Outline - Lecture 3

Introduction

1. Chern Number Measurement
2. 'Aharonov-Bohm' Interferometry in Bloch Bands
3. Many-Body Localisation with Ultracold Atoms

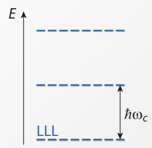
Outlook

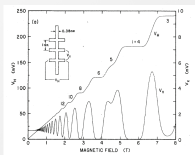
Probing Bloch Band Topology (Single Band Case)

Topology

Band Topology

- **Integer Quantum Hall Effect**



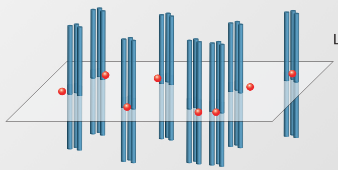


$$\sigma_{xy} = \nu e^2/h$$

ν Integer

Chern Insulators

- **Topological Insulators** (e.g. due to Quantum Spin Hall Effect - Spin-Orbit)
- **Fractional Quantum Hall Effect** - Fractional Chern Insulators (Lattice analog)



Laughlin state at $\nu = 1/3$

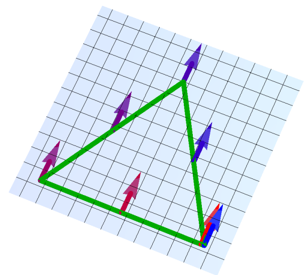
flux quantum ϕ_0=h/ec

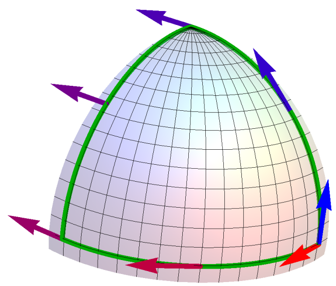
electron

Geometry

Illustrating Geometric Phases

Parallel transport on a surface





Flat manifold: $\varphi_G = 0$

Curved manifold: $\varphi_G \neq 0$

measures the **integrated Gaussian curvature** enclosed by chosen path

Berry Phase in Quantum Mechanics

$$\boxed{\Psi(R) \to e^{i(\varphi_{\text{Berry}}+\varphi_{\text{dyn}})}\ \Psi(R)}$$

Adiabatic evolution through closed loop

$$\boxed{\varphi_{\text{Berry}} = \oint_C A_n(R)dR = i\oint_C \langle n(R)|\nabla_R|n(R)\rangle\, dR}$$

$$\varphi_{\text{Berry}} = \oint_A \Omega_n(R)\, dA$$ **Berry Phase**

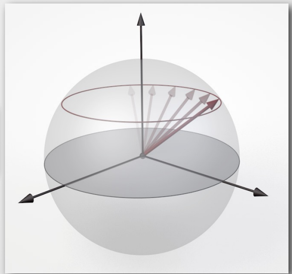
M.V. Berry, Proc. R. Soc. A (1984)

Berry connection

$$A_n(R) = i\langle n(R)|\nabla_R|n(R)\rangle$$

Berry curvature

$$\Omega_{n,\mu\nu}(R) = \frac{\partial}{\partial R^\mu}A_{n,\nu} - \frac{\partial}{\partial R^\nu}A_{n,\mu}$$



Example: Spin-1/2 particle in magnetic field

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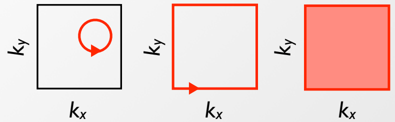
Berry Phase for Periodic Potentials

$$\Psi_k(\mathbf{r}) = e^{i\mathbf{kr}}u_k(\mathbf{r})$$ Bloch wave in periodic potential

Adiabatic motion in momentum space generates Berry phase!



Berry phase is fundamental to characterize topology of energy bands

$$\boxed{n_{\text{Chern}} = \frac{1}{2\pi}\oint_{BZ} A_k dk = \frac{1}{2\pi}\int_{BZ}\Omega_k\, d^2k \iff \sigma_{xy} = n_{\text{Chern}}\, e^2/h}$$

Chern Number (Topological Invariant) **Quantized Hall Conductance**

Thouless, Kohmoto, den Nijs, and Nightingale (TKNN), PRL 1982
Kohmoto Ann. of Phys. 1985

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Single Band Topology

Band structure characterized by scalar & geometric features!

Eigenstates: Bloch waves $\boxed{\psi_{\mathbf{q},n}(\mathbf{r}) = e^{i\mathbf{qr}}u_{\mathbf{q},n}(\mathbf{r})}$

Scalar Features

Dispersion relation

$E_{\mathbf{q},n}$

Geometric Features

Berry connection

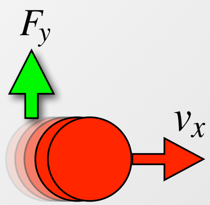
$\mathbf{A}_n(\mathbf{q}) = i\langle u_{\mathbf{q},n}|\nabla_{\mathbf{q}}|u_{\mathbf{q},n}\rangle$

Berry curvature

$\Omega_n(\mathbf{q}) = \nabla_{\mathbf{q}} \times \mathbf{A}_n(\mathbf{q})\cdot \mathbf{e}_z$

How to measure ?

Hall Response & Anomalous Velocity



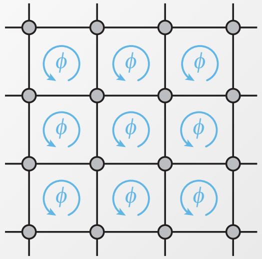
$$\hbar\frac{d\mathbf{k}_c}{dt} = -e\left(\nabla\phi(\mathbf{r}_c) + \frac{d\mathbf{r}_c}{dt}\times\mathbf{B}(\mathbf{r}_c)\right)$$
$$\frac{d\mathbf{r}_c}{dt} = \frac{1}{\hbar}\nabla_{k_c}\varepsilon_n(\mathbf{k}_c) - \underbrace{\frac{d\mathbf{k}_c}{dt}\times\Omega(\mathbf{k}_c)\hat{\mathbf{z}}}_{\text{anomalous velocity}}$$

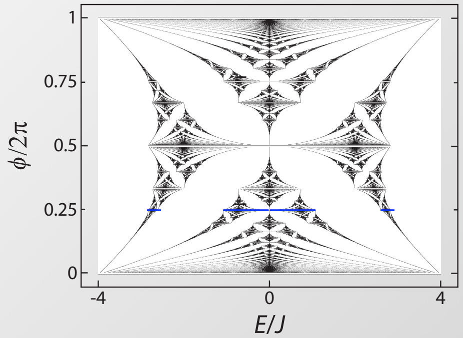
Karplus & Luttinger, Phys. Rev. (1954)
Sundaram & Niu, Phys. Rev. B (1999)

Exp: M. Aidelsburger *et al.*, Nature Physics 11, 162 (2015)
see also G. Jotzu et al. Nature (2014)

Gauge Fields — Harper Hamiltonian and Hofstadter Butterfly

Harper Hamiltonian: $J=K$ and ϕ uniform.

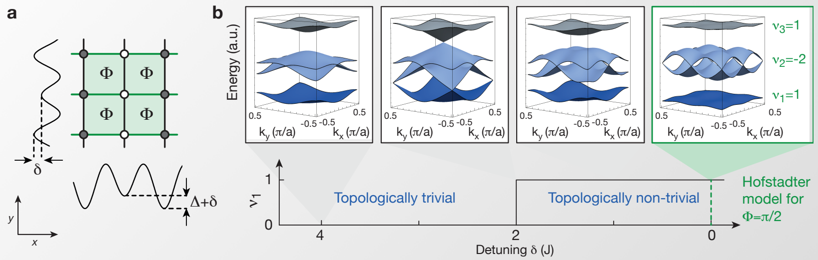




The lowest band is topologically equivalent to the lowest Landau level.

D.R. Hofstadter, Phys. Rev. B14, 2239 (1976)
see alo Y. Avron, D. Osadchy, R. Seiler, Physics Today 38, 2003

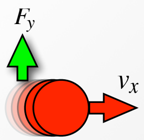
Uniform Flux — Loading and Probing Hofstadter Bands



Key insight for adiabatic loading/probing:
keep Brillouin zone
of topologically trivial & non-trivial phase matched!

Flat bands realized! $E_{gap}/E_{bw} \simeq 8$

Uniform Flux — Cloud Deflection & Chern Number



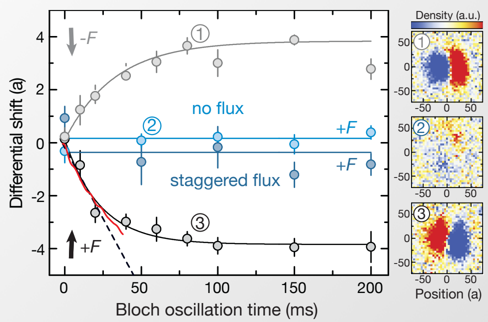
$$\mathbf{v}(\mathbf{k}_c) = \frac{d\mathbf{r}_c}{dt} = \frac{1}{\hbar}\nabla_k \cancel{\varepsilon_n}(\mathbf{k}_c) - \frac{d\mathbf{k}_c}{dt} \times \Omega(\mathbf{k}_c)\hat{\mathbf{z}}$$

Assume uniformly filled band and rational flux p/q:

$$\begin{aligned}
\langle v_x \rangle &= \frac{1}{N_{at}} \int \rho(\mathbf{k}) v_x(\mathbf{k})\, d^2k \\
&= -\frac{1}{N_{at}} \cdot 2\pi \cdot \frac{N_{at}}{\delta k_x \delta k_y} \cdot \frac{F_y}{\hbar} \cdot \frac{1}{2\pi} \int \Omega(\mathbf{k})\, d^2k \\
&= -(2\pi)^2 \cdot \frac{1}{(2\pi/a)(2\pi/qa)} \cdot \frac{F_y}{h} \cdot \nu \\
&= -\frac{F_y q a^2}{h} \nu
\end{aligned}$$

H. Price & N. Cooper PRA (2012)
A. Dauphin & N. Goldman PRL (2013)

Uniform Flux — Chern Number Measurement



$\nu_{exp} = 0.99(5)$

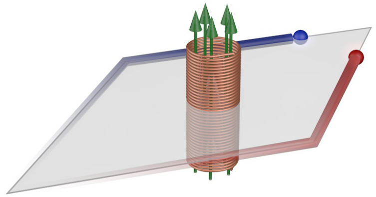
Experimentally measured Chern number

M. Aidelsburger *et al.*, Nature Physics 11, 162 (2015)

Probing Band Topology Using Atom Interferometry

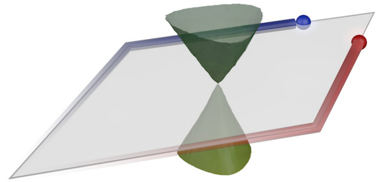
An Aharonov Bohm Interferometer for Determining Bloch Band Topology

π-Magnetic Flux



Real space

π-Berry Flux



Crystal momentum space

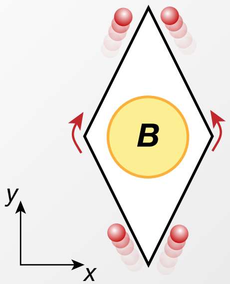
D. Abanin E. Demler

L. Duca et al. Science **347**, 288 (2015)
D. Abanin et al. PRL **110**, 165304 (2013)

AB

Aharonov-Bohm Effect

Real Space



$$\varphi_{AB} = \frac{q}{\hbar}\oint_C \mathbf{A}(\mathbf{r})\,d\mathbf{r} = \frac{q}{\hbar}\int_S \nabla\times\mathbf{A}(\mathbf{r})\,d^2r$$

$$\varphi_{AB} = \frac{q}{\hbar}\int \mathbf{B}\,d\mathbf{S} = 2\pi\Phi/\Phi_0$$

Aharonov-Bohm Phase

Y. Aharonov D. Bohm

..., contrary to the conclusions of classical mechanics, there exist effects of potentials on charged particles, even in the region where all the fields (and therefore the forces on the particles) vanish.

Y. Aharonov & D. Bohm Phys. Rev. (1959)
W. Ehrenberg & R. Siday Proc. Phys. Soc B (1949)
Exp: A. Tonomura, et al. Phys. Rev. Lett. (1986)

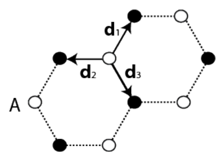
Band Topology

Hexagonal Lattices

__Lattice:__ A and B degenerate sublattices

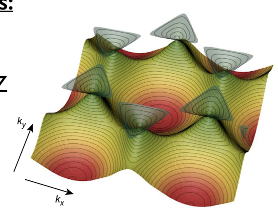
$$H = H_0 - J\sum_{\mathbf{R}}\sum_{i=1}^{3}\left(\hat{a}_{\mathbf{R}}\hat{b}^{\dagger}_{\mathbf{R}+\mathbf{d}_i} + \text{h.c.}\right)$$

Real Space

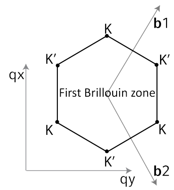


__Lowest energy bands:__

Dirac points at the corners of the first BZ



Reciprocal Space



A. Castro Neto et al., Rev. Mod. Phys. **81**, 109 (2009)
__cold atoms:__ hexagonal - K. Sengstock (Hamburg), brick wall - T. Esslinger (Zürich)

Scalar & Geometric Features

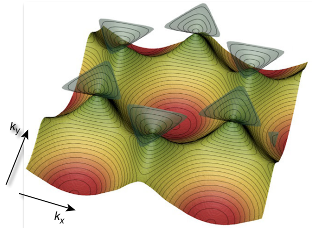
Band structure characterized by scalar & geometric features!

Eigenstates: Bloch waves $\boxed{\psi_{\mathbf{q},n}(\mathbf{r}) = e^{i\mathbf{qr}} u_{\mathbf{q},n}(\mathbf{r})}$

Scalar Features

Dispersion relation

$E_{\mathbf{q},n}$



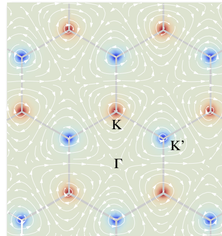
Geometric Features

Berry connection

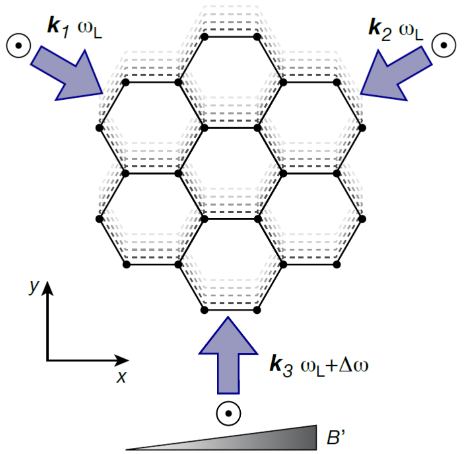
$\mathbf{A}_n(\mathbf{q}) = i\langle u_{\mathbf{q},n}|\nabla_{\mathbf{q}}|u_{\mathbf{q},n}\rangle$

Berry curvature

$\Omega_n(\mathbf{q}) = \nabla_{\mathbf{q}} \times \mathbf{A_n}(\mathbf{q}) \cdot \mathbf{e}_z$



Accelerating the Lattice

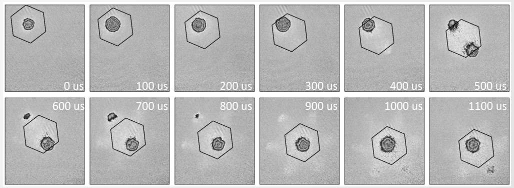


Arbitrary accelerations in any direction can be applied!

Bloch Oscillations

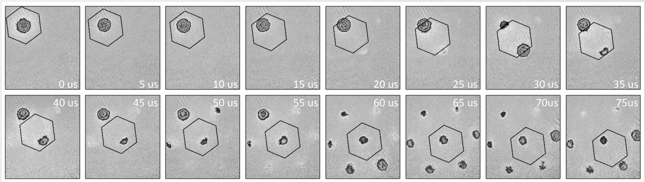
Bloch oscillations induced by accelerating the lattice

weaker force



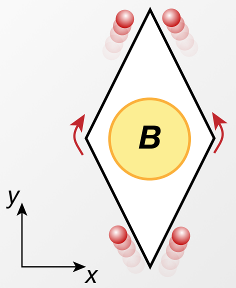
quasi-momentum distribution

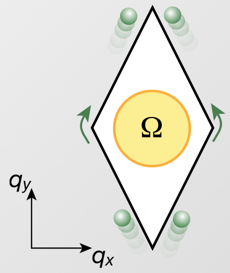
stronger force



'Aharonov Bohm' Interferometer in Momentum Space

Real Space **Momentum Space**





$$\varphi_{AB} = \frac{q}{\hbar}\oint_C \mathbf{A}(\mathbf{r})\,d\mathbf{r} = \frac{q}{\hbar}\int_S \nabla\times\mathbf{A}(\mathbf{r})\,d^2r$$

$$\varphi_{\text{Berry}} = \oint_C \mathbf{A}_n(\mathbf{q})\,d\mathbf{q} = \int_{S_q} \nabla\times\mathbf{A}_n(\mathbf{r})\,d\mathbf{S}_q$$

$$\boxed{\varphi_{AB} = \frac{q}{\hbar}\int \mathbf{B}\,d\mathbf{S} = 2\pi\Phi/\Phi_0}$$

Aharonov-Bohm Phase

$$\boxed{\varphi_{\text{Berry}} = \int \Omega_n(\mathbf{q})\,d\mathbf{S}_q}$$

Berry Phase

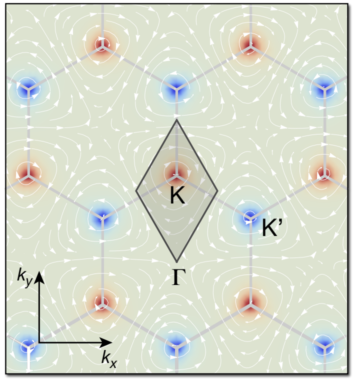
Berry Phases in Graphene

Berry Phase around K-Dirac cone

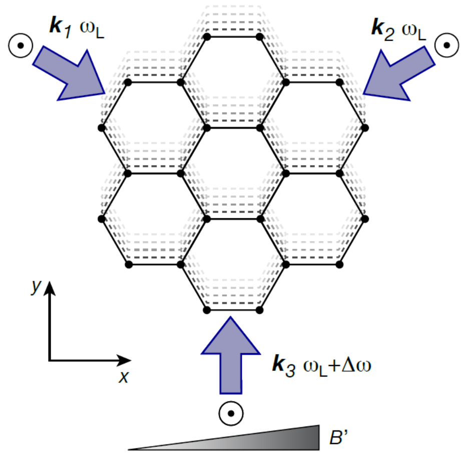
$$\boxed{\varphi_{\text{Berry},\mathbf{K}} = \oint_C \mathbf{A}(\mathbf{q})\,d\mathbf{q} = \pi}$$

Berry Phase around K'-Dirac cone

$$\boxed{\varphi_{\text{Berry},\mathbf{K}'} = -\pi}$$



Accelerating the Lattice

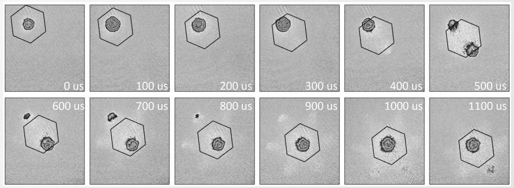


Arbitrary accelerations in any direction can be applied!

Bloch Oscillations

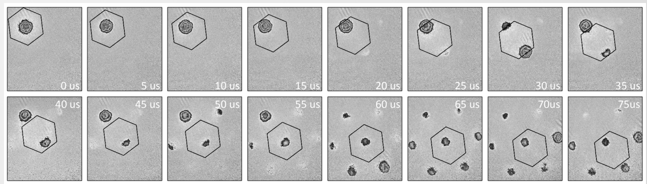
Bloch oscillations induced by accelerating the lattice

weaker force

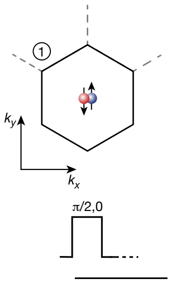


quasi-momentum distribution

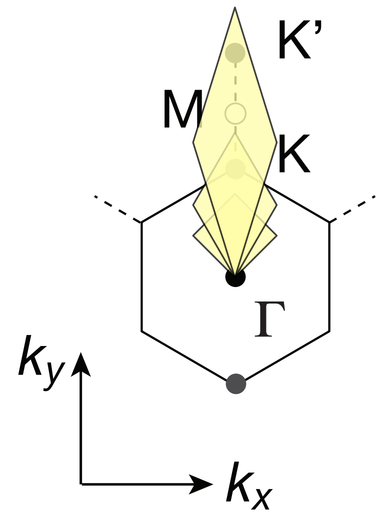
stronger force



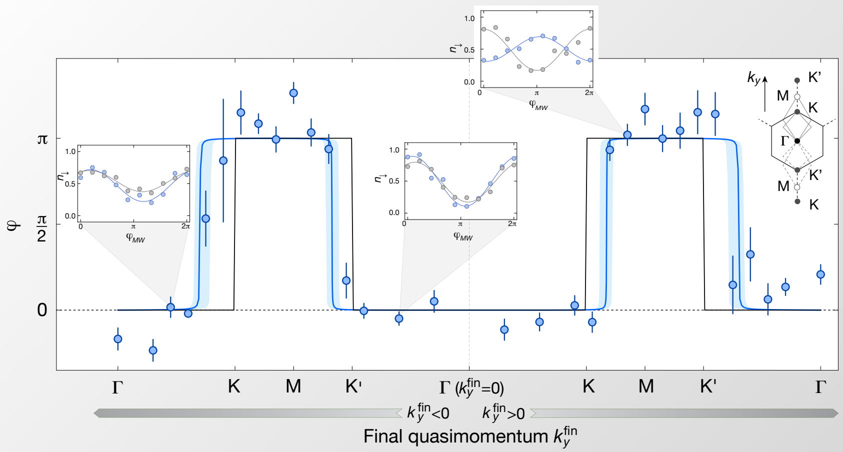
The Interferometer



Forces applied by **lattice acceleration** and **magnetic gradients!**



Interferometry Results

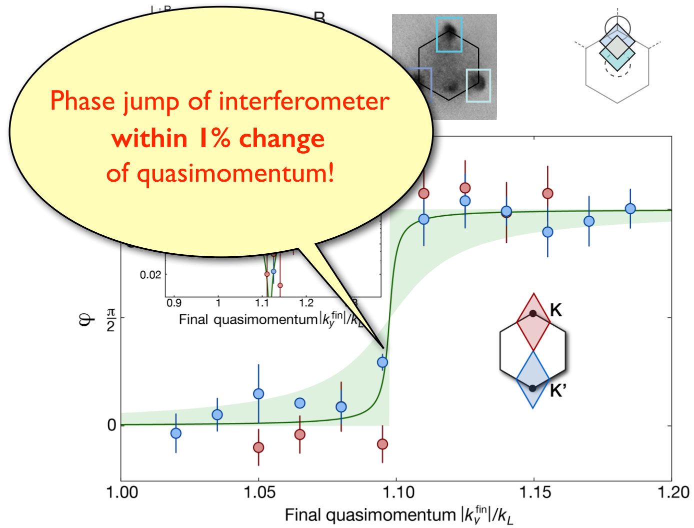


Zooming into the Edge

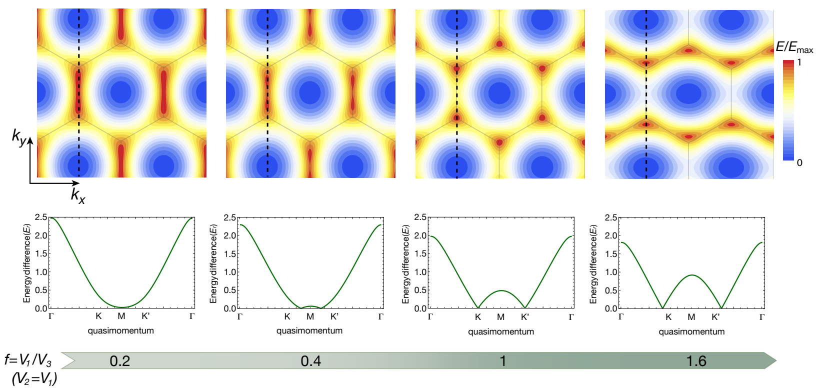
Interferometer path

Location after band-mapping

Differential area

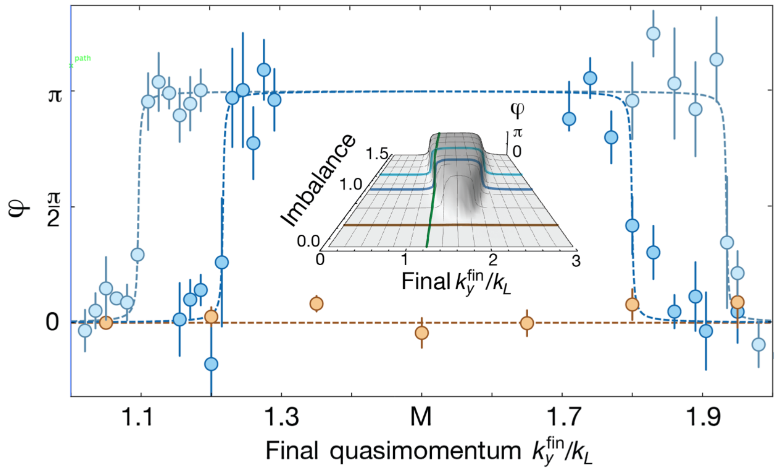


Changing Topology



As long as *T* and *I* symmetry are preserved, Dirac cones are protected!

Robust Berry Flux at Dirac Cone



Interferometer Performance

Berry curvature spread	$< 6 \times 10^{-4}$ pi-flux localized to 10^{-6} of Brillouin zone
A-B site offset	$\Delta < h \times 12\,\mathrm{Hz}$
Energy gap/Band width	3×10^{-3}

Probing Many-Body Localisation with Ultracold Atoms

M. Schreiber et al. Science **349**, 842 (2015)
J.-Y. Choi et al. Science **352**, 1547 (2016)

MBL

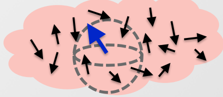
Motivation

Thermalization

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

Classical hydro description of remaining slow modes (conserved quantities, and order parameters).

Many-body localization



Local quantum information persists indefinitely.

Need a fully **quantum** description of the long time dynamics!

?

The many-body localization transition = elusive interface between quantum and classical worlds

from E. Altman

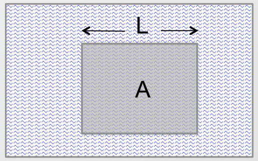
MBL

Eigenstate Thermalisation Hypothesis

Deutsch (91), Srednicki (94,98), Rigol, Dunjko & Olshanii (2009)

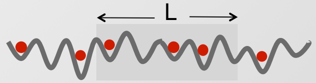
$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$

$$S_A \equiv \mathrm{tr}\,[\rho_A \ln \rho_A] \propto L^d$$



Anderson localization is an example where ETH fails:

"Area law" entropy even in high energy eigenstates



$S_A \propto L^{d-1}$

Many body localization = stability of such localized states to interactions

System fails to act as its own heat bath!

Approaching Many-Body Localization from Disordered Luttinger Liquids
C. Karrasch, J. E. Moore
Subjects: Strongly Correlated Electrons (cond-mat.str-el)
28. arXiv:1506.00592 [pdf, other]
Protection of topological order by symmetry and many-body loc
Andrew C. Potter, Ashvin Vishwanath
Comments: 17 pages, 4 figures
Subjects: Disordered Systems and Neural Networks (cond-mat.
29. arXiv:1505.07089 [pdf, other]
Dynamics of many-body localisation in a translation invariant qu
Merlijn van Horssen, Emanuele Levi, Juan P. Garrahan
Comments: 5 pages, 4 figures
Subjects: Statistical Mechanics (cond-mat.stat-mech); Quantum
30. arXiv:1505.06343 [pdf, ps, other]
Many-body ground state localization and coexistence of localize
Yucheng Wang, Haiping Hu, Shu Chen
Comments: 5 pages, 6 figures
Subjects: Disordered Systems and Neural Networks (cond-mat.
31. arXiv:1505.05386 [pdf, other]
Revisiting Many-body Localization with Random Networks of Te
Benoît Descamps, Frank Verstraete
Comments: 3 figures
Subjects: Quantum Physics (quant-ph)
32. arXiv:1505.05147 [pdf, other]
Many-Body Localization of Symmetry Protected Topological States
Kevin Slagle, Zhen Bi, Yi-Zhuang You, Cenke Xu
Comments: 5 pages 2 figures
Su
33.
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L.
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34.
Tot
J. Goold, C. Gogolin, S. R. Clark, J. Eisert, A. Scardicchio, A. Silva
Comments: Slight Restructuring of the manuscript and additional analysis performed
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn); Quantum Physics (quant-ph)
35. arXiv:1504.00016 [pdf, other]
Many body localization and quantum non-ergodicity in a model with a single-particle mobility edge
Xiaopeng Li, Sriram Ganeshan, J. H. Pixley, S. Das Sarma
Comments: 5+6 pages, 7 figures, added entanglement entropy results
Subjects: Strongly Correlated Electrons (cond-mat.str-el); Quantum Gases (cond-mat.quant-gas); Quantum Physics (quant-ph)
36. arXiv:1503.07620 [pdf, ps, other]
Many body localization in the presence of a single particle mobility edge
Ranjan Modak, Subroto Mukerjee
Comments: 5 pages, 6 figures
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn); Statistical Mechanics (cond-mat.stat-mech); Strongly Correlated Electrons (cond-mat.str-el)
37. arXiv:1503.06508 [pdf, ps, other]
Localization in a random $x-y$ model with the long-range interaction: Intermediate case between single particle and many-body problems
Alexander L. Burin
Comments: Modified version after review
Subjects: Disordered Systems and Neural Networks (cond-mat.dis-nn)
38. arXiv:1503.06147 [pdf, other]
Many-body localization characterized from a one-particle perspective

Pioneering work:
D. M. Basko, I. L. Aleiner, B. L. Altschuler, Ann. Phys. (2006).

Good review/intro:
D.A. Huse, R. Nandkishore, V. Oganesyan, Annu. Rev. Cond. Mat. 6, 15 (2015)

R. Vosk & E. Altman, Annu. Rev. Cond. Mat. 6, 383 (2015)

No Experiments!

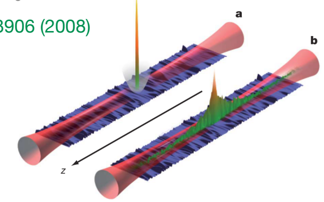
MBL

Measuring Localisation

Anderson localization:

Ready…Set…Go!

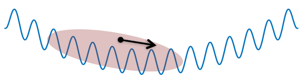
T. Schwartz et al. Nature **446**, 52 (2007)
Y. Lahini, et al. Phys. Rev. Lett. **100**, 013906 (2008)
J. Billy et. al. Nature 2008 (Inst. Opt.)
G. Roati et. al. Nature 2008 (LENS)



Many-body localization:

Ready…Set…Go!

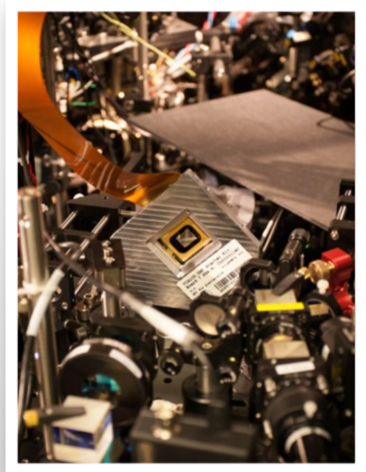
Fastest timescale: local probe!

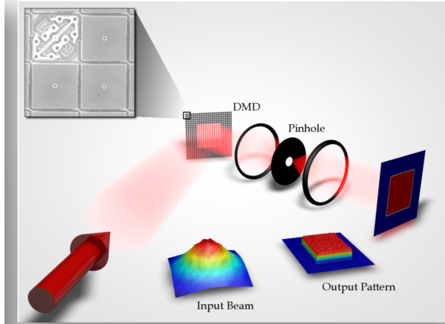


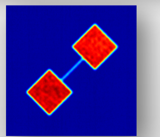
Slowest timescale: global probe

Kondov et al. (DeMarco) Phys. Rev. Lett. 114, 083002 (2015)

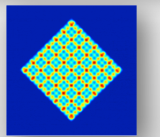
Arbitrary Light Patterns





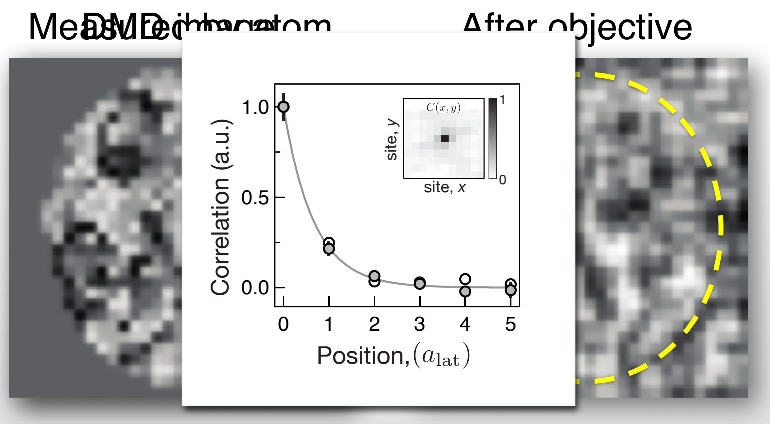


Quantum wires



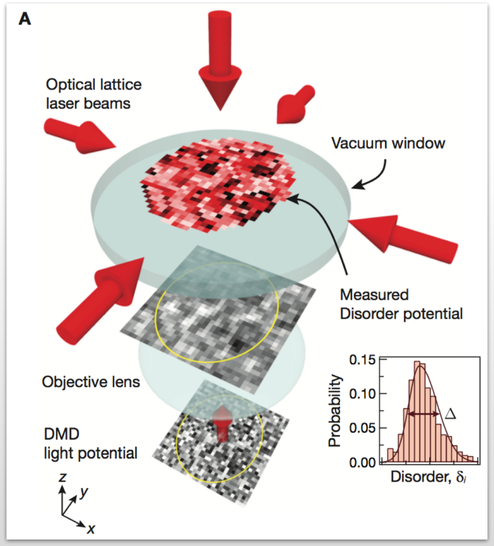
Exotic lattice

Disorder Potential



Excellent characterization of disorder !!

System Summary

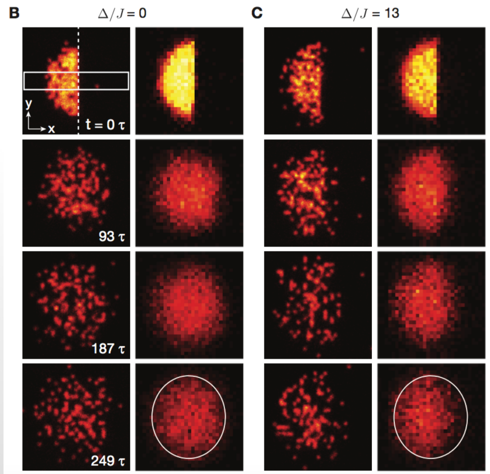


1. Prepare Domain Wall
   (no tunneling dynamics)
2. Turn on disorder potential
3. Lower the lattice depth
   (near critical point)
4. Measure atomic distribution

* Tunneling time is 6.4 ms.
* Disorder is changed for each image.
* Take 100 picture for averaging.

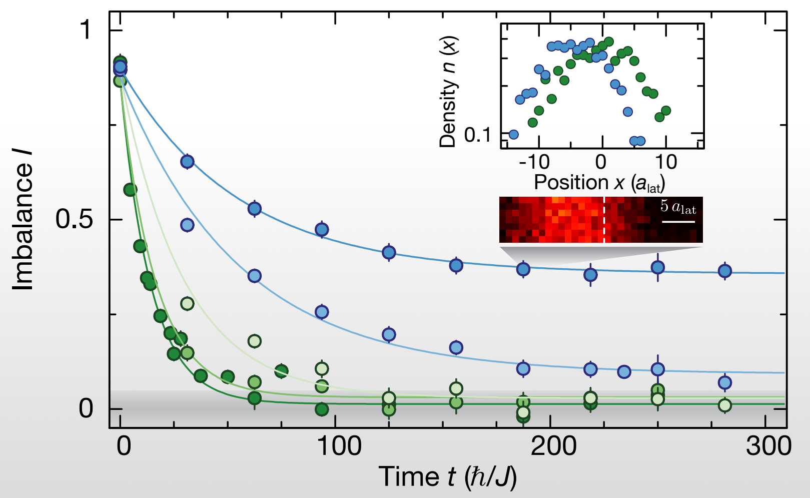
$$U = 24J$$
$$\Delta = 0 - 20J$$

Domain Wall Dynamics

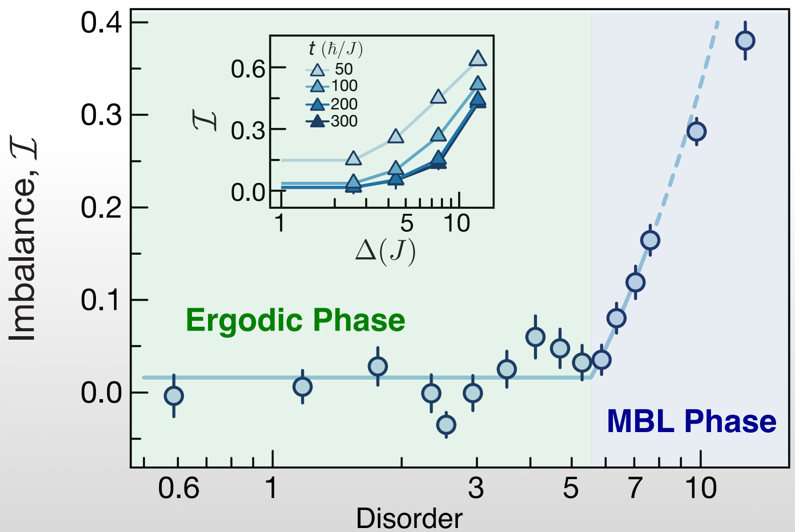


Without disorder   With disorder

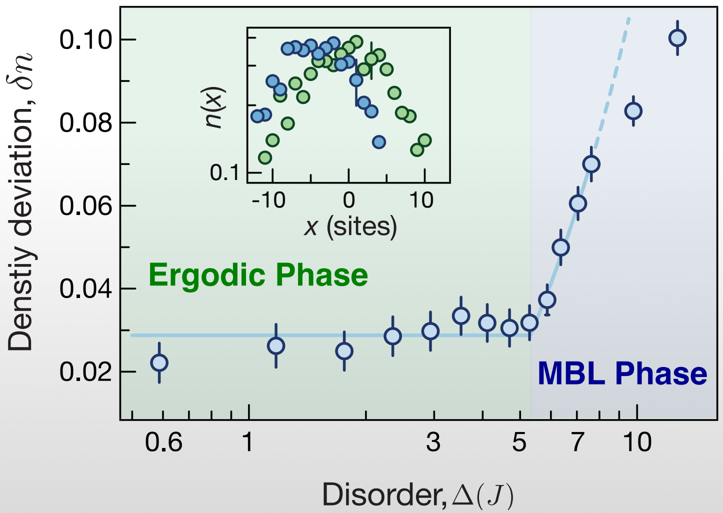
Imbalance of domain wall



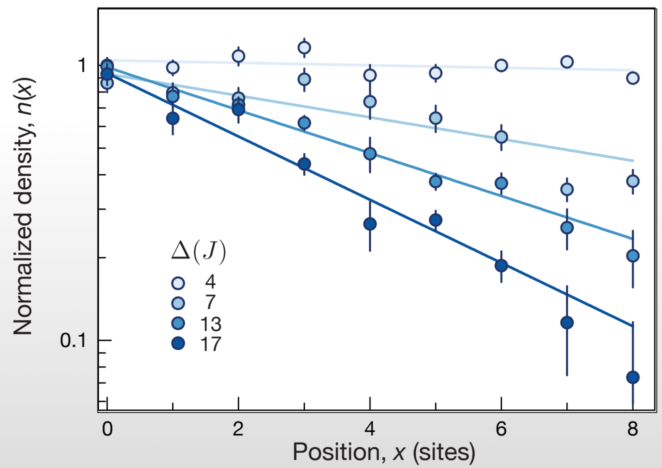
Delocalization-to-Localization



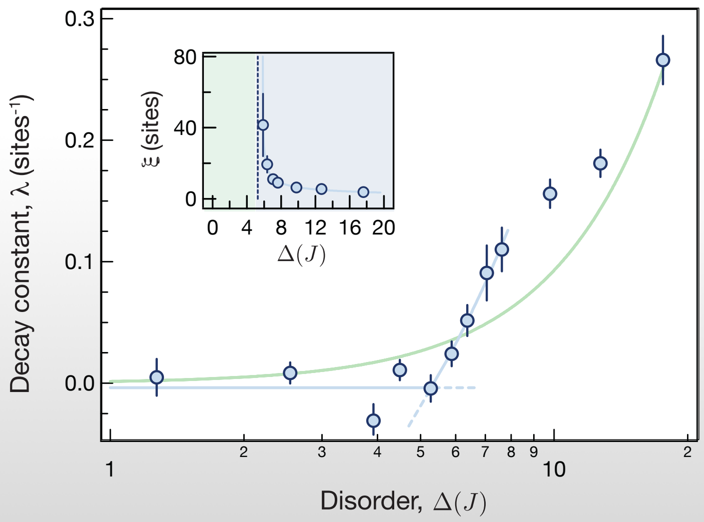
Delocalization-to-Localization



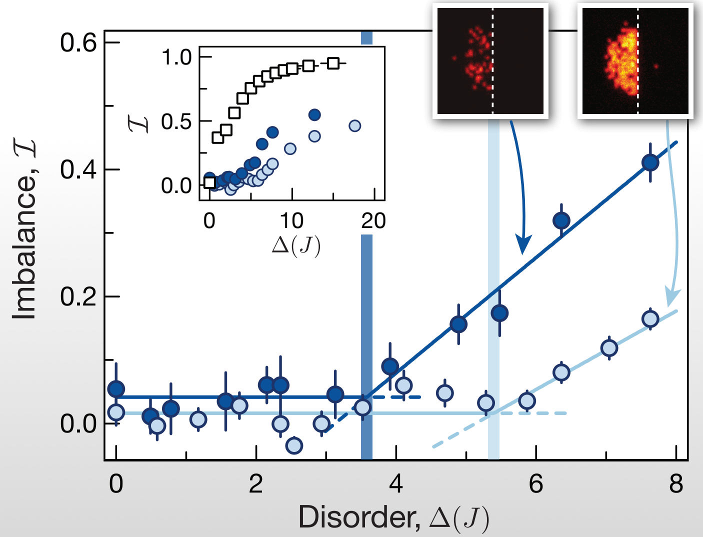
Disorder Effect in Real Space II



Diverging Length Scale



Role of Interaction

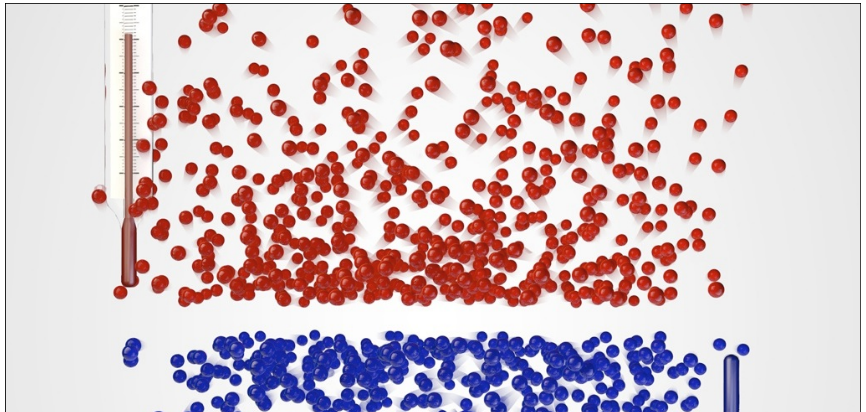


MBL §

So far: good qualitative and in parts quantitative understanding!

- MBL for different dimensionalities? 1D/2D/3D - Disorder Dimension
- **Coupling to outside world** - Photon Scattering destruction of MBL?
- Optical Conductivity - Ergodic vs MBL phase
- Local fluctuation measurements with Quantum Gas Microscopes
- Measuring **localization length**? dynamical (domain walls)? impurities?
- Critical slowing down at transition
- Entanglement Entropy growth?
- MBL in driven systems

LMU



Quantum Matter at Negative Absolute Temperature

S. Braun, J.-P. Ronzheimer, M. Schreiber, S. Hodgman, T. Rom, D. Garbe, IB, U. Schneider

S. Braun et al. Science **339**, 52 (2013)
A. Mosk, PRL **95**, 040403 (2005) ,A. Rapp, S. Mandt & A. Rosch, PRL **105**, 220405 (2010)

Thermodynamic Definition of Temperature

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right.$$

...erature

...ith Energy

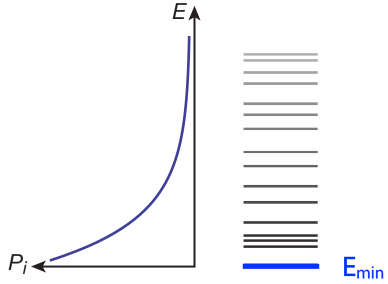
...ure

...s with Energy

Warning:
Temperature does not measure energy content!!!

Thermodynamic theorems apply in negative as well as positive temperature regime!

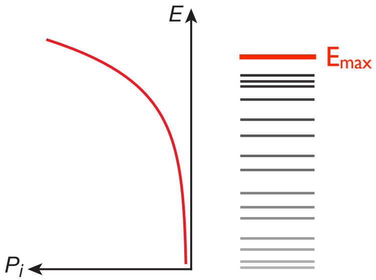
Requirements



$$P_i \propto e^{-\frac{E_i}{k_B T}}$$

For positive temperatures, we require lower energy bound E_{min}!

Requirements



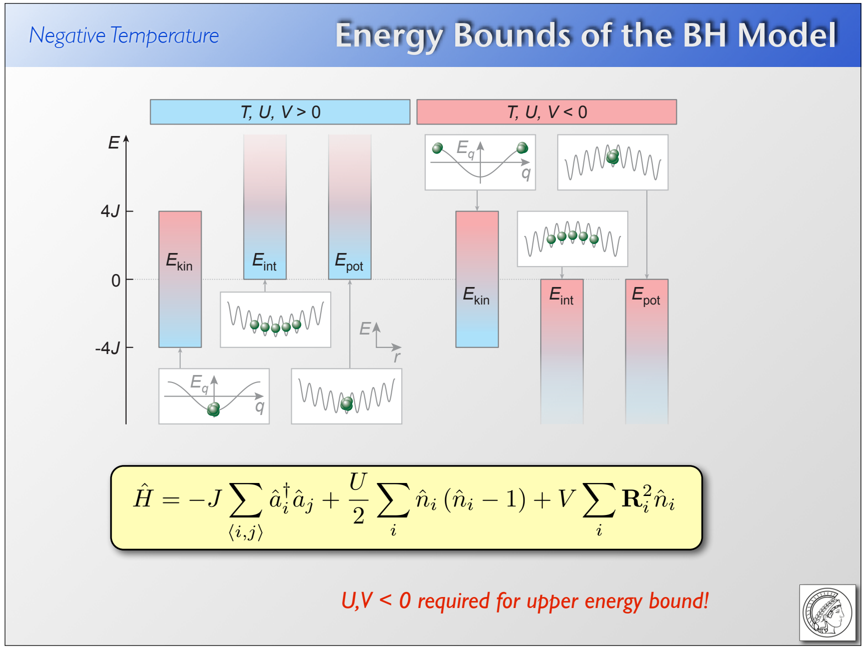
$$P_i \propto e^{-\frac{E_i}{k_B (-T)}}$$

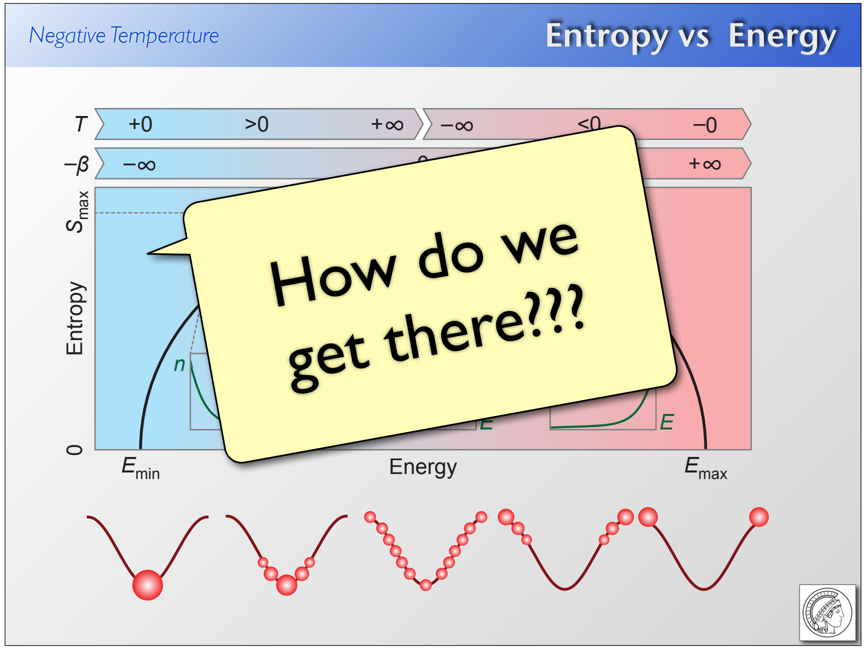
For negative temperatures, we require upper energy bound E_{max}!

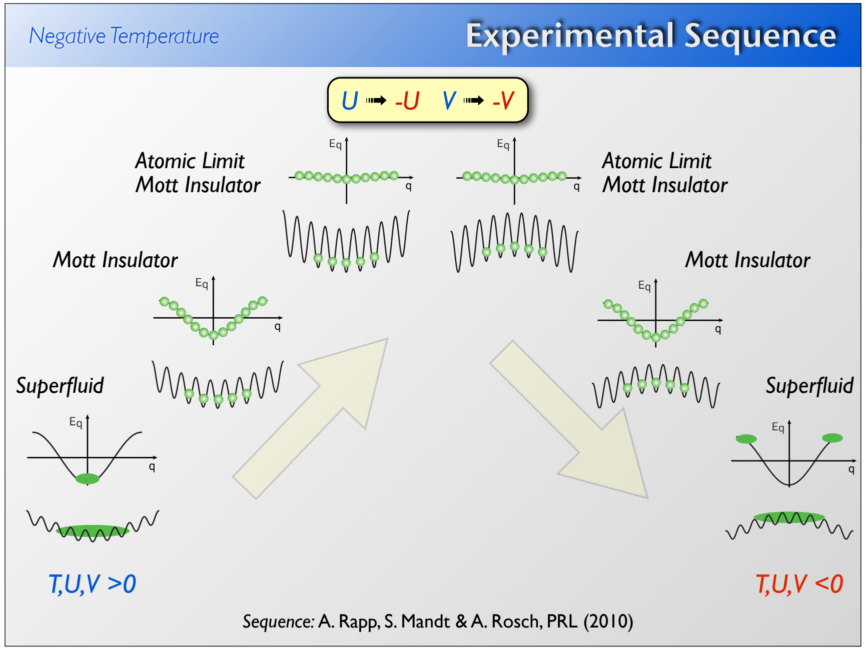
Requirements

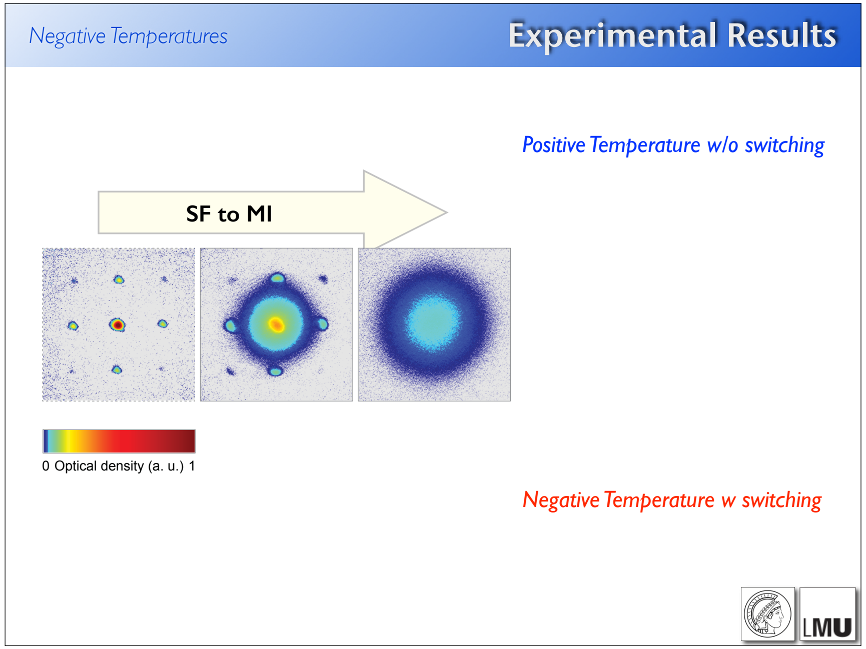
But how to realise in gas of moving atoms, for motional states???

L... (1949)
E.M. Purcell & R.V. Pound, Phys. Rev. **81**, 27...
N. Ramsey, Phys. Rev. **103**, 20 (1956)
M.J. Klein, Phys. Rev. **104**, 589 (1956)
P. Hakonen & O. Lounasmaa, Science **265**...
P. Medley et al, Phys. Rev. Lett **106**, 195301 (2011)


Negative Temperature
Energy Bounds of the BH Model
T, U, V > 0
T, U, V < 0
E
4J
0
-4J
E_{kin}
E_{int}
E_{pot}
E_q
q
r
$\hat{H} = -J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + V \sum_i \mathbf{R}_i^2 \hat{n}_i$
U,V < 0 required for upper energy bound!

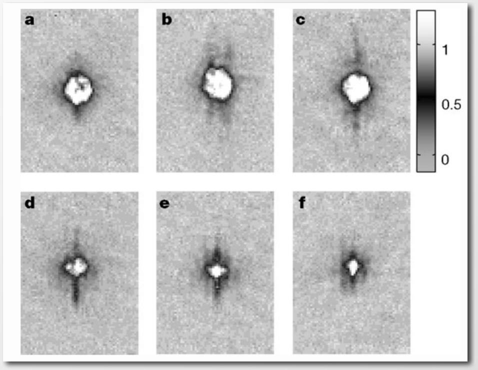

Negative Temperature
Entropy vs Energy
T
+0
>0
+∞
−∞
<0
−0
−β
−∞
+∞
S_{max}
Entropy
0
n
E
E_{min}
Energy
E_{max}
How do we get there???


Negative Temperature
Experimental Sequence
U ➟ -U V ➟ -V
Atomic Limit
Mott Insulator
Atomic Limit
Mott Insulator
Mott Insulator
Mott Insulator
Superfluid
Superfluid
Eq
q
T,U,V >0
T,U,V <0
Sequence: A. Rapp, S. Mandt & A. Rosch, PRL (2010)


Negative Temperatures
Experimental Results
Positive Temperature w/o switching
SF to MI
0 Optical density (a. u.) 1
Negative Temperature w switching
LMU

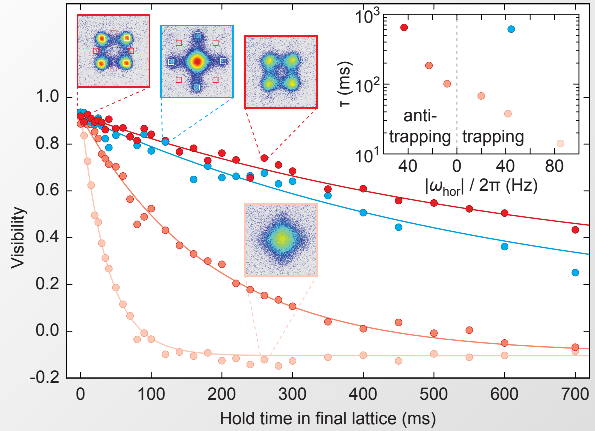
Collapse of Condensate

For attractive interactions (a<0), condensate collapses!



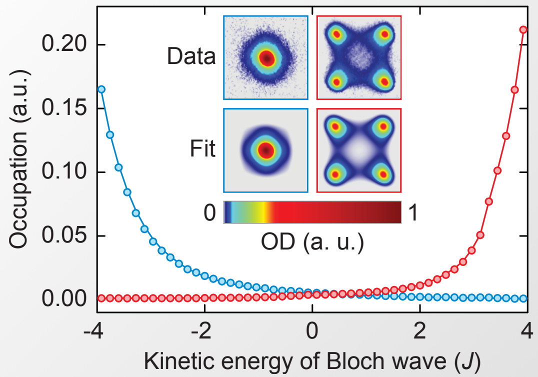
E.A. Donley et al. *Nature* **412**, 295-299 (2001)
J. M. Gerton et al. *Nature* 408, 692 (2000)

Stability



Negative Temperature State as Stable as Positive Temperature State!

Occupation of Energy States



T=-2.2J/k$_B$

Kinetic energy well fitted by Bose-Einstein distribution

$$n(q_x, q_y) = \frac{1}{e^{(E_{kin}(q_x,q_y)-\mu)/k_B T} - 1}$$

$$E_{kin}(q_x, q_y) = -2J\,[\cos(q_x d) + \cos(q_y d)]$$

Implications

Gases with negative temperature possess negative pressure!

$$\left.\frac{\partial S}{\partial V}\right|_E \geq 0 \qquad \text{and} \qquad dE = TdS - PdV$$

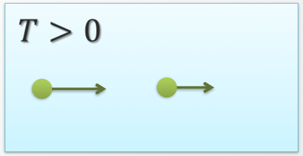
$$\Rightarrow \left.\frac{\partial S}{\partial V}\right|_E = \frac{P}{T} \geq 0$$

Carnot engines above unit efficiency! **(but no perpetuum mobile!)**

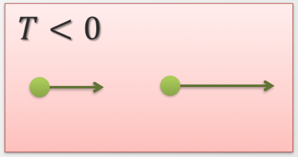
$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

Some statements for the second law of thermodynamics become invalid!

Neg T — Anti-Friction at Negative Temperature



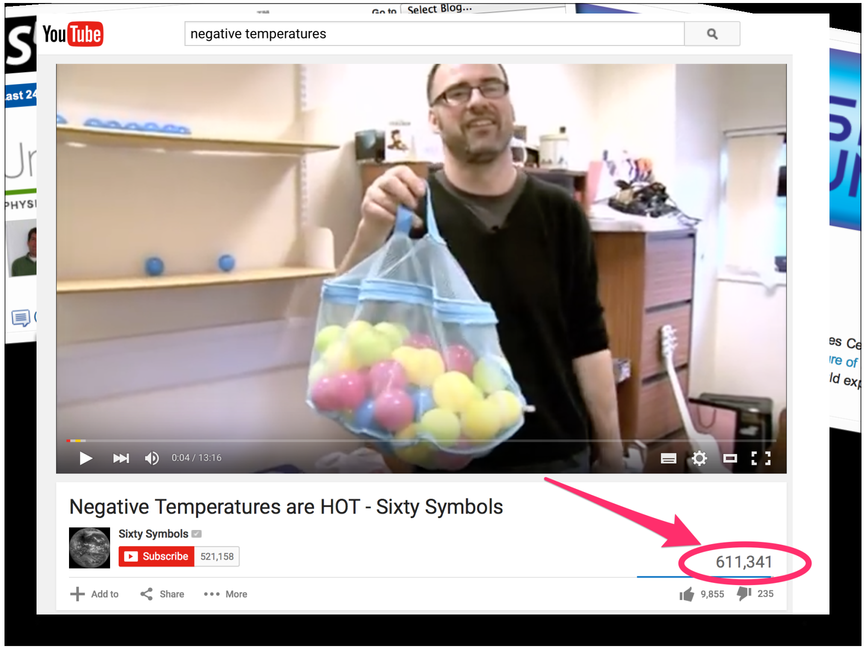
Friction:
- entropy increases
  → Medium heats up
- Particle slows down



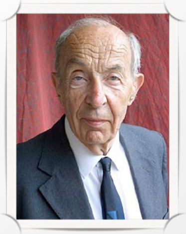
Anti-Friction:
- entropy increases
  → Medium cools down
- Particle accelerates
  (but direction is randomized in long-term limit)

particle spectrum is assumed to be unbounded

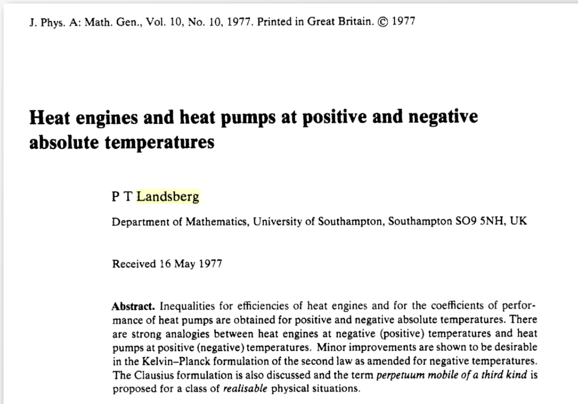
LMU



Negative Temperatures — Thermodynamics at Negative Temperatures



Peter Landsberg
(1922-2010)



J. Phys. A: Math. Gen., Vol. 10, No. 10, 1977. Printed in Great Britain. © 1977

Heat engines and heat pumps at positive and negative absolute temperatures

P T Landsberg

Department of Mathematics, University of Southampton, Southampton SO9 5NH, UK

Received 16 May 1977

Abstract. Inequalities for efficiencies of heat engines and for the coefficients of performance of heat pumps are obtained for positive and negative absolute temperatures. There are strong analogies between heat engines at negative (positive) temperatures and heat pumps at positive (negative) temperatures. Minor improvements are shown to be desirable in the Kelvin–Planck formulation of the second law as amended for negative temperatures. The Clausius formulation is also discussed and the term *perpetuum mobile of a third kind* is proposed for a class of *realisable* physical situations.

N. Ramsey, Phys. Rev. (1956)
M.J. Klein, Phys. Rev. (1956)
J. Dunning-Davies, J. Phys. A (1961)
A.M. Tremblay, Am. J. Phys. (1976)
P.T. Landsberg, J. Phys A (1977)
P.T. Landsberg, R.J. Tykodi & A.M. Tremblay (1979)....

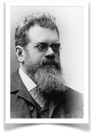
LMU

What is the correct form of the entropy?

- Observation: Cold atoms are thermally isolated
  → *microcanonical ensemble?*
- Equivalence of ensembles not a priori clear for bounded systems.
- Two possible entropy definitions:

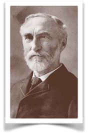
Boltzmann / Surface entropy:

$$S_B = k_B \log(\rho(E)dE)$$



Gibbs / Hertz / Volume entropy:

$$S_G = k_B \log(\int_0^E \rho(E')\ dE')$$



- Typically in unbounded systems:

$$\rho(E) \propto \exp(E) \rightarrow \int \rho(E)\ dE \propto \exp(E)$$

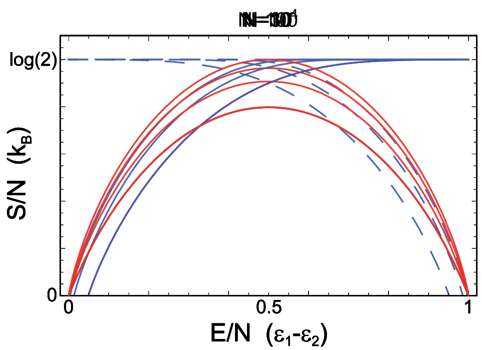
→ *no real difference*

What is the correct form of the entropy?

- Neccesary condition for consistent thermodynamics:
  dS=... must be a total differential (needed for e.g. Maxwell relations)
- Boltzmann entropy: $S_B = k_B \log(\rho(E)dE)$ does ***not*** fulfill above requirement for the ***microcanonical ensemble***
- Need to use Gibbs / Hertz entropy: $S_G = k_B \log(\int_0^E \rho(E')\ dE')$
- $\rho(E) \geq 0$ → S_G monotonously increasing ➔ $T \geq 0$ ☹ ??

Dunkel, & Hilbert Nat. Phys. **10**, 67 (2014)

Example: N two-level atoms



- Boltzmann entropy:
  $S_B = k_B \log(\rho(E)\ dE)$
- Gibbs / Hertz entropy:
  $S_G = k_B \log(\int_0^E \rho(E')\ dE')$
- New: Inverted Gibbs:
  $\overline{S_G} = k_B \log(\int_E^{E_{max}} \rho(E')\ dE')$

→ $d\overline{S_G}$ is also total differential!

Proposal: $\boldsymbol{S_m = \min\{S_G, \overline{S_G}\}}$

- dS_m is also total differential (except at $E = \frac{E_{max}}{2}$)
- thermodynamic limit: $\lim_{N\to\infty} S_m = \lim_{N\to\infty} S_B$

➔ Equivalence of Ensembles

Stability?

Nonexistence of equilibrium states at absolute negative temperatures

Víctor Romero-Rochín* Phys. Rev. E 88, 022144 (2013)

We show that states of macroscopic systems with purported absolute negative temperatures are not stable under small, yet arbitrary, perturbations. We prove the previous statement using the fact that, in equilibrium, the

Observation:

Couple small ideal gas thermometer (e.g. single bulk atom $\mathrm{H} \propto p^2$) to a large negative T system.

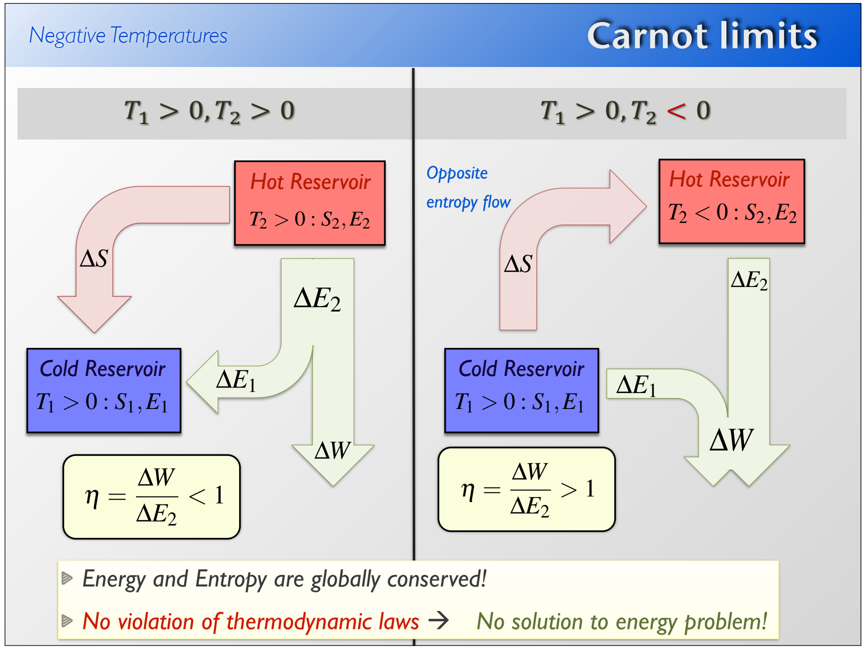
→Common equilibration temperatur will be positive ☹ ??

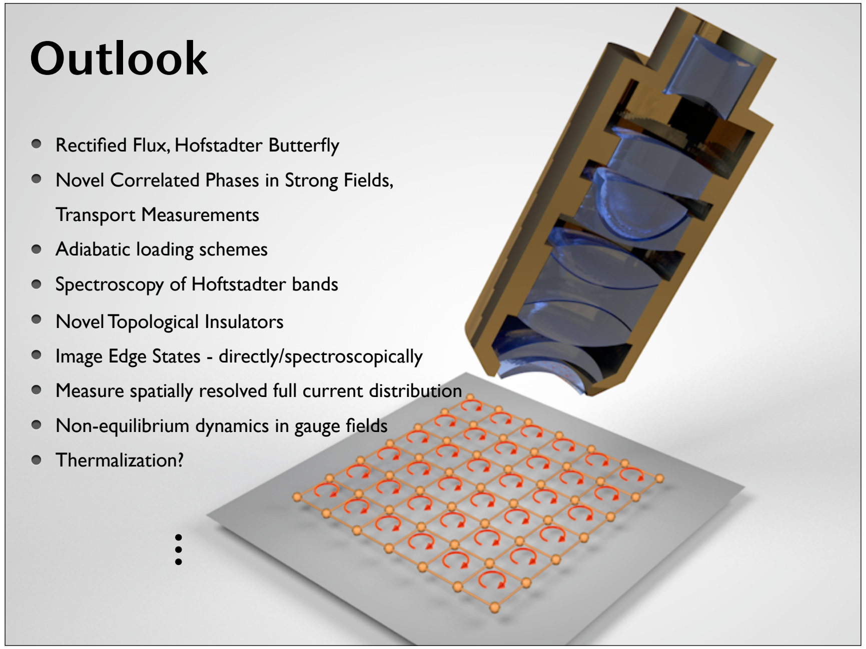
What is small?

Not $N_1 \ll N_2$, but rather: $|\int_{T_1}^{T_2} C_{v,1}| \ll |\int_{T_2}^{T_1} C_{v,2}|$

➔Classical ideal gas $\mathrm{T} \propto \langle E \rangle$ ➔ $\int_{T_1}^{\infty} C_b = \infty$ ➔ not a *small* perturbation

➔Ideal gas thermometer ill suited, use instead e.g. an ideal lattice gas


Negative Temperatures
Carnot limits
$T_1 > 0, T_2 > 0$
$T_1 > 0, T_2 < 0$
Hot Reservoir
$T_2 > 0 : S_2, E_2$
ΔS
ΔE_2
Cold Reservoir
$T_1 > 0 : S_1, E_1$
ΔE_1
ΔW
$\eta = \frac{\Delta W}{\Delta E_2} < 1$
Opposite
entropy flow
Hot Reservoir
$T_2 < 0 : S_2, E_2$
ΔS
ΔE_2
Cold Reservoir
$T_1 > 0 : S_1, E_1$
ΔE_1
ΔW
$\eta = \frac{\Delta W}{\Delta E_2} > 1$
Energy and Entropy are globally conserved!
No violation of thermodynamic laws → No solution to energy problem!


Outlook
Rectified Flux, Hofstadter Butterfly
Novel Correlated Phases in Strong Fields,
Transport Measurements
Adiabatic loading schemes
Spectroscopy of Hoftstadter bands
Novel Topological Insulators
Image Edge States - directly/spectroscopically
Measure spatially resolved full current distribution
Non-equilibrium dynamics in gauge fields
Thermalization?
⋮