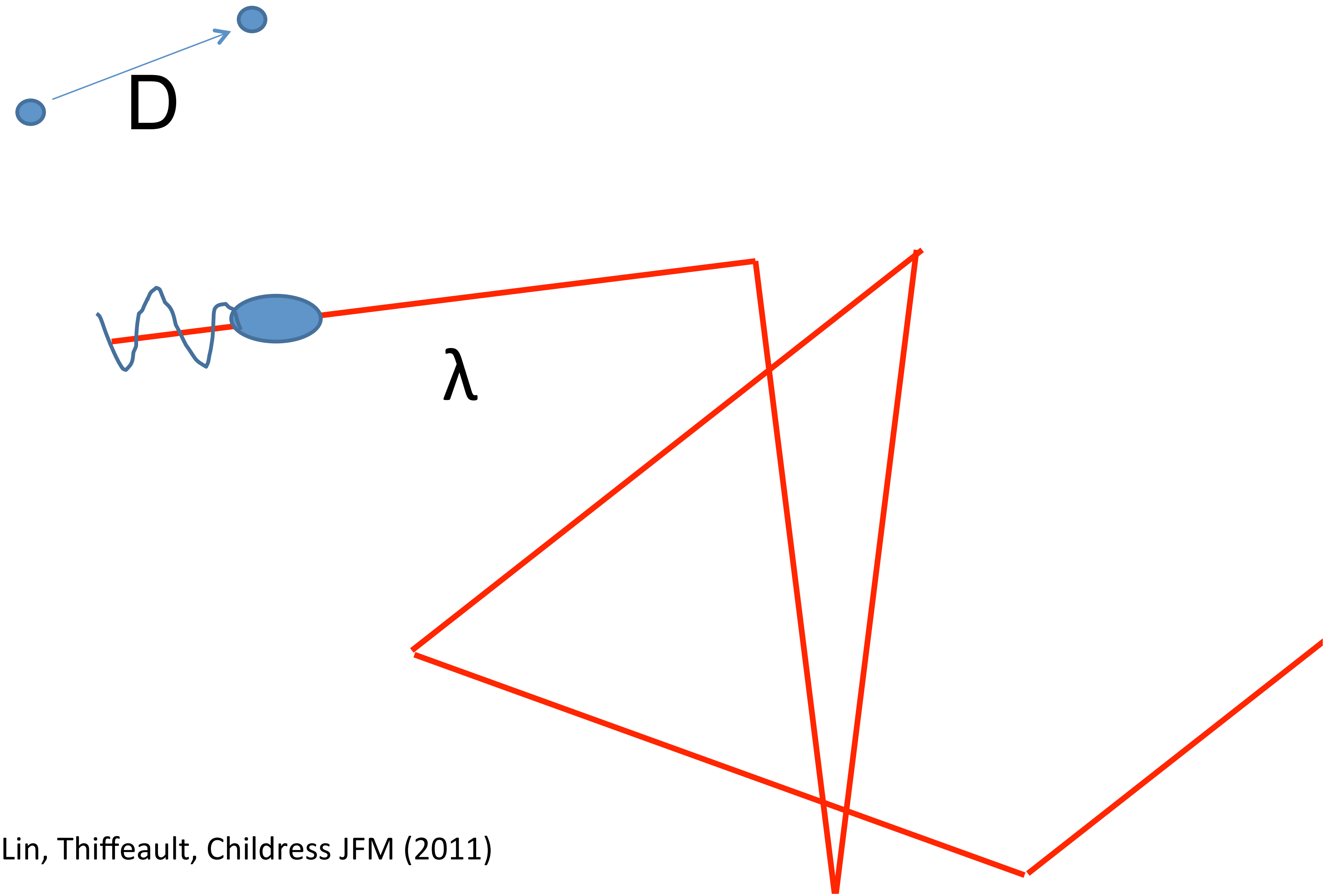
E-coli tracks: Howard Berg

!!!!!   Bacterial Olympics: 100 micrometres   !!!!



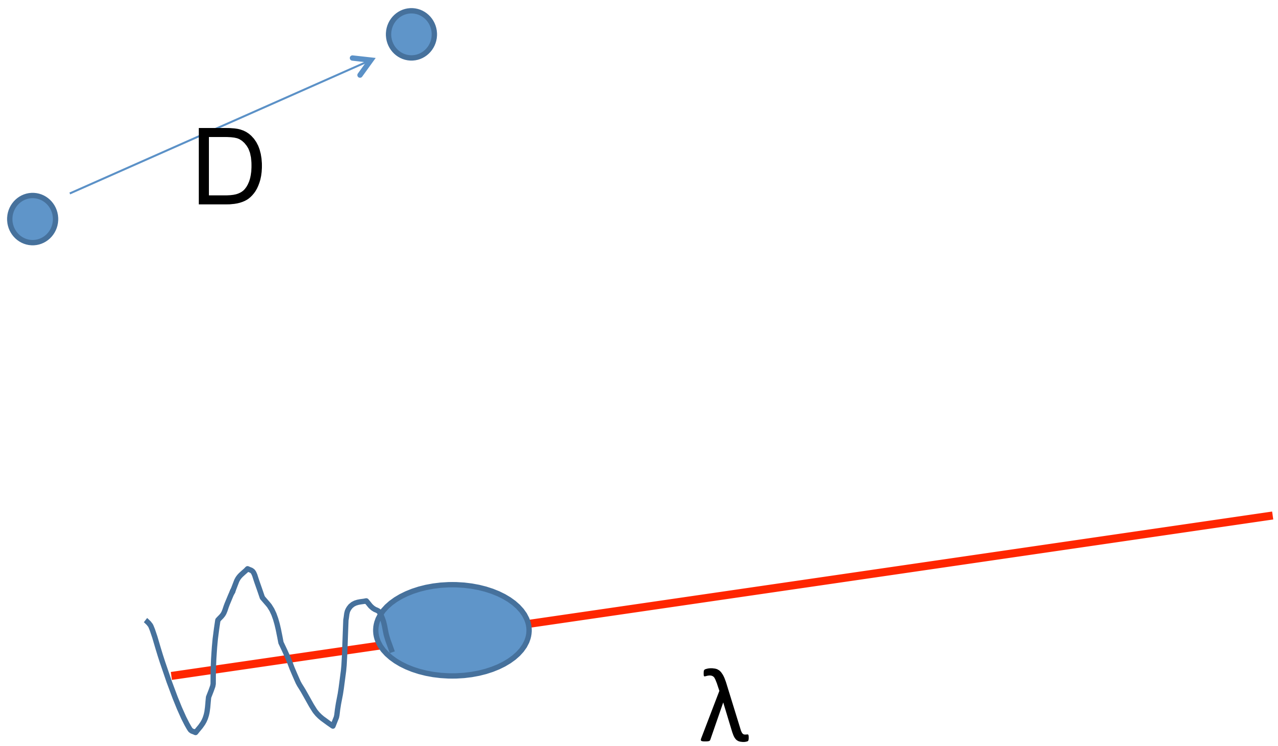
Judith Armitage, Oxford biophysics

Random reorientations



Lin, Thiffeault, Childress JFM (2011)

Random reorientations



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Diffusion constant for random reorientations

Dipolar swimmer 3D

$$D_{rr} = \frac{4\pi}{3} \kappa_m^2 \, n \, V \, a^4$$

- Independent of swimmer run length for dipolar swimmers in 3D
- Distribution of tracer run lengths converges to a Gaussian

Fluid mixing by curved trajectories of microswimmers, D.O. Pushkin and J.M. Yeomans, Phys. Rev. Lett. **111** 188101 (2013).

Entrainment	Random reorientations

Dipolar swimmer, d=3

$$\frac{D_{rr}}{D_{entr}} \approx \frac{\tilde{\kappa}^2\, n\, V\, a^4}{n\, V\, a^4} \sim \tilde{\kappa}^2$$

What is missing?

Swimmer-swimmer interactions
Fluctuations
Diffusion in films
Experiments

Diffusion of particles in an active suspension has contribution from entrainment, random swimmer reorientations and thermal fluctuations.

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