The Hydrodynamics of Active Matter

Julia Yeomans
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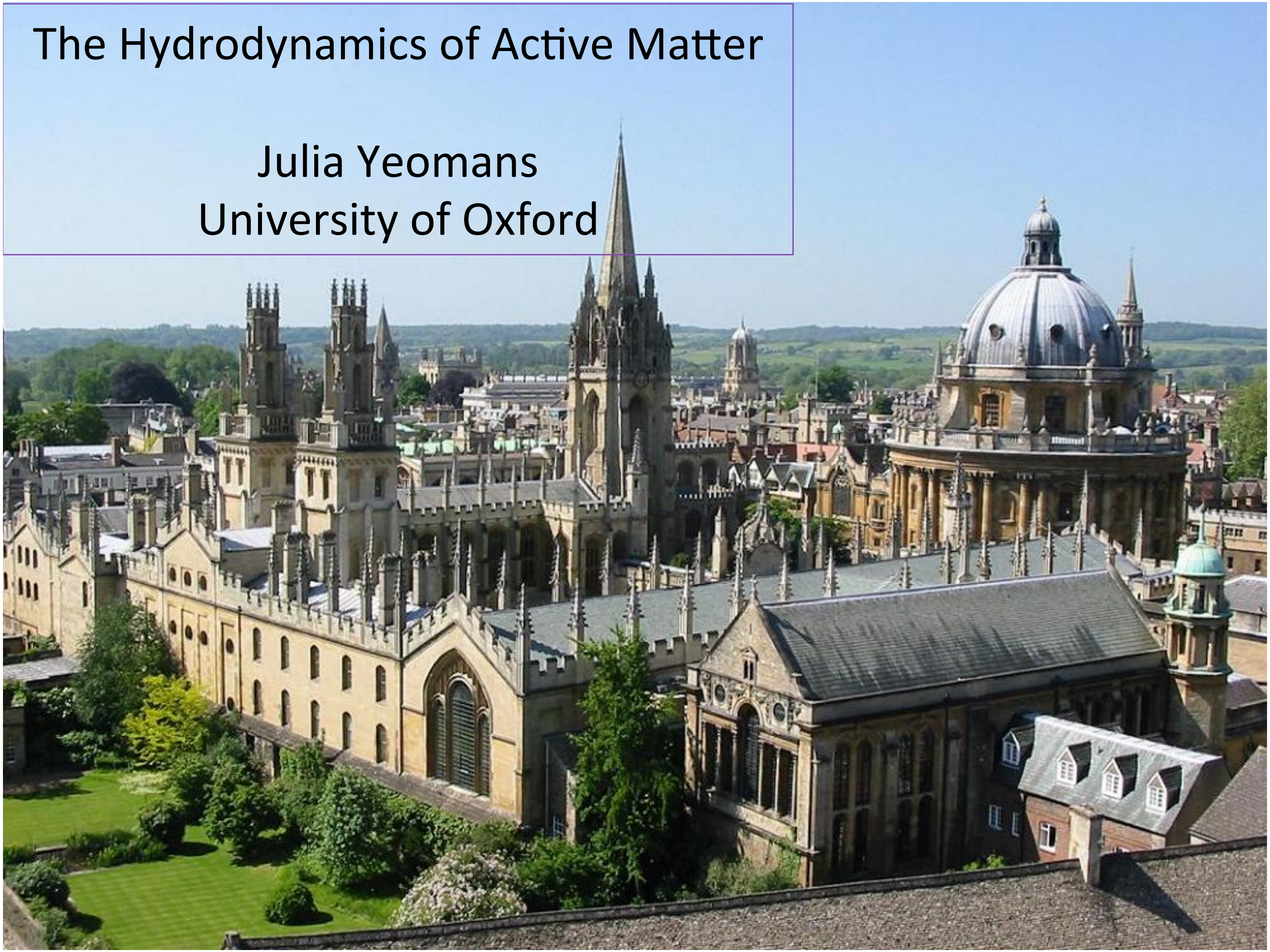


Lecture 1: The mathematics and physics of bacterial swimming

one active particle

Lecture 2: Applications

Lecture 3: Continuum models of dense active matter

lots of active particles

Lecture 4: Active turbulence and lyotropic active nematics

1. Introduction: a zoo of active systems

2. Liquid crystals and Q

3. Active equations of motion

4. Instabilities

5. Active turbulence + topological defects

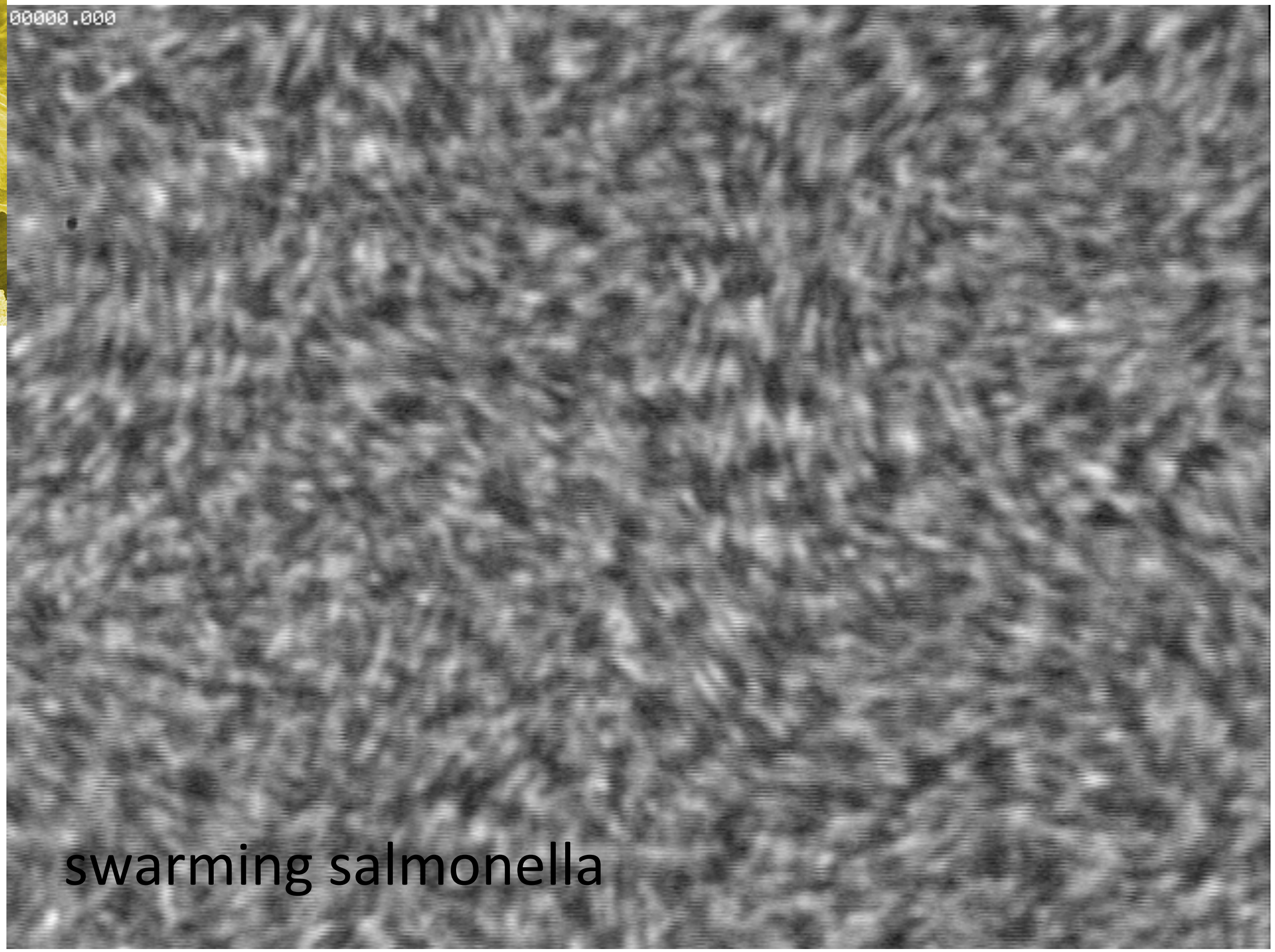
6. Molecular motors and Dogic experiments

7. Lyotropic active nematics

8. Dividing cells

Bacteria 1

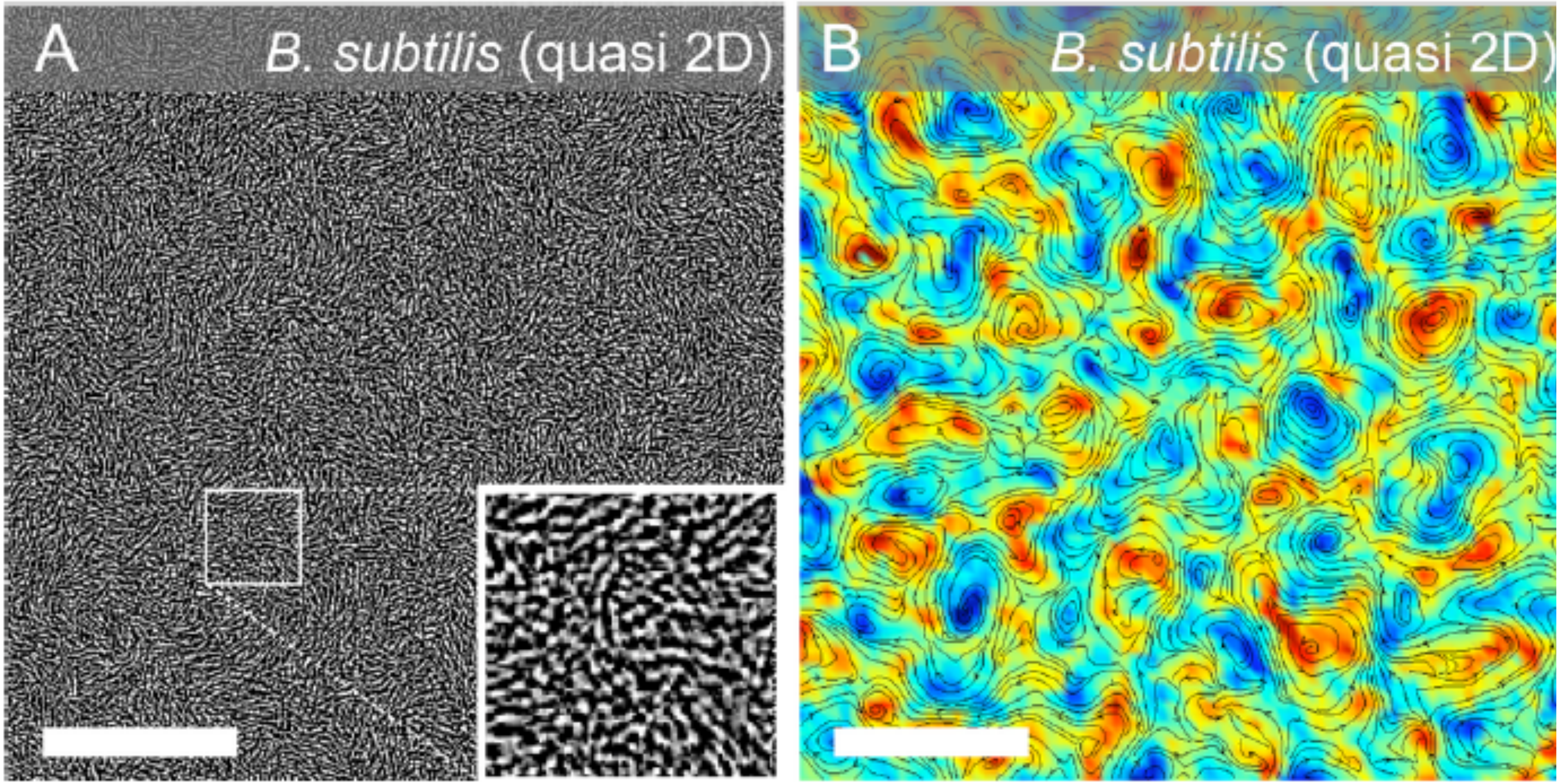
http://webmac.rowland.org/labs/bacteria/index.html



Chakrabartu, Das, Dasgupta,Ramaswamy, Sood PRL (2004)

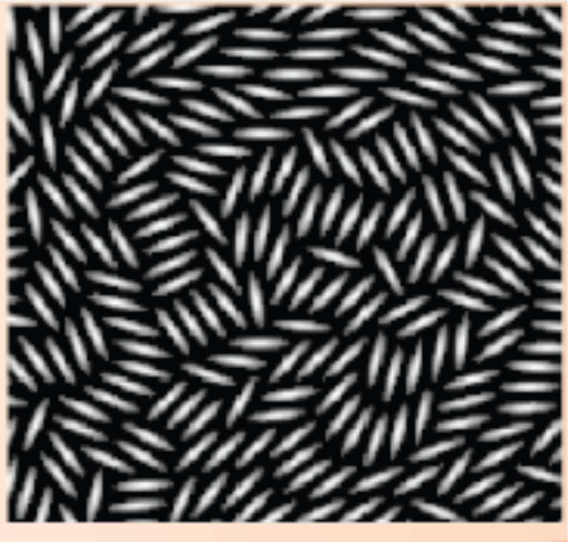
Bacteria 2

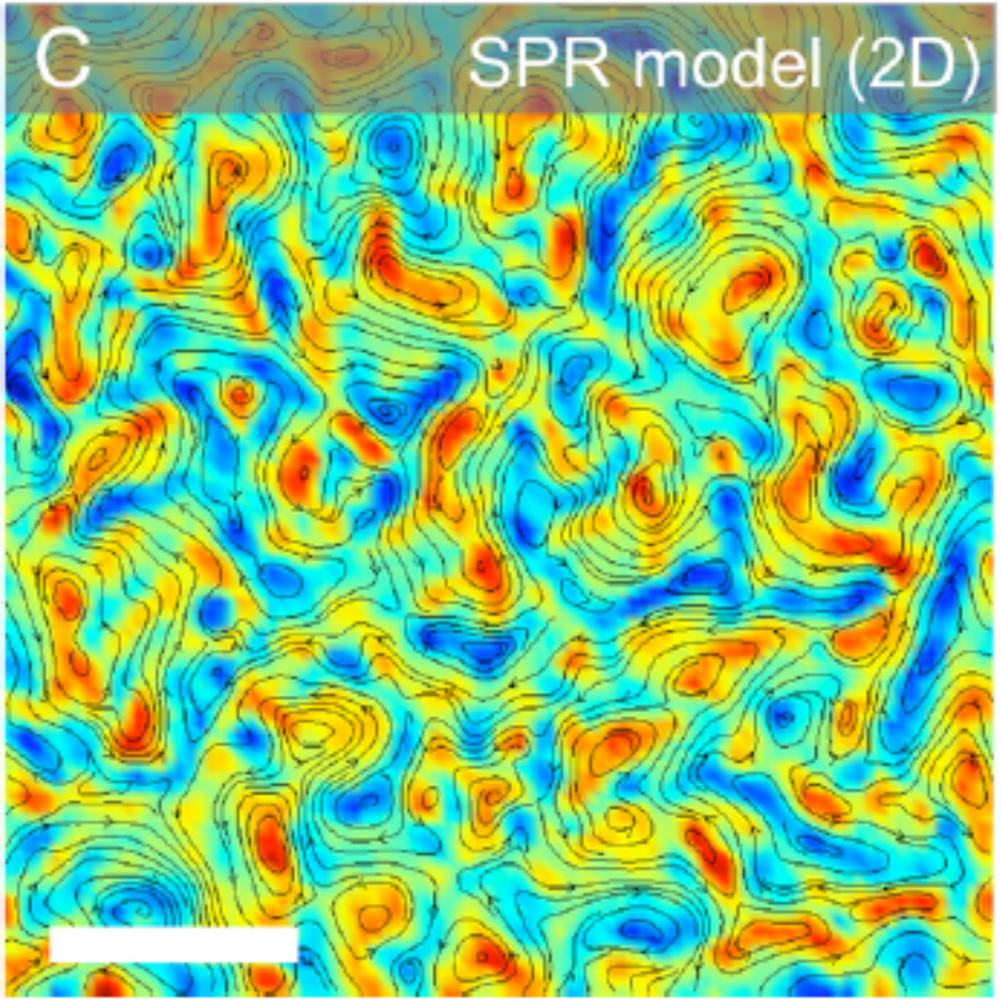
vorticity



Wensink, Dunkel, Heidenreich, Dresher, Goldstein, Lowen, Yeomans, PNAS 2012

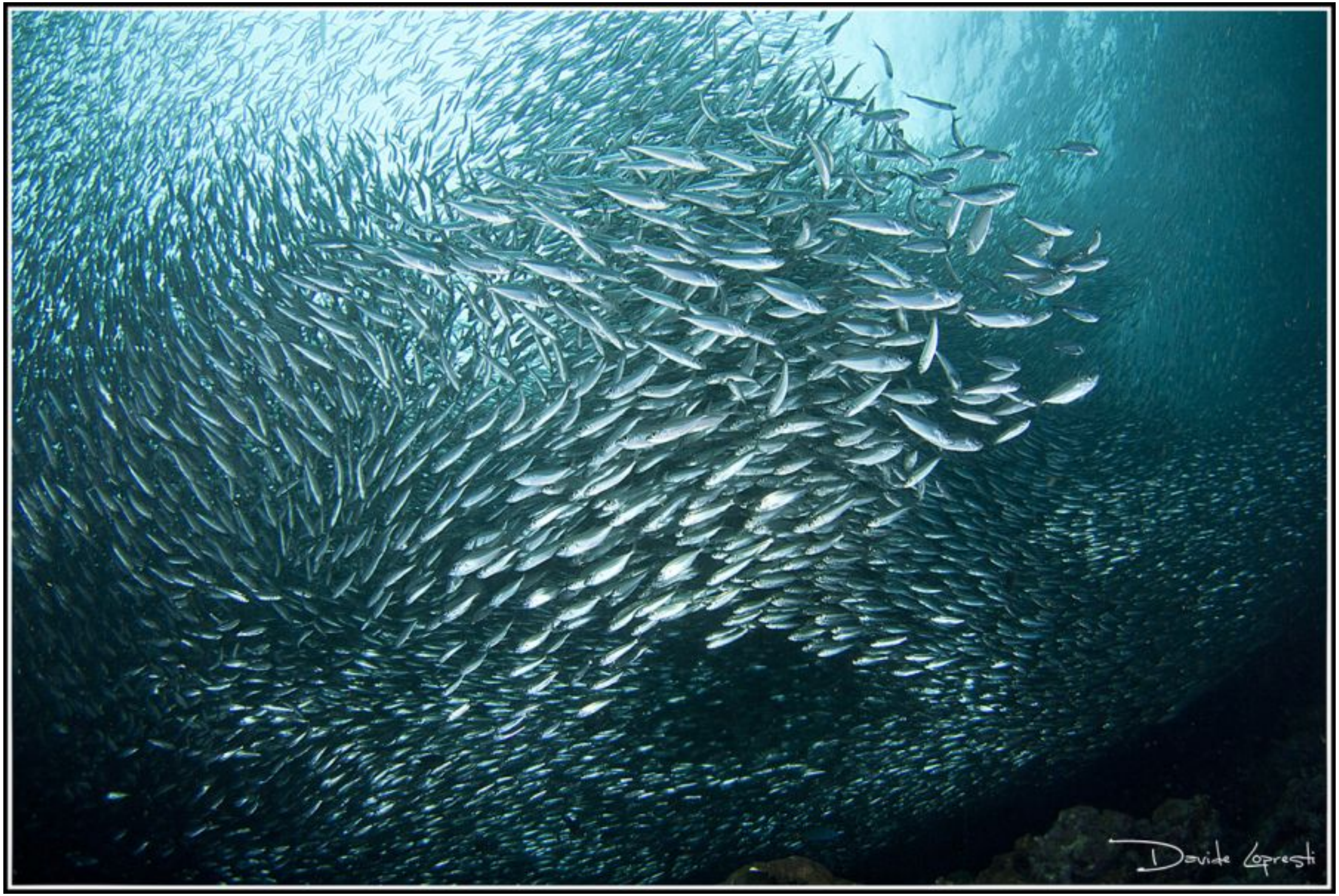
Discrete simulations



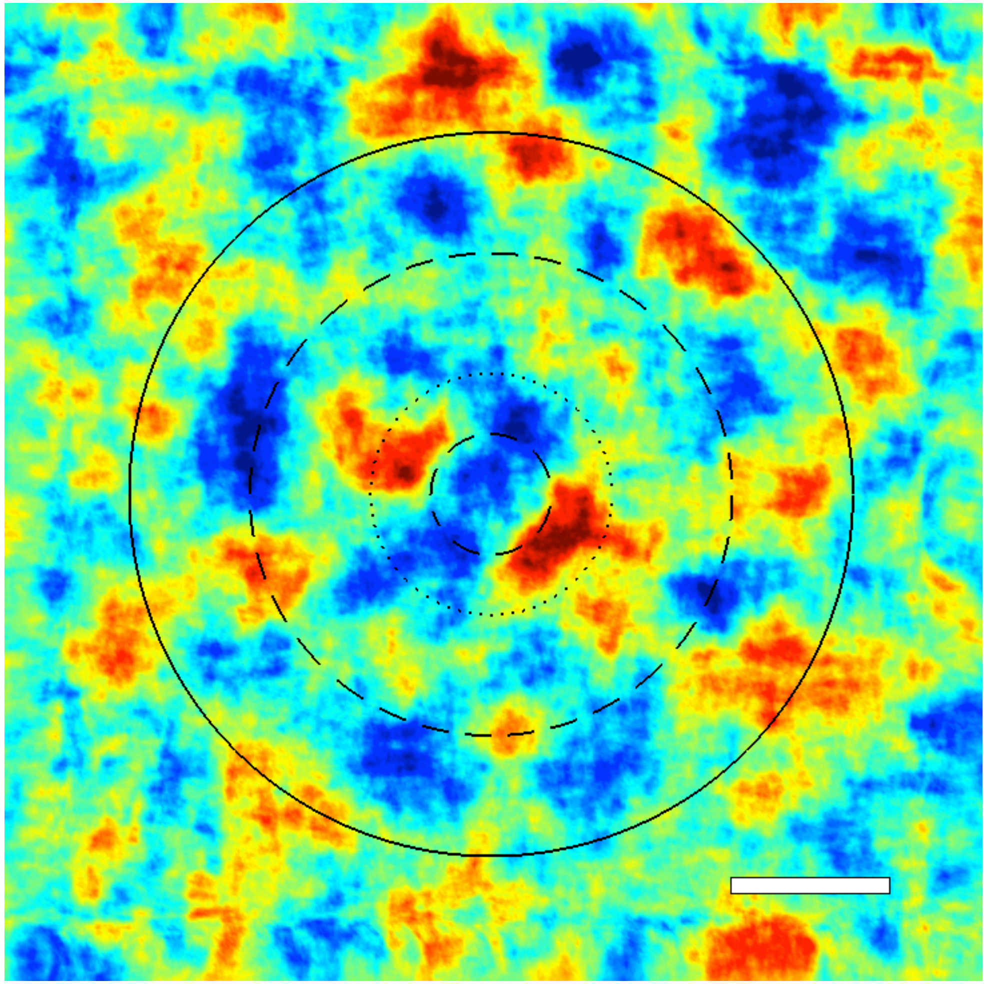


Wensink, Dunkel, Heidenreich, Dresher, Goldstein, Lowen, Yeomans, PNAS 2012

Fish?



Endothelial cells

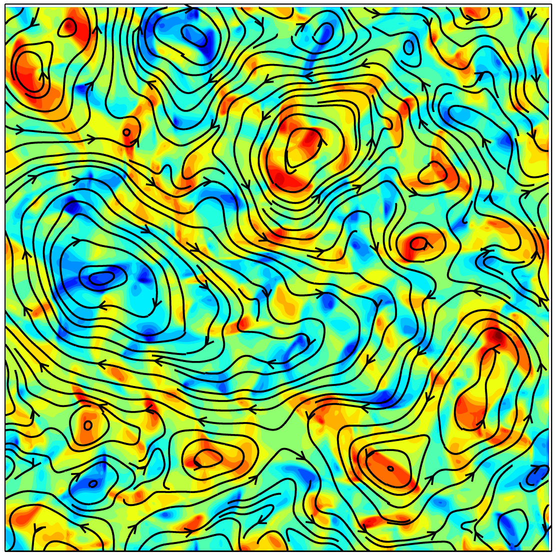


vorticity plots
near a dividing cell

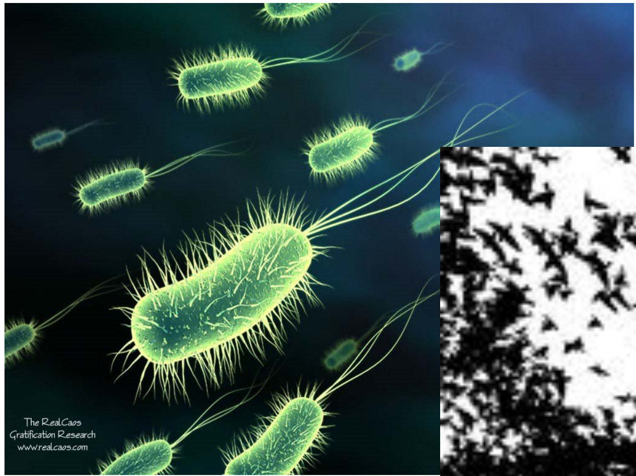
Lene B. Oddershede
Niels Bohr Institute
University of Copenhagen
Denmark

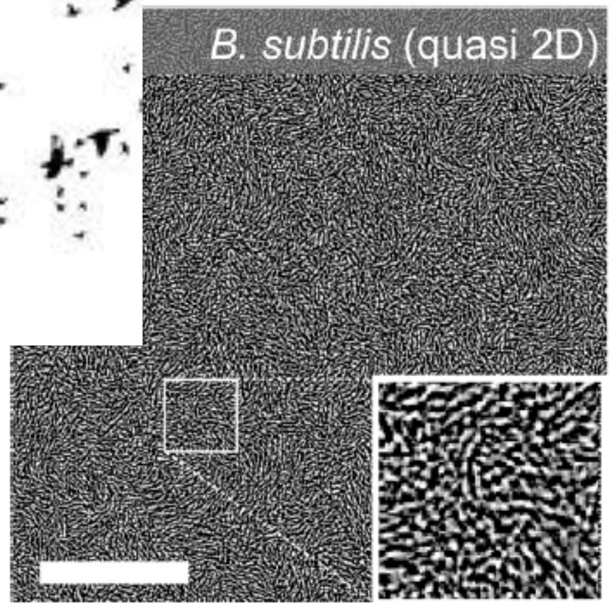
Active turbulence

- What are the mechanisms that drive active turbulence?
- What determines the flow patterns and characteristics?
- Can we identify generic properties and universal features among different systems



concentration





“There's a door."
"Where does it go?"
"It stays where it is, I think.”
— [Terry Pratchett, *Eric*]

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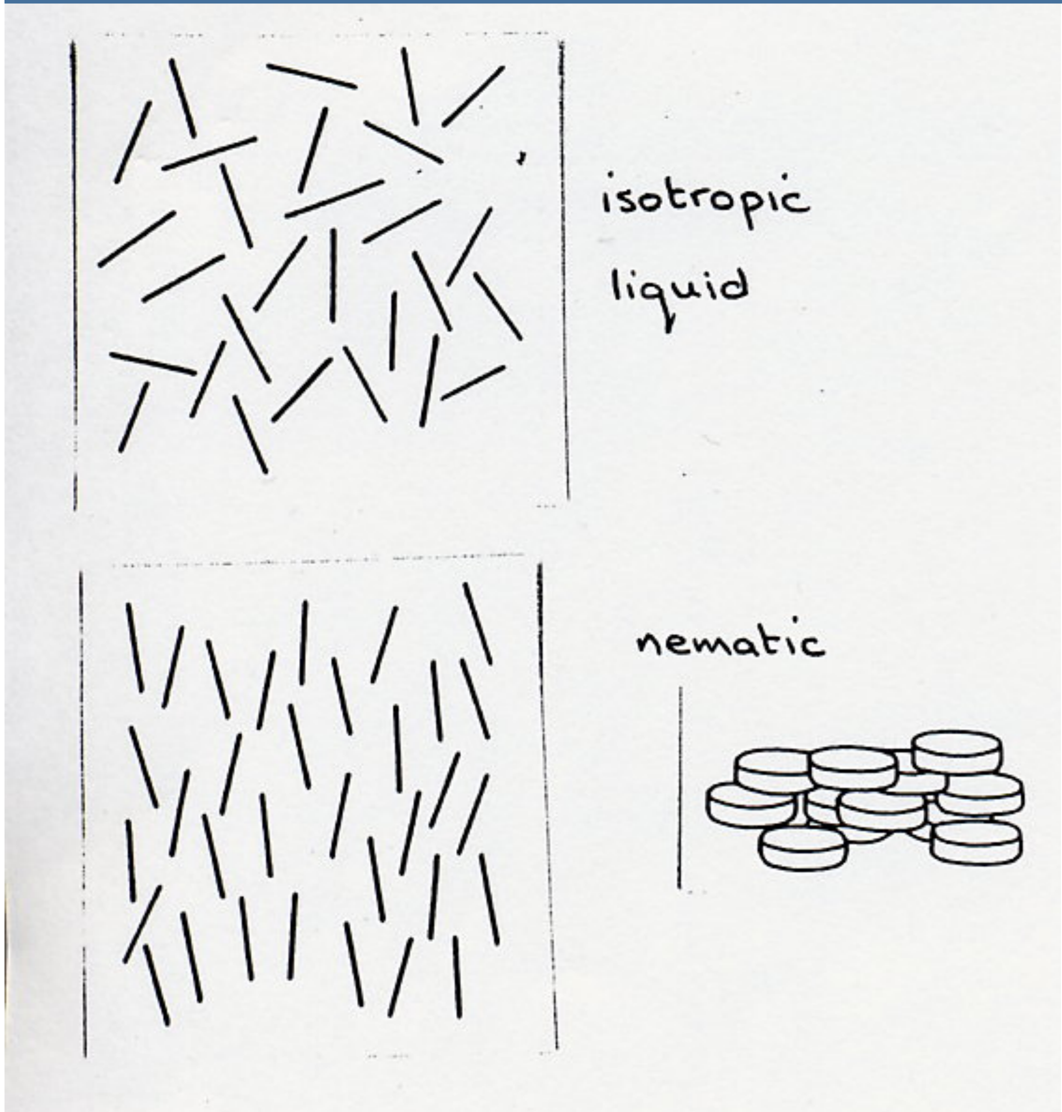
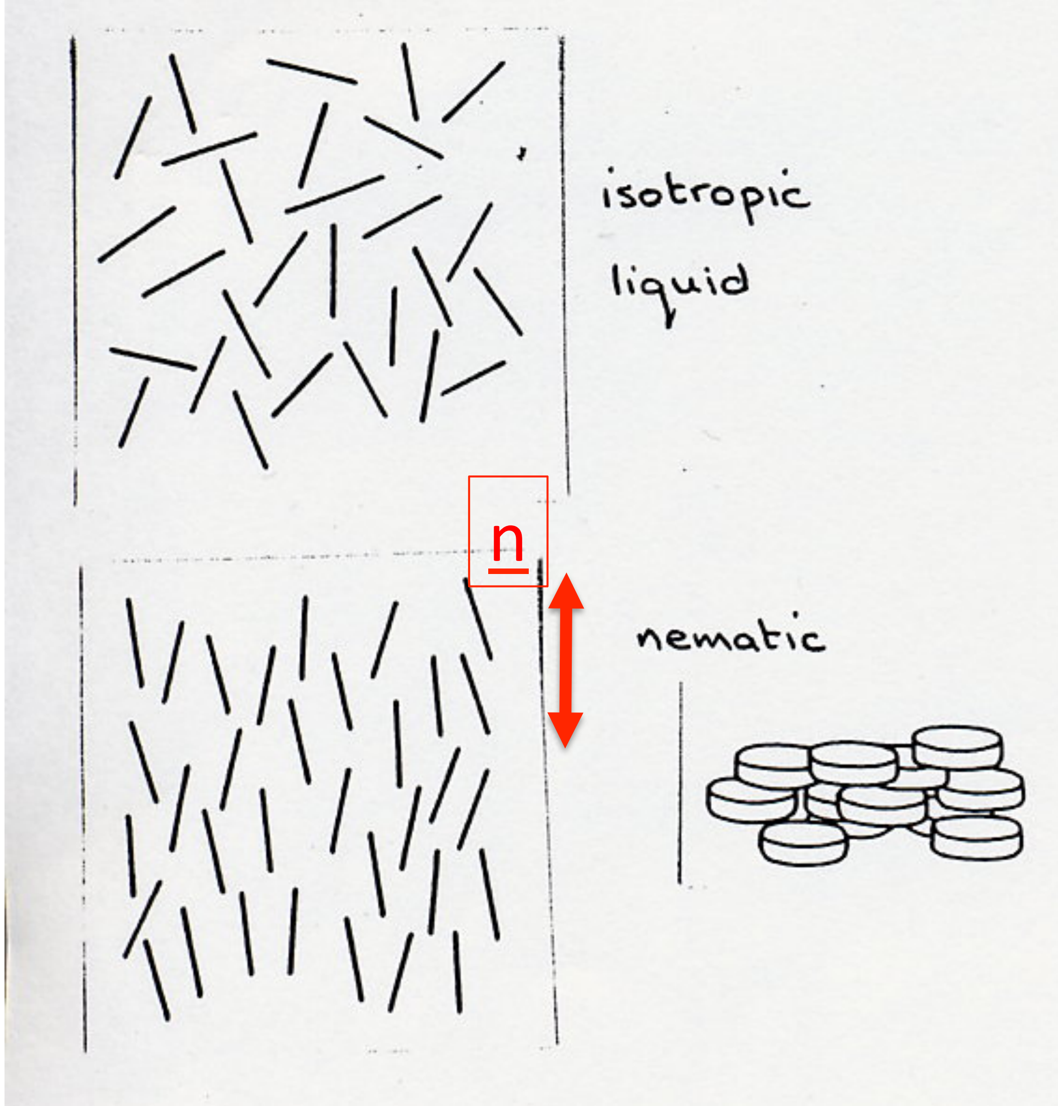
Liquid crystals



nematic symmetry

nematic order parameter $\underline{n}$
tensor order parameter Q

Liquid crystals



nematic symmetry

nematic order parameter n
tensor order parameter Q

Tensor order parameter, Q

$$Q_{ij} = \langle n_i n_j - \frac{\delta_{ij}}{3} \rangle$$

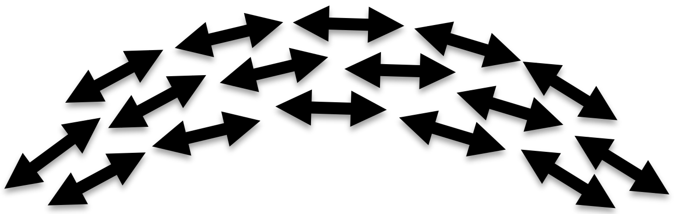
nematic with director aligned along z:

$$\mathbf{Q} = \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 2/3 \end{pmatrix}$$

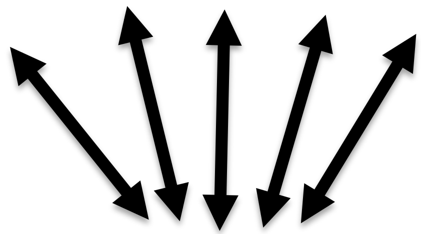
Landau-de Gennes free energy

$$\mathcal{F} = \frac{K}{2}(\partial_k Q_{ij})^2 + \frac{A}{2} Q_{ij} Q_{ji} + \frac{B}{3} Q_{ij} Q_{jk} Q_{ki} + \frac{C}{4}(Q_{ij} Q_{ji})^2$$

Elasticity of liquid crystals



Bend

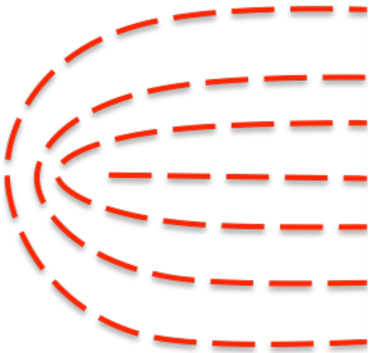


Splay

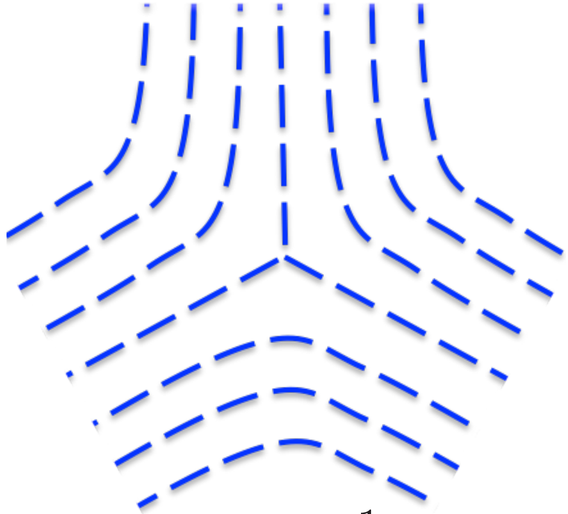
Landau-de Gennes free energy

$$\mathcal{F} = \frac{K}{2}(\partial_k Q_{ij})^2 + \frac{A}{2}Q_{ij}Q_{ji} + \frac{B}{3}Q_{ij}Q_{jk}Q_{ki} + \frac{C}{4}(Q_{ij}Q_{ji})^2$$

Topological defects in nematic liquid crystals

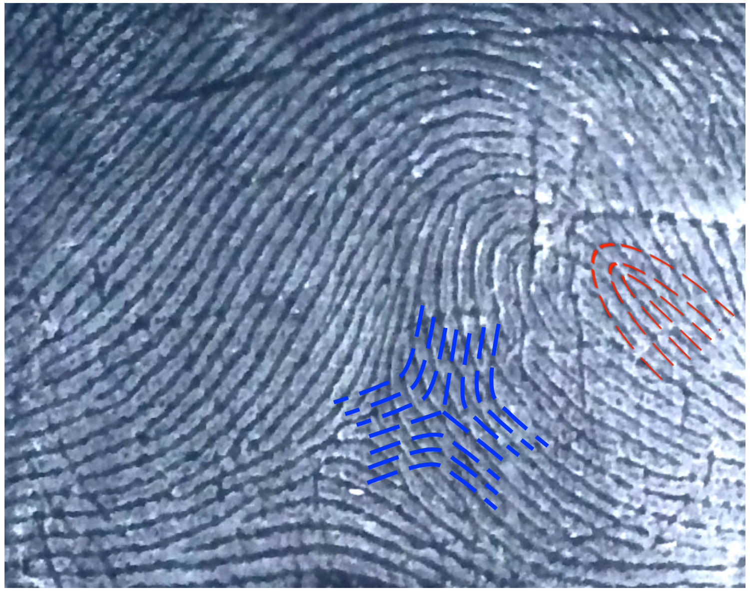


$$m = +\frac{1}{2}$$

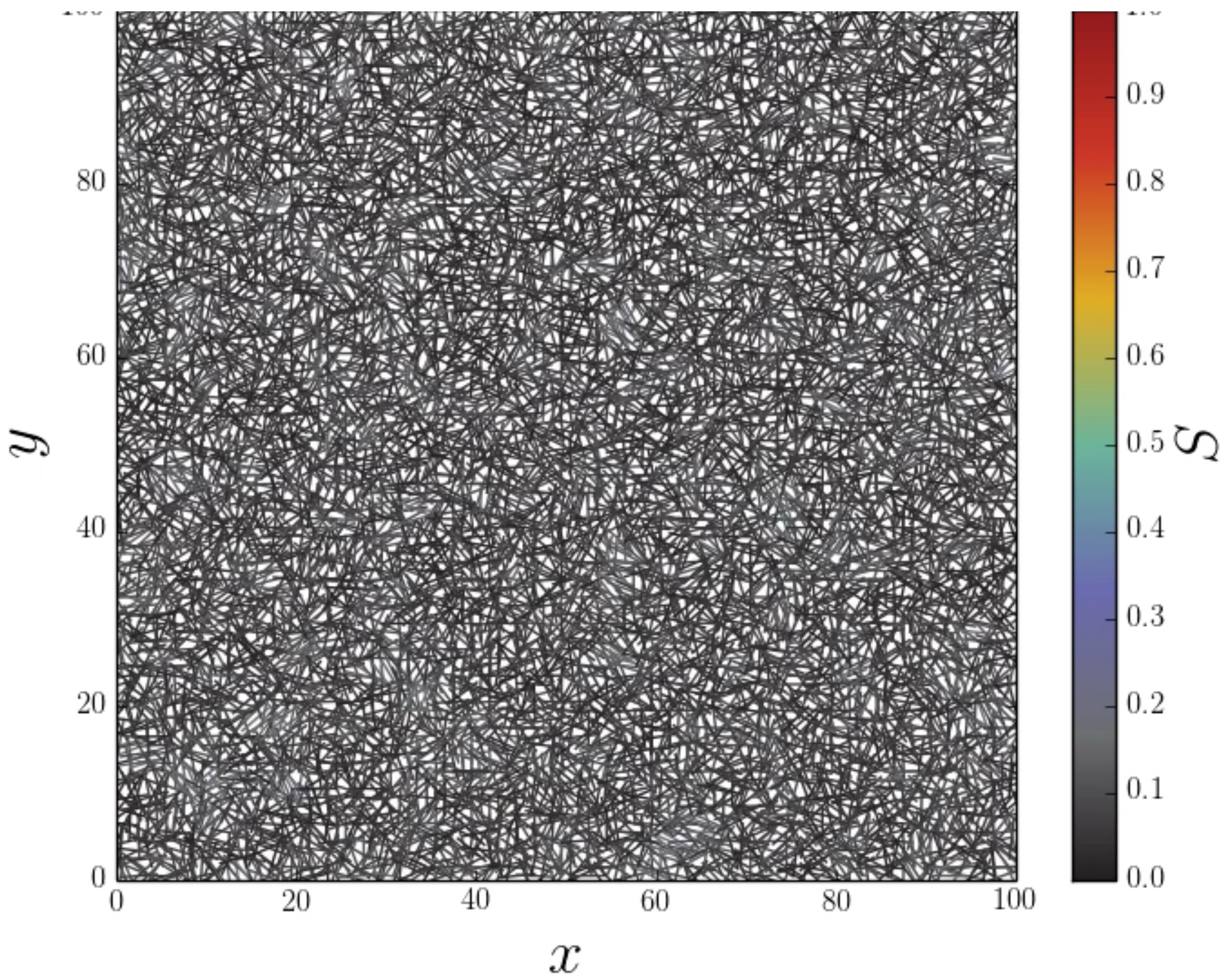


$$m = -\frac{1}{2}$$

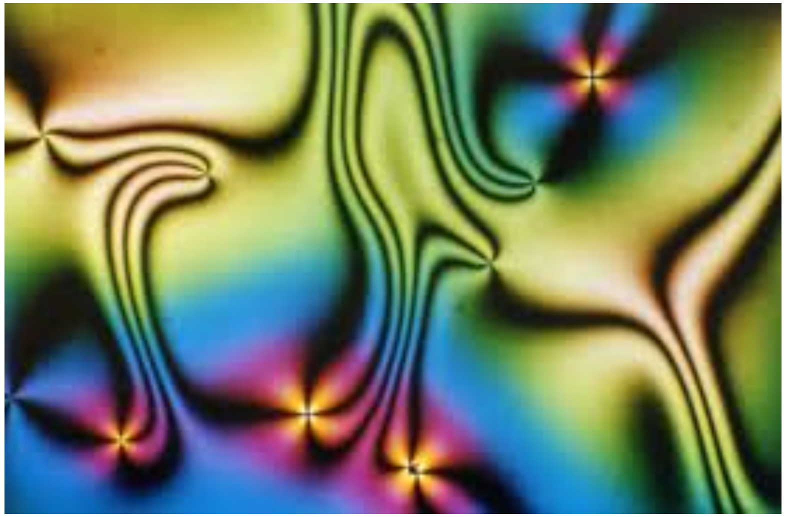
$$\boxed{\begin{array}{c}\text{topological charge}\\ m = \dfrac{1}{2\pi}\displaystyle\int_{dS} d\theta\end{array}}$$



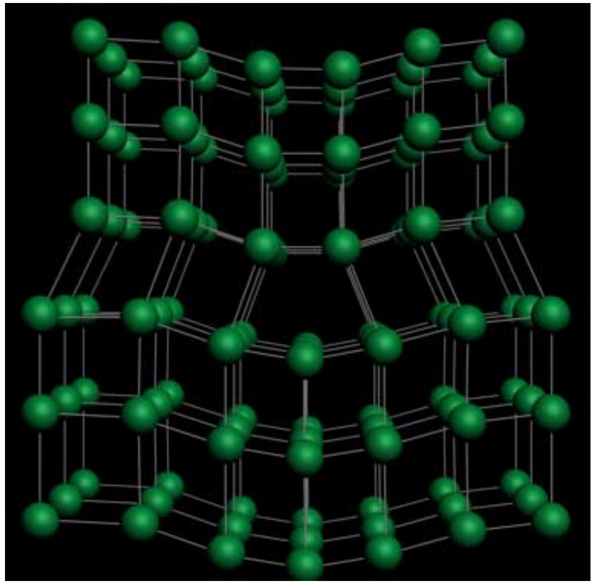
Idea courtesy: A Sengupta, MIT



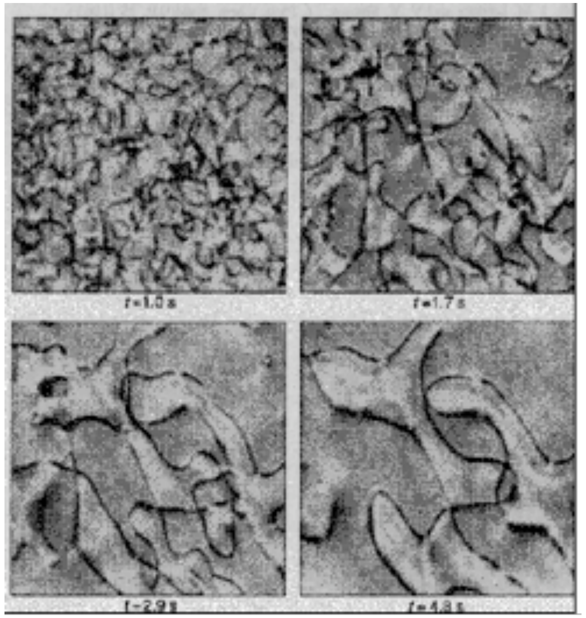
Tyler Shendruk, MPCD



liquid crystals



crystal dislocations



magnetic monopoles in spin ice

topological insulators

quantum vortex in a superfluid

cosmic strings in the early universe

Tumbling and aligning liquid crystals: response to a shear

Continuum equations of liquid crystal hydrodynamics

Continuum equations of liquid crystal hydrodynamics

$$(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij}$$

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

$$\rho(\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij}$$

viscous + passive stress

Continuum equations of active liquid crystal hydrodynamics

$$\boxed{(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij}}$$

couples nematic order and shear flows

relaxation to minimum of Landau-de Gennes free energy

$$\boxed{\rho(\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij}}$$

viscous + passive + active stress

$$\boxed{\Pi_{ij}^{active} = -\zeta Q_{ij}}$$

Punchline

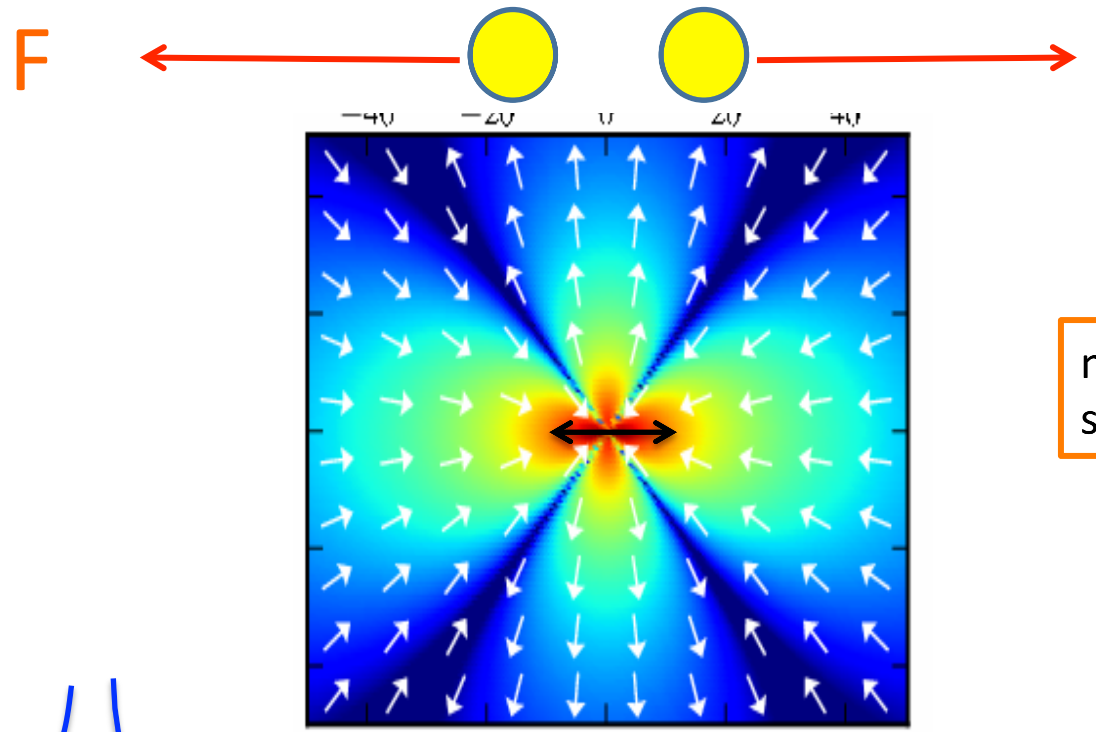
Active contribution to the stress $-\zeta\mathbf{Q}$

$\boxed{\text{Gradients in the magnitude or direction of the order parameter induce flow.}}$

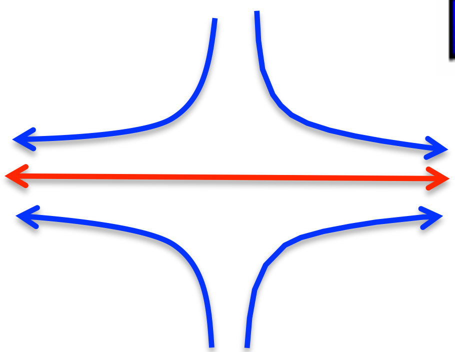
Hatwalne, Ramaswamy, Rao, Simha, PRL 2004

Hydrodynamics of active systems

Swimmers are force free => flow field is dipolar

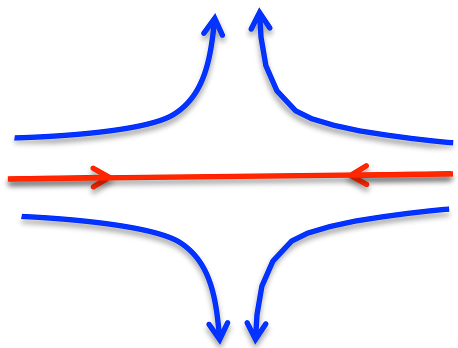


nematic symmetry

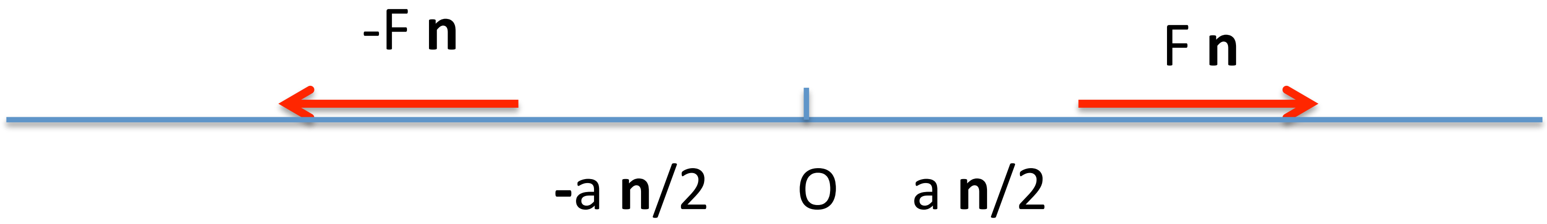


extensile

contractile



$$S_{jk} = \frac{1}{2}\int (\xi_k f_j + \xi_j f_k)\; d\xi - \frac{1}{3}\int \xi_i f_i\; \delta_{jk}\; d\xi$$



$$S_{jk} = \frac{1}{2} Fa(n_k n_j + n_j n_k) - \frac{1}{3} Fa n_i n_i\; \delta_{jk} = Fa(n_j n_k - \frac{\delta_{jk}}{3})$$

$$\Sigma_p \frac{F_p a_p}{V}(n_j n_k - \frac{\delta_{jk}}{3}) = -\zeta Q_{jk}$$

density of dipoles

“In fact, the mere act of opening the box will determine the state of the cat, although in this case there were three determinate states the cat could be in: these being Alive, Dead, and Bloody Furious.”

— Terry Pratchett, *Lords and Ladies*

Punchline

Active contribution to the stress $-\zeta \mathbf{Q}$

Gradients in the magnitude or direction of the order parameter induce flow.

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004

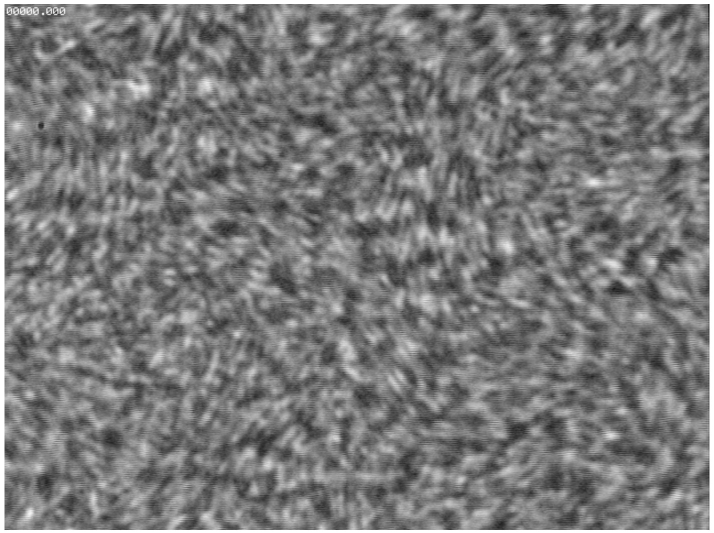
Summary

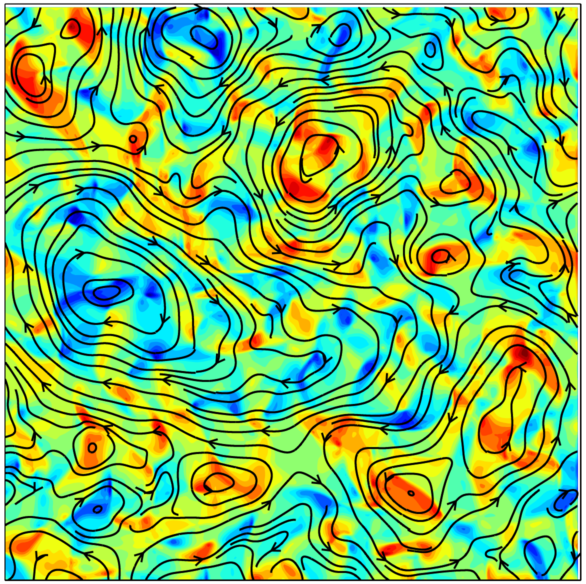
Active contribution to the stress $$-\zeta \mathbf{Q}$$

Gradients in the magnitude or direction of the order parameter induce flow.

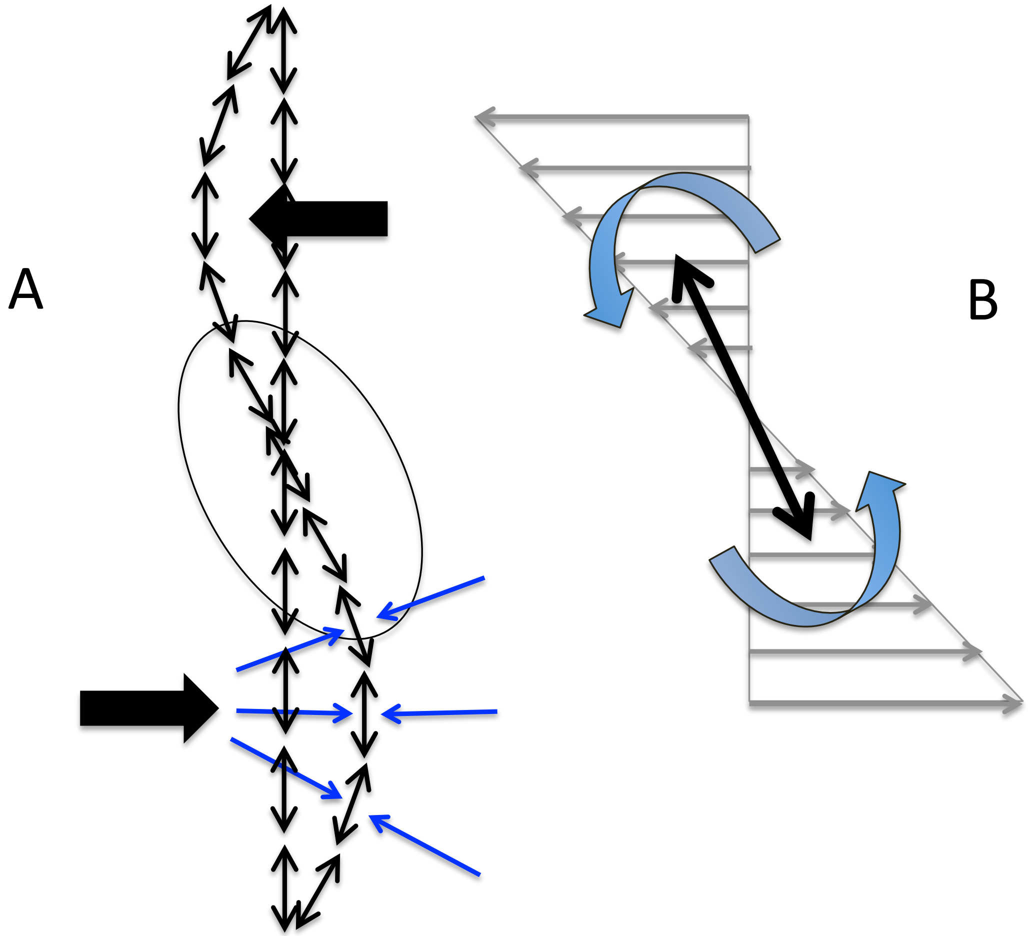
nematic state is unstable to vortical flows

Hatwalne, Ramaswamy, Rao, Simha, PRL 2004





Instabilities in active nematics



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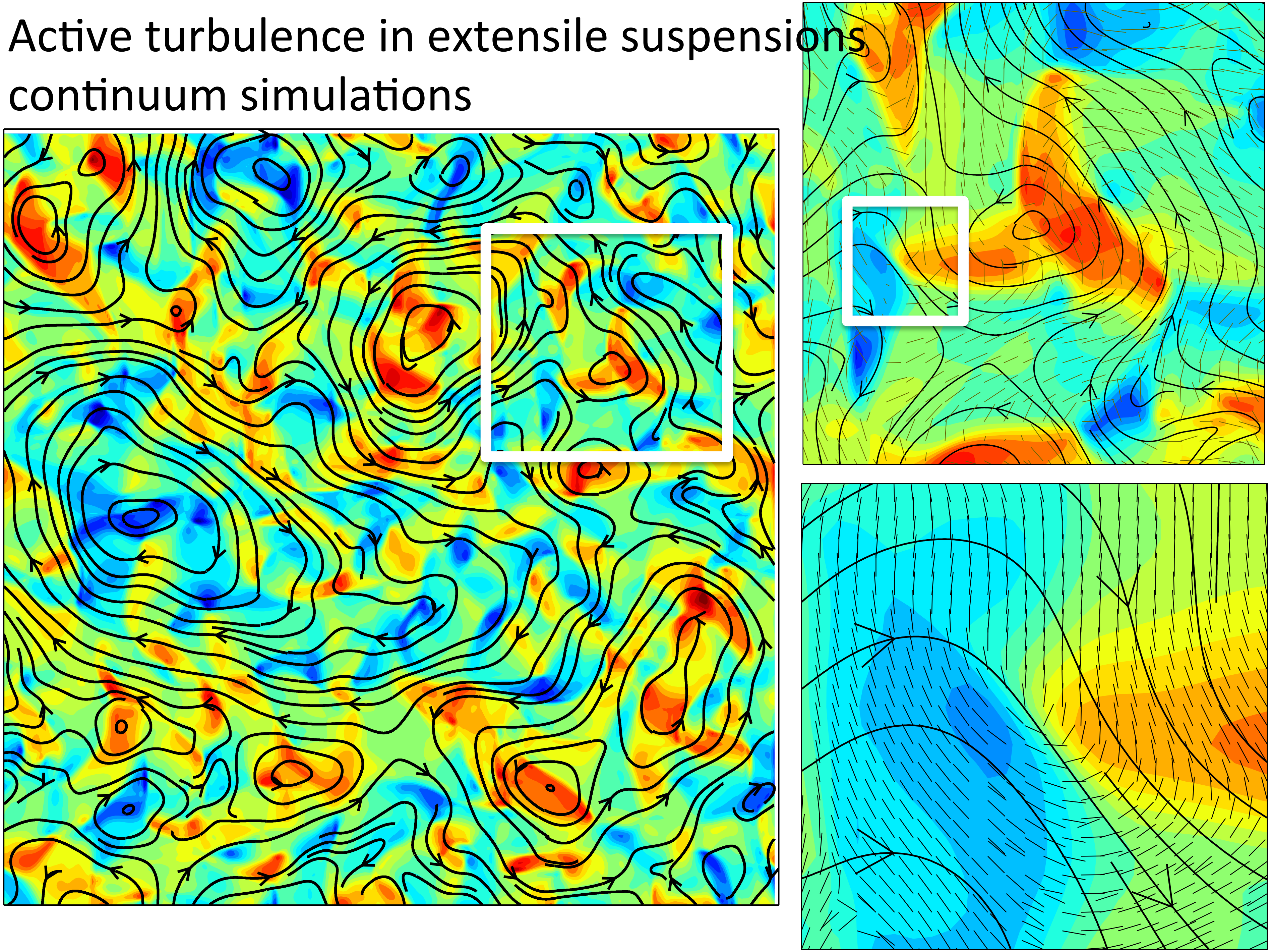
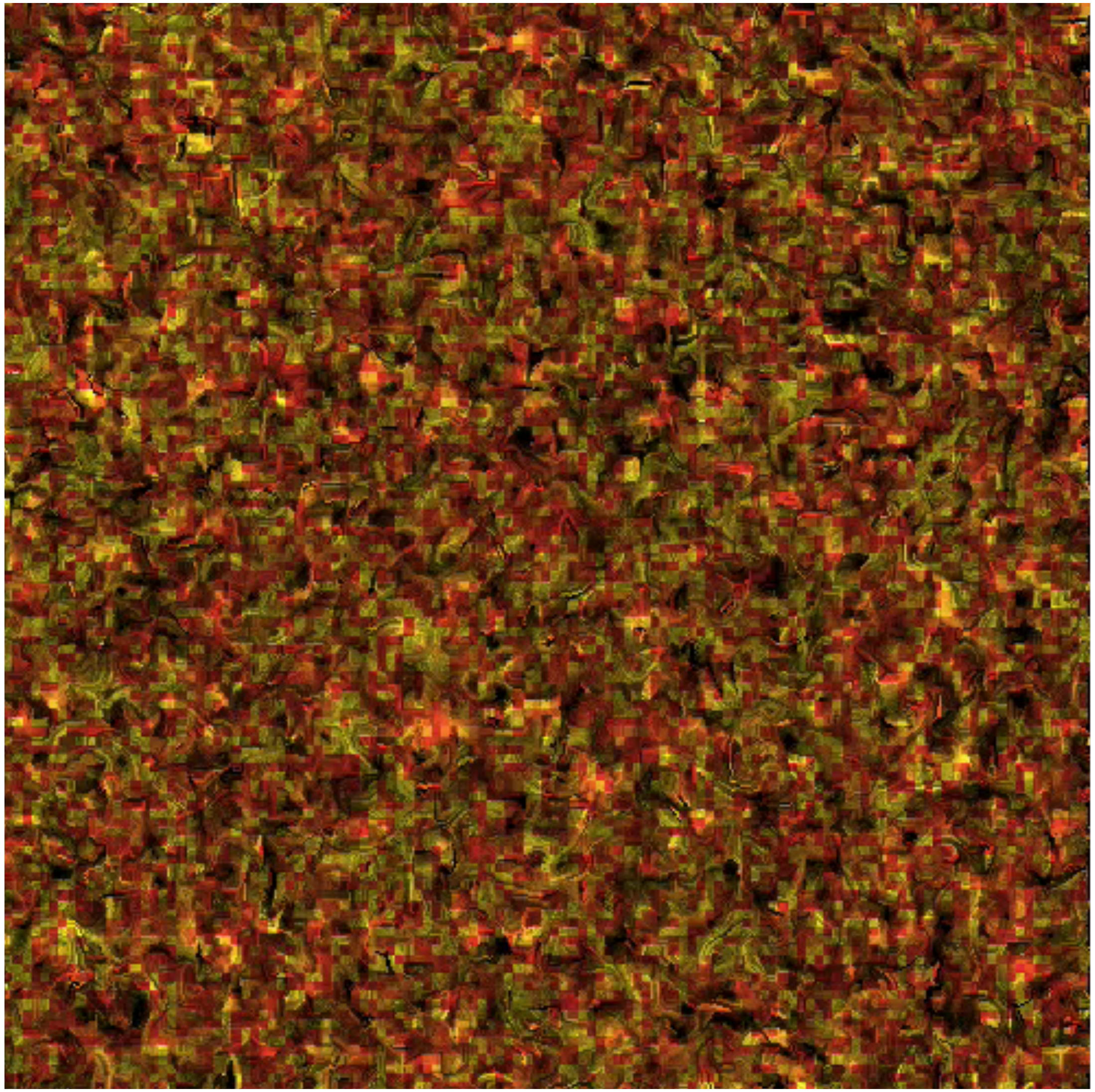
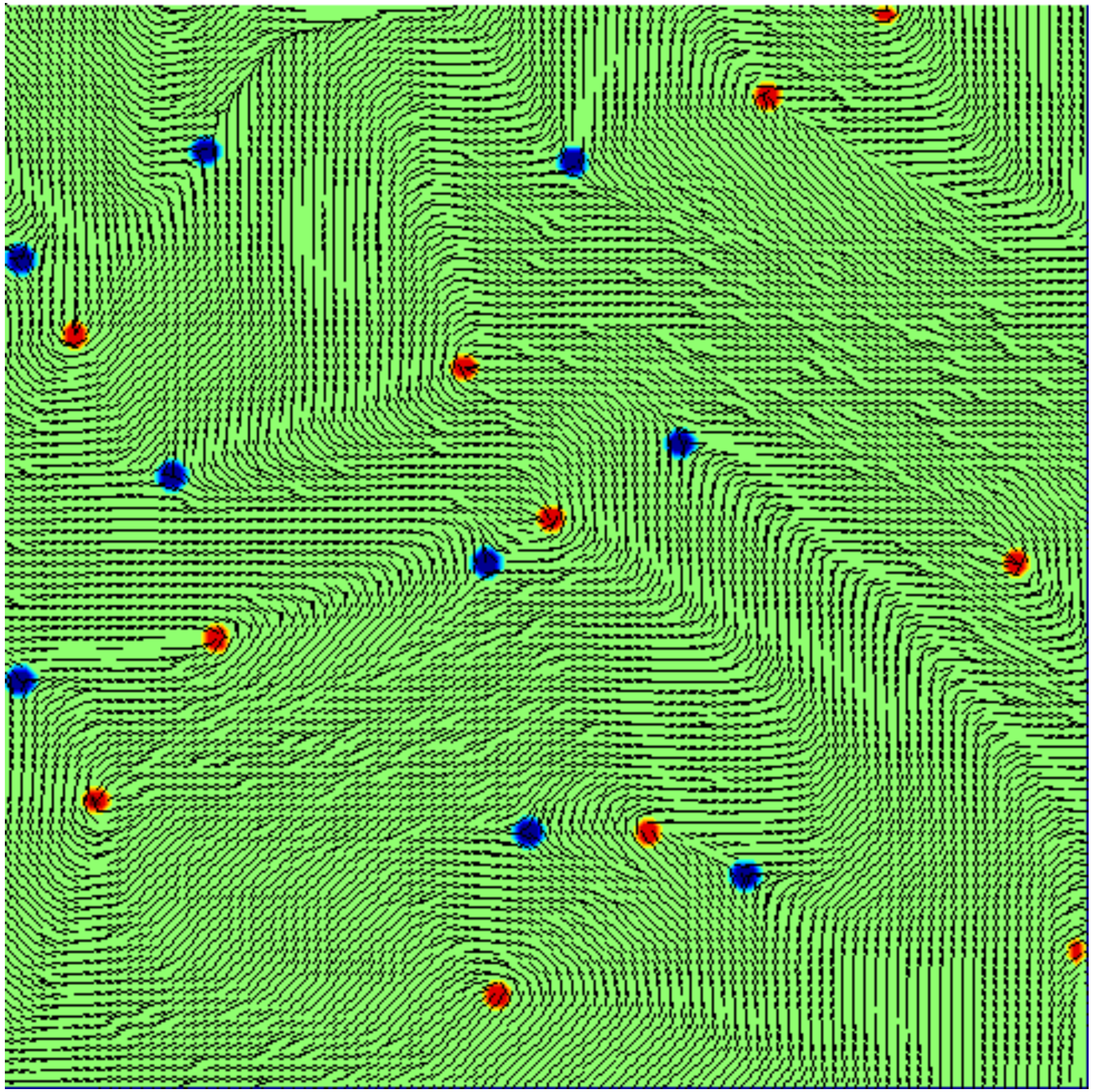
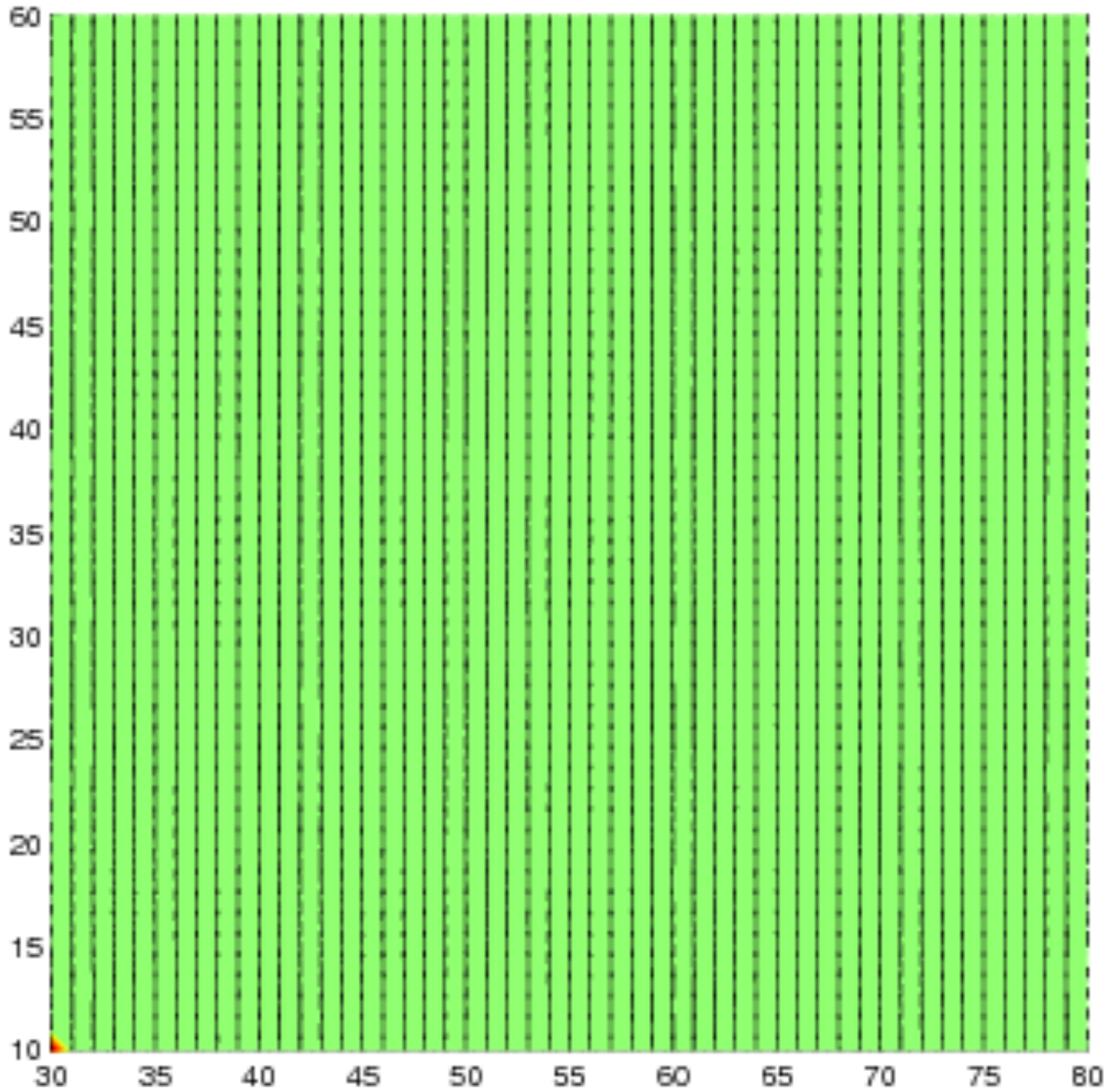
Active turbulence in extensile suspensions
continuum simulations





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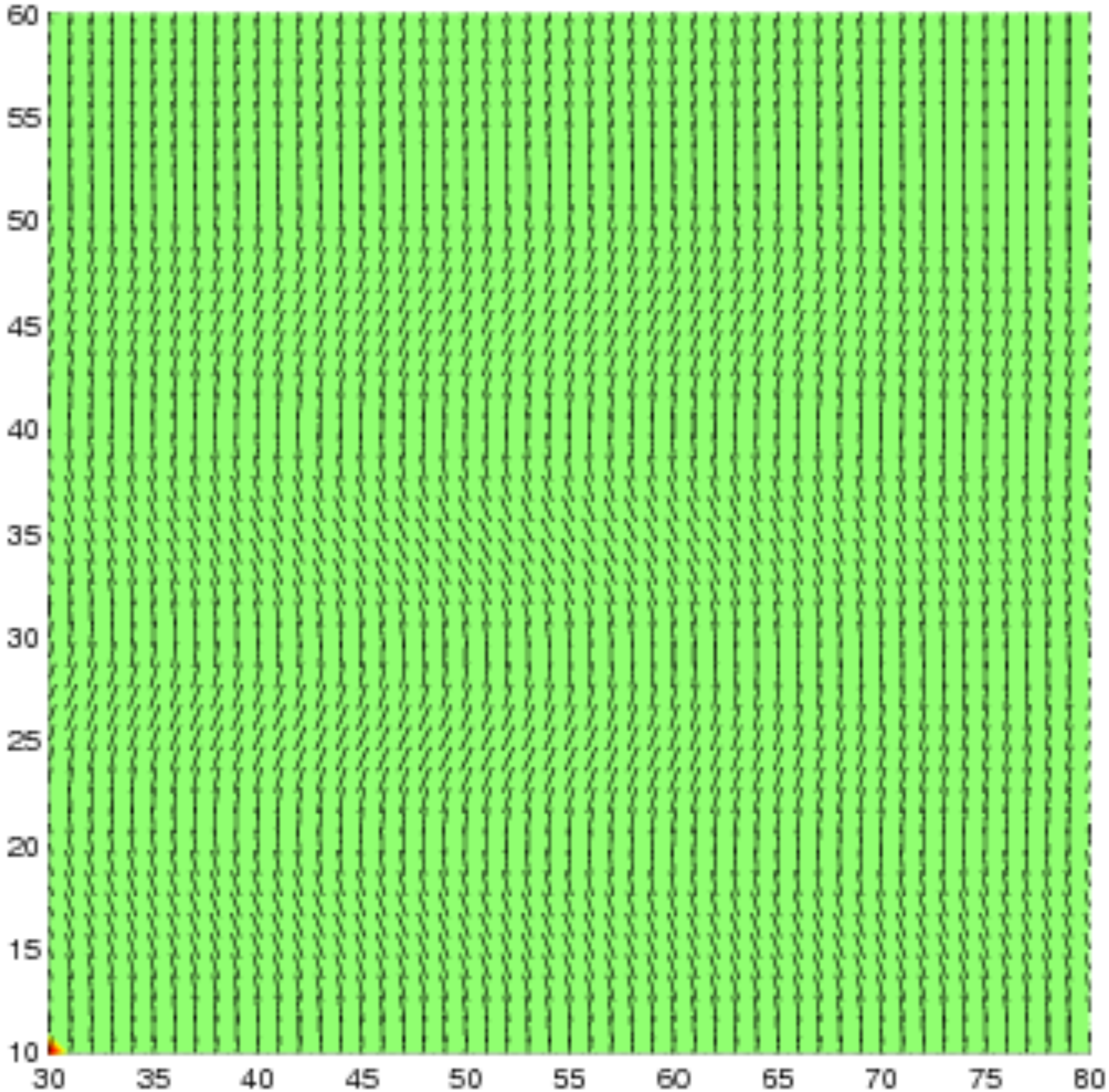
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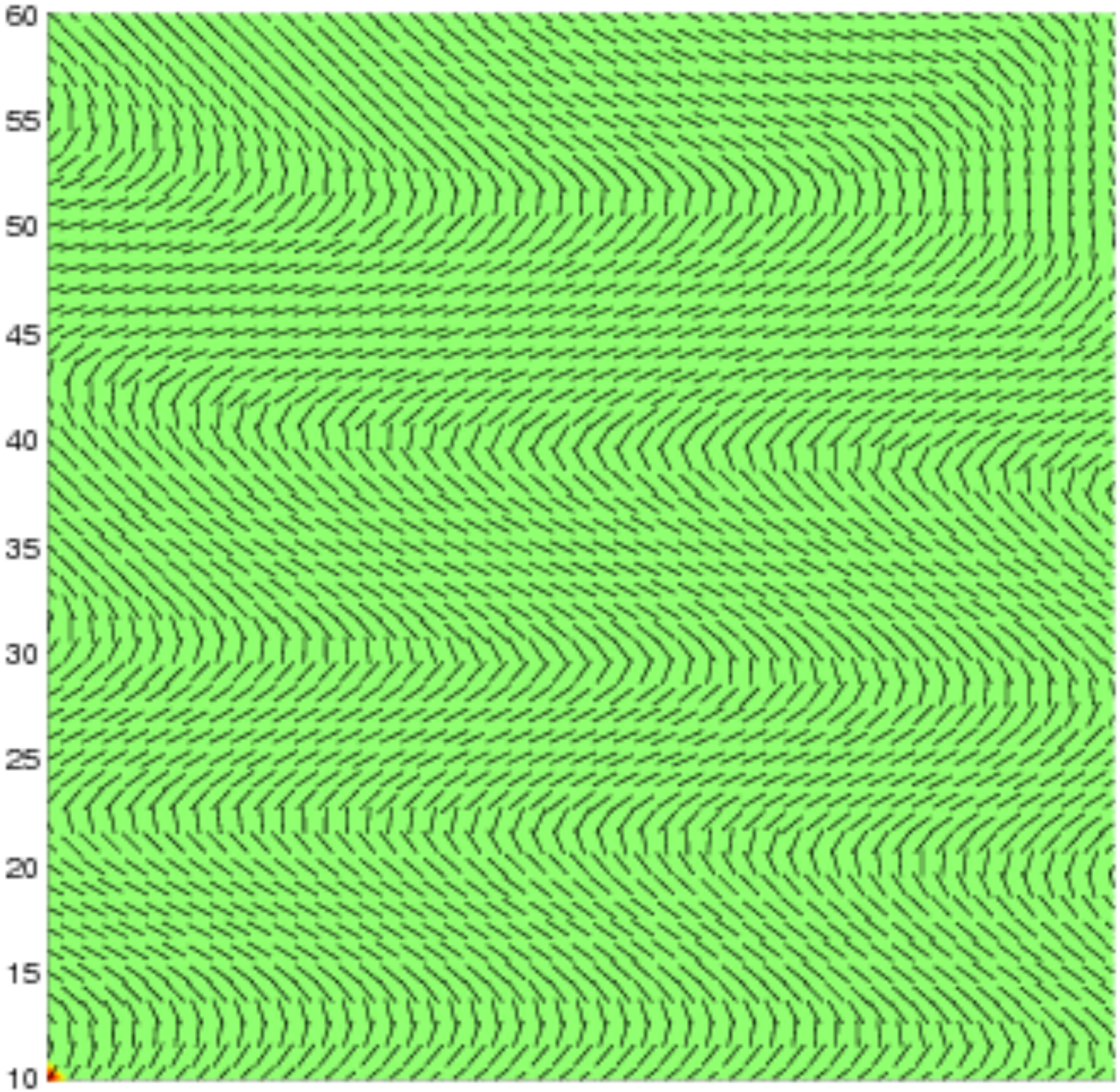
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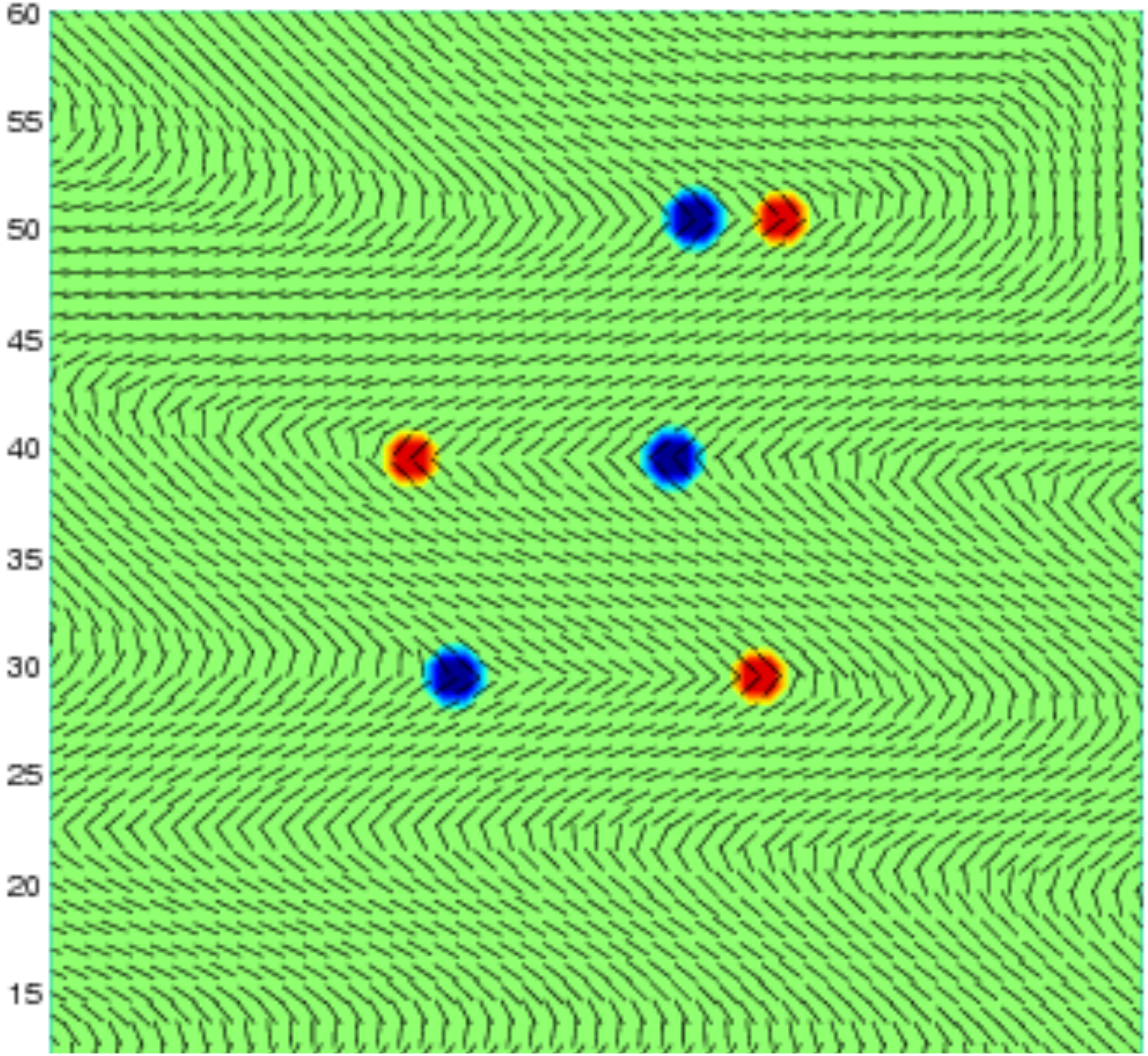

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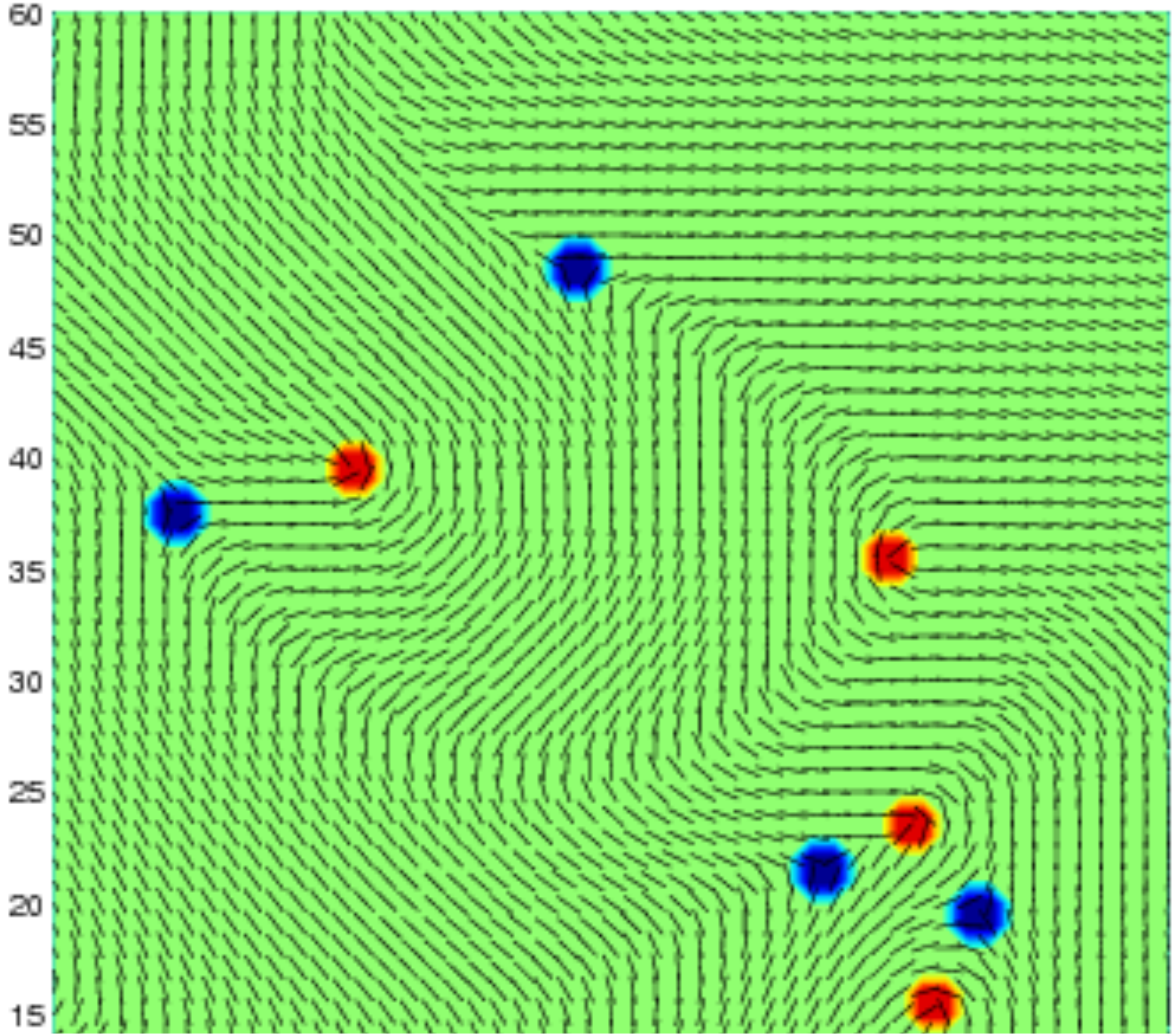
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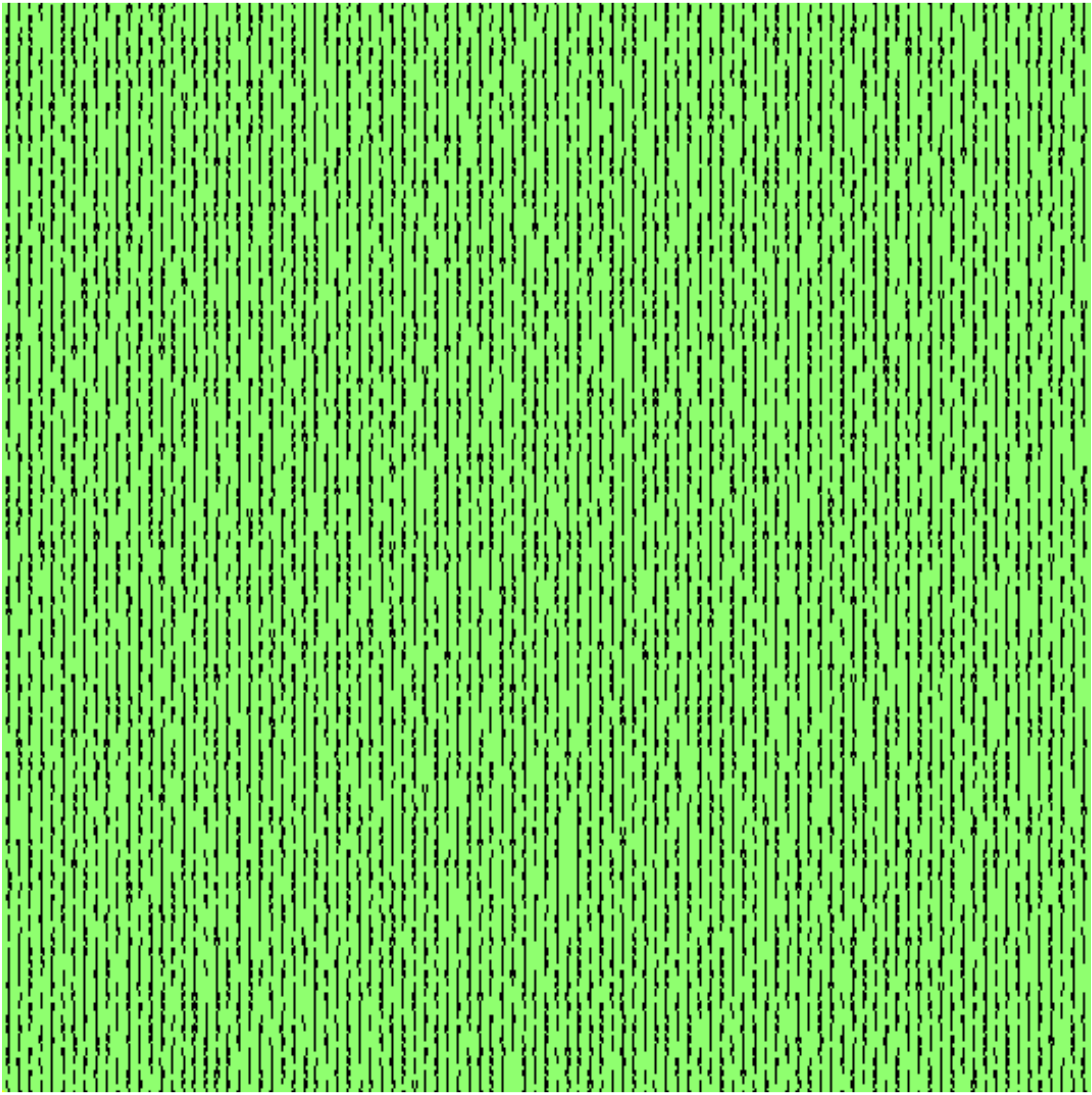
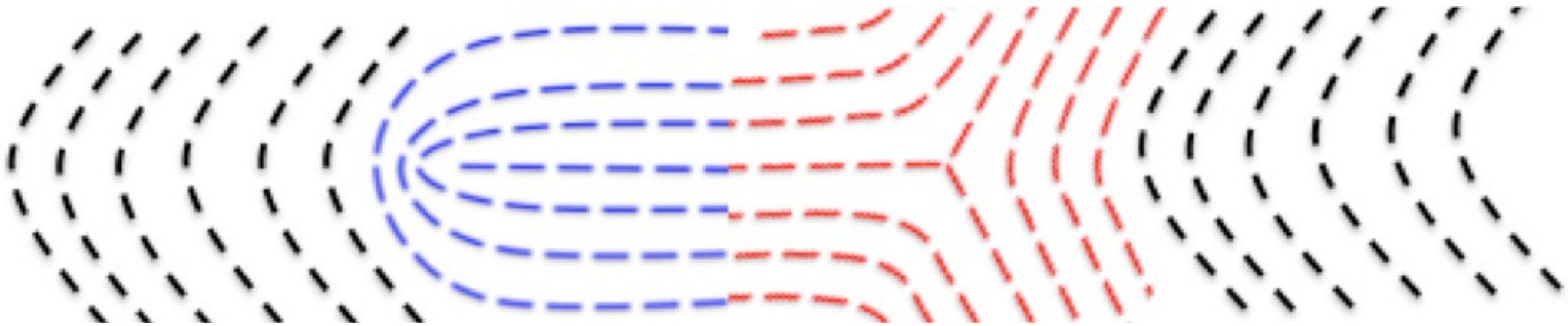
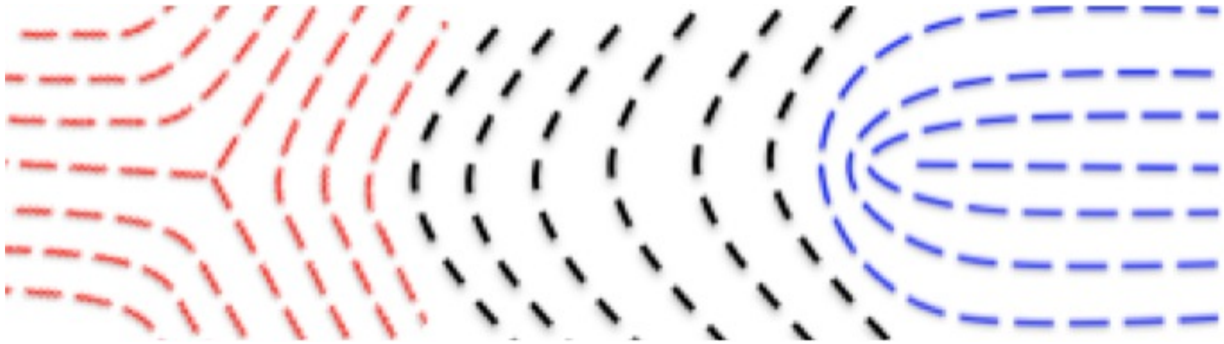
Onset of active turbulence



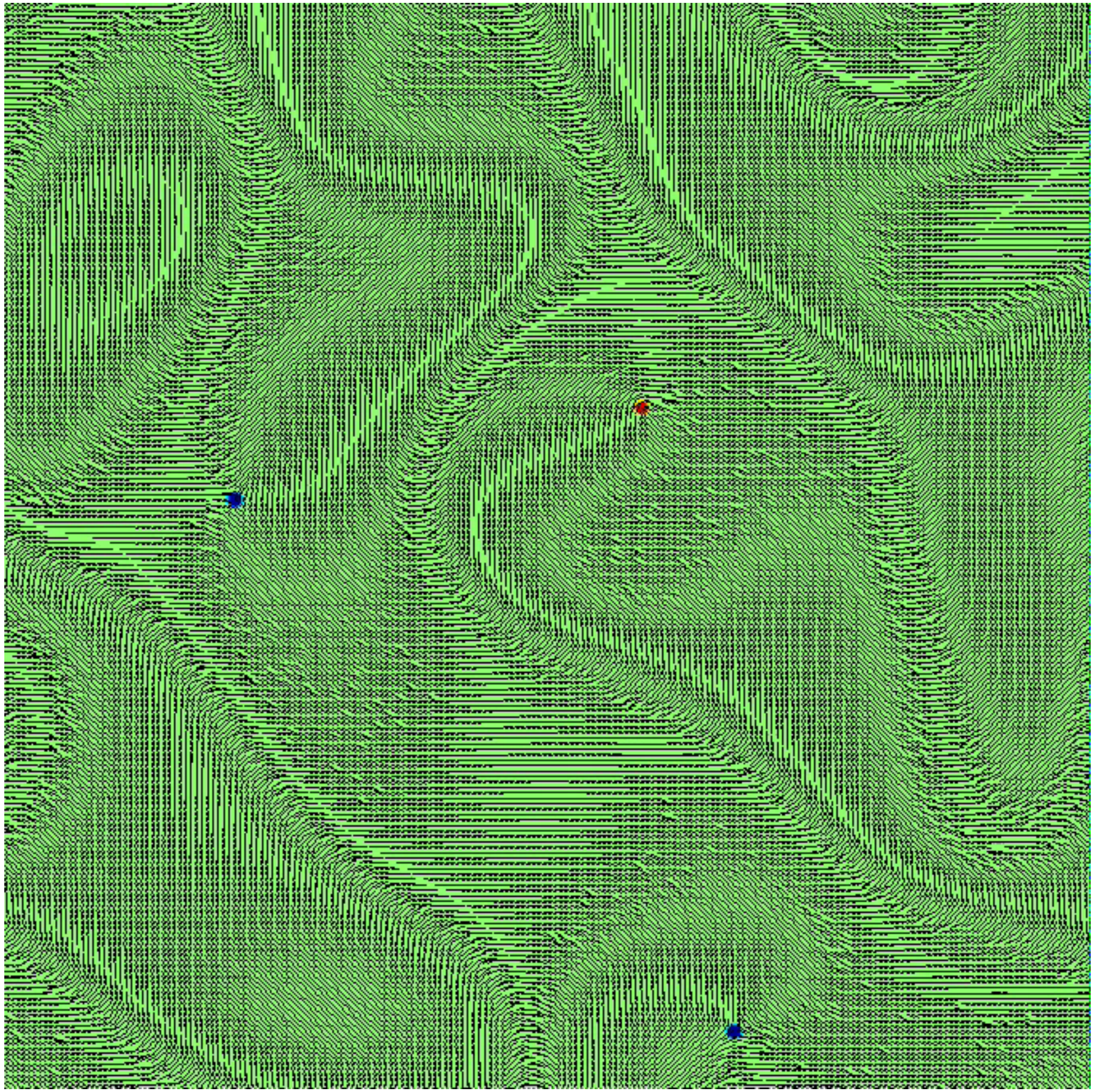


defect creation

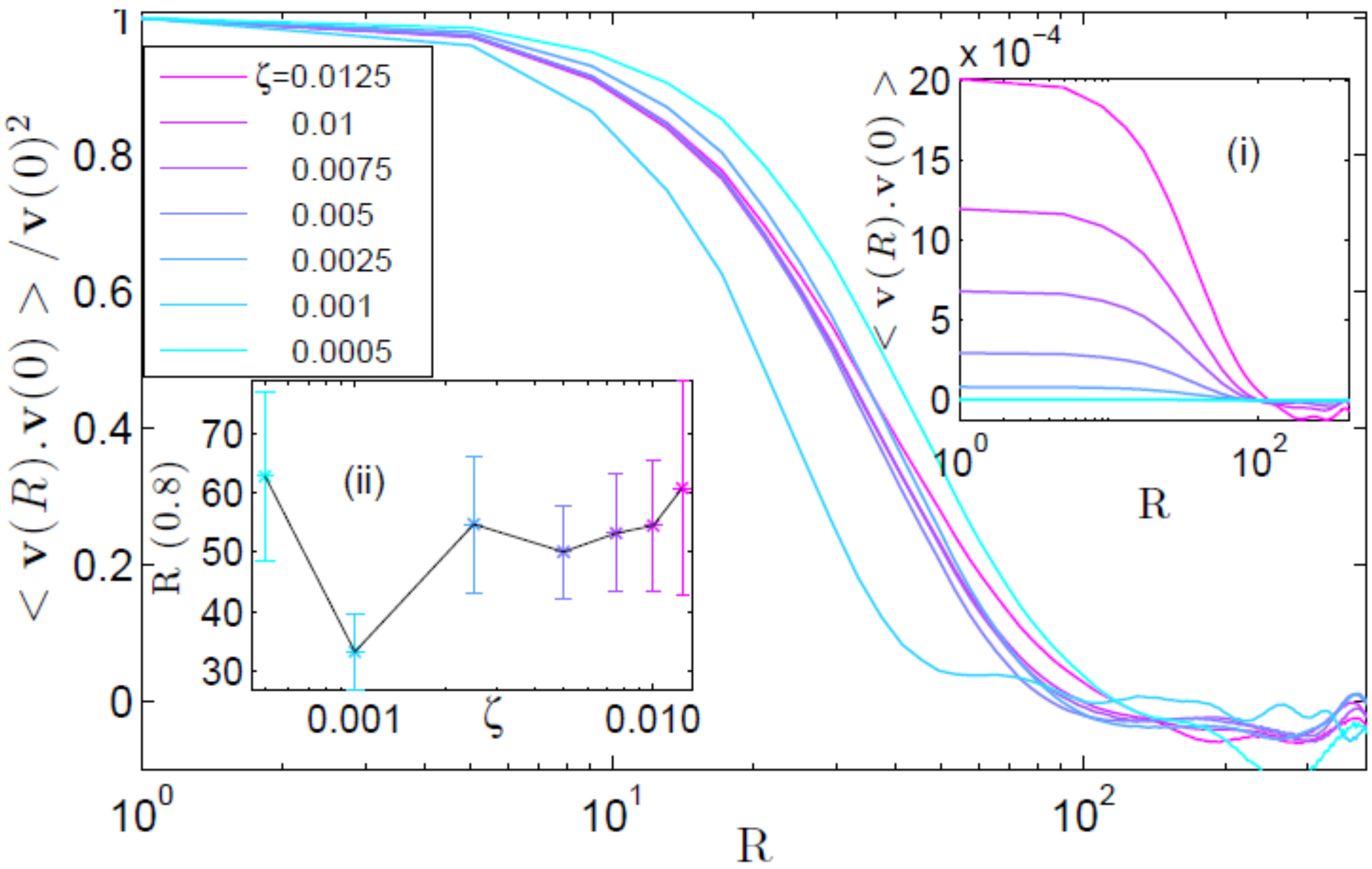


defect annihilation

Low activities



<vv>: simulations



Summary of active turbulence

Active contribution to the stress $-\zeta \mathbf{Q}$