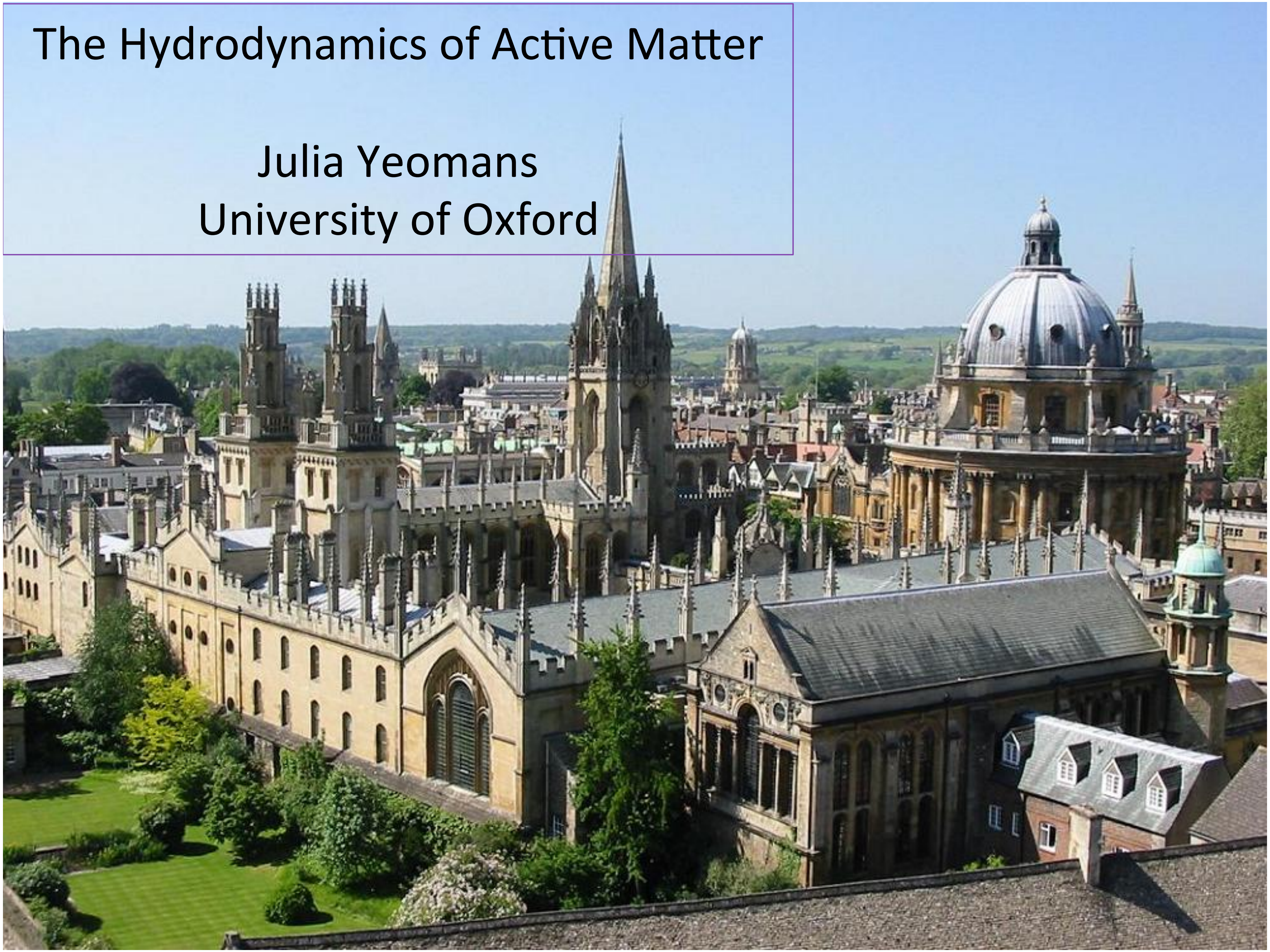
The Hydrodynamics of Active Matter

Julia Yeomans
University of Oxford

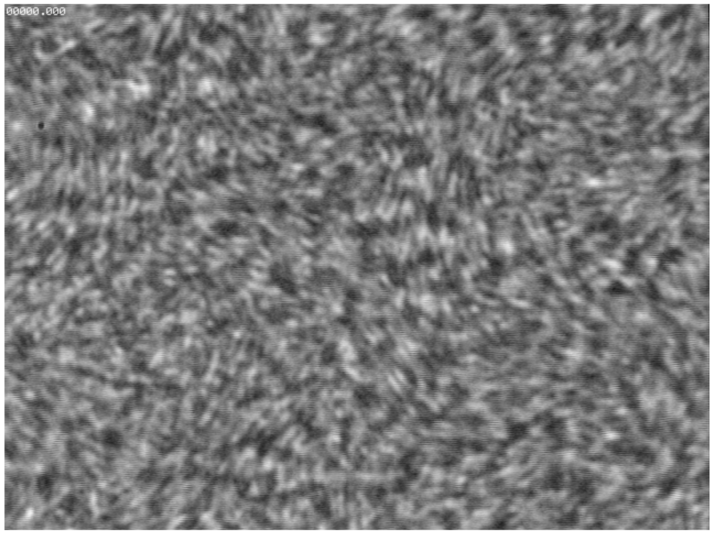


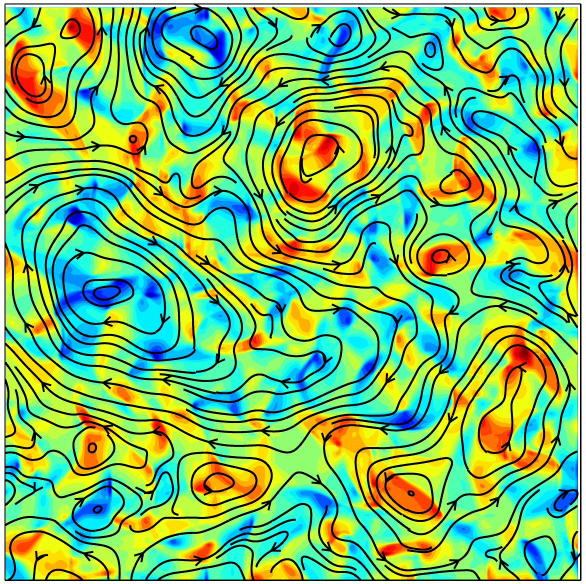
Summary

Active contribution to the stress $-\zeta \mathbf{Q}$

Gradients in the magnitude or direction of the order parameter induce flow.

→ nematic state is unstable to vortical flows

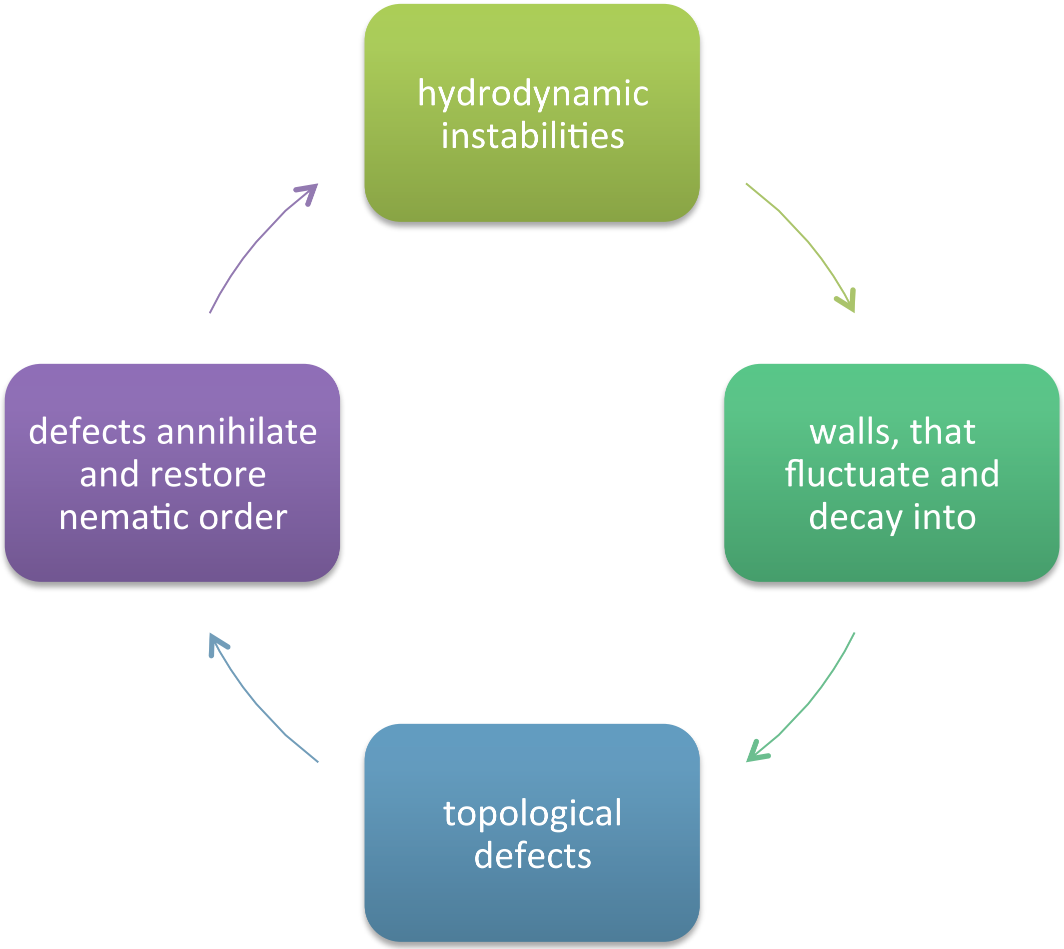




Hatwalne, Ramaswamy, Rao, Simha, PRL 2004

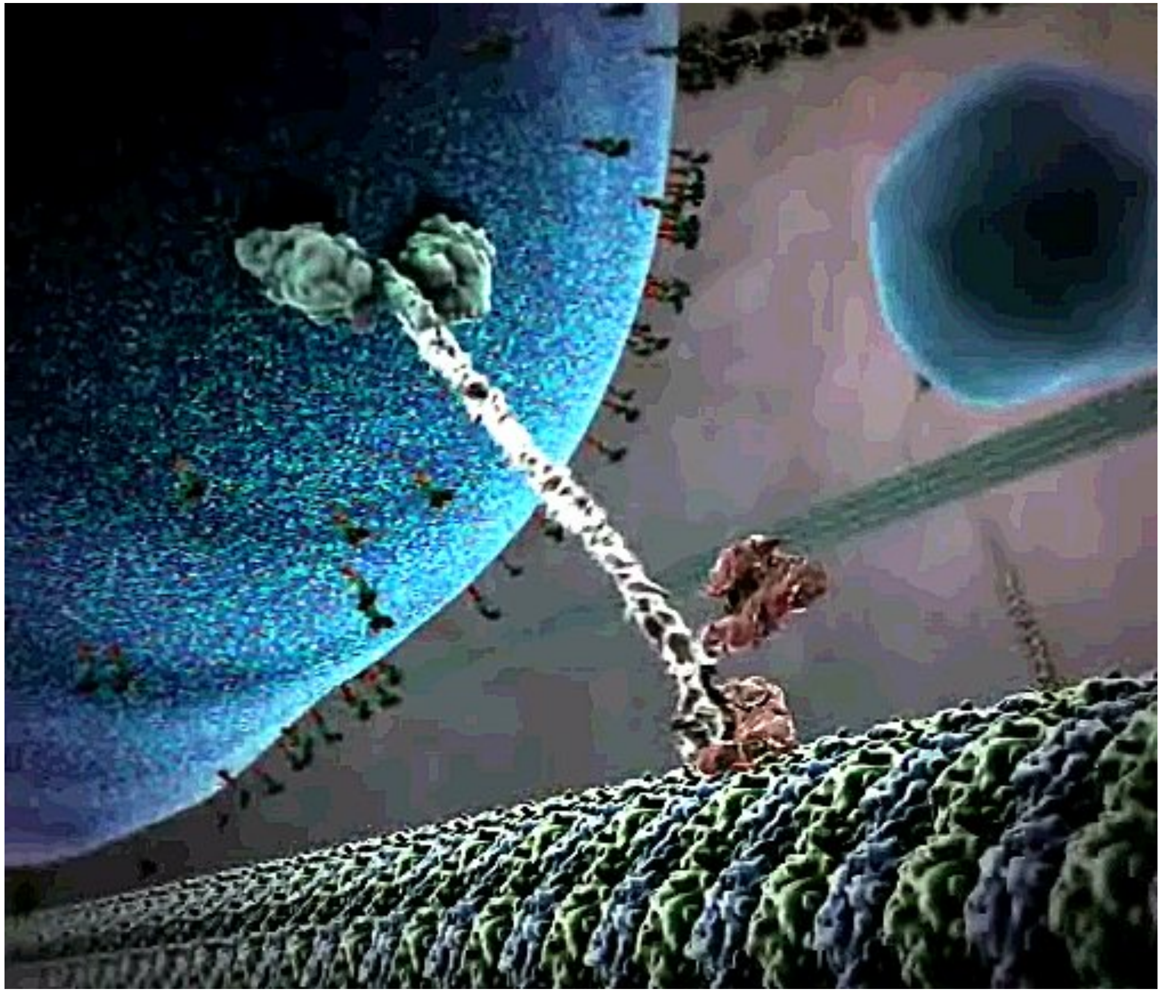
Summary of active turbulence

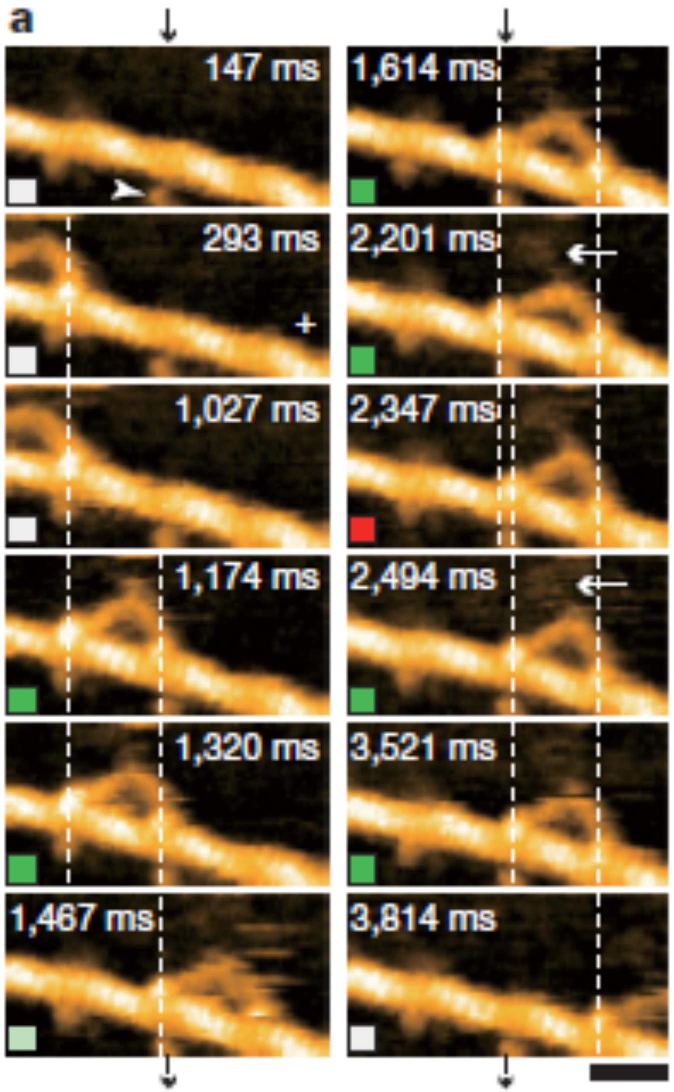
Active contribution to the stress $-\zeta \mathbf{Q}$



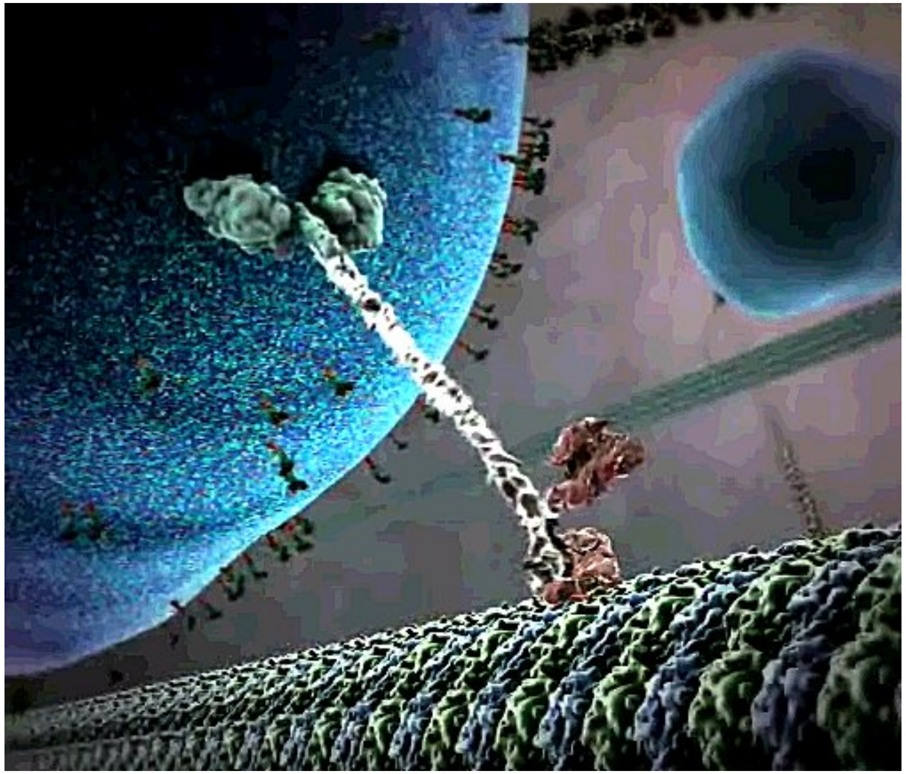
1. Introduction: a zoo of active systems
2. Liquid crystals and Q
3. Active equations of motion
4. Instabilities
5. Active turbulence + topological defects
6. Molecular motors and Dogic experiments
7. Lyotropic active nematics
8. Dividing cells

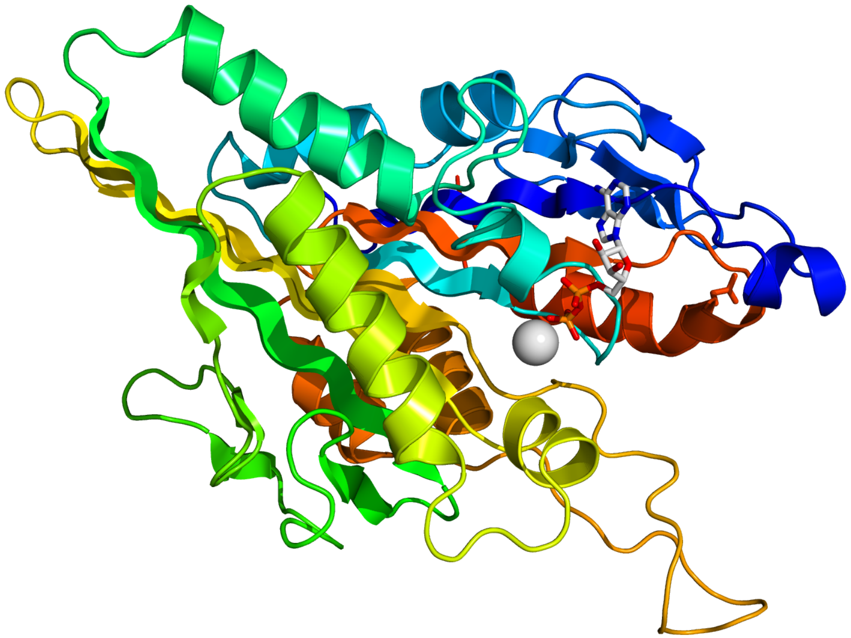
Molecular motors





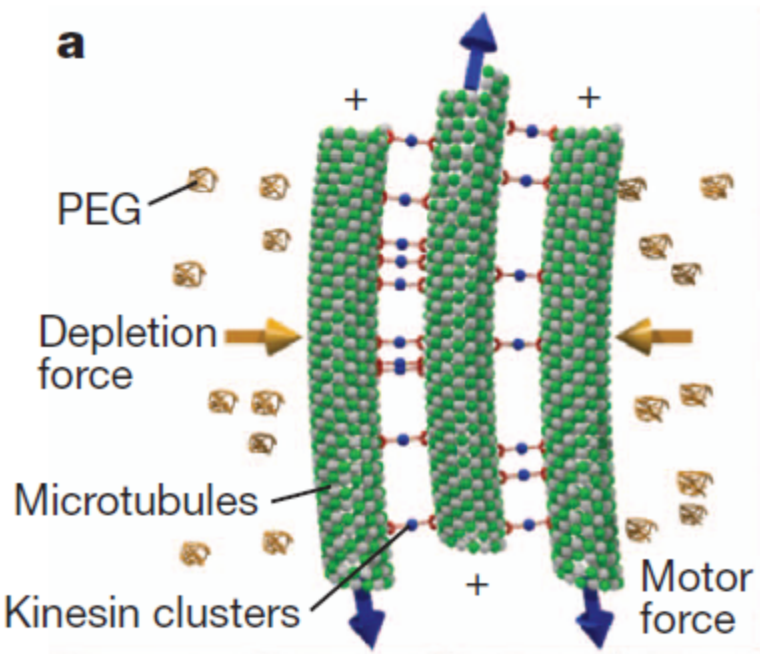
Molecular motors

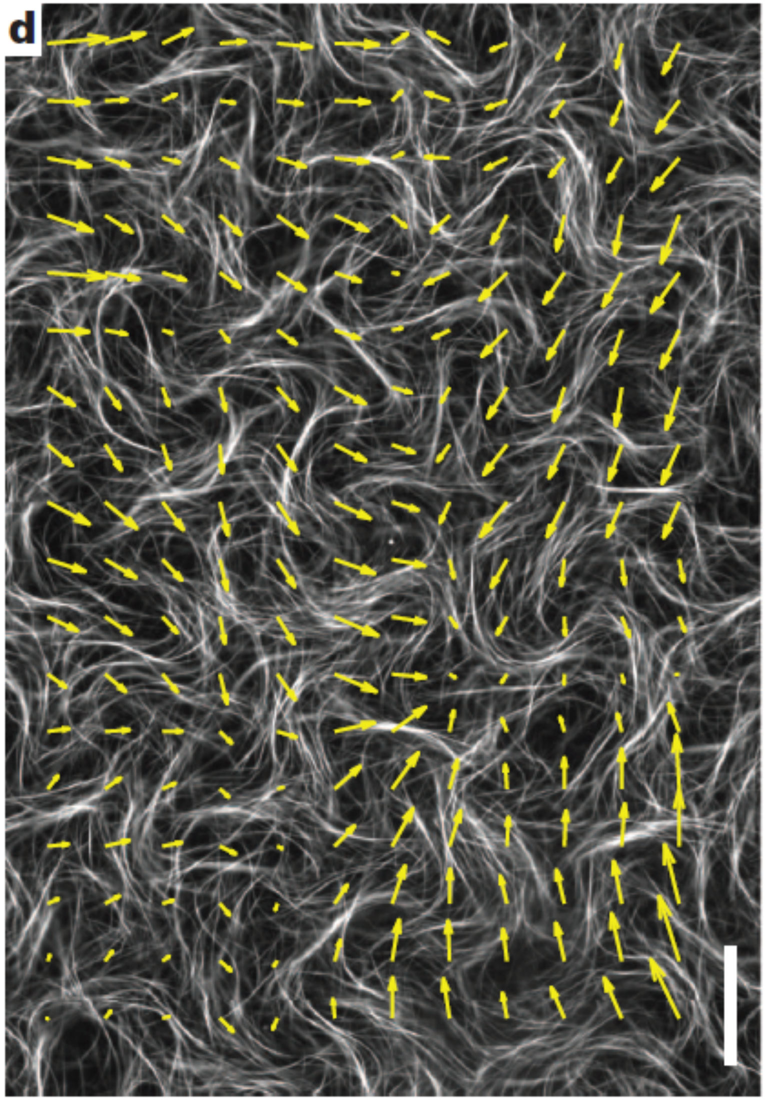




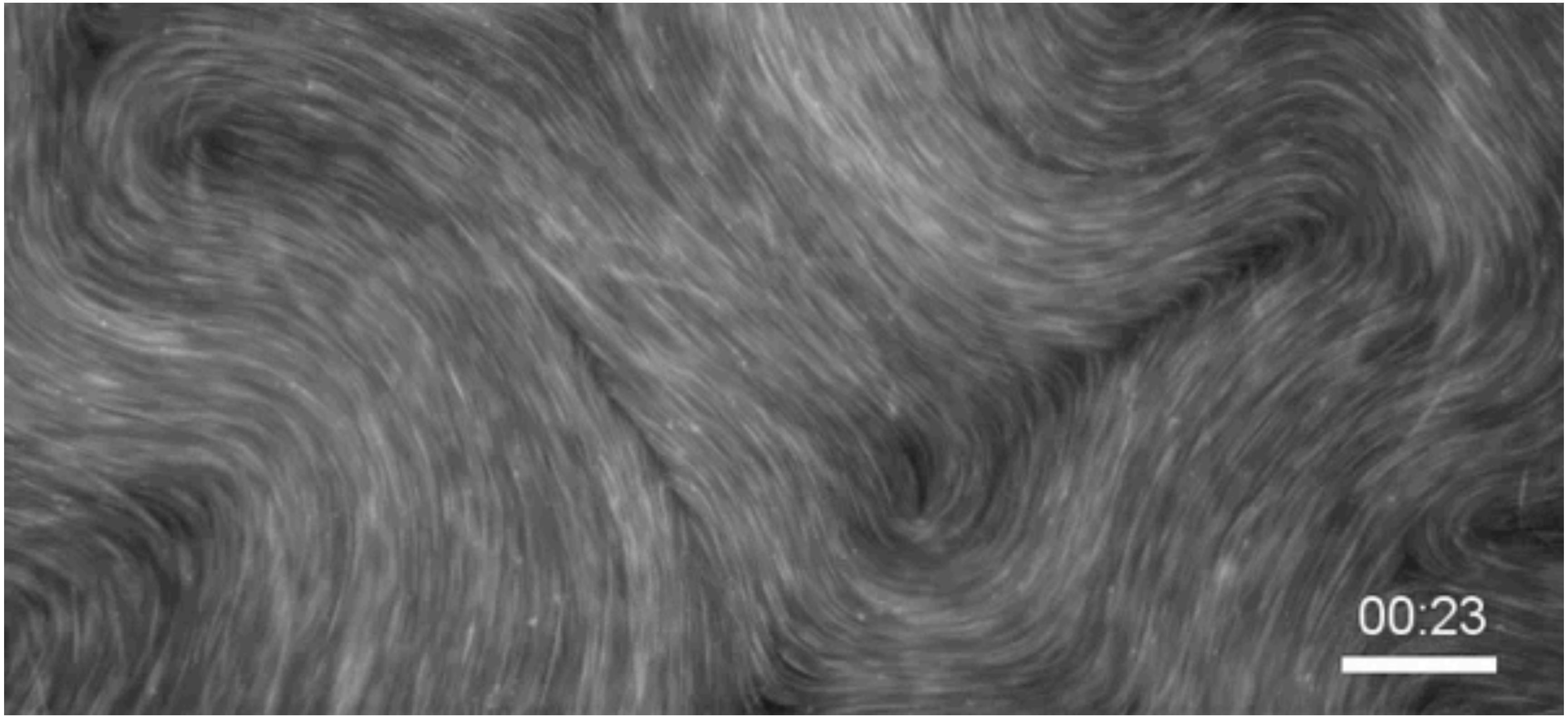
N-terminus = blue, C-terminus = red) complexed with ADP (stick diagram, carbon = white, oxygen = red, nitrogen = blue, phosphorus = orange) and a magnesium ion (grey sphere)

Molecular motors





Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012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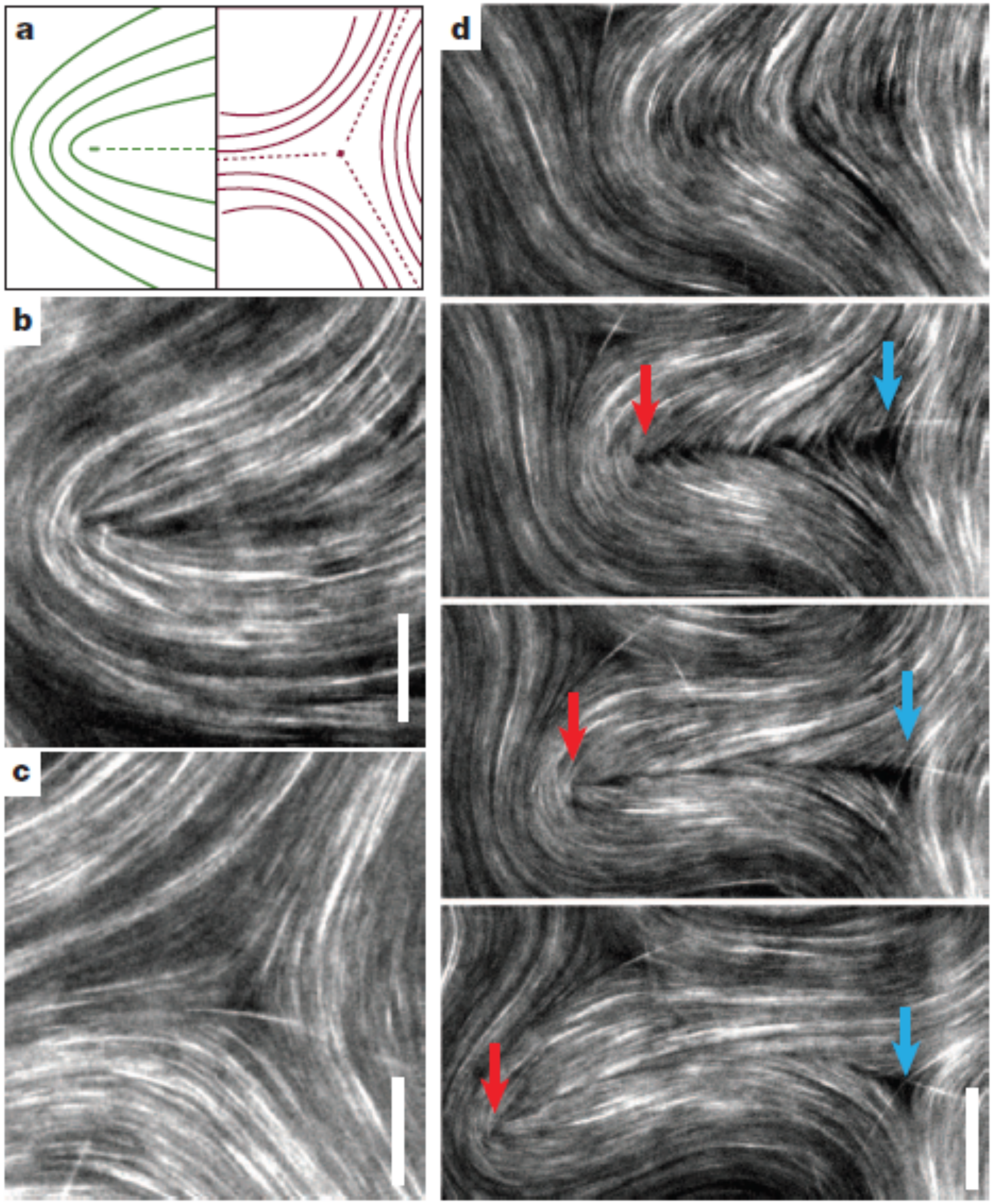
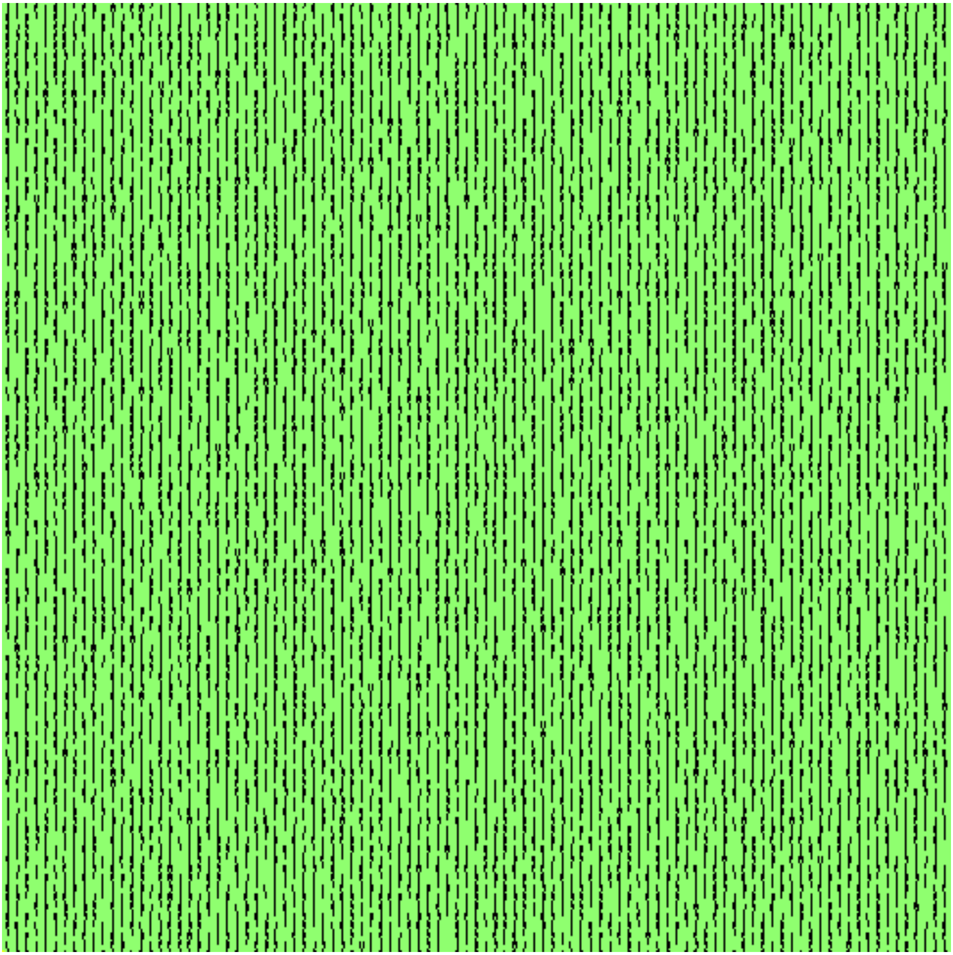
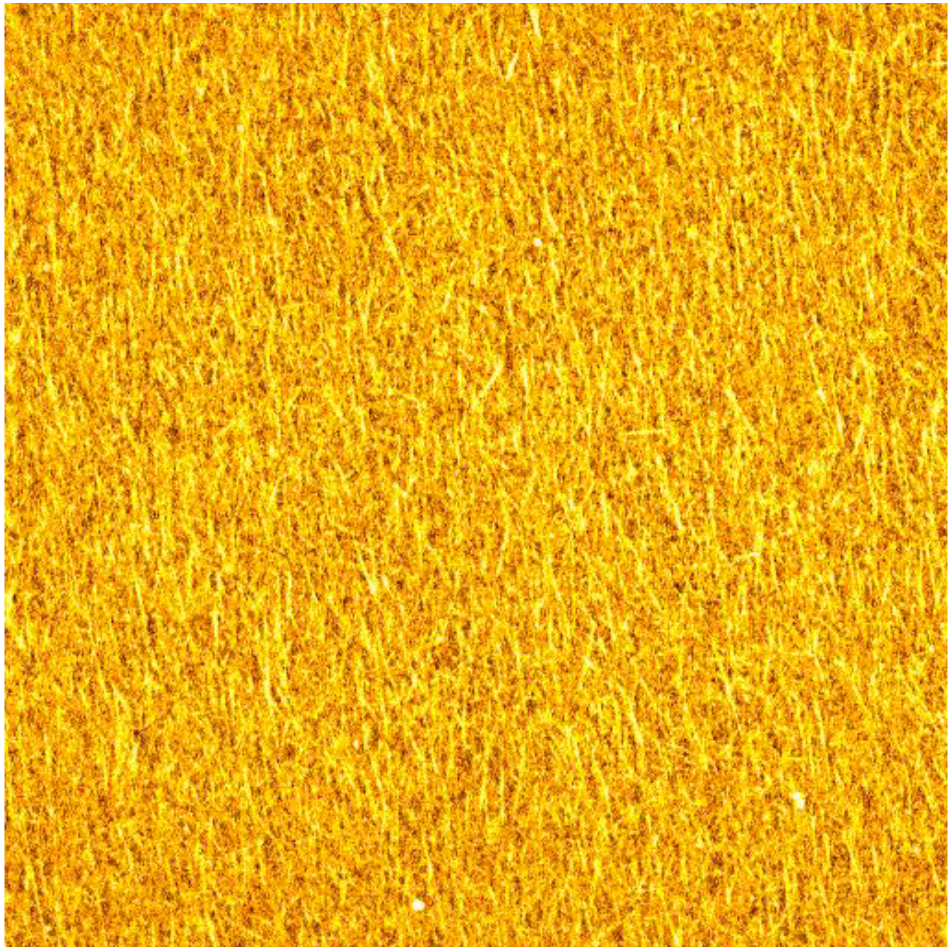


Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012
L. Giomi, M.J. Bowick, Ma Xu, M.C. Marchetti, PRL 110, 228101

Onset of active turbulence:

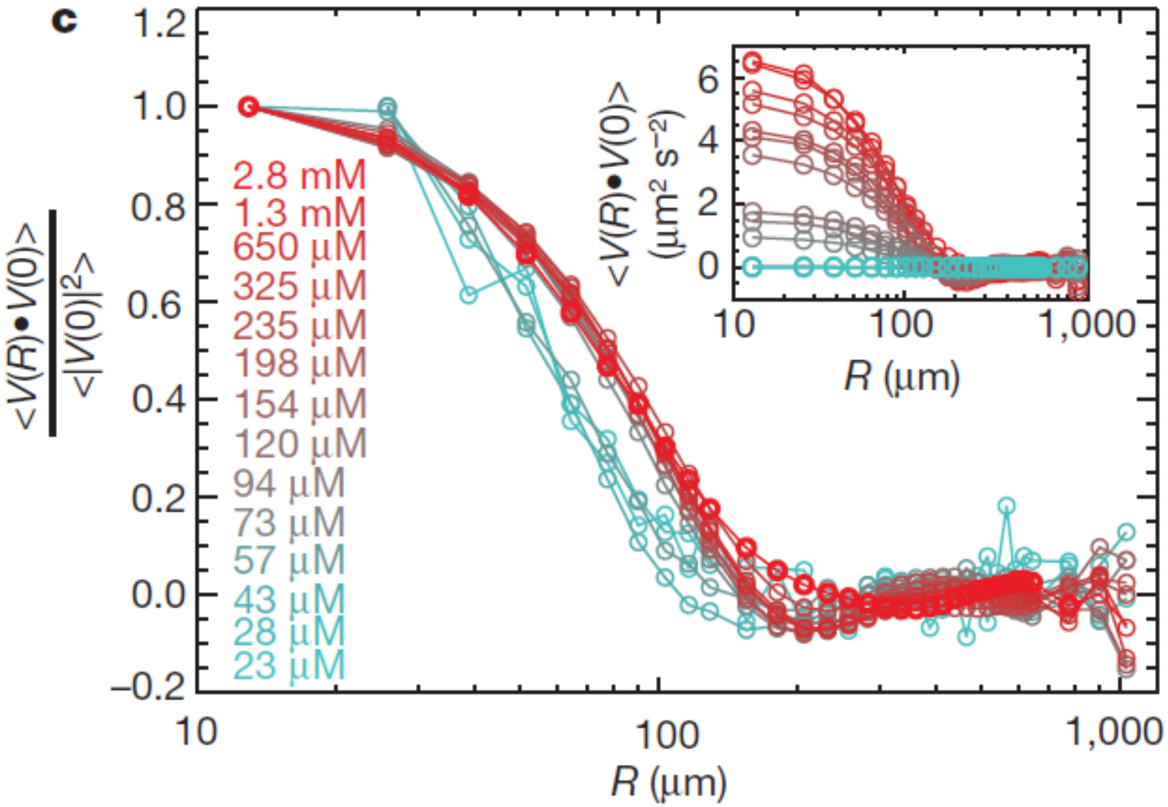




Microtubule bundles, D. Chen, Brandeis University

- Stage-1: Walls – driven by hydrodynamic instabilities
- Stage-2: Defects – driven by elasticity and flow

Molecular motors

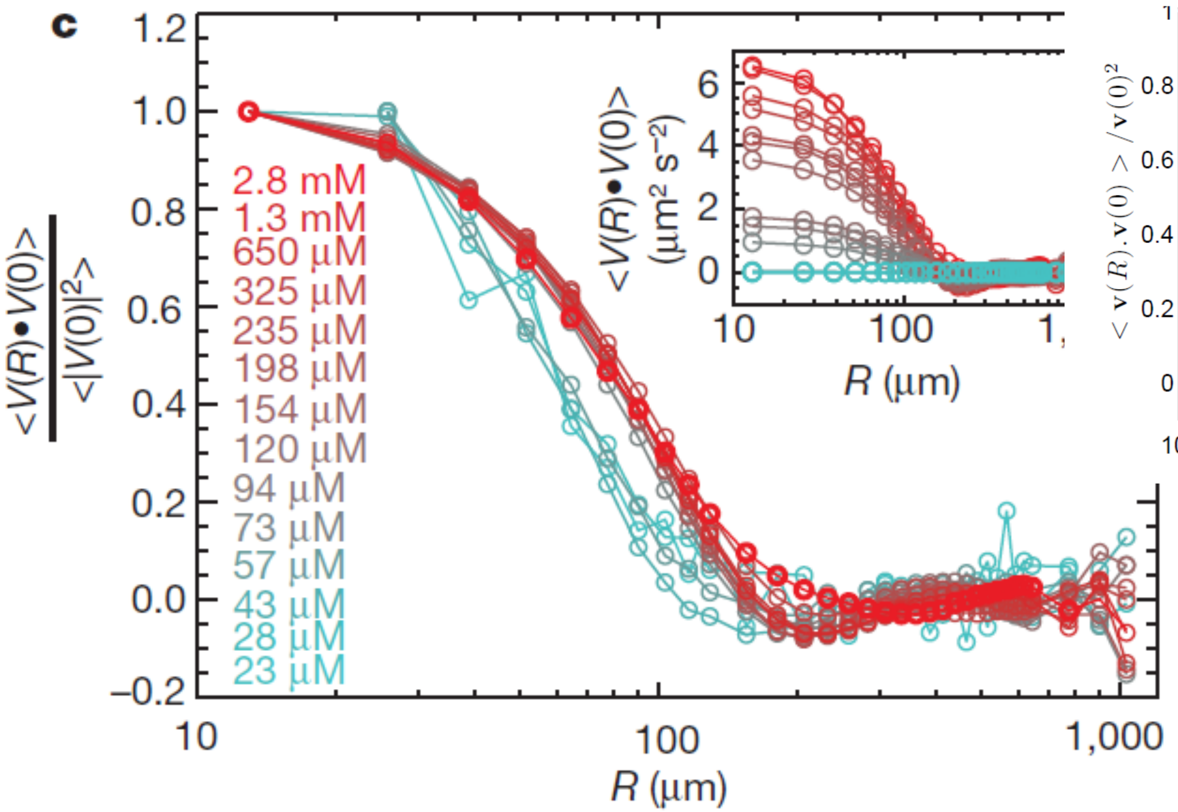


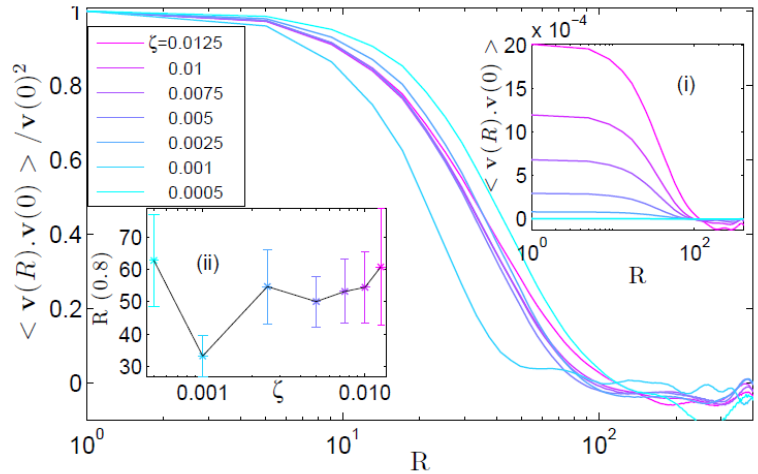
Velocity increases with activity

Length scale controlling decay of <vv> independent of activity

Sanchez, Chen, DeCamp, Heymann, Dogic, Nature 2012

Molecular motors





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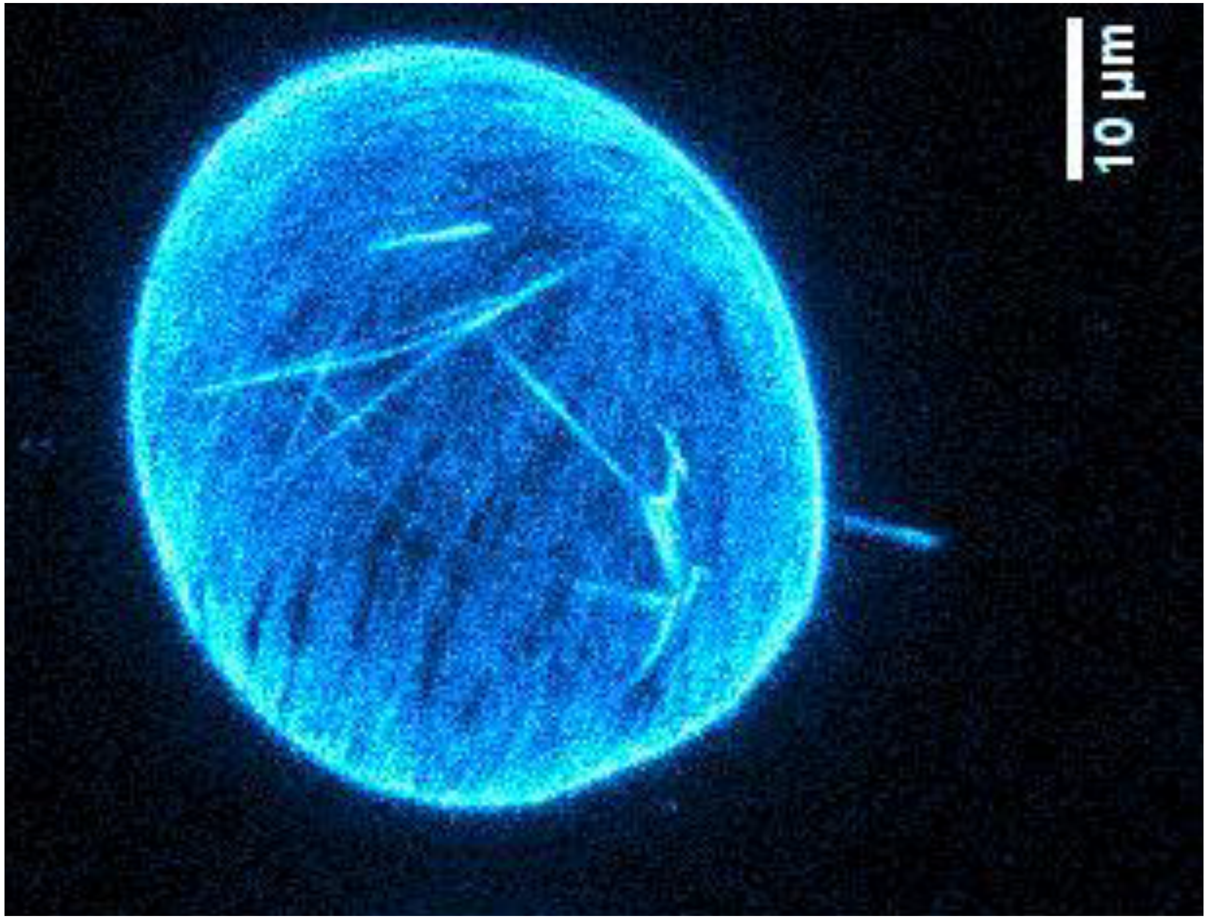

FC Keber et al, Science, 2014

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D. Nelson

What do we need to describe cells?

1. Lyotropics: adding an interface

2. Adding cell division

3. Adding friction

Equations of motion: active lyotropic

$$(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij},$$

$$\partial_t \rho + \partial_i (\rho u_i) = 0,$$

$$\rho (\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij},$$

$$\Pi_{ij}^{active} = -\zeta Q_{ij},$$

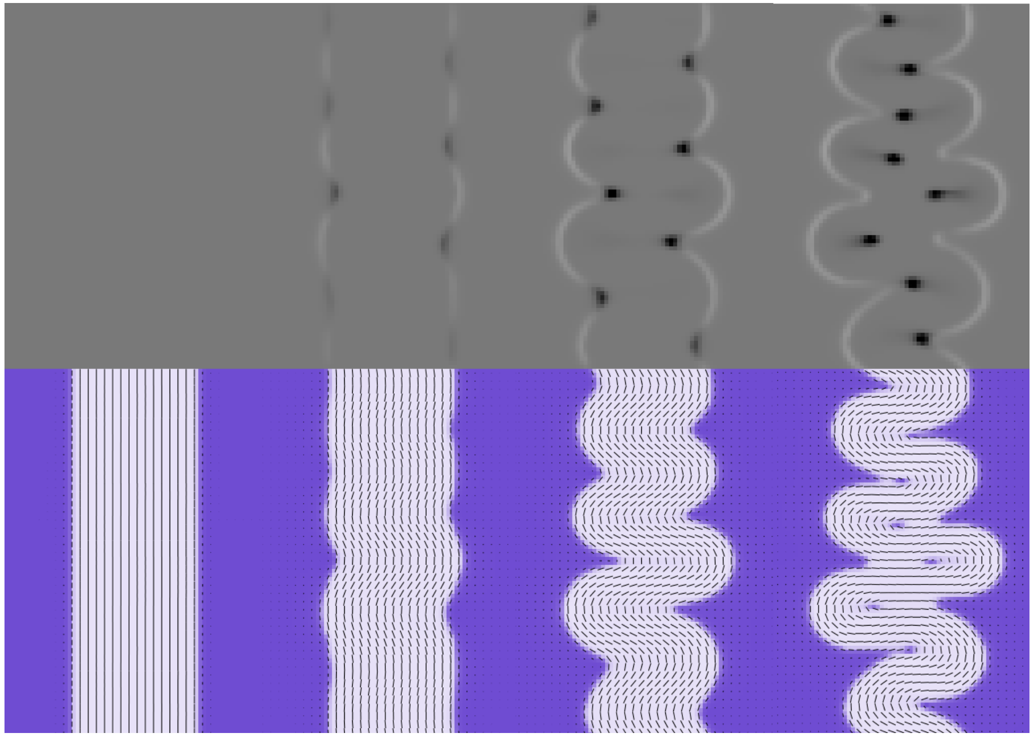
$$\boxed{\partial_t \phi + \partial_i (u_i \phi) = \Gamma_\phi \nabla^2 \mu}$$

concentration

$$\mathscr{F} = \frac{A_\phi}{2} \phi^2 (1-\phi)^2 + \frac{1}{2} A (S^2 \phi - \frac{1}{2} Q_{ij} Q_{ji})^2$$

$$+ \frac{1}{2} \kappa_\phi (\partial_k \phi)^2 + \frac{1}{2} K (\partial_k Q_{ij})^2,$$

The active interface



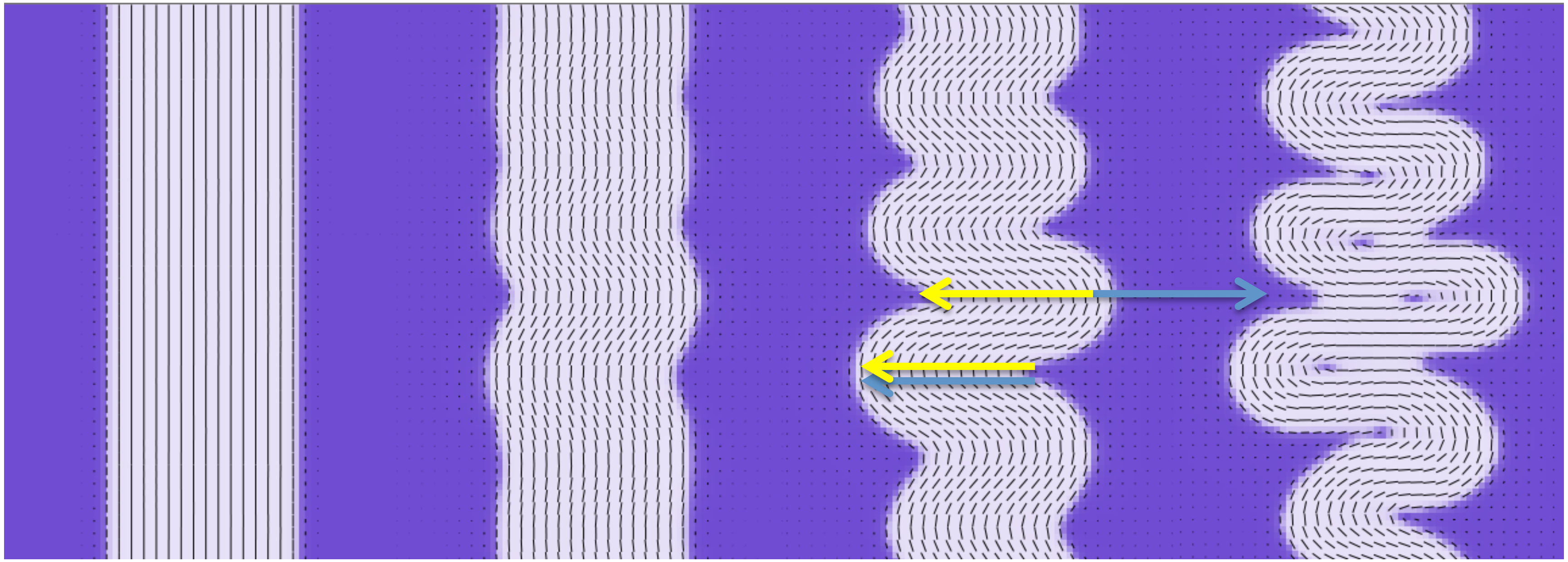
time →

Interface instability is asymmetric

+1/2 topological defects originate from the interface
and move into the bulk

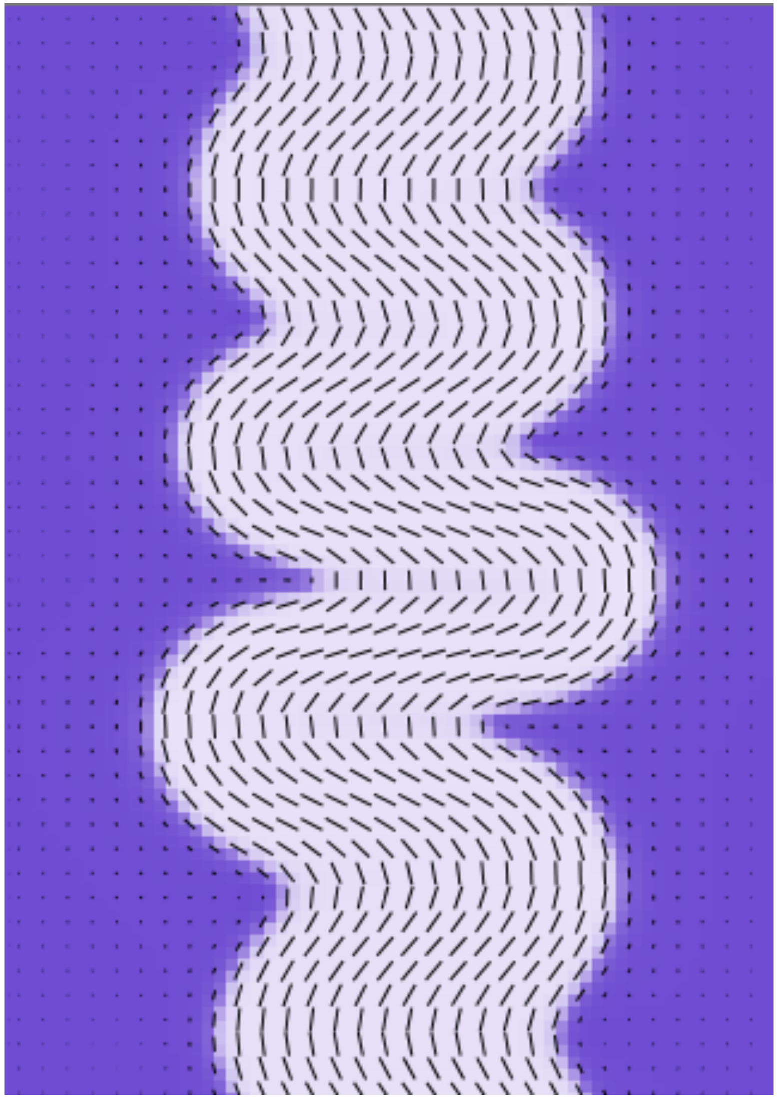
Consequence of the active stress

$$\Pi_{ij}^{active} = -\zeta Q_{ij},$$



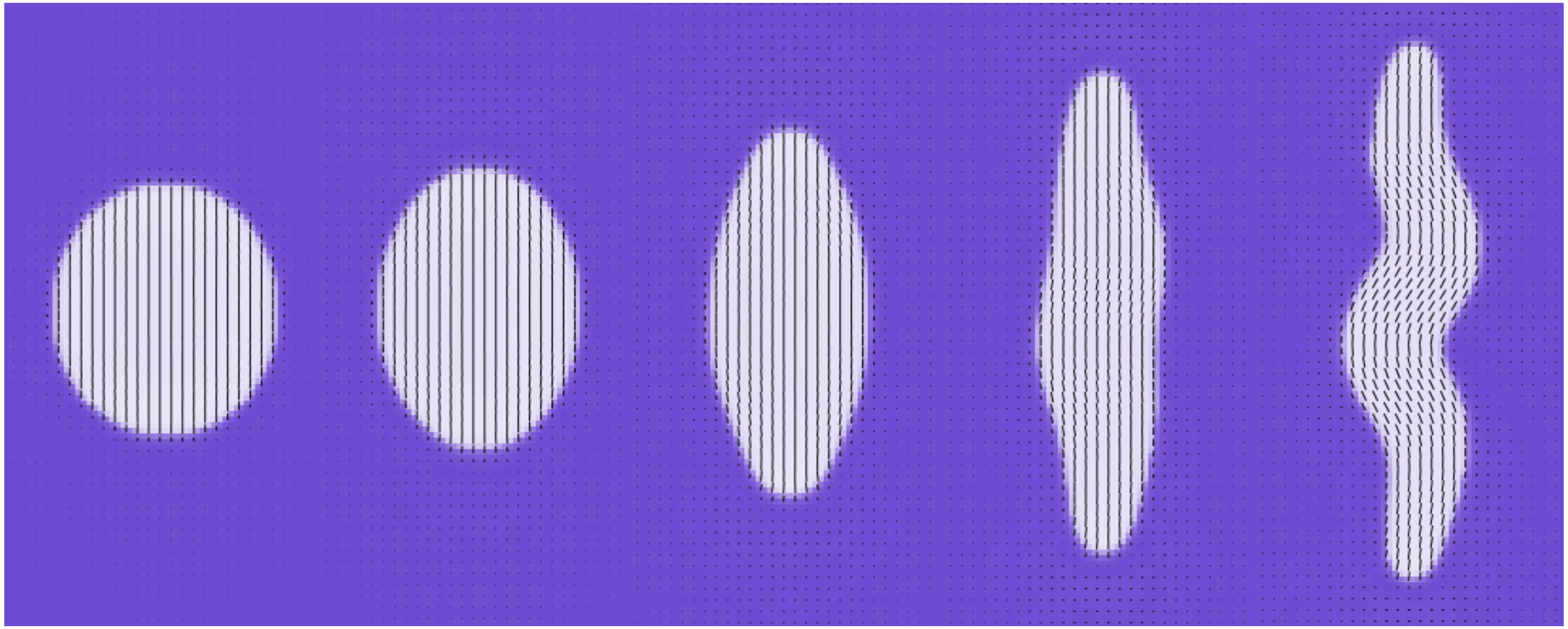
force due to gradient in direction of order parameter
force due to gradient in magnitude of order parameter

Active anchoring



extensile ⇔ planar
contractile ⇔ homeotropic

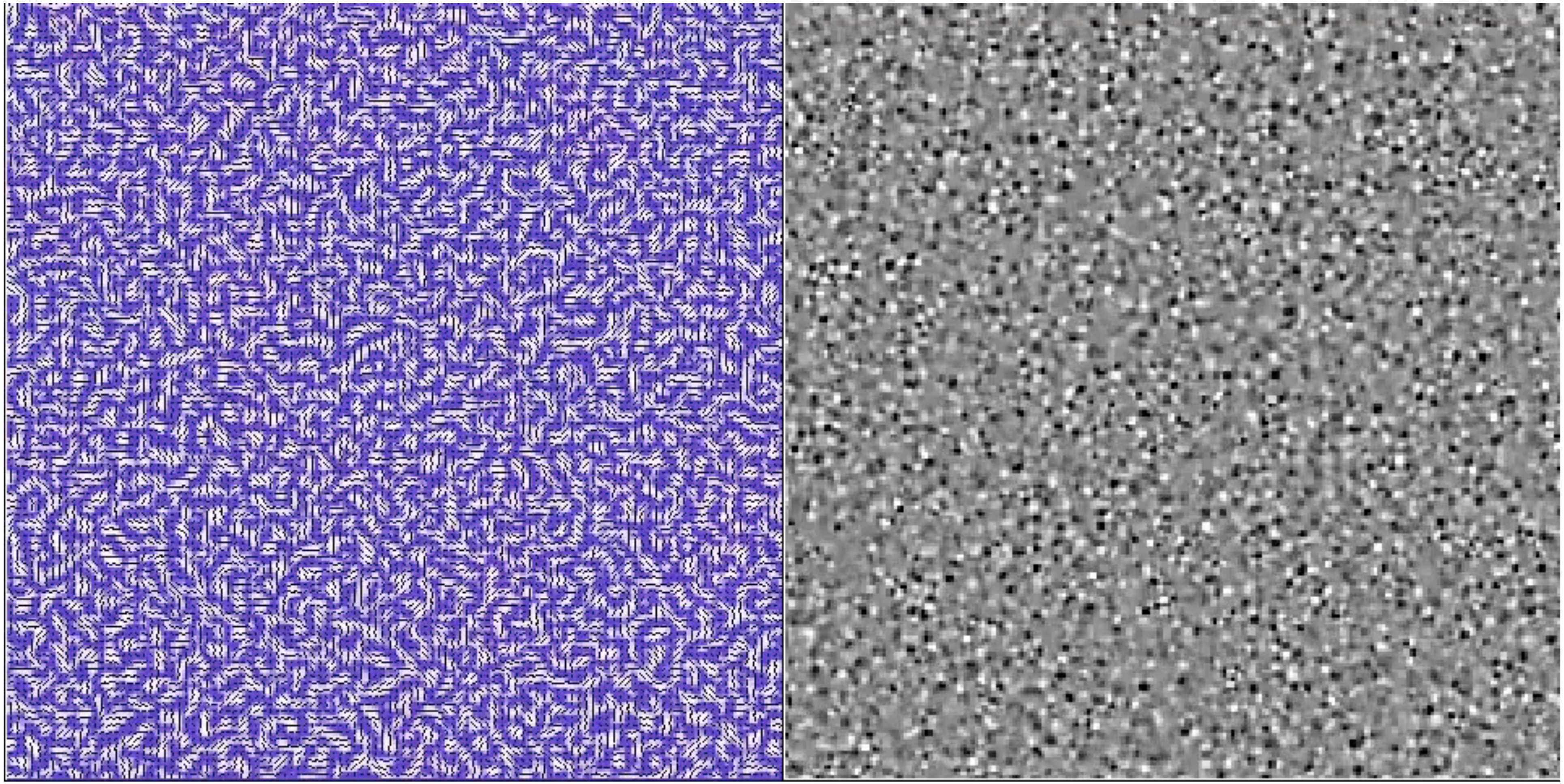
NB interface shape,
topological defects



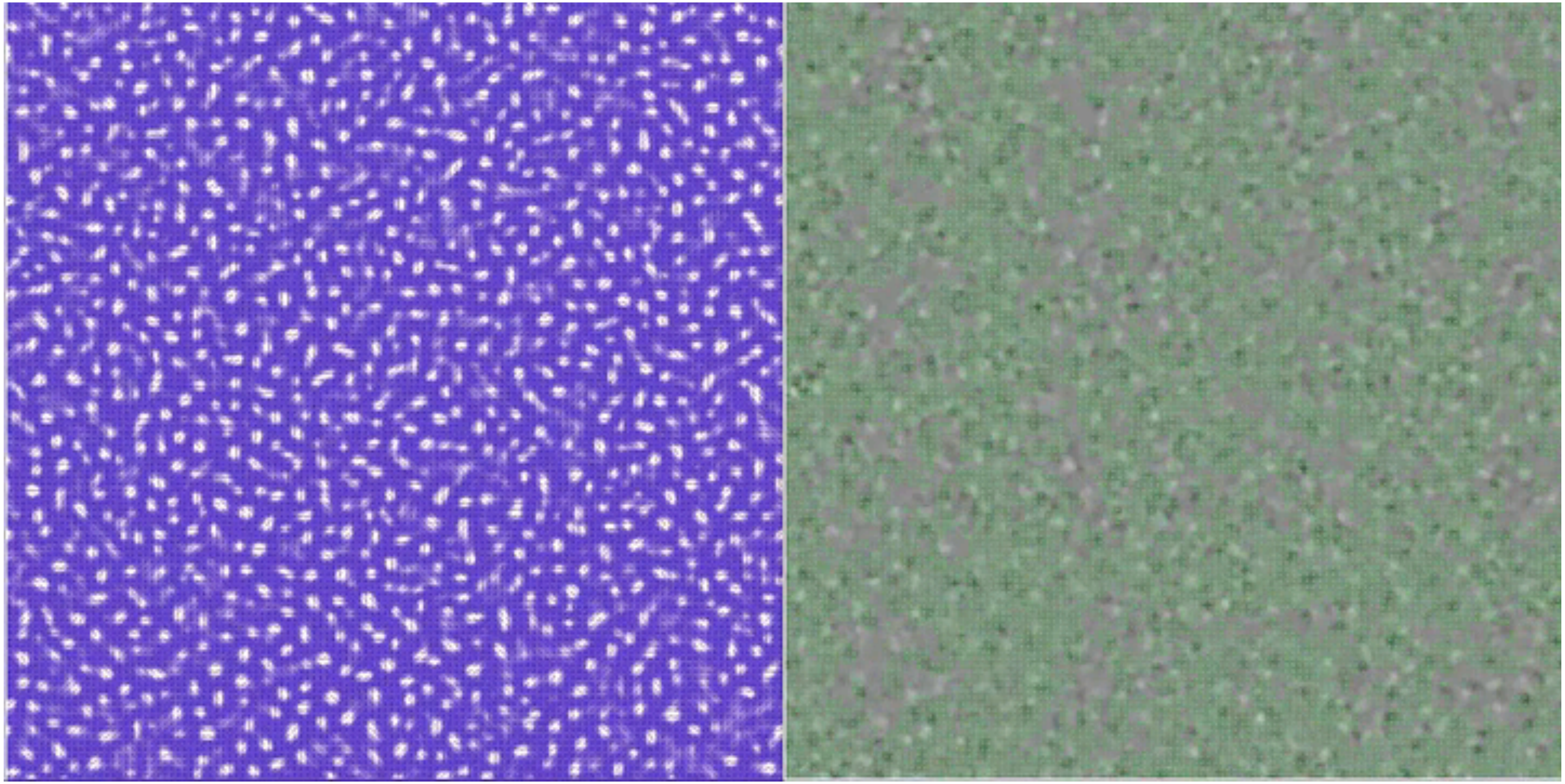
time

preferred anchoring at the interface => elongation of domains

arrested ordering: competition between the ordering effect of the free energy
and the disordering effect of the activity



arrested ordering: competition between the ordering effect of the free energy and the disordering effect of the activity: flow tumbling



Summary

1. Lyotropics: adding an interface

active anchoring
cusp-like interface shapes
+1/2 defects migrate from surface to bulk

2. Adding cell division

3. Adding friction

“Ponder had invented a little system he'd called, in the privacy of his head, Lies-to-Wizards. It was for their own good, he told himself. There was no point in telling your bosses everything; they were busy women, they didn't want explanations. There was no point in burdening them. What they wanted was little stories that they felt they could understand, and then they'd go away and stop worrying.”
— Terry Pratchett, *The Science of Discworld*

(tr: wizard = supervisor; Ponder = grad student)

Adding cell division

$$(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij},$$

$$\partial_t \rho + \partial_i (\rho u_i) = 0,$$

$$\rho (\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij},$$

activity due to motility

$$\Pi_{ij}^{active} = -\zeta Q_{ij},$$

$$\partial_t \phi + \partial_i (u_i \phi) = \Gamma_\phi \nabla^2 \mu + \alpha \phi$$

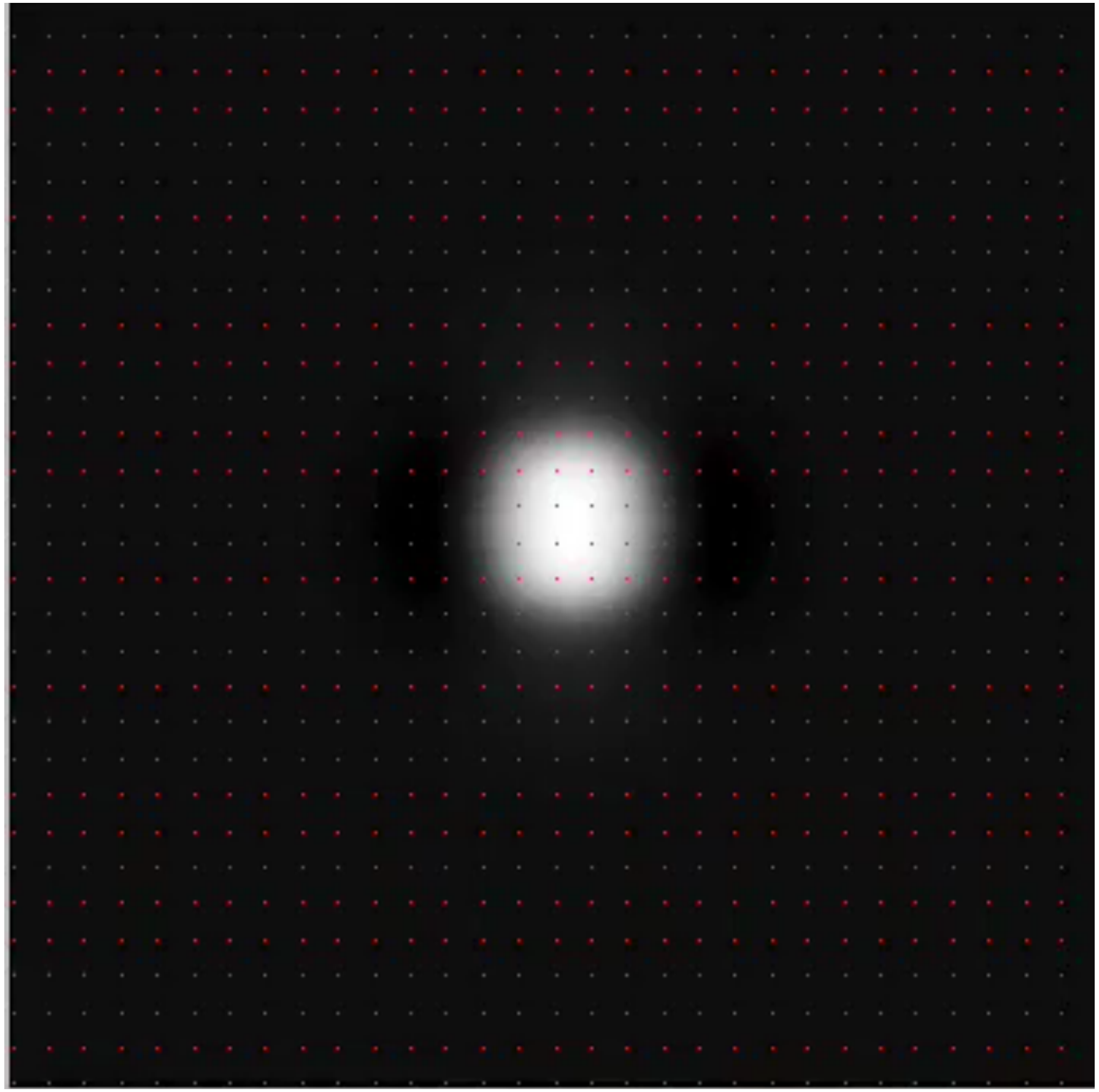
activity due to cell division

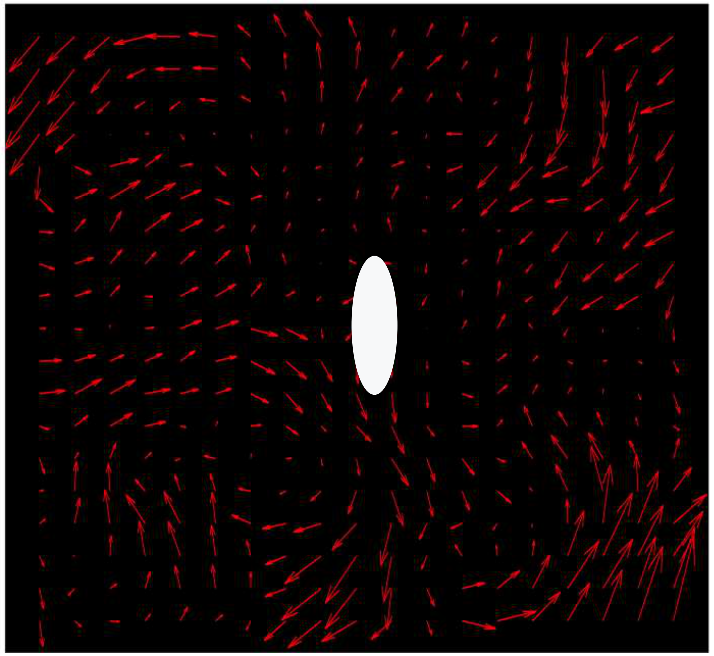
$$\mathscr{F} = \frac{A_\phi}{2} \phi^2 (1-\phi)^2 + \frac{1}{2} A (S^2 \phi - \frac{1}{2} Q_{ij} Q_{ji})^2$$

$$+ \frac{1}{2} \kappa_\phi (\partial_k \phi)^2 + \frac{1}{2} K (\partial_k Q_{ij})^2,$$

Joanny, Prost,
Julicher, Brochard-Wyatt

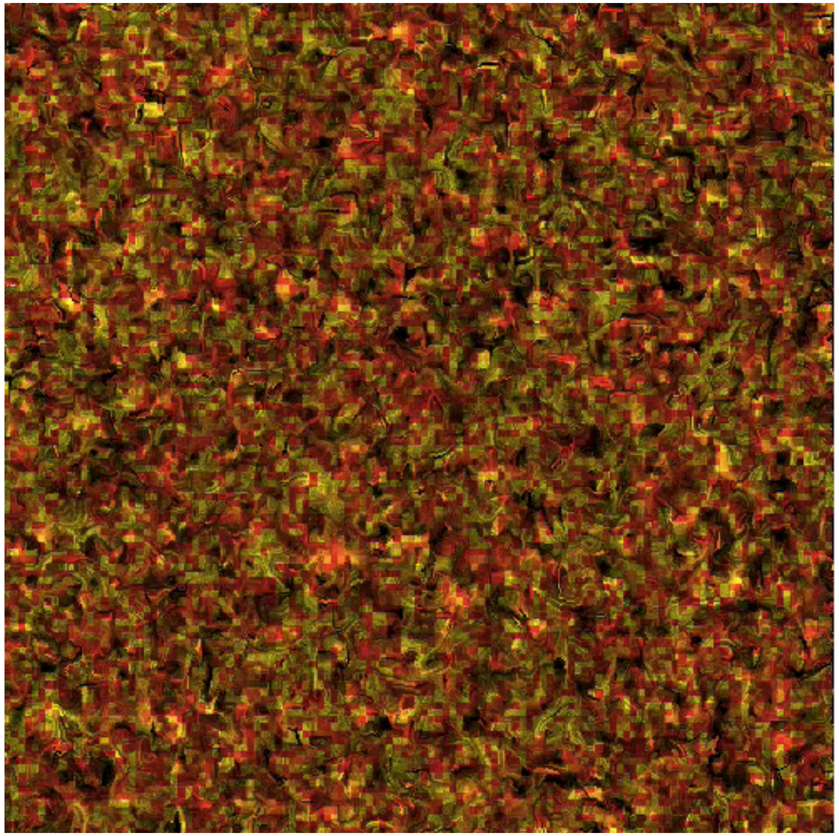
Flow fields around a dividing cell

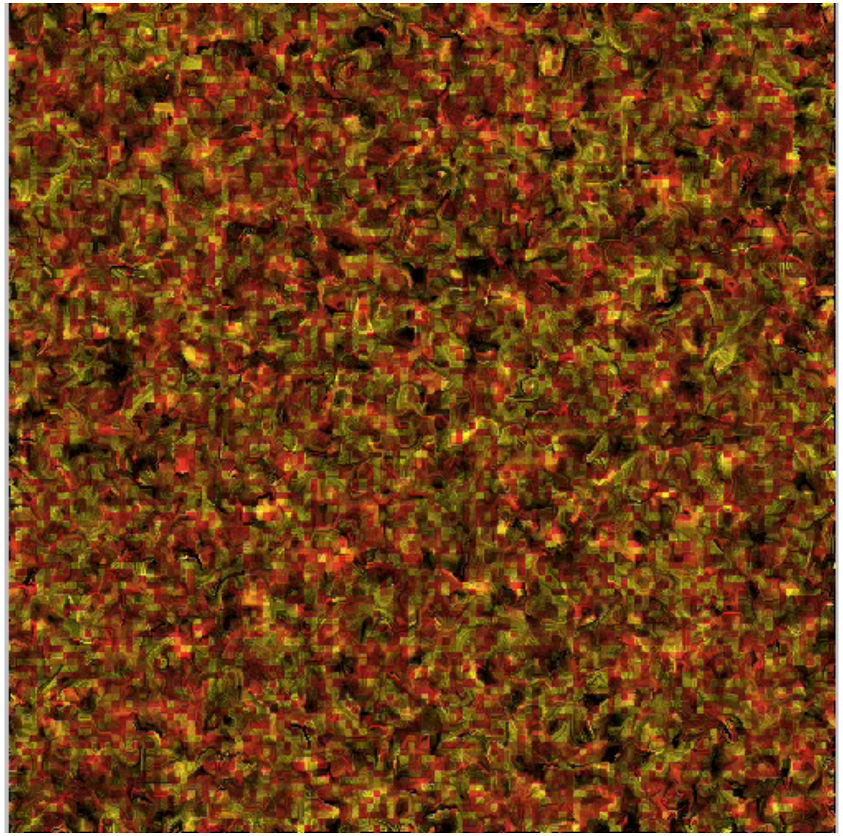




Experiments
Thuan B. Saw, Chwee T. Lim,
Benoit Ladoux

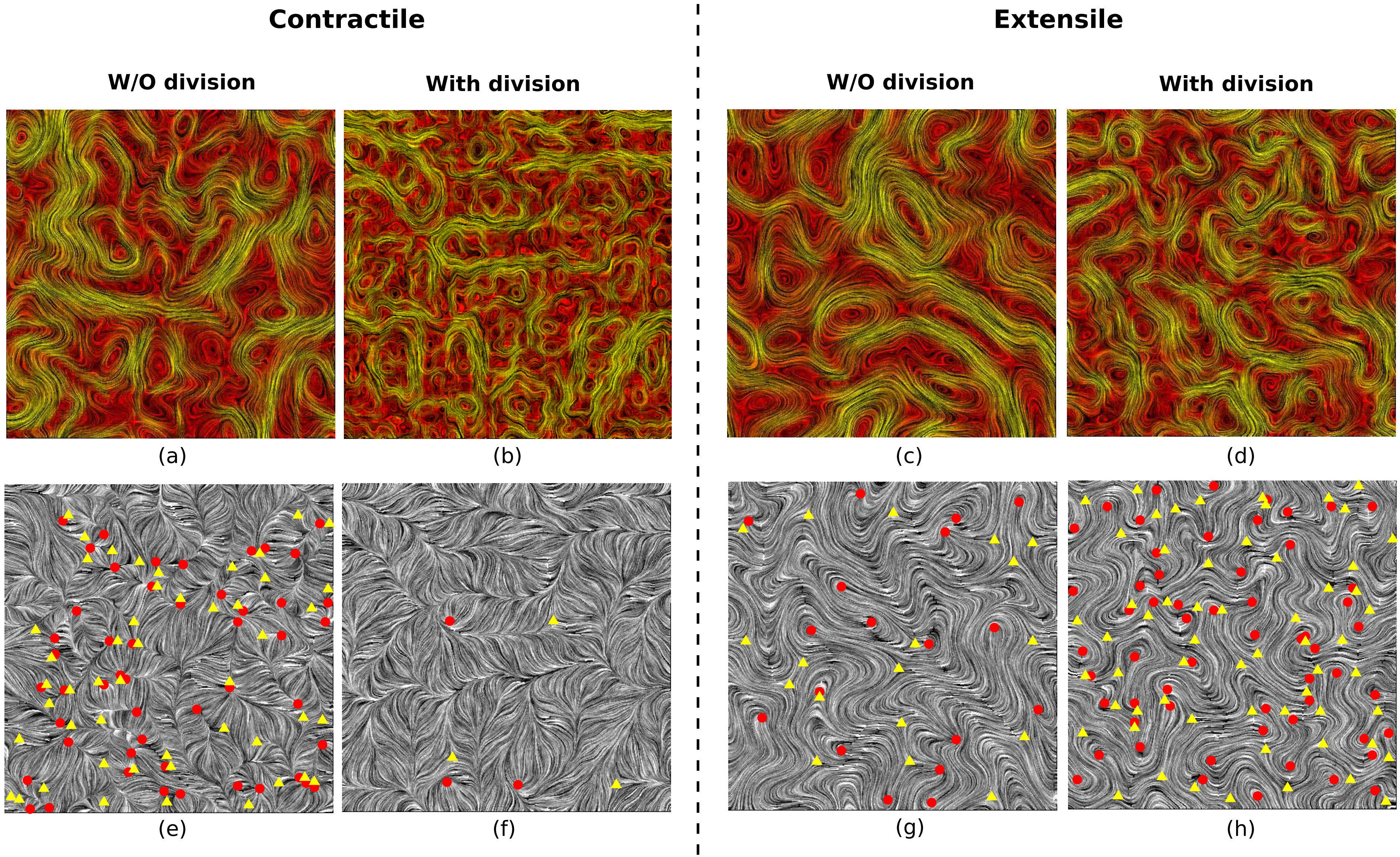
Cell division



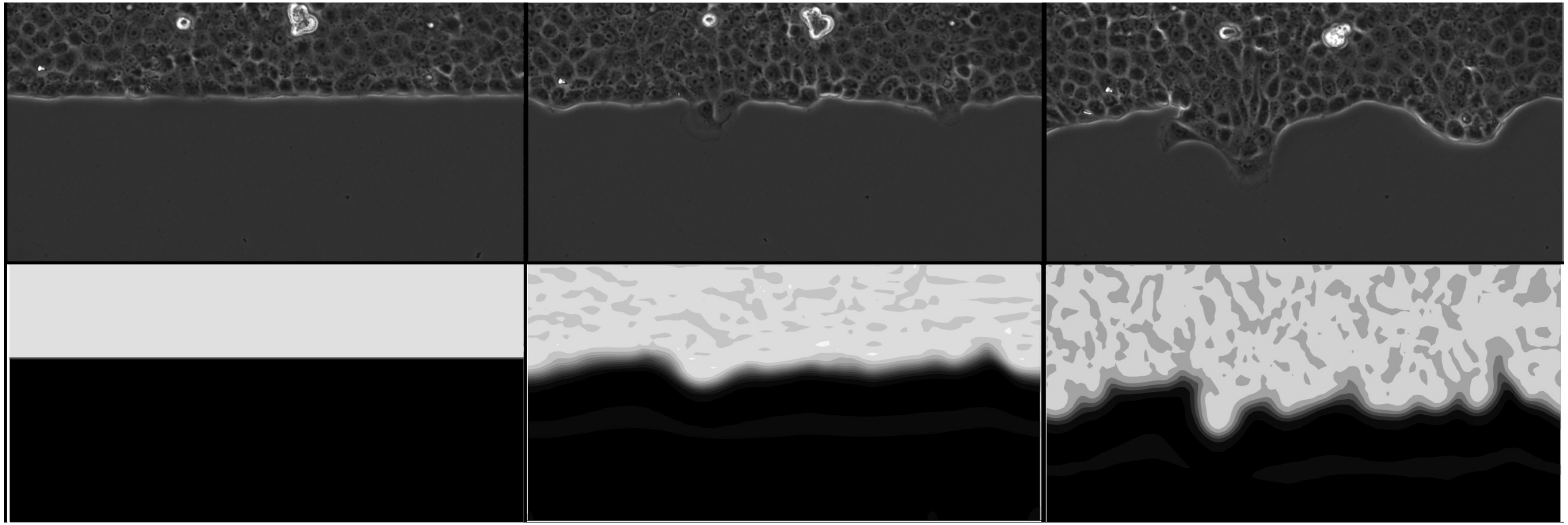


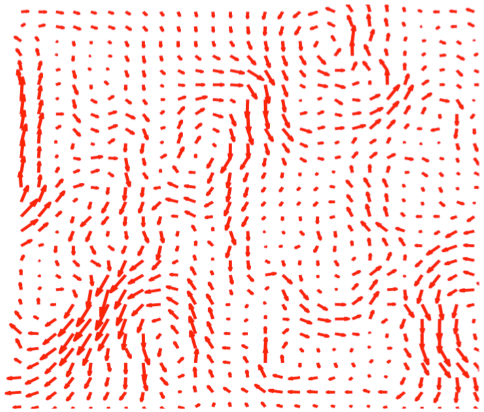
(more division events)

Cell division and motility



Cell division: an interface

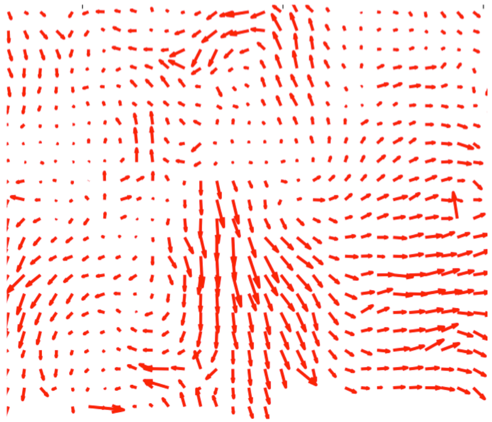




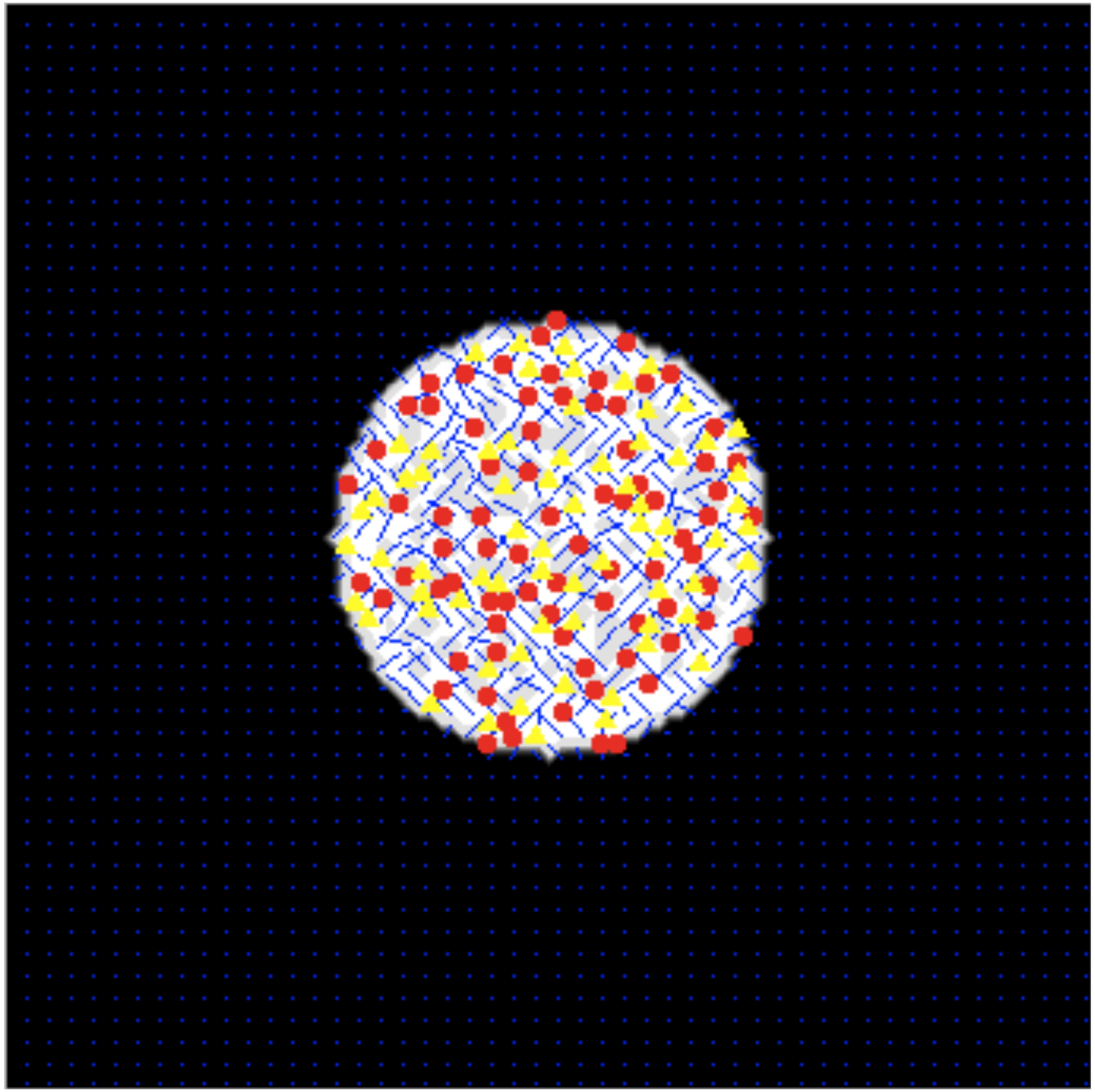
top: experiments

Thuan B. Saw, Chwee T. Lim, Benoit Ladoux

bottom: simulations



Cell colony



Adding friction

$$(\partial_t + u_k \partial_k) Q_{ij} - S_{ij} = \Gamma H_{ij},$$

$$\partial_t \rho + \partial_i (\rho u_i) = 0,$$

$$\rho (\partial_t + u_k \partial_k) u_i = \partial_j \Pi_{ij},$$

$$\Pi_{ij}^{active} = -\zeta Q_{ij},$$

$$f_0 u_i$$

$$\partial_t \phi + \partial_i (u_i \phi) = \Gamma_\phi \nabla^2 \mu + \alpha \phi$$

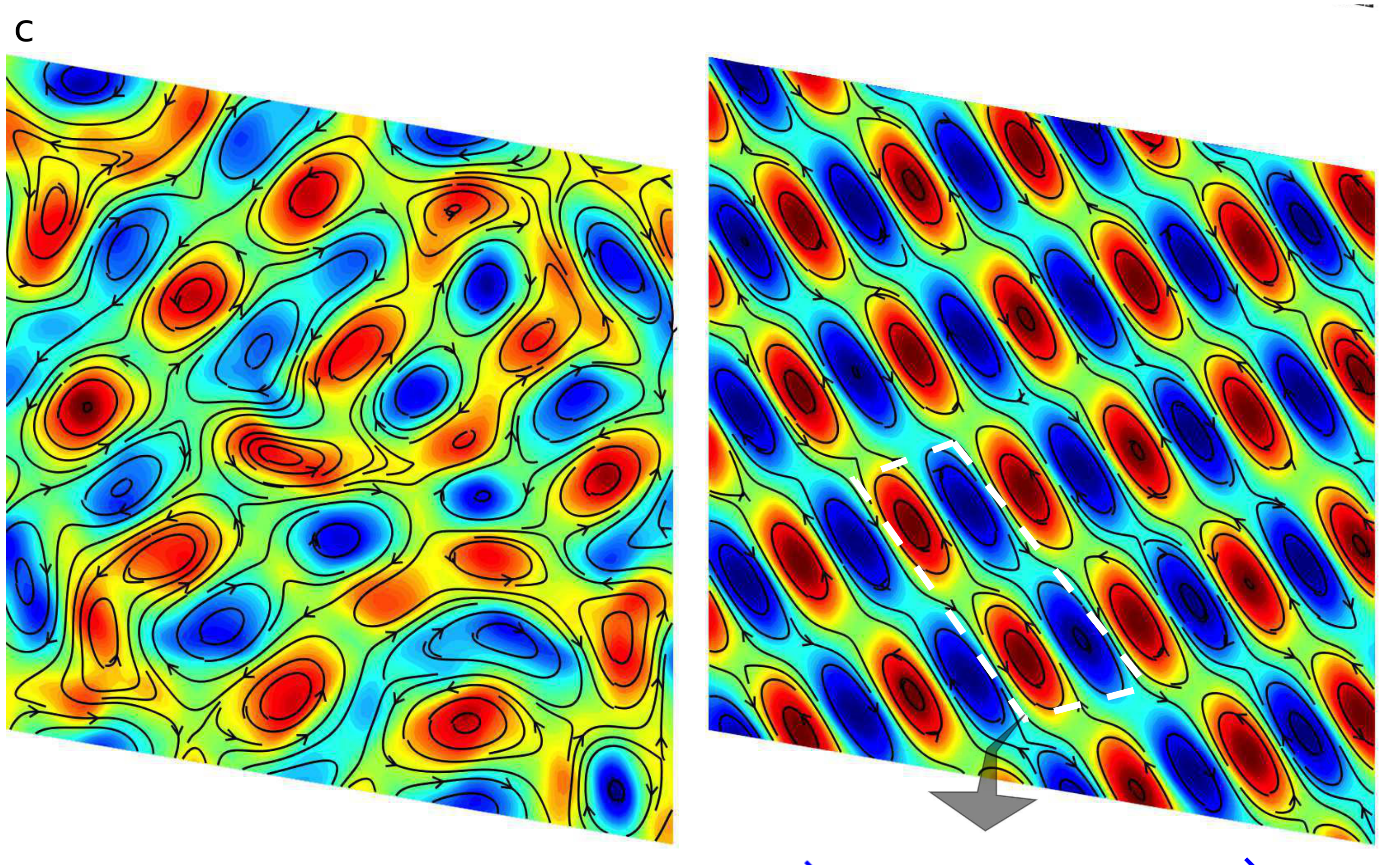
$$\mathcal{F} = \frac{A_\phi}{2} \phi^2 (1-\phi)^2 + \frac{1}{2} A (S^2 \phi - \frac{1}{2} Q_{ij} Q_{ji})^2$$

$$+ \frac{1}{2} \kappa_\phi (\partial_k \phi)^2 + \frac{1}{2} K (\partial_k Q_{ij})^2,$$

"...what would humans be without love?"
RARE, said Death."
— [Terry Pratchett, *Sourcery*]

vortex lattice

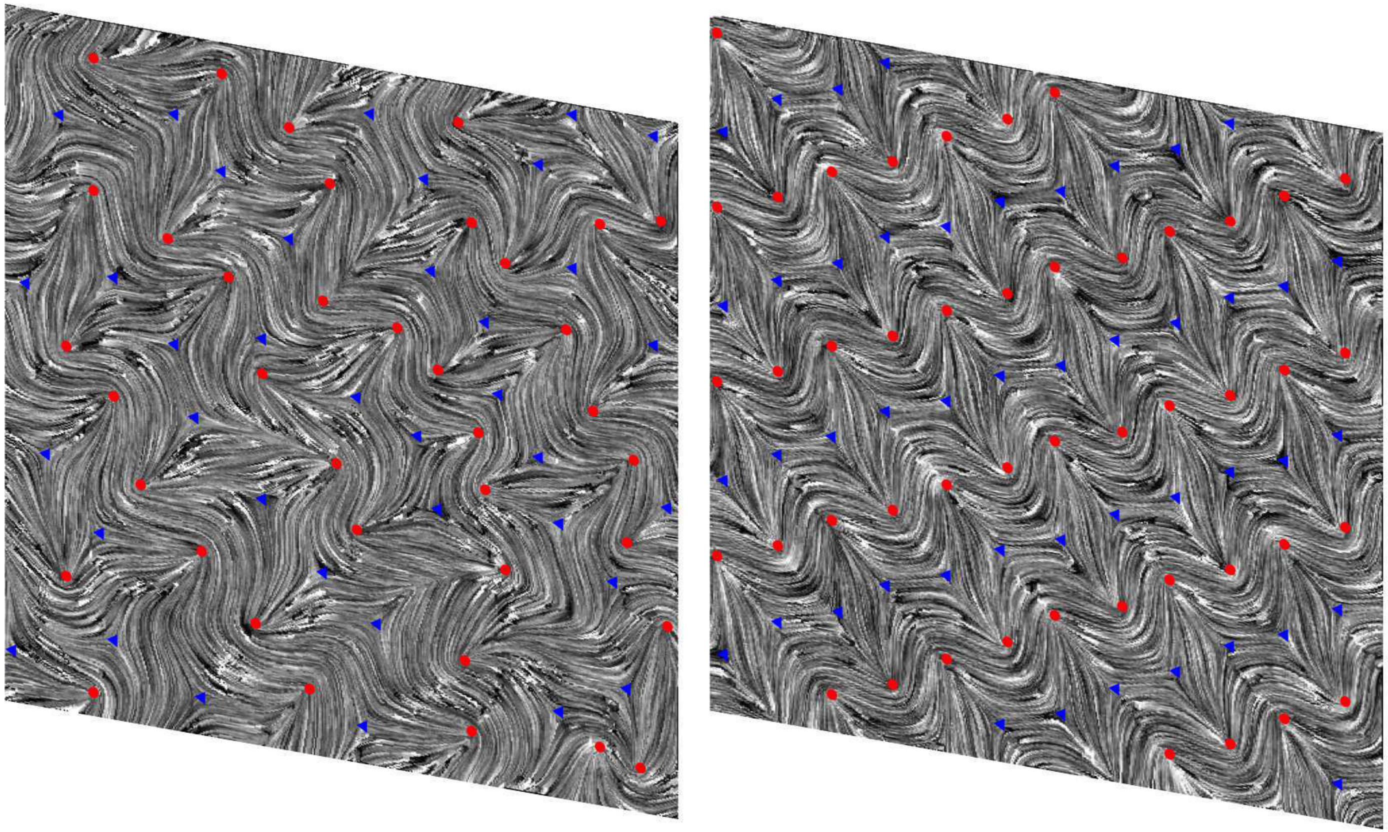
c

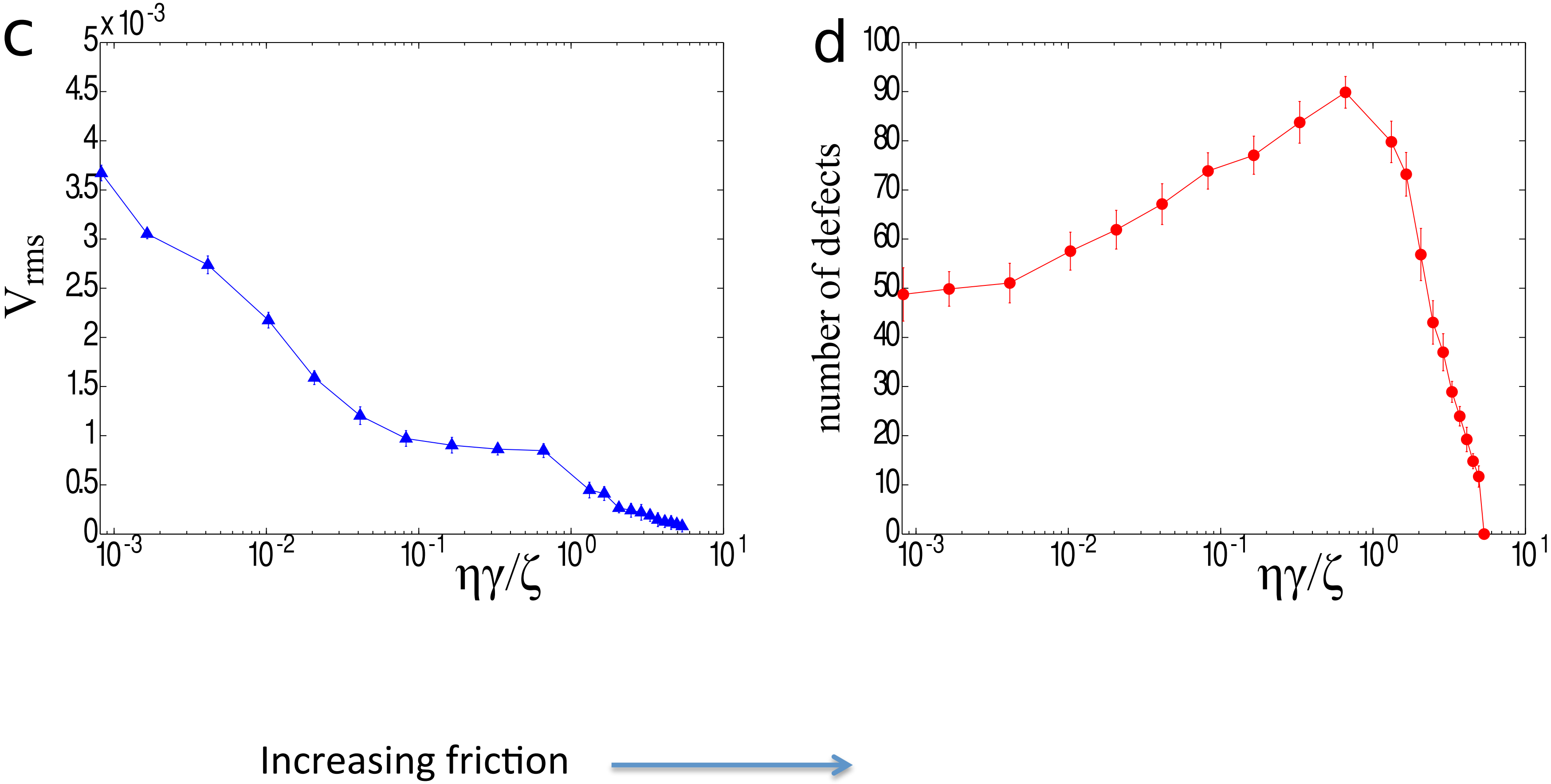


friction --------------------------------→

Topological defects in the vortex lattice

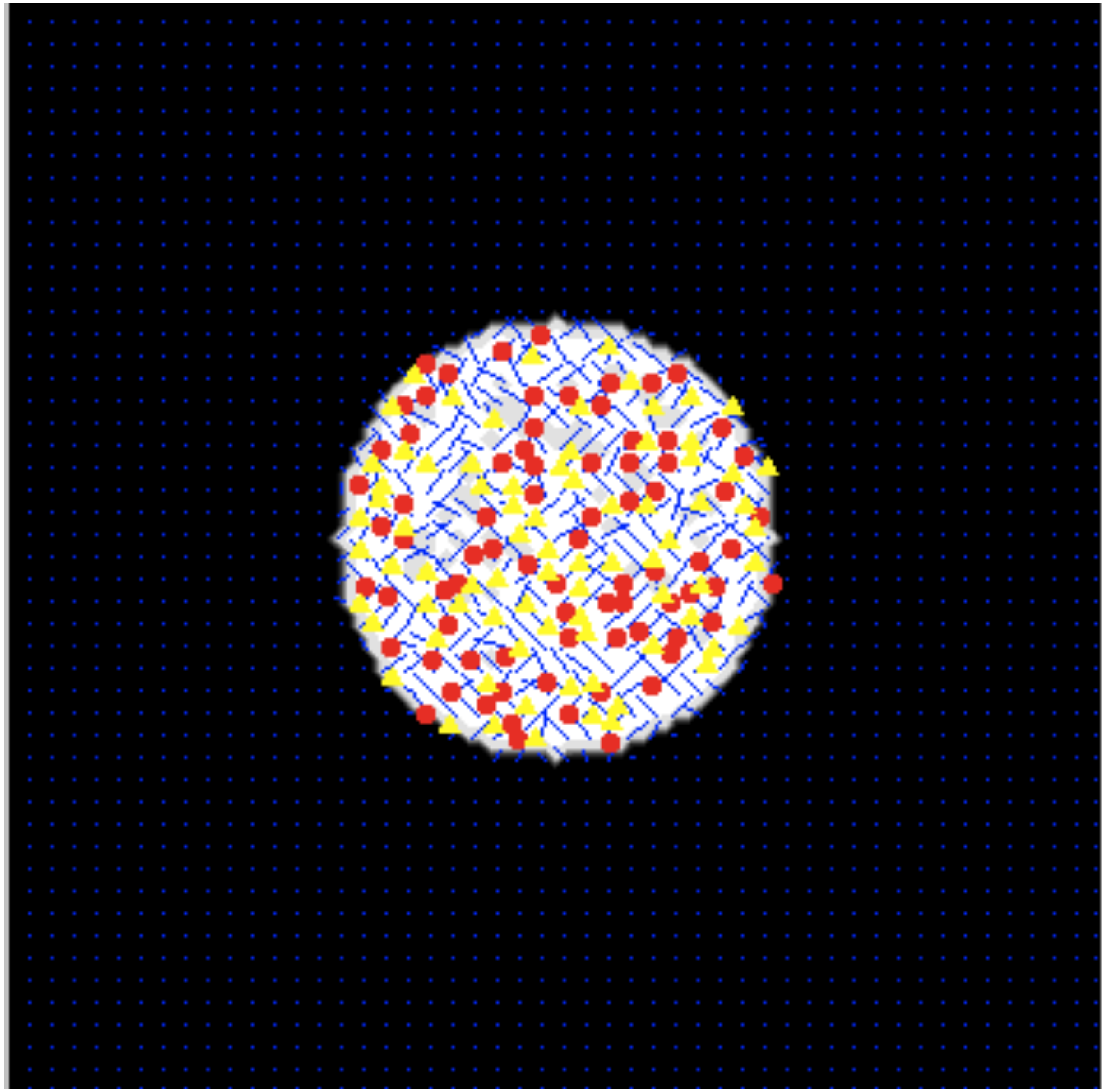
a





c
x 10^-3
V_rms
ηγ/ζ
d
number of defects
ηγ/ζ
Increasing friction

High friction



Summary

1. Lyotropics: adding an interface

active anchoring
cusp-like interface shapes
+1/2 defects migrate from surface to bulk

2. Adding cell division

behaves like extensile activity

3. Adding friction

friction increases the number of defects
defects can migrate from bulk to surface

in the bulk system friction can stabilise a
vortex lattice

Flocking
Dry active systems
Active self assembly
Active colloids
Viscoelasticity
Synchronisation
Intra-cellular mechanics
….

Swimming at low Reynolds number

1. The Scallop theorem

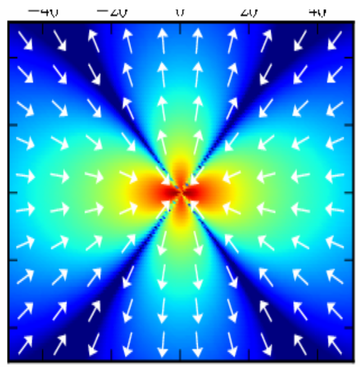
When can a creature swim at low Re?

Swimming stroke must be different forwards and backwards in time

2. Dipolar flow fields

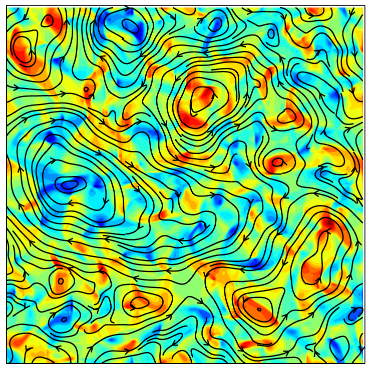
What do the flow fields look like?

The far flow fields are generically dipolar



3. Collective behaviour of active nematics

Flow instabilities => active turbulence



Amin Doostmohammadi
University of Oxford

Henry Shum
University of Pittsburgh

Jorn Dunkel
MIT

Sumesh Thampi
University of Oxford

Dmitri (Mitya) Pushkin
University of York

Gareth Alexander
Anna Balazs
Ramin Golestanian
Rodrigo Ledesma-Aguilar
Arnold Mathijssen
Vic Putz
Miha Ravnik
Tyler Shendruk

Matthew Blow
University of Edinburgh

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