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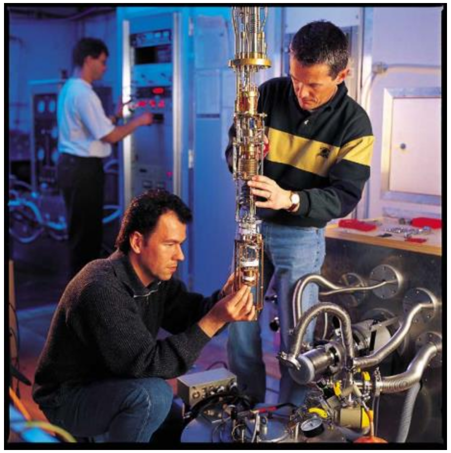
**Federal Institute of Metrology METAS**

# Electrical Standards based on quantum effects: Part II

Beat Jeckelmann

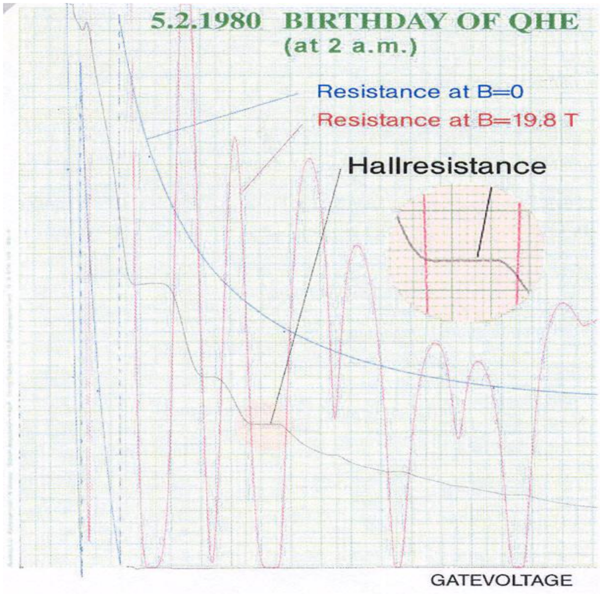
# Part II: The Quantum Hall Effect Overview

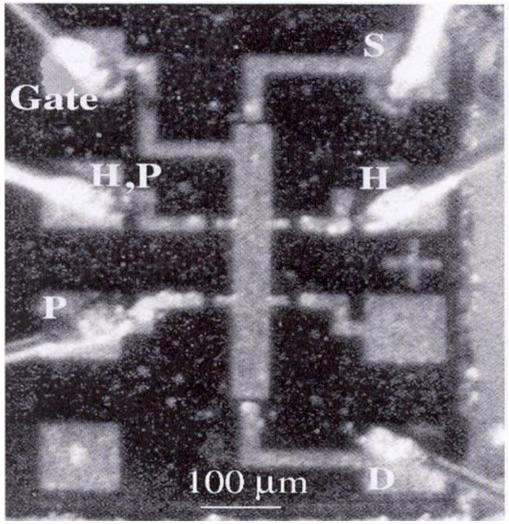
- Classical Hall effect
- Two-dimensional electron gas
- Landau levels
- Measurement technique
- Accuracy of the quantized Hall resistance
- Applications in dc and ac electrical metrology



# Discovery of the Quantum Hall Effect

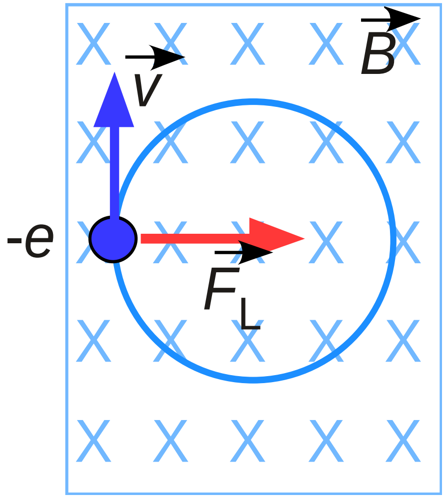
K. Von Klitzing discovers the quantum Hall effect in on 5 February 1980 in Grenoble




K. v. Klitzing et al, Phys. Rev. Lett., 45, 494 (1980)

# Classical Hall effect (1879)

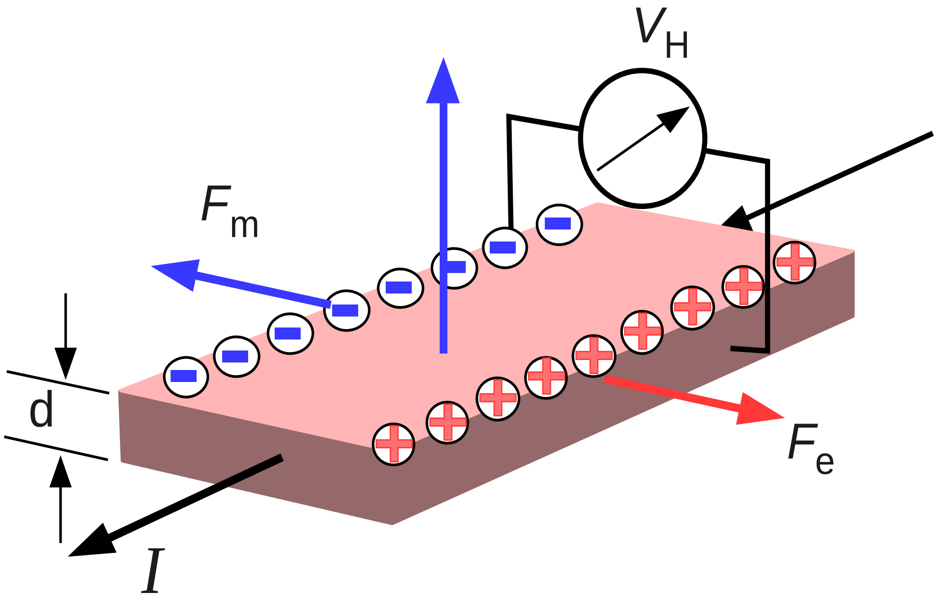



Lorentz Force

$$\vec{F}_L = q\left(\vec{v} \times \vec{B}\right)$$

Edwin Hall
(1855 – 1938)

# Classical Hall effect (2)




$$U_H = \frac{B \cdot I}{n_{3D} \cdot e \cdot d}$$

$n_{3D}$: carrier concentration

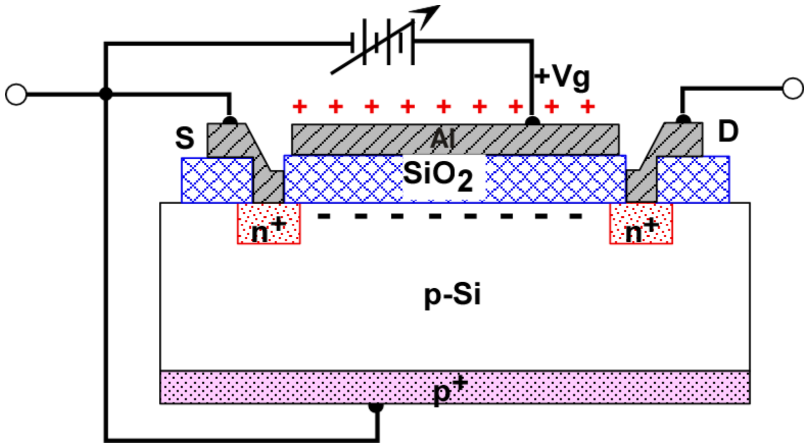
**2D-system:**

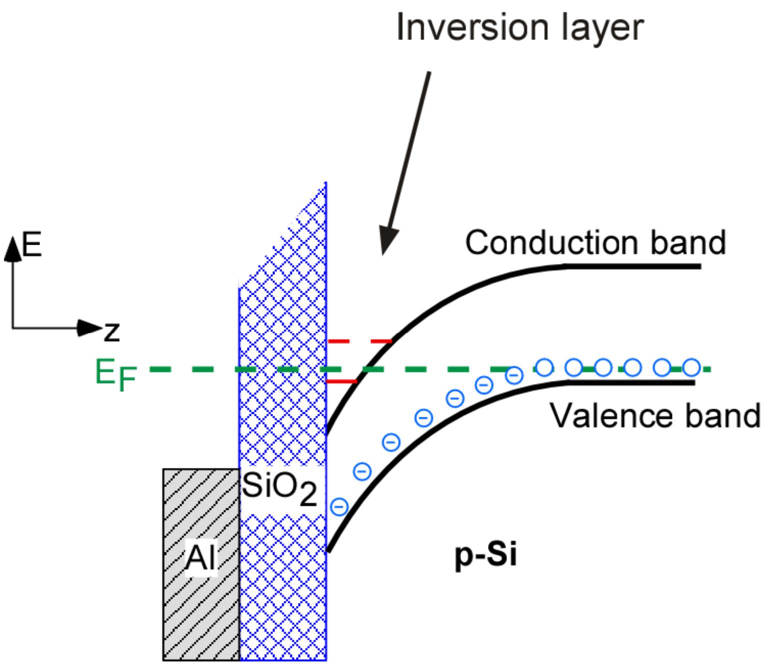
$$U_H = \frac{B \cdot I}{n \cdot e}$$

Hall effect **independent of geometrical dimensions** !

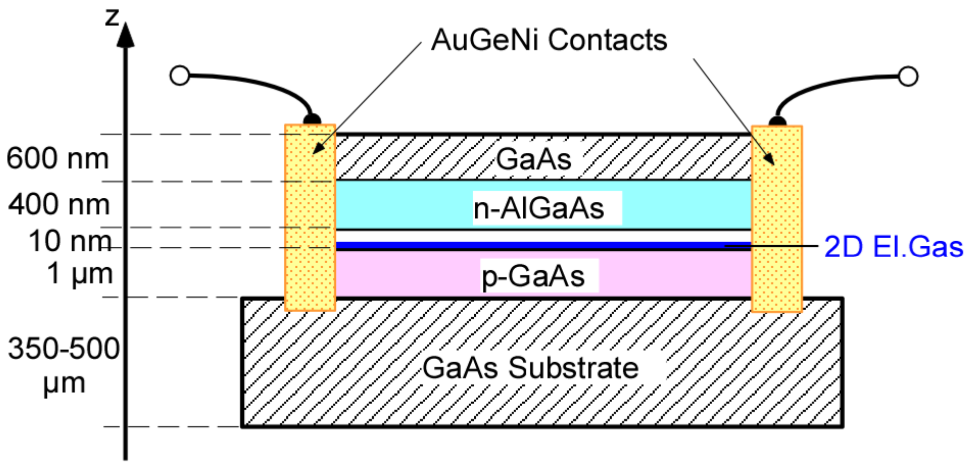
# Realisation of a 2D electron gas

Si MOSFET

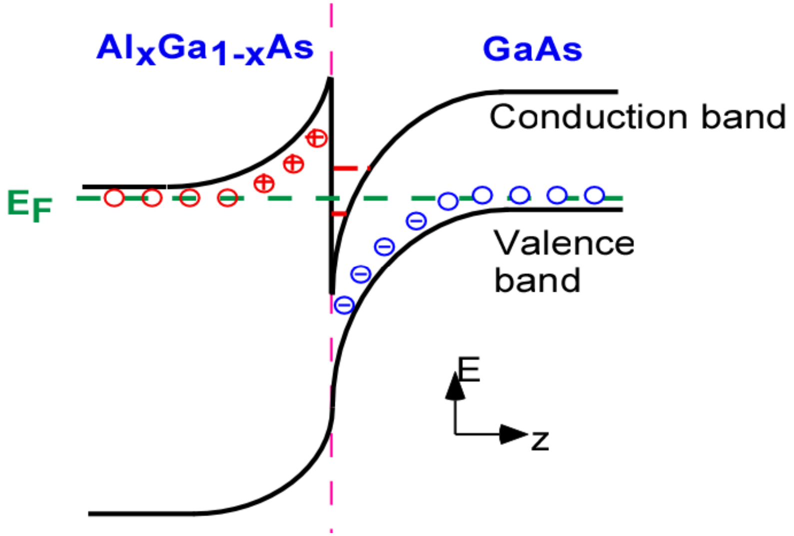


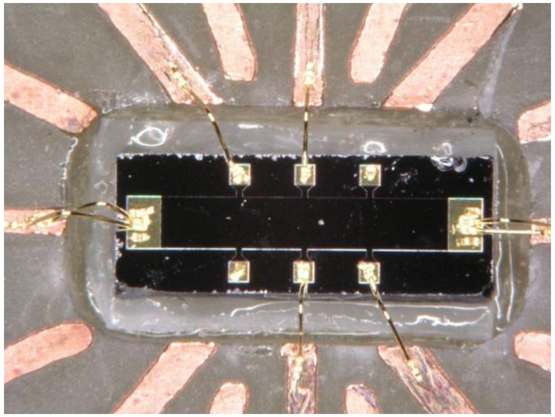



# GaAs Heterostructure



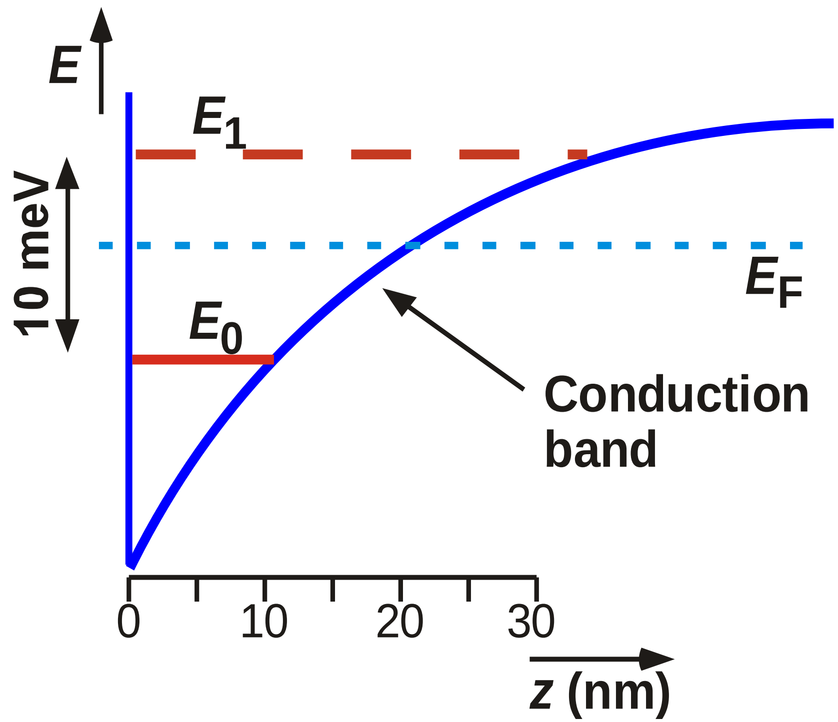

Layers grown by:
Molecular Beam Epitaxy (MBE)
or Metal Organic Chemical
Beam Epitaxy (MOVCD)



4 mm

# Inversion Layer



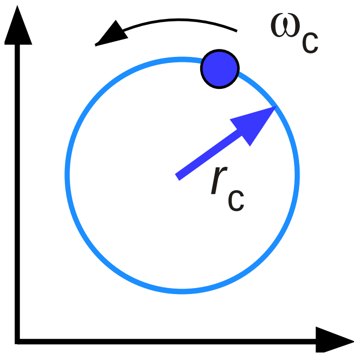

de Broglie wavelength:
typically 100 nm (GaAs)

level spacing at 10 T:

| | |
|---|---|
| Si: | 7.9 meV |
| GaAs: | 17 meV |
| Graphene: | 120 meV |

thermal energy:
$T$ = 4 K: kT = 0.36 meV

# Landau levels




**cyclotron motion in a strong magnetic field**
Classical:

$$\omega_c = \frac{v}{r_c} = \frac{e \cdot B_z}{m^*}$$

**Quantum mechanics**:
Energy values of closed orbits

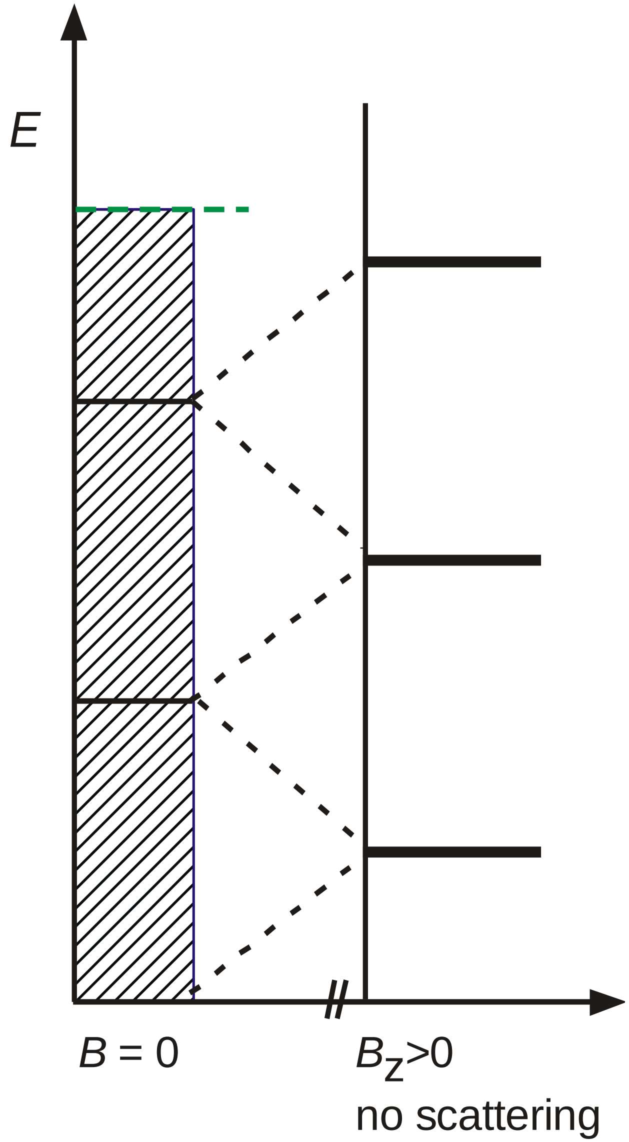
$$E = E_0 + \hbar\omega_c\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, 3...$$

(spin neglected)

magnetic length:
(8 nm @ 10 T)

$$r_c = l\sqrt{2n+1}, \quad l = \sqrt{\frac{\hbar}{e \cdot B}}$$

# Landau quantization (2)




**Orbital degeneracy**

$$N = \frac{L \cdot w}{2\pi l^2}$$

Number of states in a Landau level

**State density**

$$n_B = \frac{1}{2\pi l^2} = \frac{eB}{h}$$

Number of flux quanta within the area of the sample

**Filling factor:**

$$i = \frac{n_s}{n_B}$$

# Quantum Hall effect ?

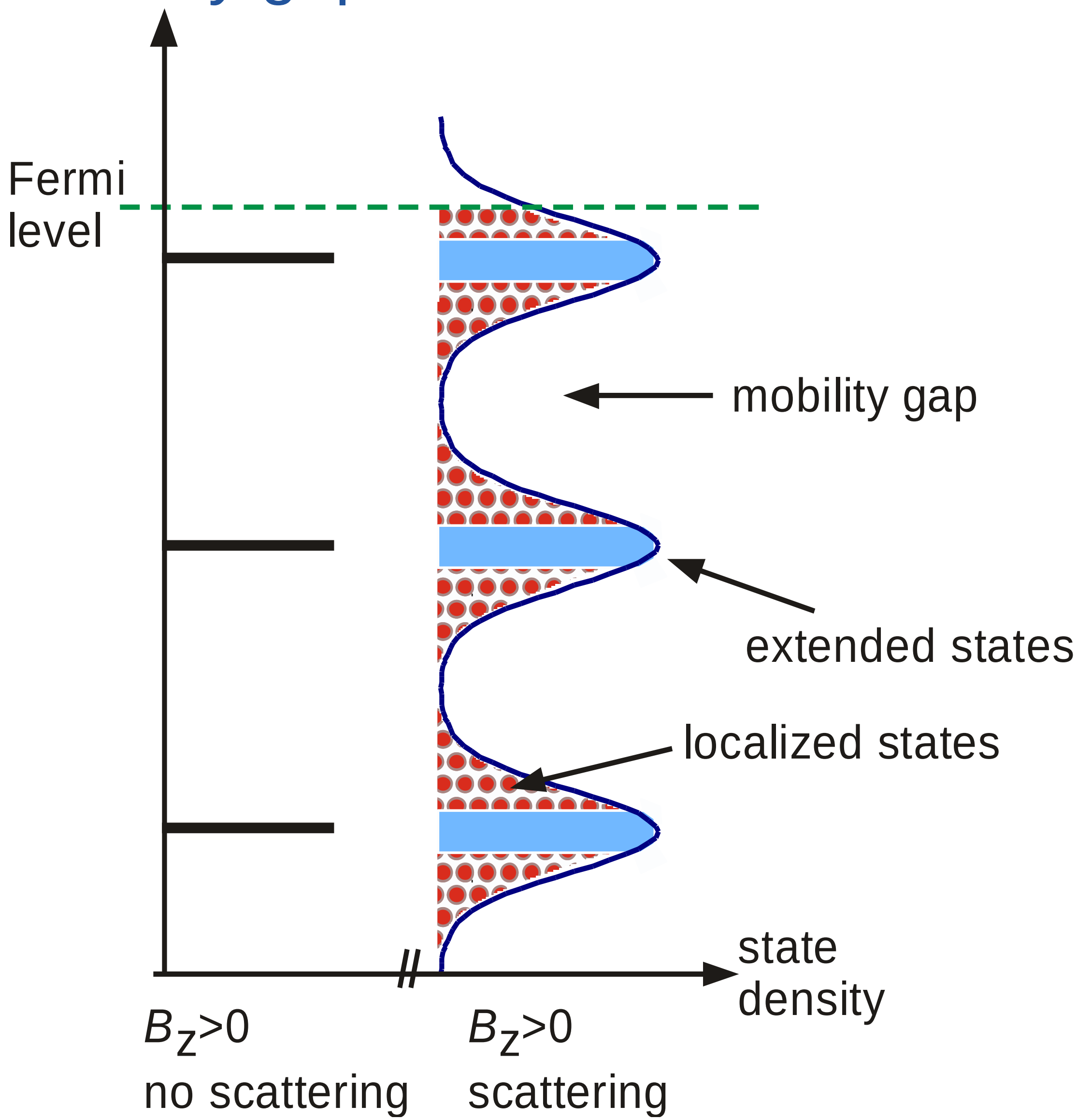
Hall voltage in a 2D-system:

$$U_H = \frac{B \cdot I}{n_s \cdot e}$$

$$R_H = \frac{B}{i \cdot n_B \cdot e} = \frac{h}{e^2 \cdot i}, \quad i = 1, 2, 3...$$

**observed when *i* levels are fully occupied !**

# Mobility gap

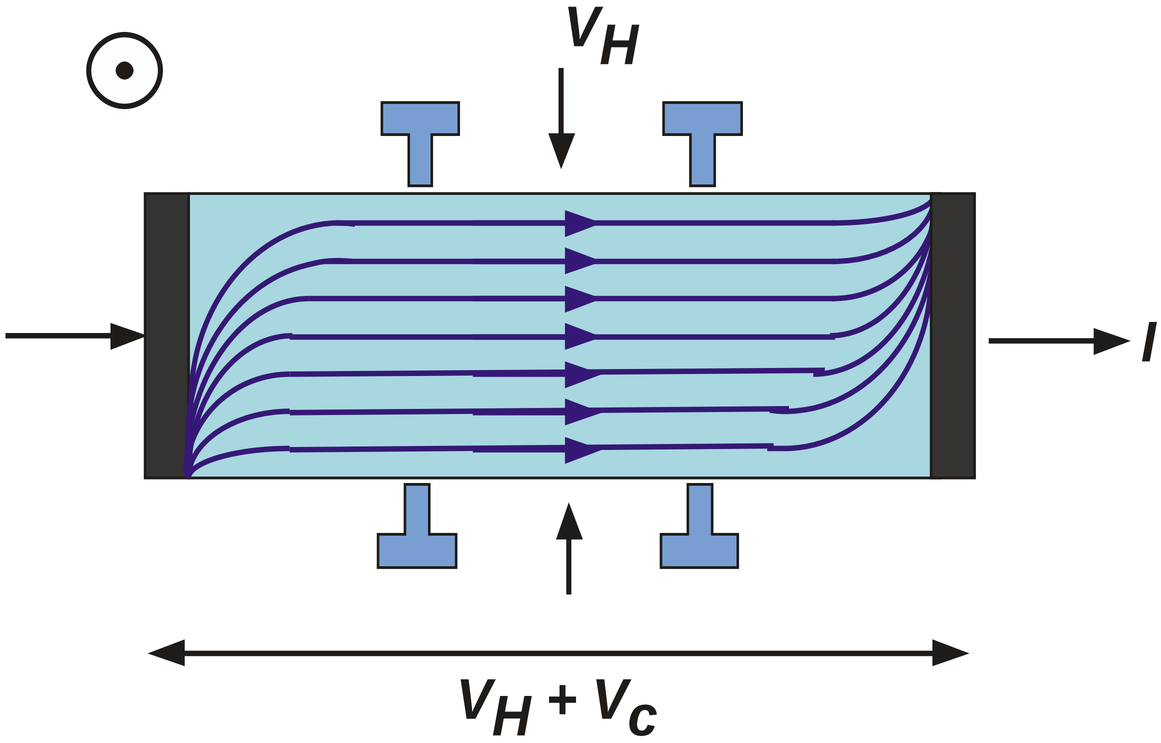



Disorder and scattering removes orbital degeneracy

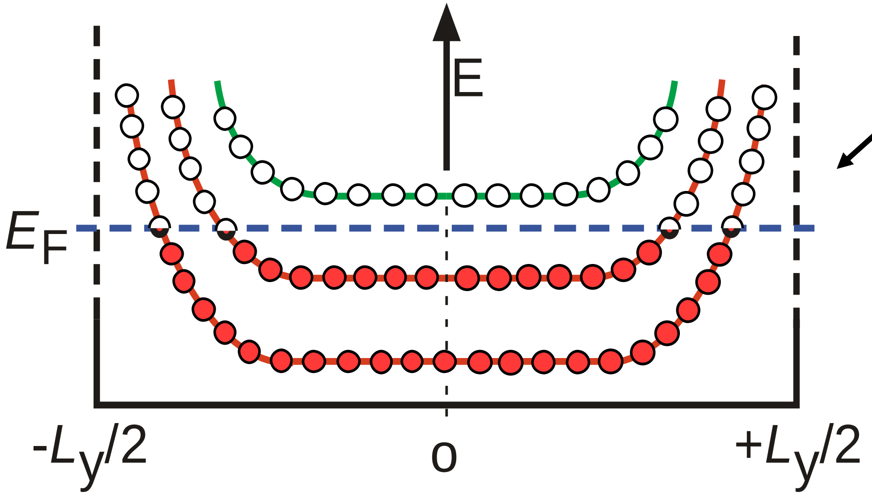
Localized states do not carry current

Plateau forms when $E_F$ resides within the localized states

# QHE in a real device



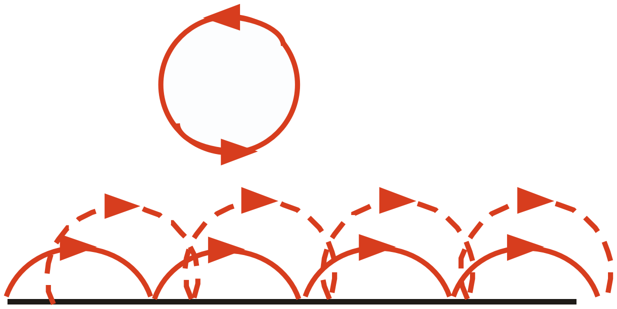
# QHE in a real device (2)




electronic states at Fermi energy

**No energy gaps** for real devices with finite width

# Edge state picture

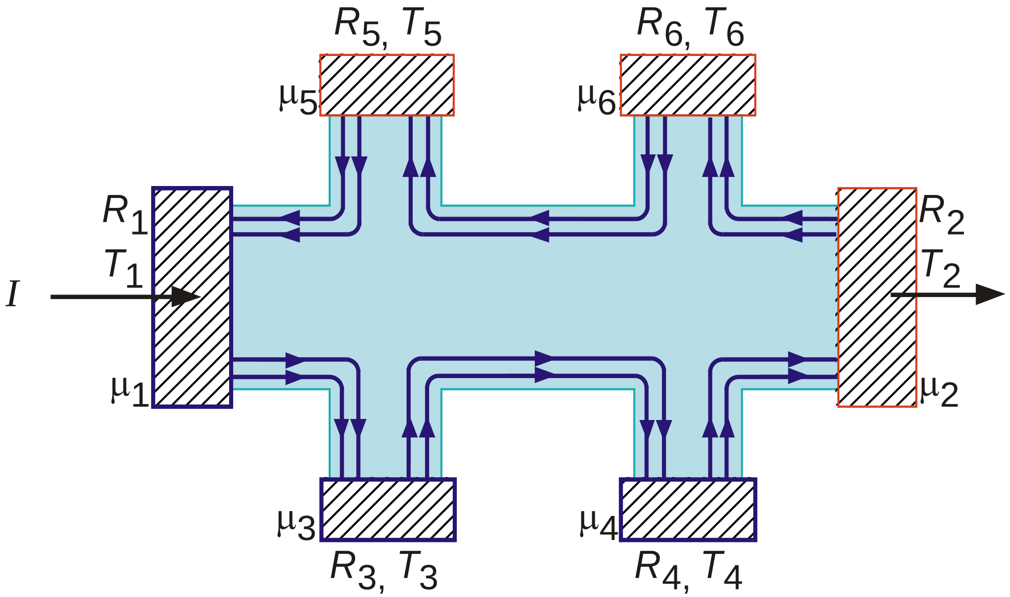


**Skipping Orbits**
source and drain contact are connected by a common edge

one-dimensional edge channels carry the current

**Landauer formalism**
current = driving force of electronic transport

# Büttiker formalism




**Current in 1D channels**:

$$I = \frac{e}{h}\Delta\mu$$

Δμ: difference electrochemical potential

*T*: transmission
*R*: reflection coefficient
$T_i = 1$, $R_i = 0$

$$\mu_1 = \mu_3 = \mu_4; \ \ \mu_2 = \mu_5 = \mu_6$$

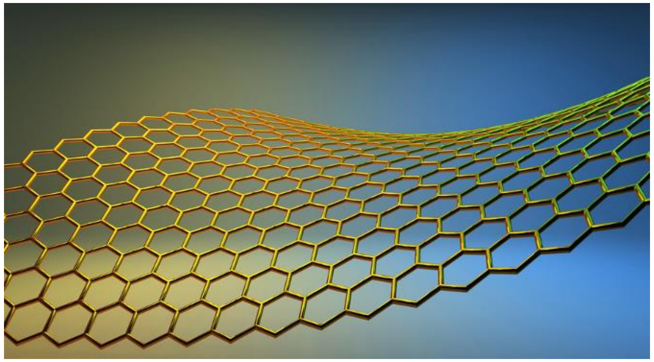
$$R_H(i) = \frac{(\mu_5 - \mu_3)}{e}\frac{h}{i \cdot e(\mu_2 - \mu_1)} = \frac{h}{i \cdot e^2}$$

$$R_{xx} = \frac{(\mu_4 - \mu_3)}{e}\frac{h}{i \cdot e(\mu_2 - \mu_1)} = 0$$

# QHE Model

- The QHE is a collective phenomenon, it can not be explained by a microscopic model
- A vanishing longitudinal resistivity indicated the absence of backscattering
- Under this condition, the QHE is the direct consequence of the transmission of one-dimensional channels.

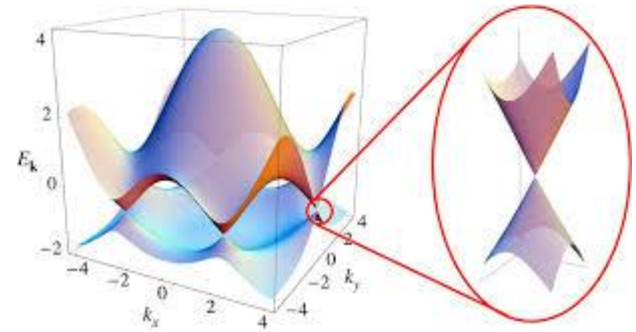
# QHE in Graphene



Graphene: 2D crystal of carbon atoms with charge carriers » massless relativistic particles

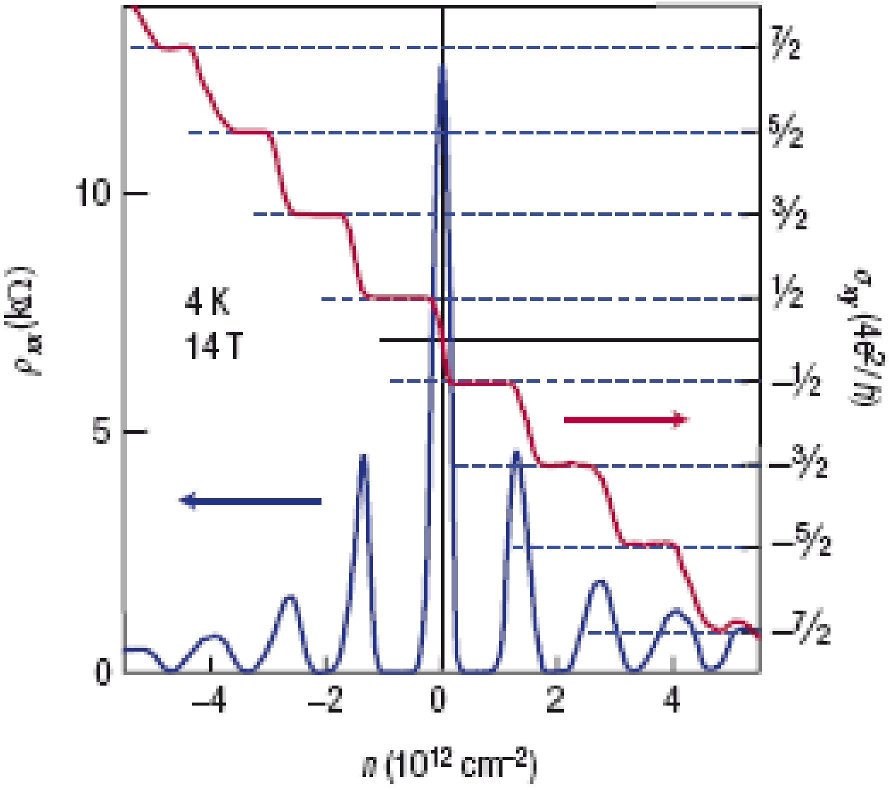

Geim & Novoselov
Nobel Prize in Physics (2010)

# Unconventional quantum Hall effect

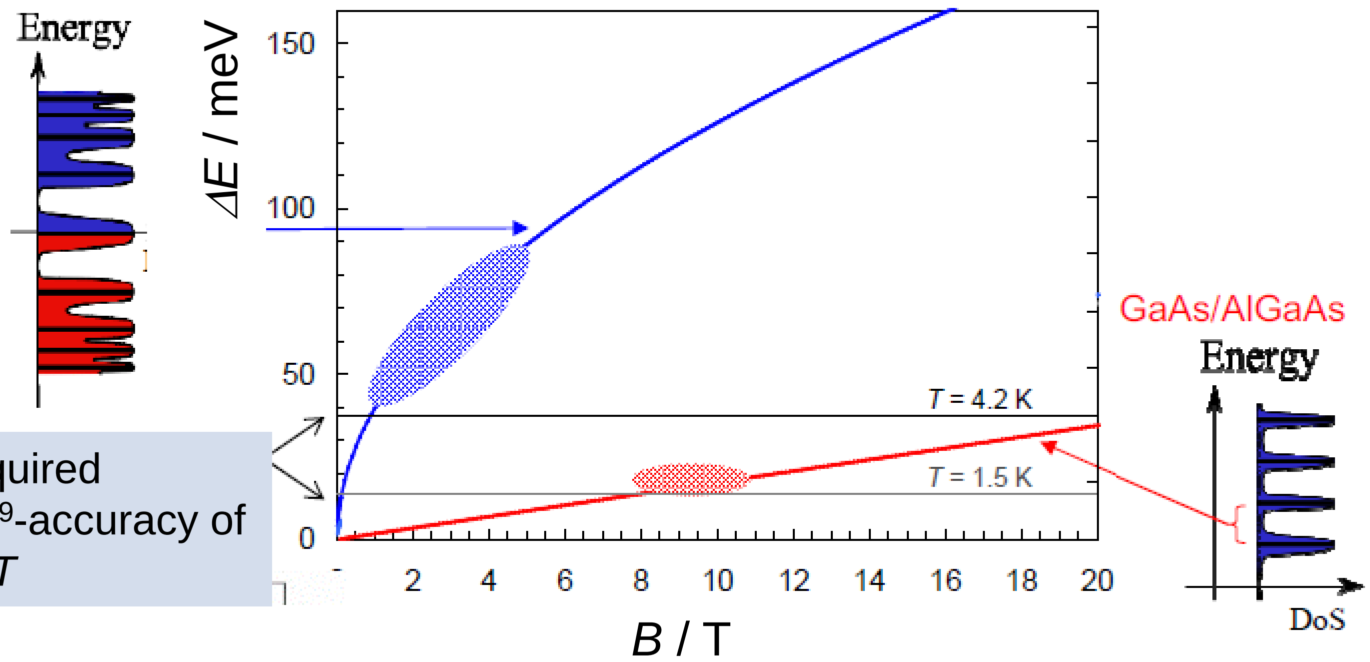


$$\sigma_{xy} = \left(4e^2/h\right)\left(N + 1/2\right)$$

*Novoselov, et al, Nature 438, 2005*



# Interest for graphene




$\Delta E$ required for $10^{-9}$-accuracy of $R_H$ at $T$

Potential to develop a quantum Hall resistance standard
At $T$ >5 K and $B$ < 5 T

⇨ **Cryogen-free and cheap**

# Fine structure constant

$$R_K \equiv \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

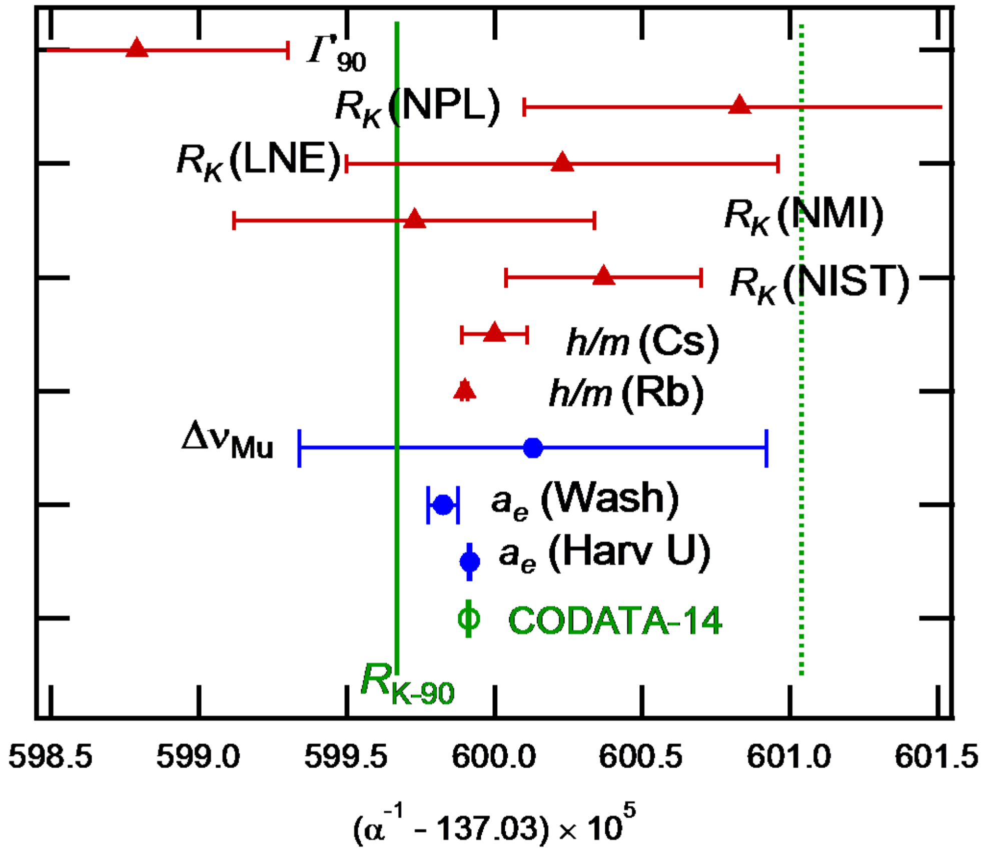
Test the validity of $R_K = i \cdot R_H(i)$

or: additional route to the determination of $\alpha$ (independent of QED)

most accurate value for $\alpha$:

measurement of $a_e$ + theoretical expansion of $a_e$ in a series expansion of $\alpha$ (numerical computations: Kinoshita et al.)

# Fine structure constant




$$R_K \equiv \frac{h}{e^2} = \frac{\mu_0 c}{2\alpha}$$

| | | |
|---|---|---|
| $a_e$: | Anomalous magnetic moment of *e* | QED theory necessary |
| $\Delta\mu_{Mu}$: | Ground state hyperfine splitting | |
| $\Gamma_{90}$: | Gyro-magnetic moment of the proton | without QED |
| ***h/m***: | Neutron diffraction; cold atoms.. | |
| $R_K$: | QHE | |

# Metrological application

- Ideal systems: $T$ = 0 K, $I$ = 0 A

- No dissipation: $R_{xx} = 0$

$$R_H(i) = \frac{h}{i \cdot e^2} = \frac{R_K}{i}$$

$R_K$ is a universal quantity

Localization theory,

Edge state model

- Real experiment: $T$ > 0.3 K, $I$ = 40 µA
  Non-ideal samples

- Dissipation: $R_{xx} > 0$

$$R_H(i, R_{xx} \rightarrow 0) = \frac{h}{i \cdot e^2} ??$$

Is $R_H(i)$ a universal quantity?

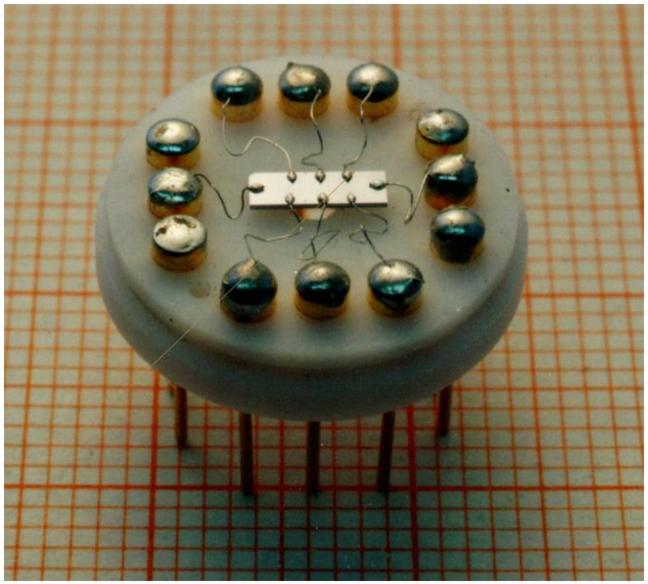
Independent of device material, mobility, carrier density, plateau index, contact properties.....?

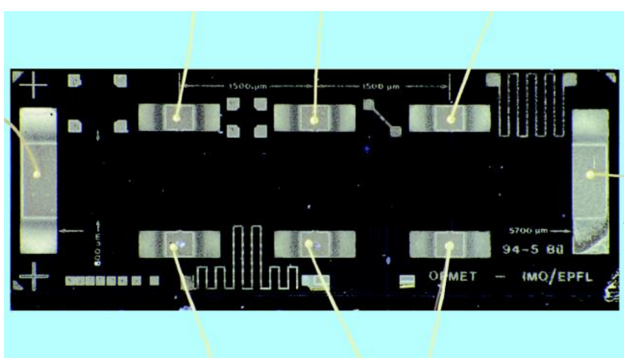
Few quantitative theoretical models available

→ empirical approach,

→ precision measurements

# QHE devices for metrology






**Carrier concentration**

$$B(i) = n_s \cdot e \cdot R_H(i)$$

$2\times10^{15}\ \mathrm{m}^{-2} < n_s < 7\times10^{15}\ \mathrm{m}^{-2}$

$n_s > 7\times10^{15}\ \mathrm{m}^{-2}$: 2nd subband fills up

**Mobility**:

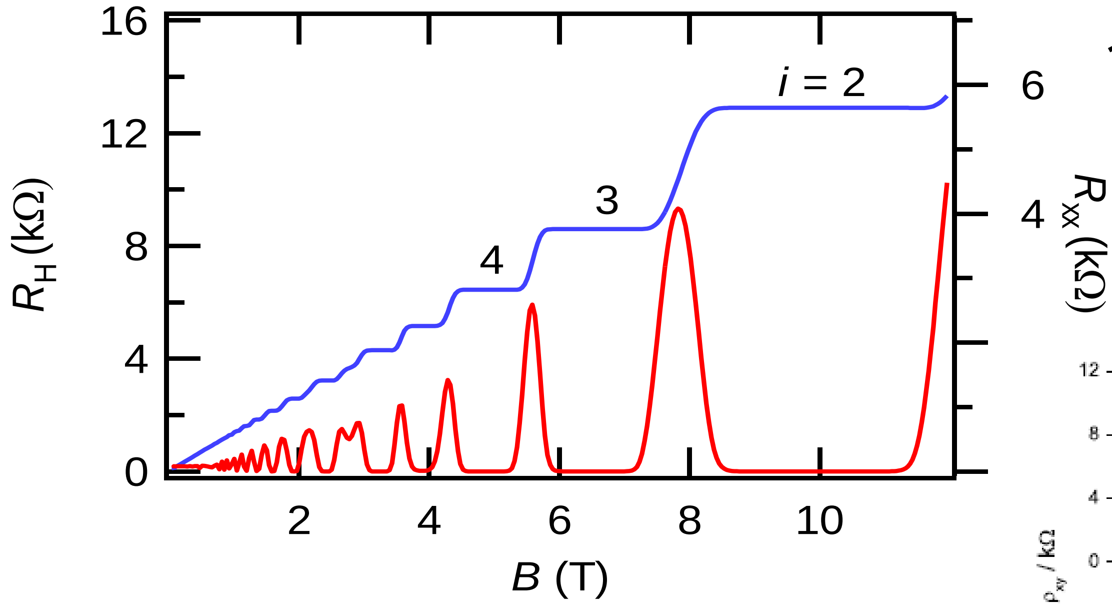
$\mu > 10\ \mathrm{T}^{-1}$ to have clear separation of the LL up to plateau 4

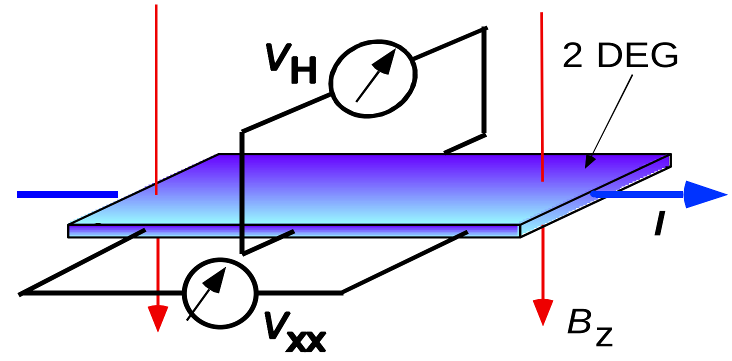
Hall bar defined by photolithography and wet etching techniques

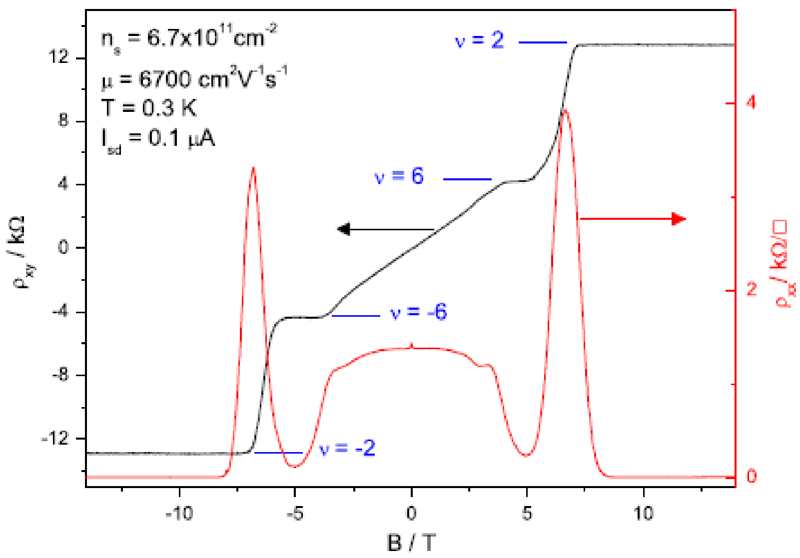
Alloyed AuGeNi contacts

# QHE in a real device

$$R_H = \frac{h}{i \cdot e^2}$$



# Temperature dependence

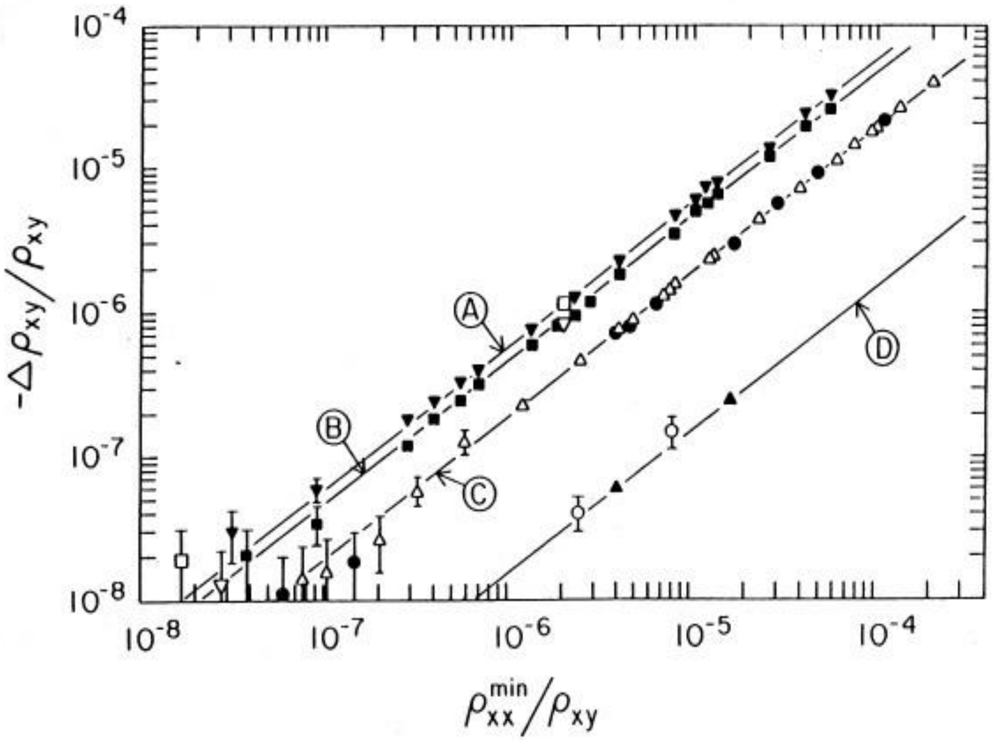
- Thermal activation:
  1 K ≤ $T$ ≤ 10 K
  electrons thermally activated
  to the nearest extended states

$$\sigma_{xx}(T) = \sigma_{xx}^0 \cdot e^{-\Delta/kT}$$

$$\Delta = E_F - E_{LL}$$

$$\delta\sigma_{xy}(T) = \sigma_{xy}(T) - \frac{ie^2}{h}$$

$$\boxed{\delta\rho_{xy}(T) = s\,\rho_{xx}(T)}$$



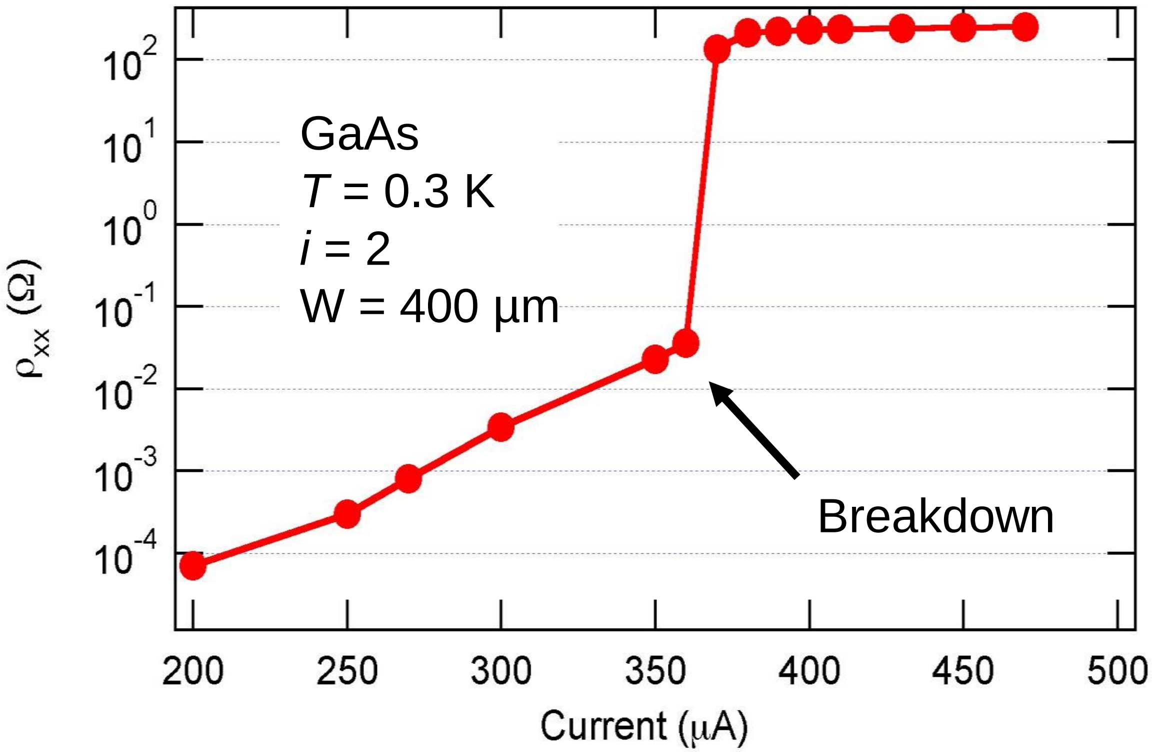

Cage et al. 1984:

1.2 K < $T$ < 4.2 K,

2 GaAs samples, $i$ = 4

-0.01 < $s$ < -0.51

# Current dependence



# Measurement technique

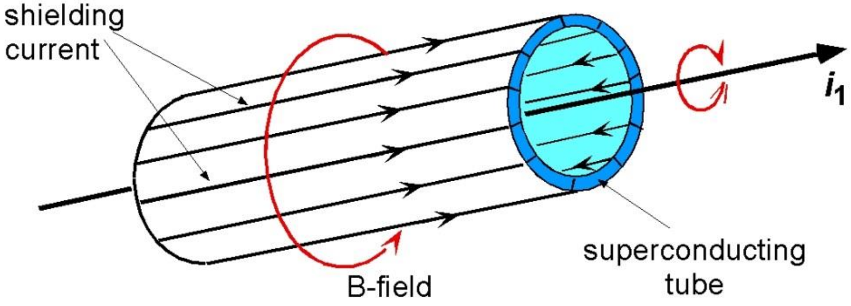
- To make use of the QHE for metrological applications, a measurement technique, capable of transferring the QHR to room temperature resistance standards must be available.

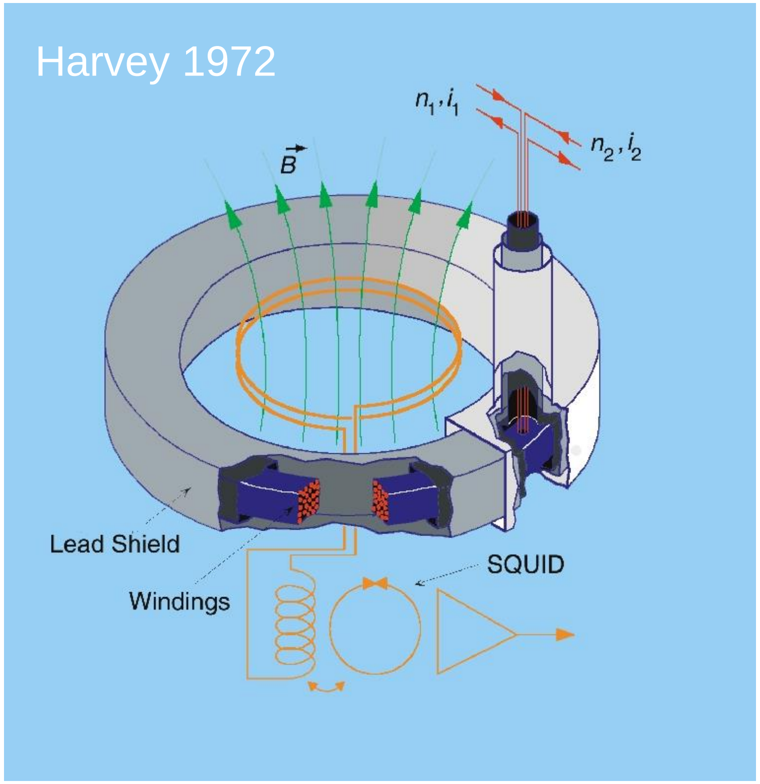
- Most accurate resistance bridge technique:

→ **Cryogenic Current Comparator (CCC)**

# The cryogenic current comparator (CCC): Principles

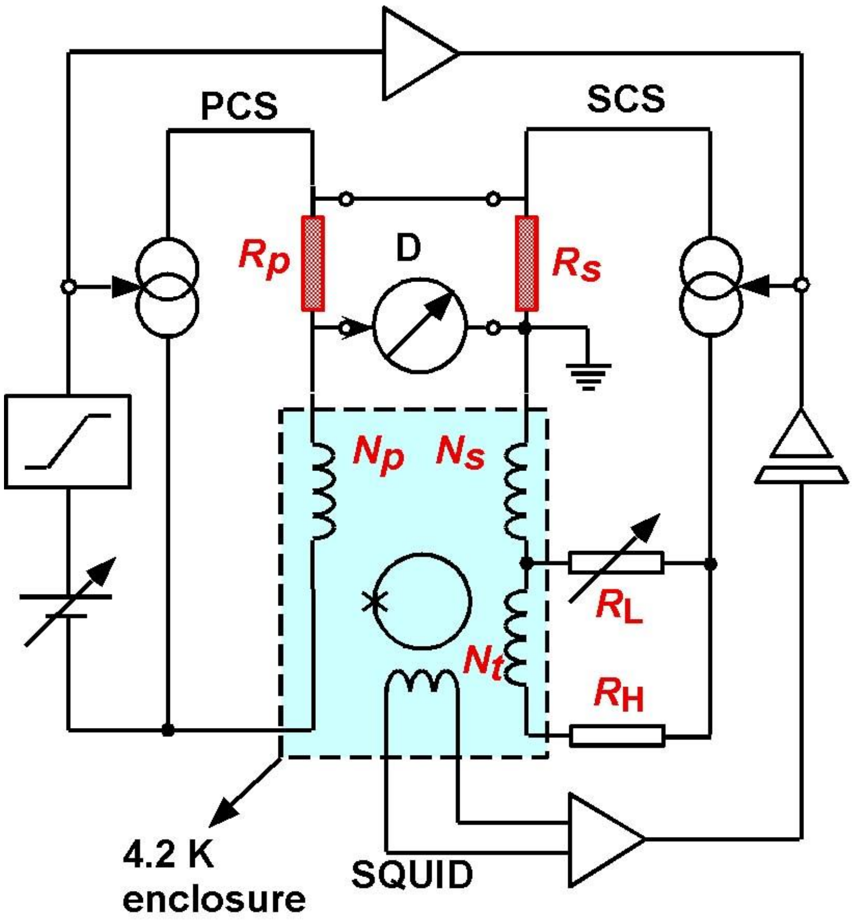
Meissner effect:




$$SQUID \propto n_1 I_1 - n_2 I_2$$

# The CCC bridge:




SQUID:

$$N_P \cdot I_P = N_S \cdot I_S \cdot (1+d)$$

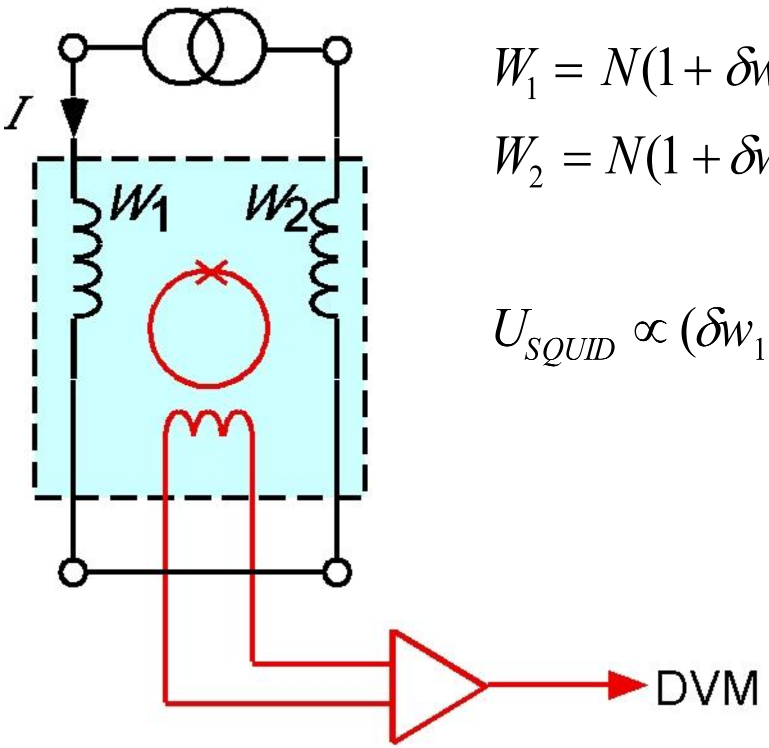
with $\quad d = \frac{N_t}{N_S} \cdot \frac{R_L}{R_L + R_H}$

Detector:

$$U_m = R_S \cdot I_S - R_P \cdot I_P$$

$$\frac{R_P}{R_S} = \frac{N_P}{N_S} \cdot \frac{1}{1+d} \cdot \frac{1}{1 + U_m / U}$$

# Ratio accuracy:




$$W_1 = N(1+\delta w_1)$$

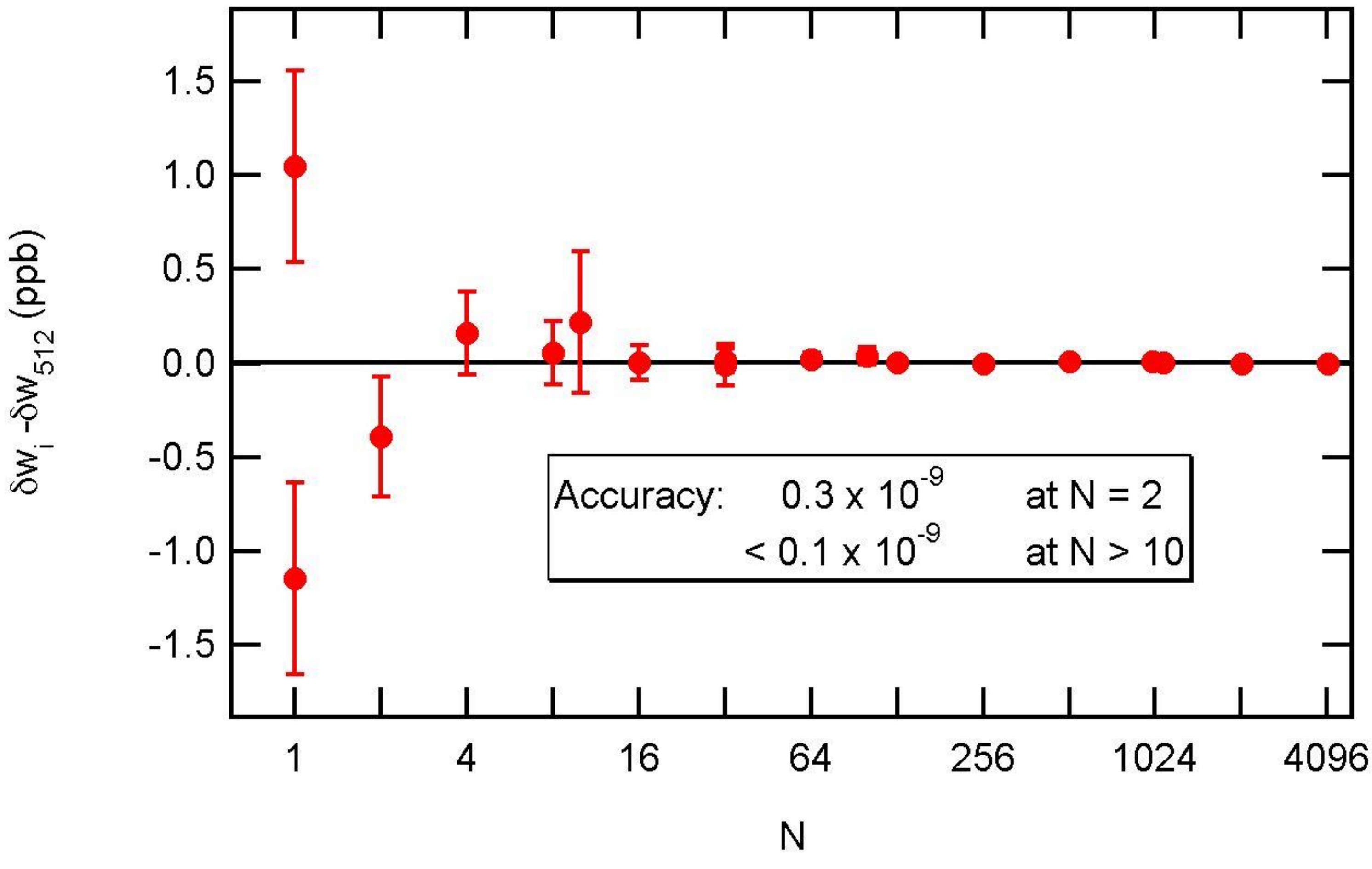
$$W_2 = N(1+\delta w_2)$$

$$U_{SQUID} \propto (\delta w_1 - \delta w_2)$$

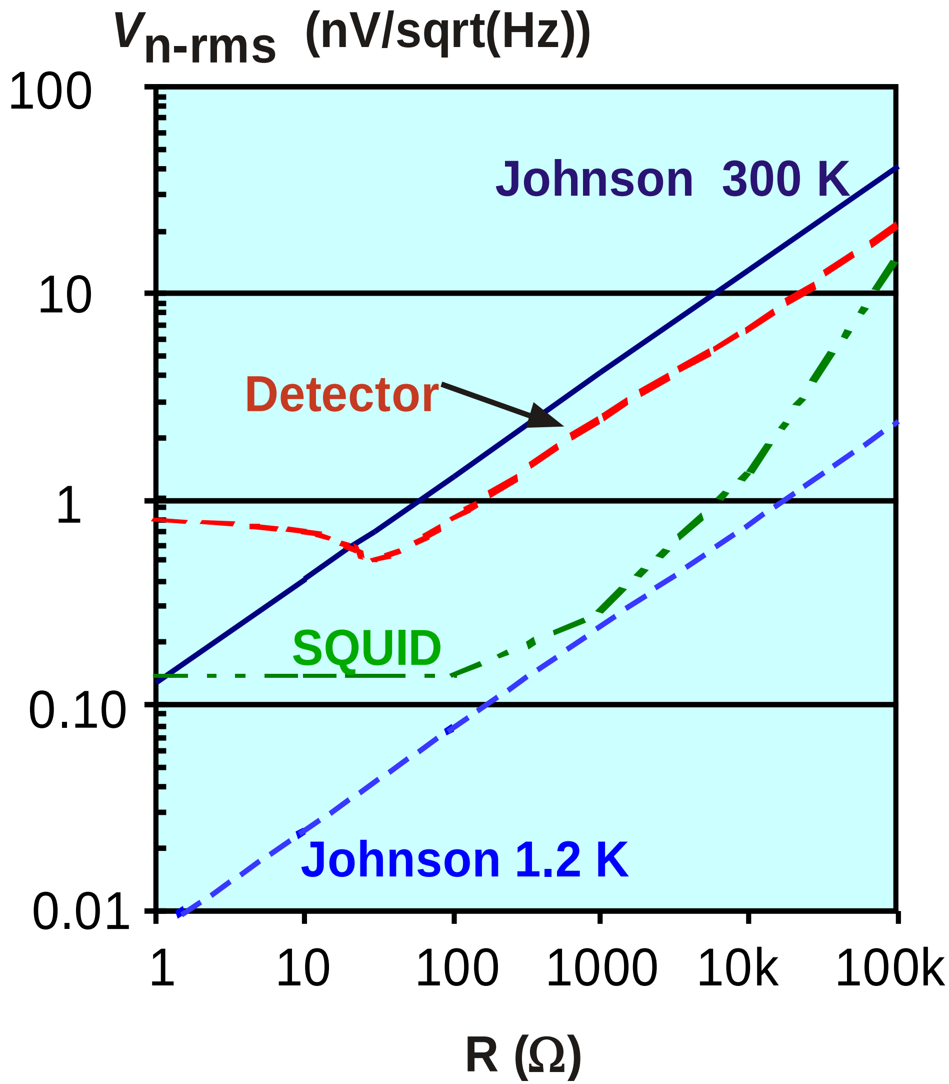
<u>Windings in a binary series</u>:

1, 1, 2, 4, 8, 10, 16, 32, 32, 64, 100, 128, 256, 512, 1000, 1097, 2065, 4130

# Ratio accuracy



# Performance




**SQUID noise**

$$7\times10^{-5}\,\phi_0/\sqrt{\mathrm{Hz}}$$

**Transfer function**

$$\frac{\Delta I_{CC}}{\Delta\phi_s}\approx 4\,\mu\mathrm{A}\cdot\mathrm{turn}/\phi_0$$

**Thermal noise**

$$V_{n-th}=\sqrt{4k_BT\cdot R\cdot B}$$

**$R_H$ (2) : 100 Ω**

$N_P$ = 2065, $N_S$ = 16, $I_P$ = 50 µA

$V_{n\text{-}rms}$: 7 nV/√Hz

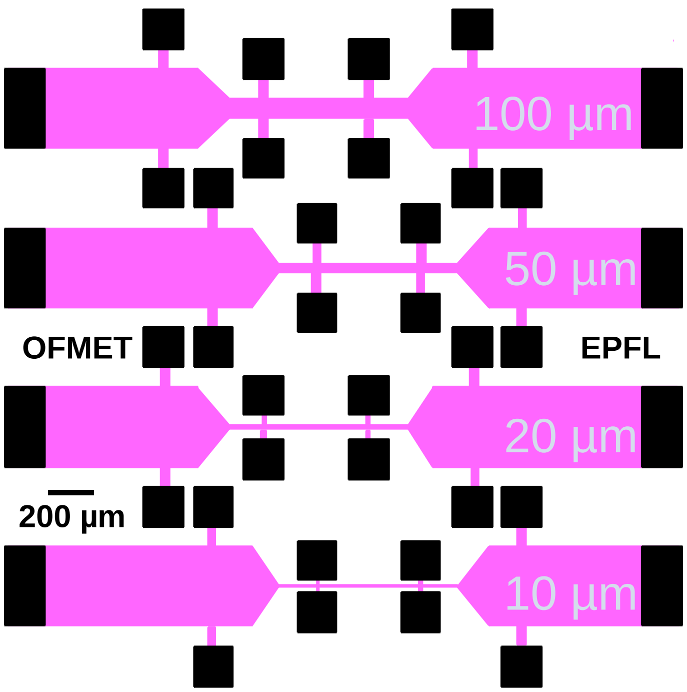
$u_A$: 1 nΩ/Ω in 2 min

# Universality of the quantum Hall effect

- Width dependence
- Contact resistance
- Device mobility
- Plateau index
- Device material: MOSFET-GaAs - Graphene

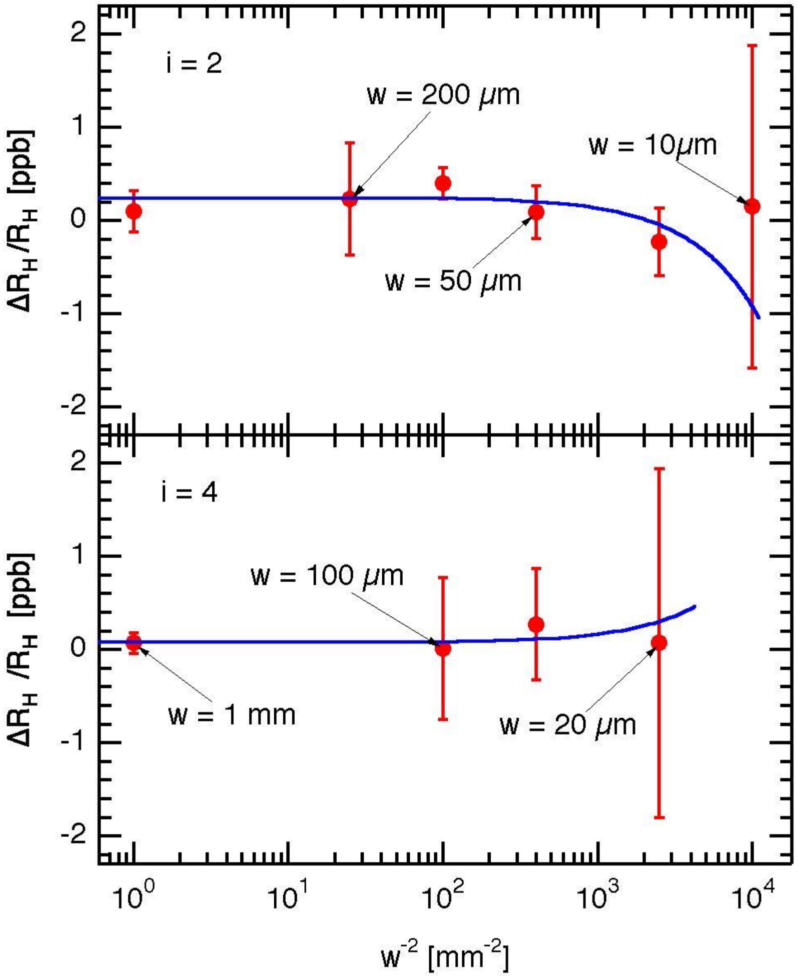
# Geometry of the QHE device

Some theories predict a **width dependence** of the QHR



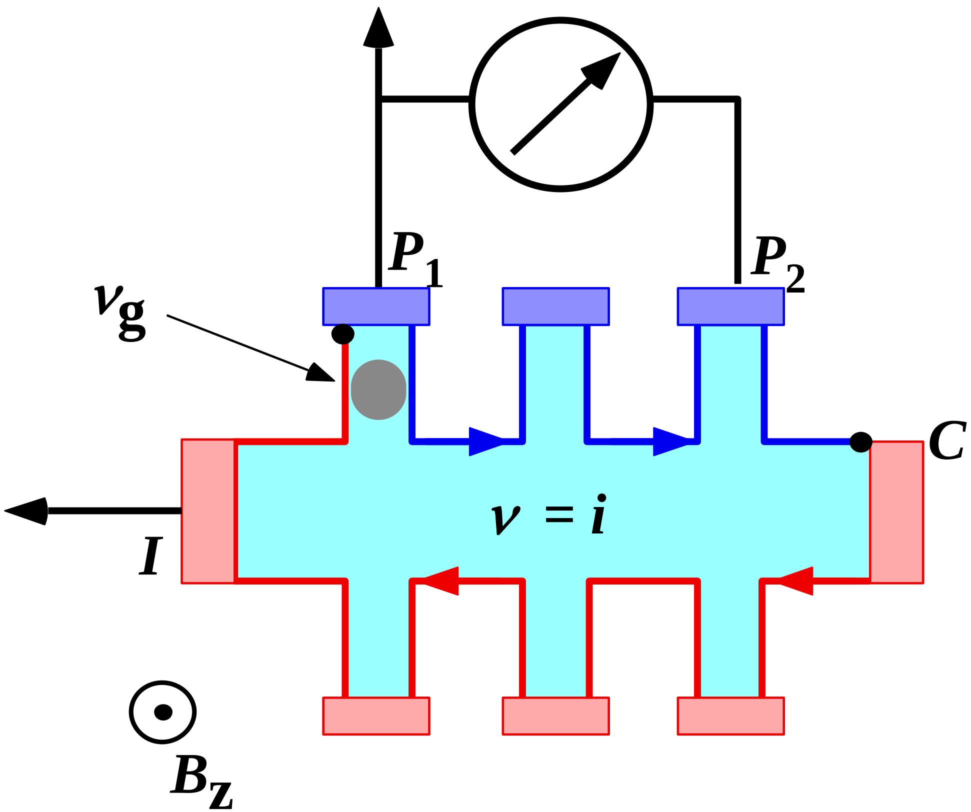

No size effect observed within the measurement uncertainty

Jeanneret et al., 95



# Effect of the contact resistance Rc

M. Büttiker, 1992: "...*It is likely, therefore, that in the future, contacts will play an essential role in assessing the accuracy of the QHE*."



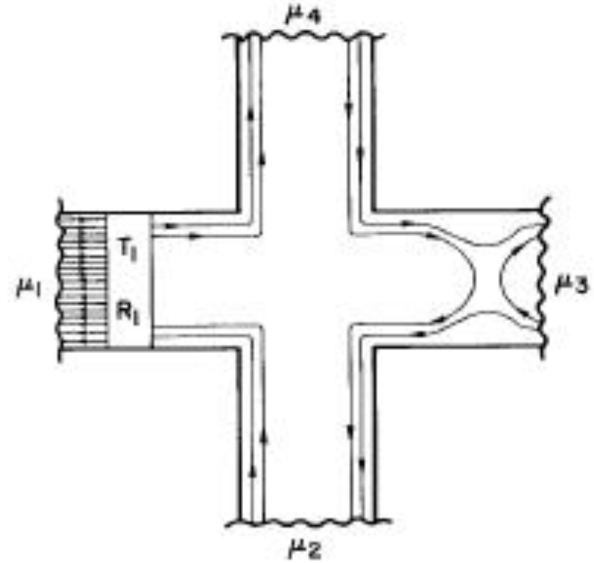

On a perfectly quantised plateau:

$$R_c(P_1) = \Delta V_{P_1P_2} / I$$

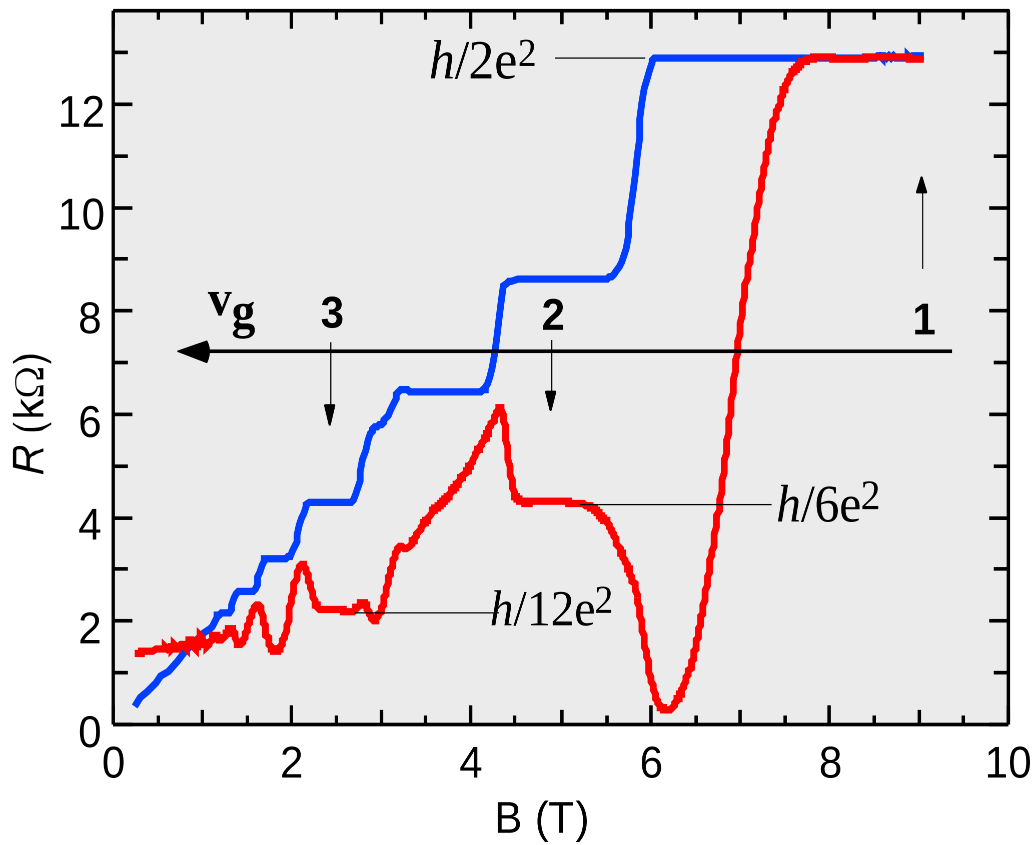
**Real sample**

- $R_c > 0$
  $\rightarrow$ Transmission $\neq 1$
  $\rightarrow$ Reflection $\neq 0$
- Bad contacts
  $\rightarrow$ electron gas depletion in the contact region
  $\rightarrow$ non-ohmic behaviour of metal semiconductor interface

# Contact resistance (2)

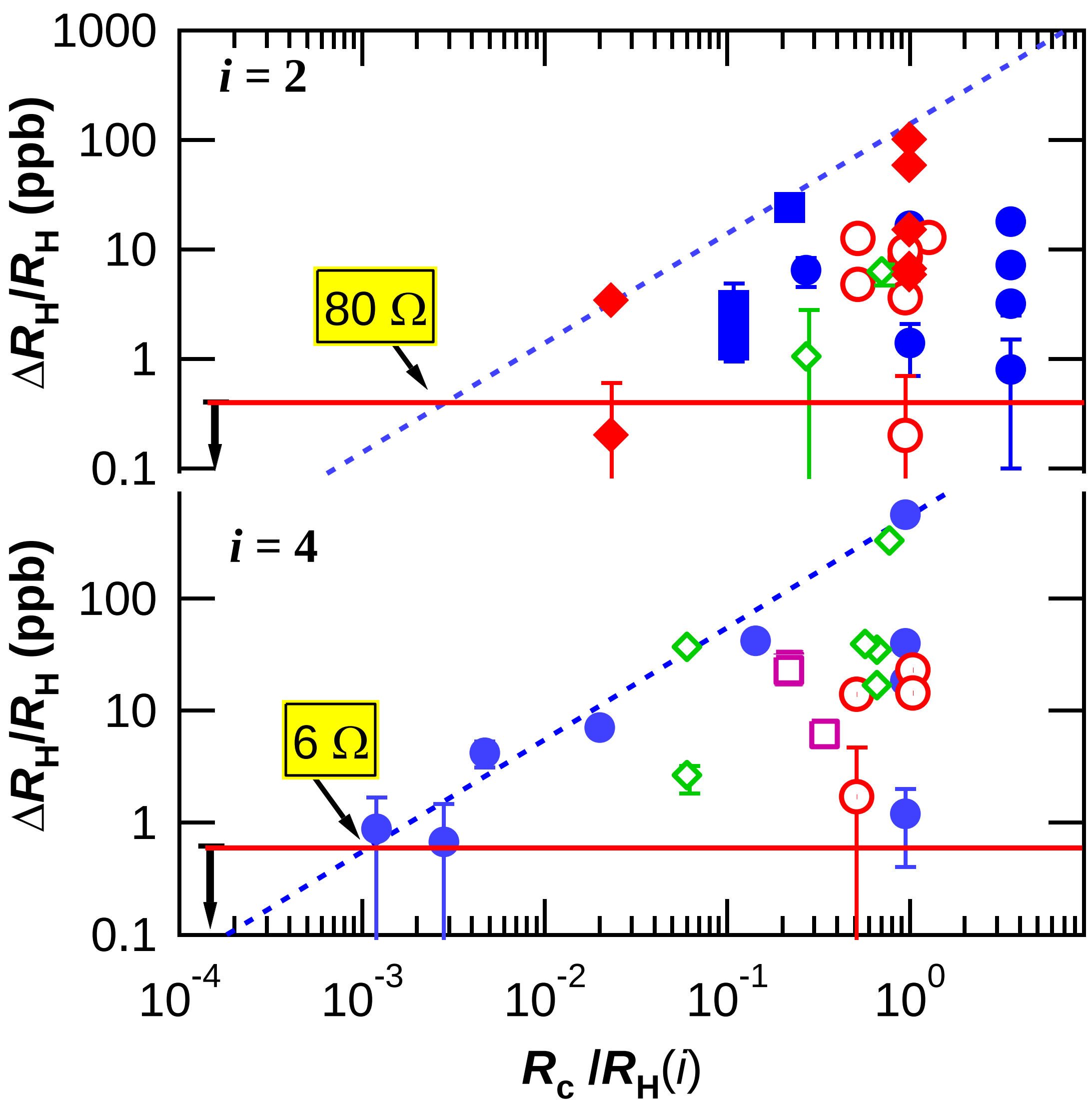



$\nu_g$: depleted region in the contact arm



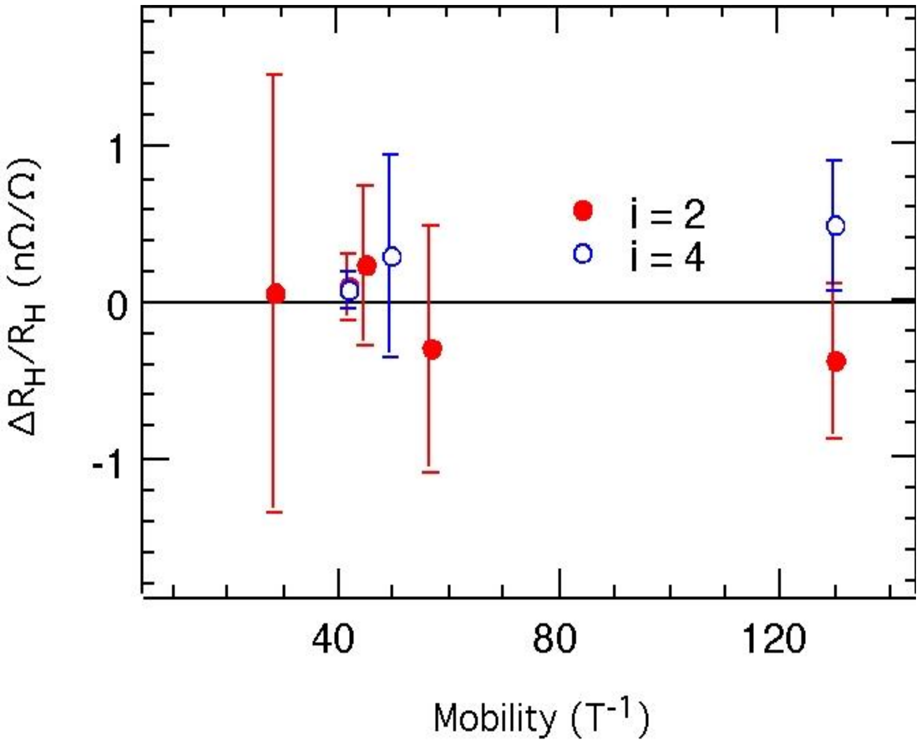

$$R_H = \frac{h}{e^2}\left(\frac{1}{\nu_g} - \frac{1}{\nu}\right)$$

# Contact resistance (3)




Deviation of $R_H$ related to finite $V_{xx}$

# Scattering parameters

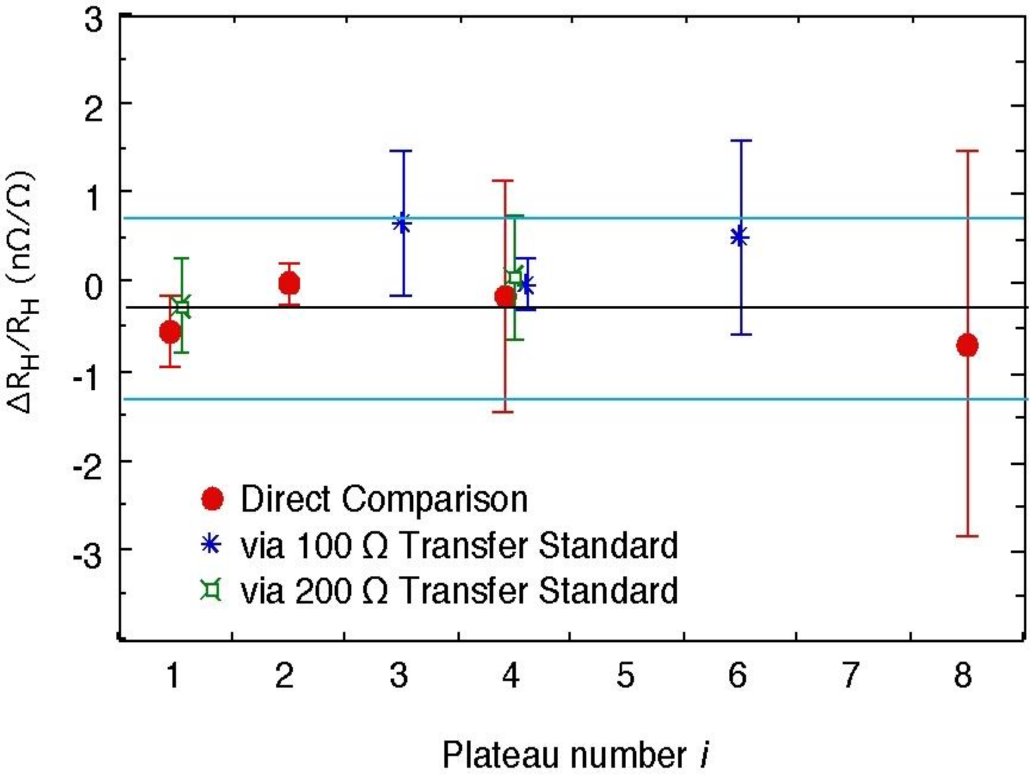



Electron mobility: measure of the electron velocity

Jeckelmann et al. 97

$R_H$ independent of the device mobility or the fabrication process to 2 parts in $10^{-10}$

# Step ratio measurements



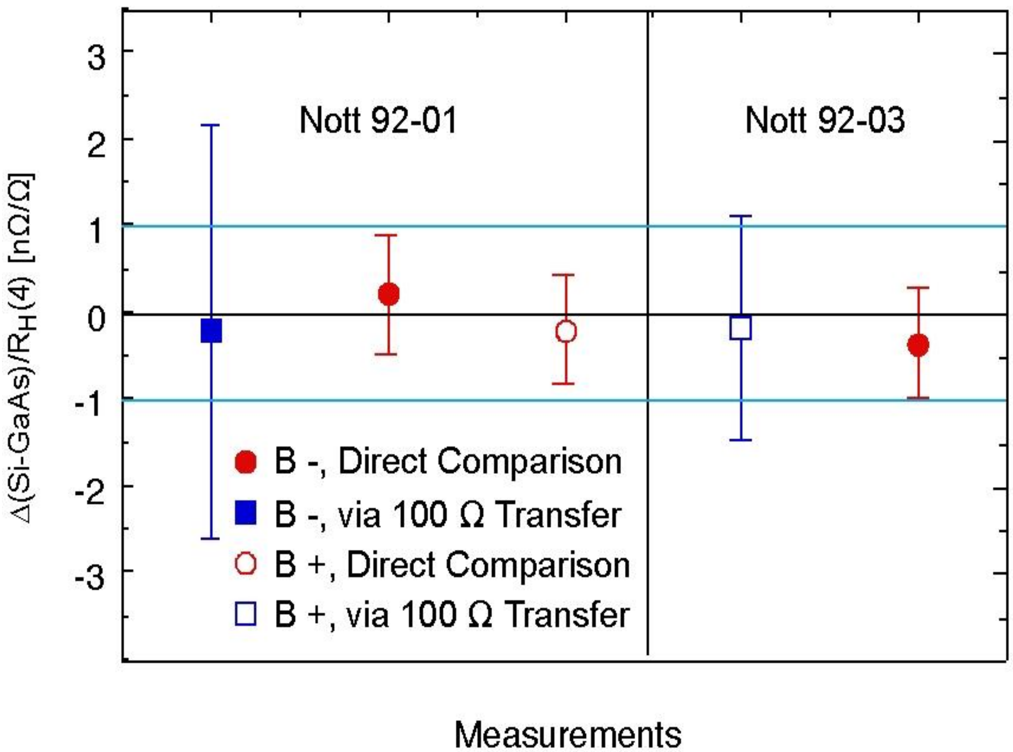

Jeckelmann et al. 97

$R_H$ independent of the plateau index to 3 parts in $10^{-10}$

$$\frac{i \cdot R_H(i)}{2 \cdot R_H(2)} = 1 - (1.2 \pm 2.9) \times 10^{-10}$$

$$i = 1, 3, 4, 6, 8$$

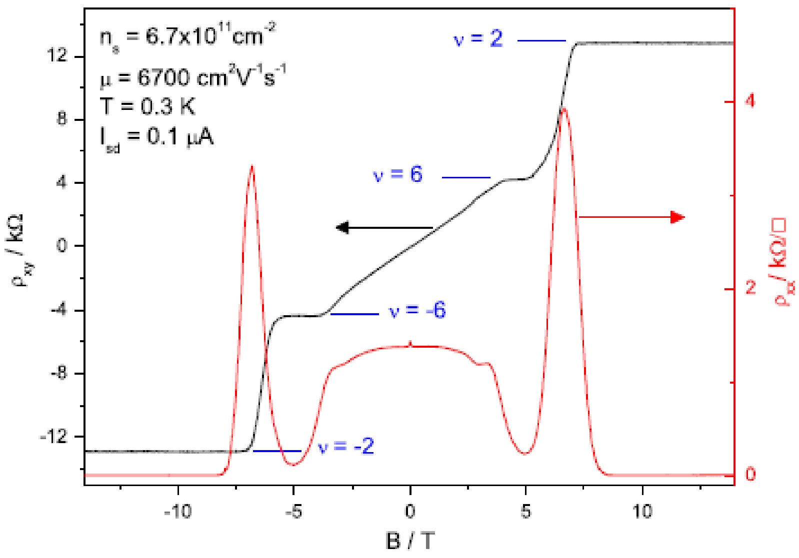
# QHR comparison GaAs - MOSFET

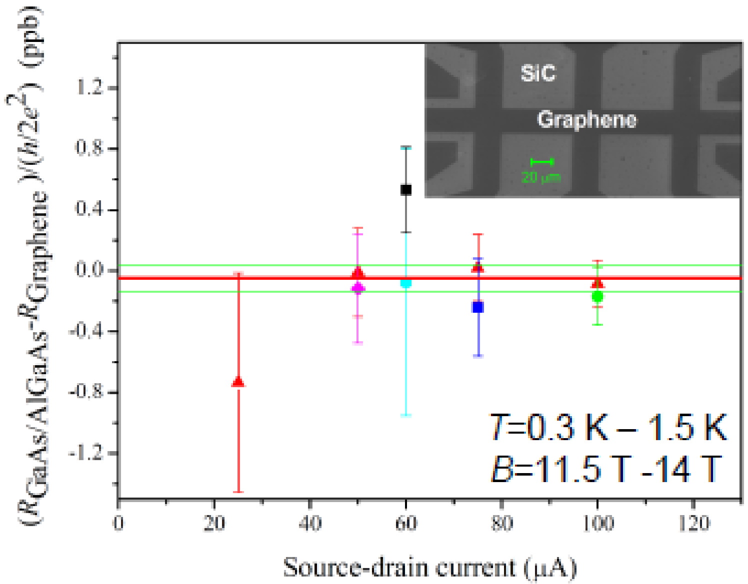



Jeckelmann et al., METAS, 1996

$$\frac{\Delta R_H(\mathrm{MOSFET}-\mathrm{GaAs})}{R_H} \leq 2.3 \times 10^{-10}$$

# QHR comparison GaAs - Graphene



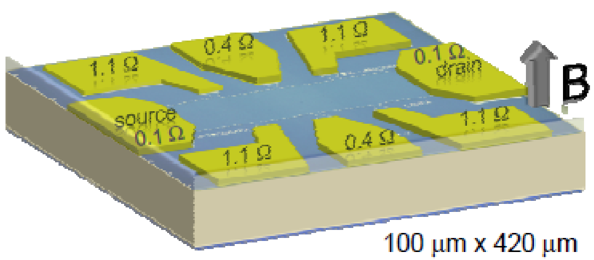




$$\Delta_{GaAs-graphene} = (-4.7 \pm 8.6) \times 10^{-11}$$

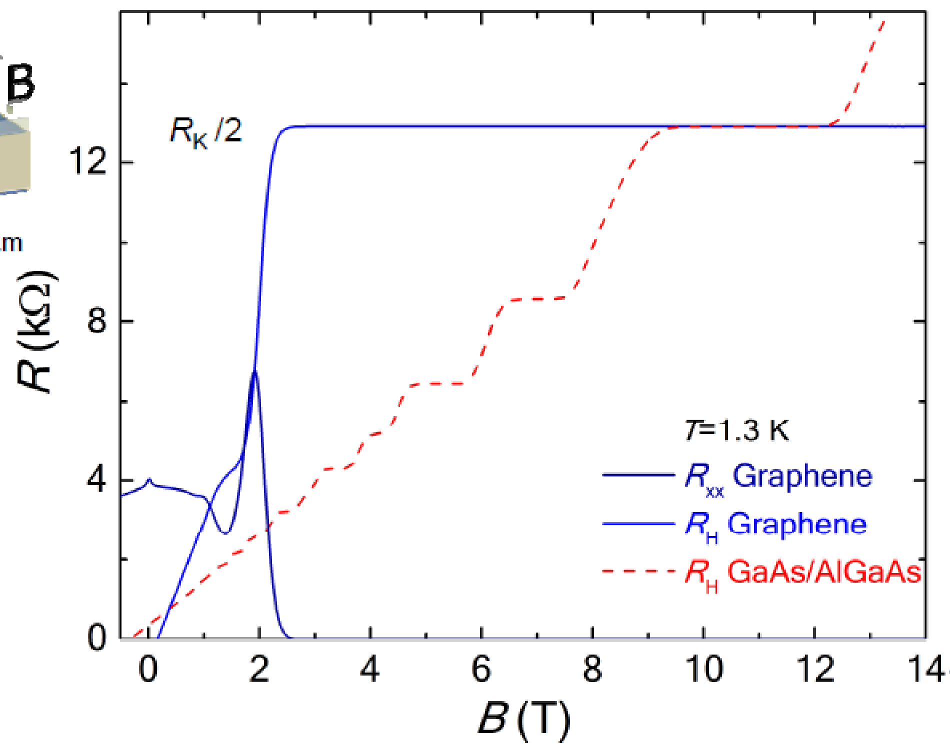
A. Tzalenchuk *et al*., Nat. Nanotech. (2010)
High precision measurements in **epitaxial graphene on SiC** at NPL
T. J. B. M. Janssen *et al.*, New J. of Phys. and Metrologia (2012)

# QHR comparison GaAs – Graphene (2)

Ti/Au contacts




Very wide plateau




$n$(electrons) = $1.8 \times 10^{11}$ cm$^{-2}$
$\mu$ = 9 400 cm$^2$V$^{-1}$s$^{-1}$

$$\Delta_{GaAs-graphene} = (-0.9 \pm 8.2) \times 10^{-11}$$

F. Lafont et al. LNE, 2014: graphene grown by CVD on SiC

# Universality: Summary

The quantum Hall resistance is a universal quantity independent of:

- Device width
- Device material: MOSFET- GaAs - Graphene
- Device mobility
- Plateau index

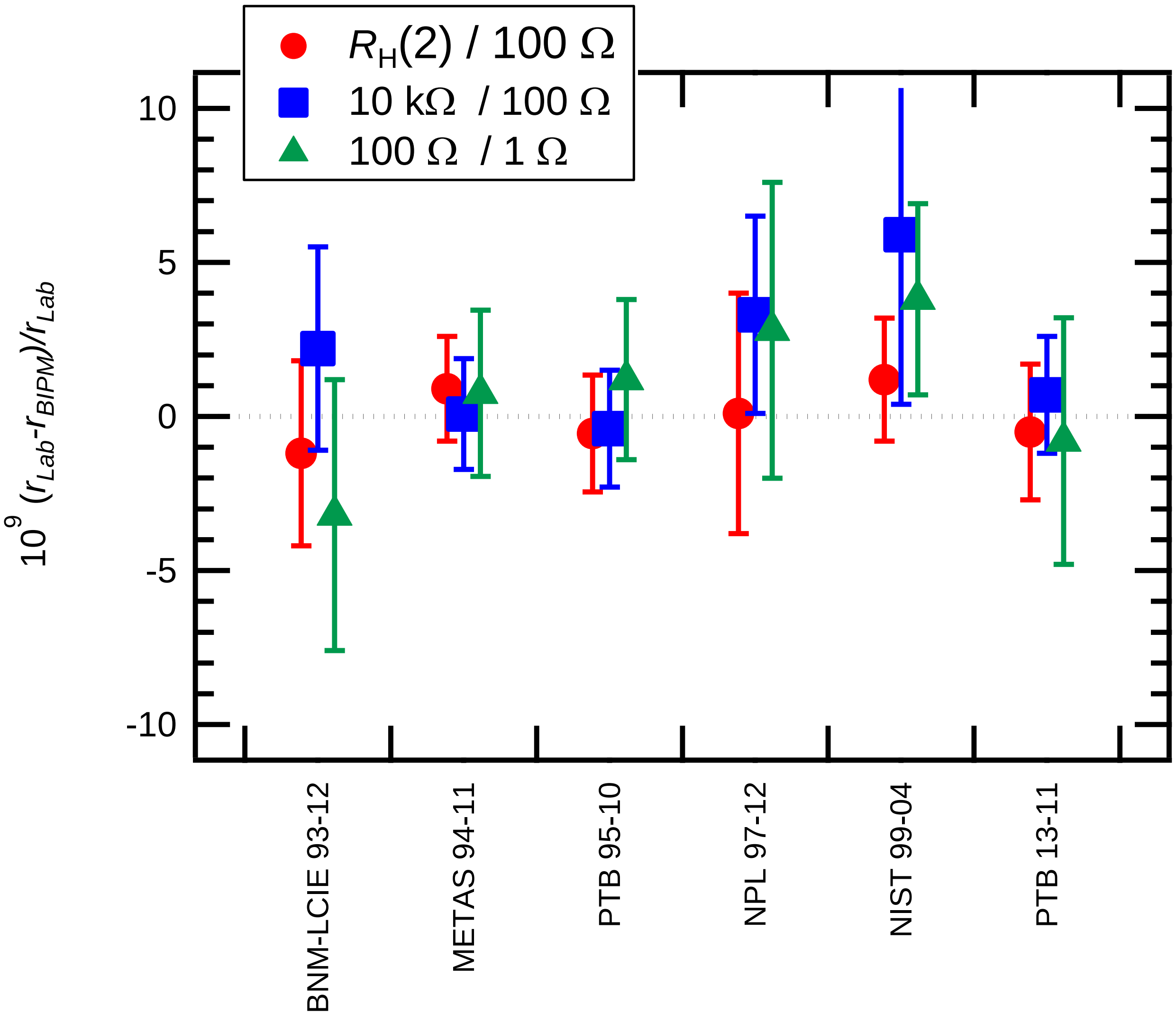
**…..to a level of < $10^{-10}$**

$$R_H\left(i, R_{xx} \to 0\right) = \frac{h}{i \cdot e^2}$$

*CCEM Technical Guidelines:*

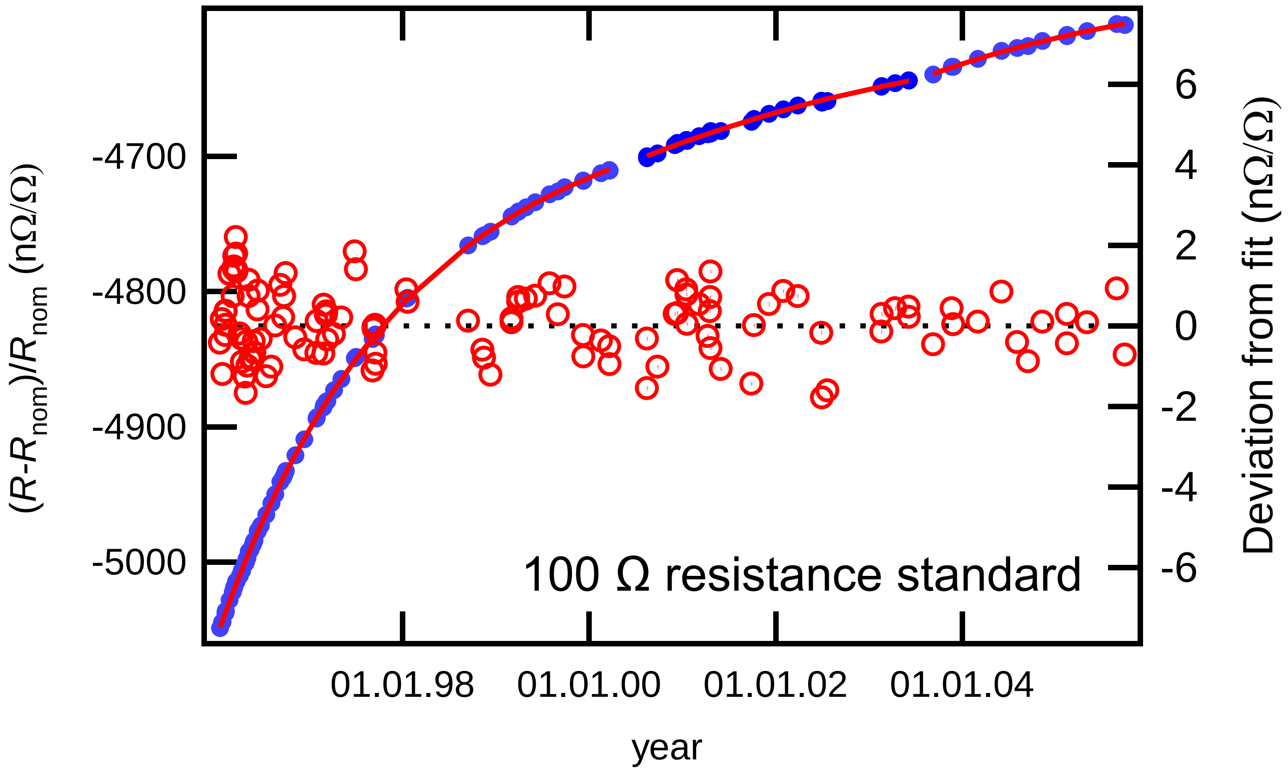
F. Delahaye and B. Jeckelmann, Metrologia 40, 217-223, (2003)

# International QHR Key-comparison




Key Comparison
BIPM.EM-K12

# Application: DC Resistance Standard
(data METAS)

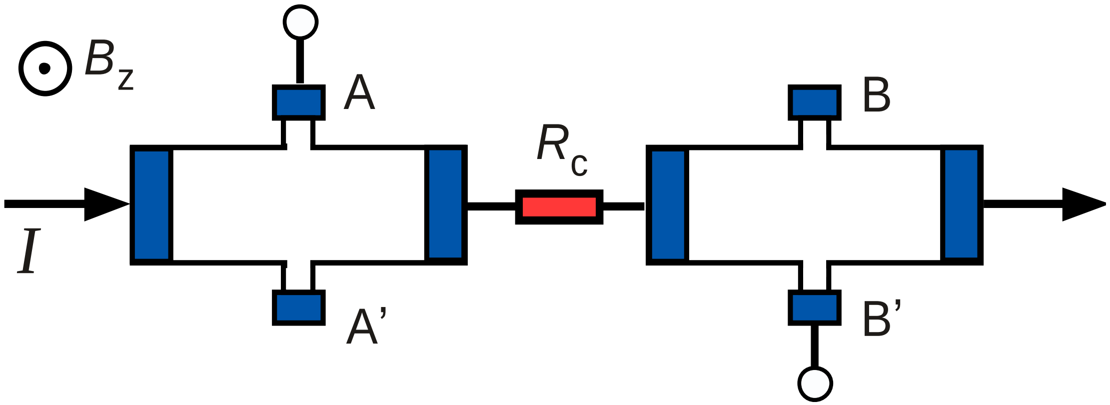



- Deviation from fit < 2 nΩ/Ω over a period of 10 years

# QHE arrays

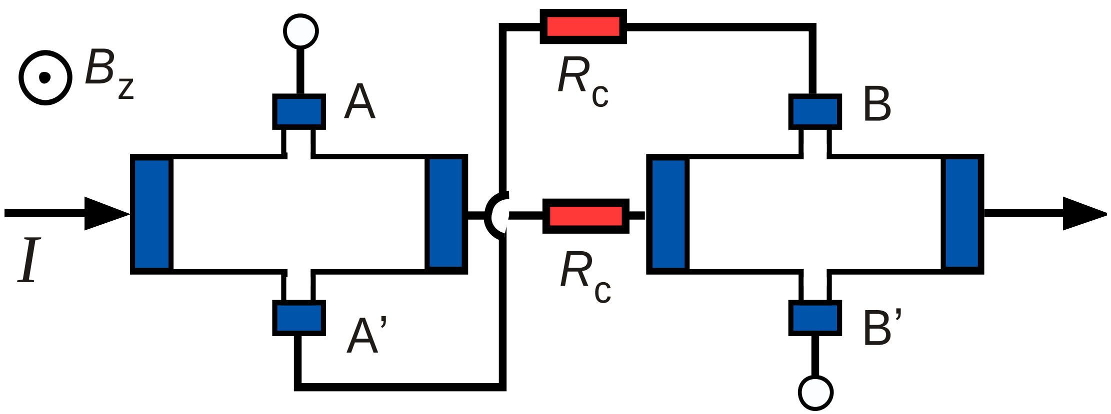
Connection of several Hall bars
(Delahaye 1993)

$$R_{AB} = 2R_H(1+\delta_c)$$




**single connection**

$$\delta_c \approx \frac{R_c}{R_H}$$

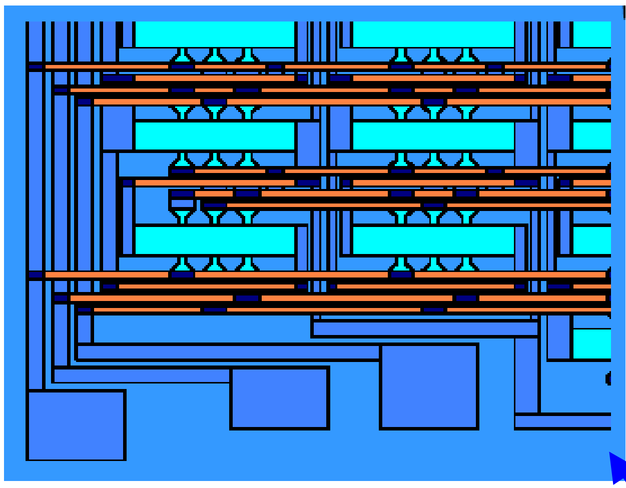


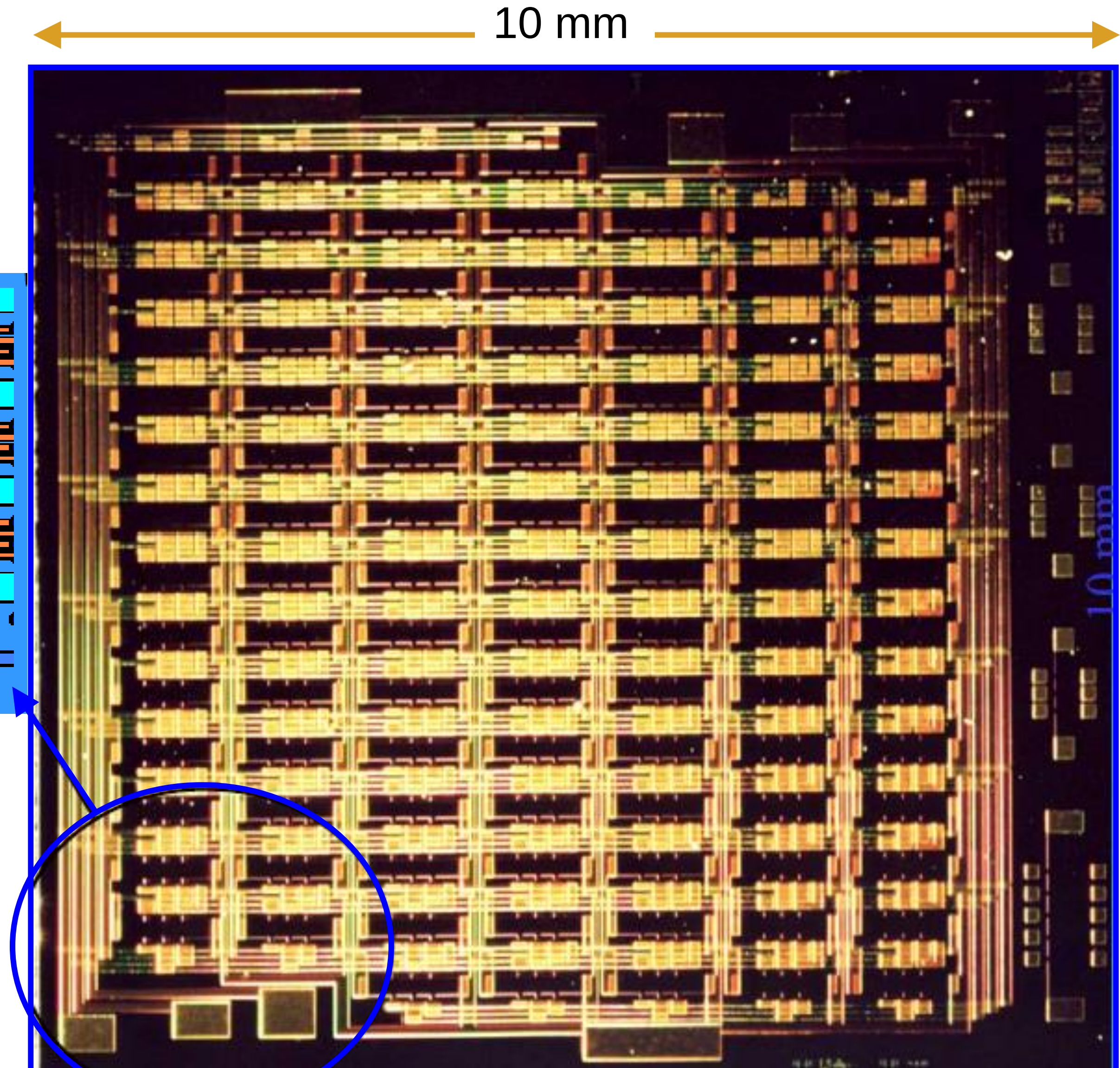

**double connection**

$$\delta_c \approx \left(\frac{R_c}{R_H}\right)^2 \leq 10^{-10}$$

# QHE array (LNE)

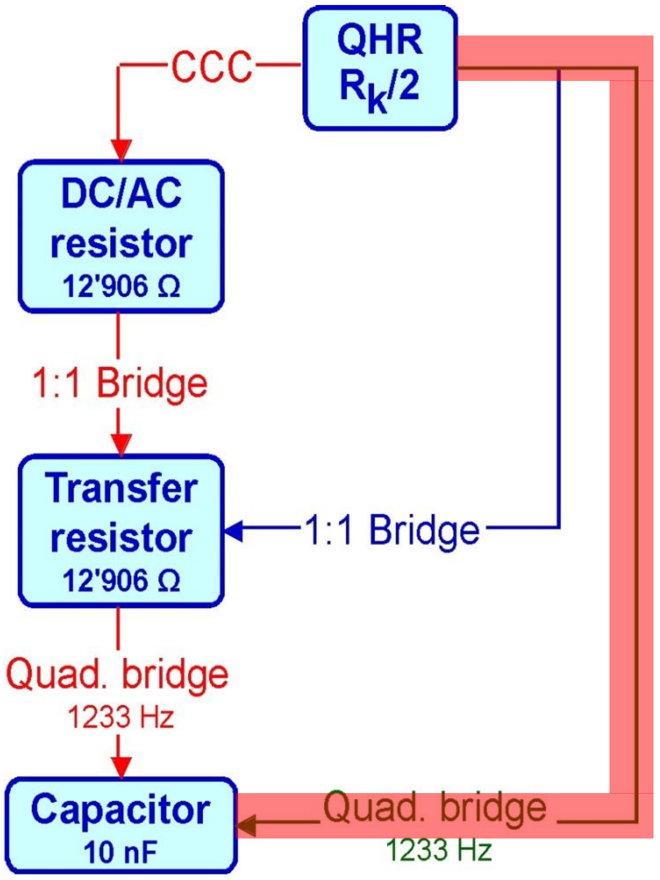
129 Ω @ $i$ = 2



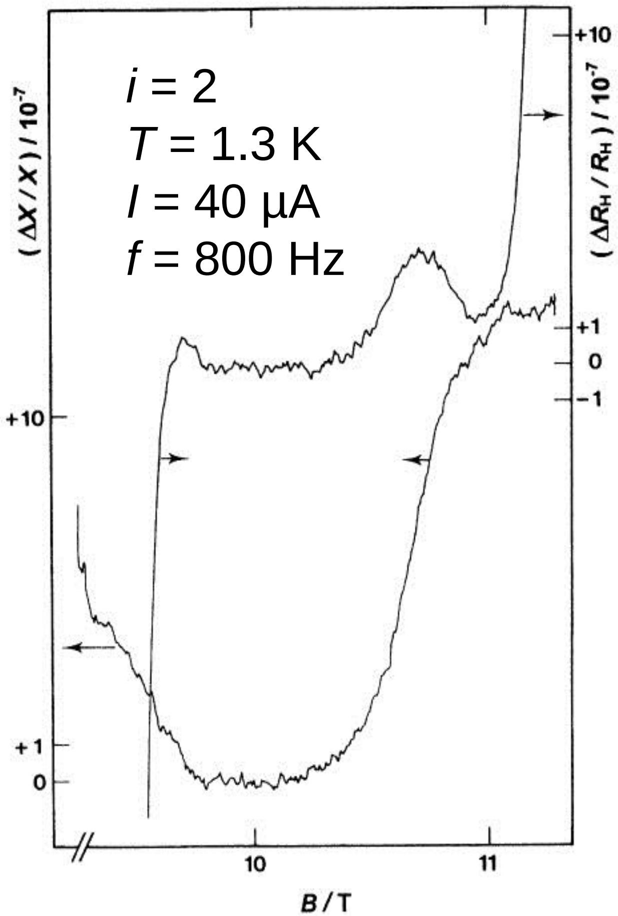


# AC-QHE: Applications

- SI realisation of the Farad: Calculable capacitor
  - complicated experiment
- Representation of the Farad: DC QHE
- **New route**: AC measurements of the QHR



# First AC measurements of the QHE




Delahaye 94

- Narrow bumpy "plateau " (PTB, NPL, NRC, BIPM)
- Frequency dependence:

$\Delta R_H(i, \omega) / R_H = \alpha\, \omega$

$\alpha = 1$ to $5\ 10^{-7}$/ kHz

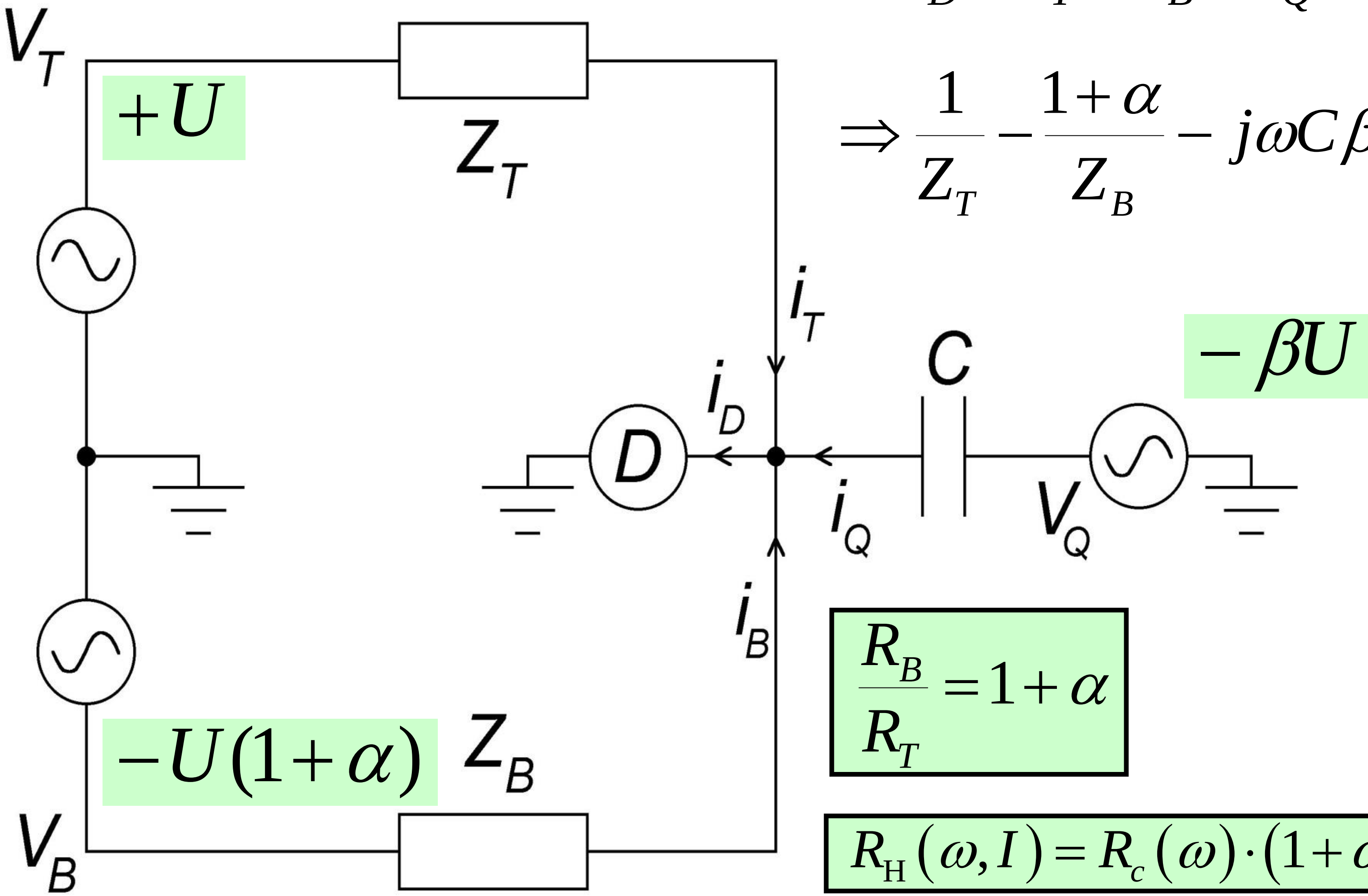
Measurement problems: AC Losses

Delahaye et al. 2000, Shurr et al. 2001, Overney et al. 2003

# Ratio Bridge: Principle

$$i_D = i_T + i_B + i_Q = 0$$

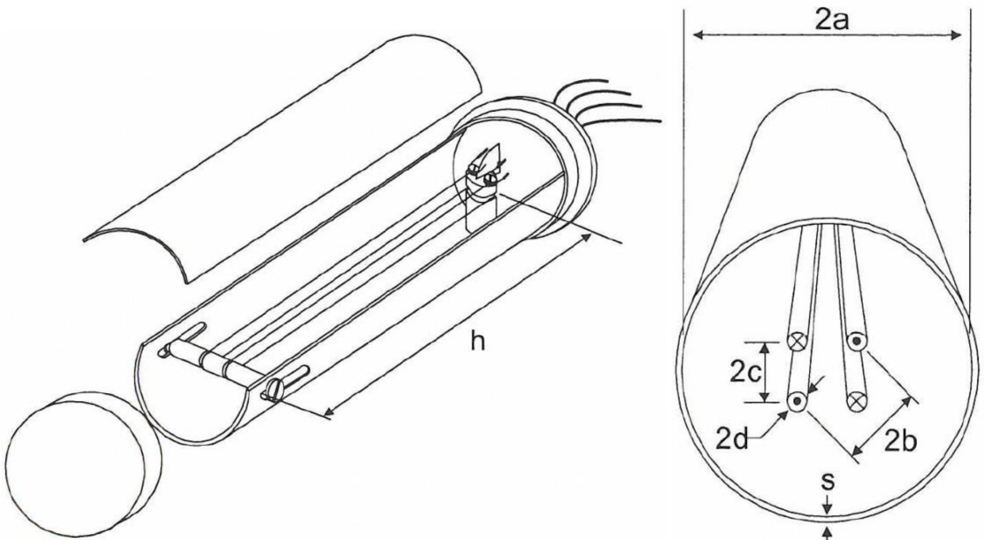
$$\Rightarrow \frac{1}{Z_T} - \frac{1+\alpha}{Z_B} - j\omega C\beta = 0$$




$$\frac{R_B}{R_T} = 1+\alpha$$

$$R_H(\omega, I) = R_c(\omega)\cdot(1+\alpha(\omega, I))$$

# Calculable Resistor: Quadrifilar Resistor (QR)




Transmission line equations

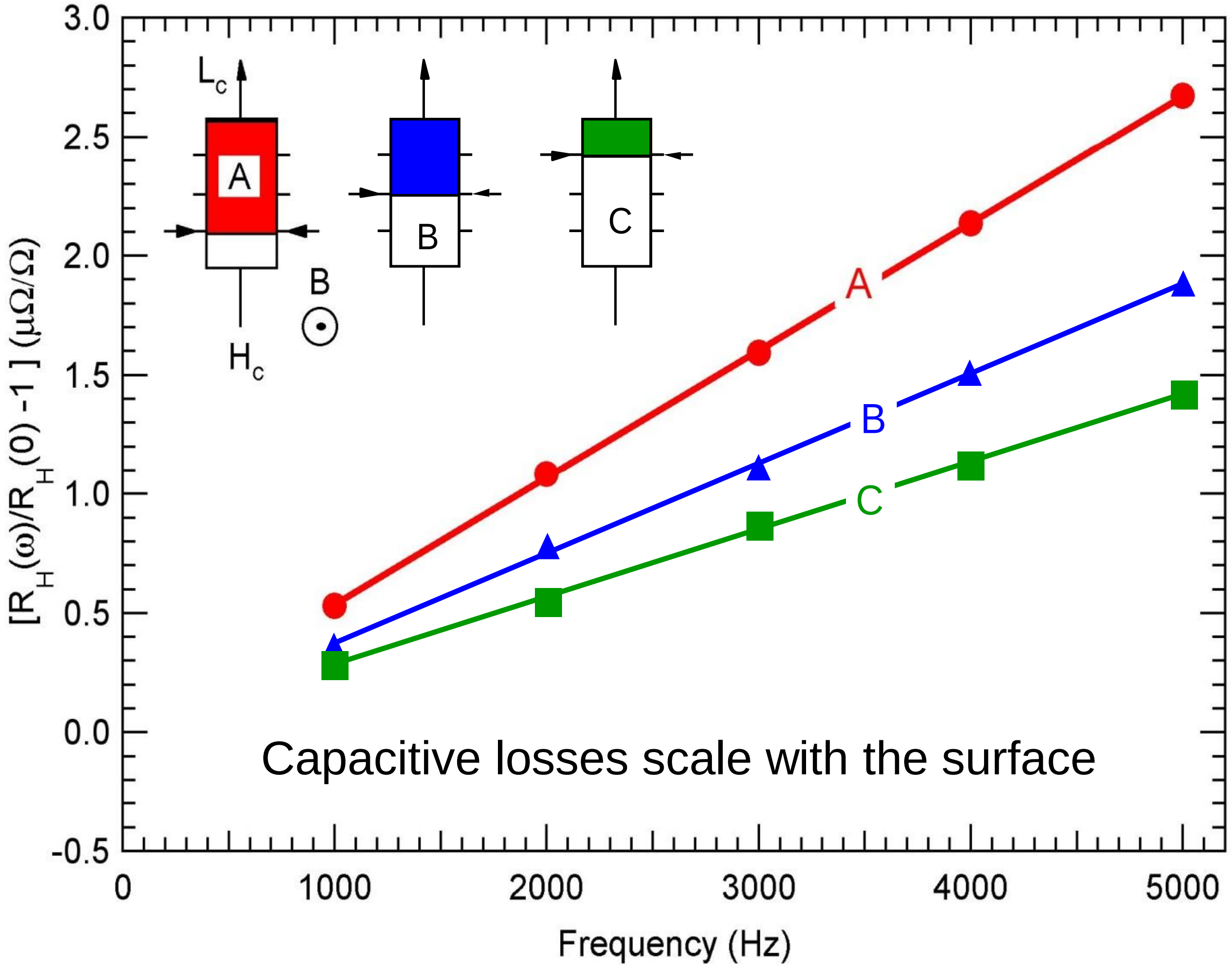
$$R_{//} = f\left(R, L, C, G, \omega\right)$$

$$R_c\left(\nu\right) = R_c^{dc}\left(1 + \delta \cdot \nu^2\right)$$

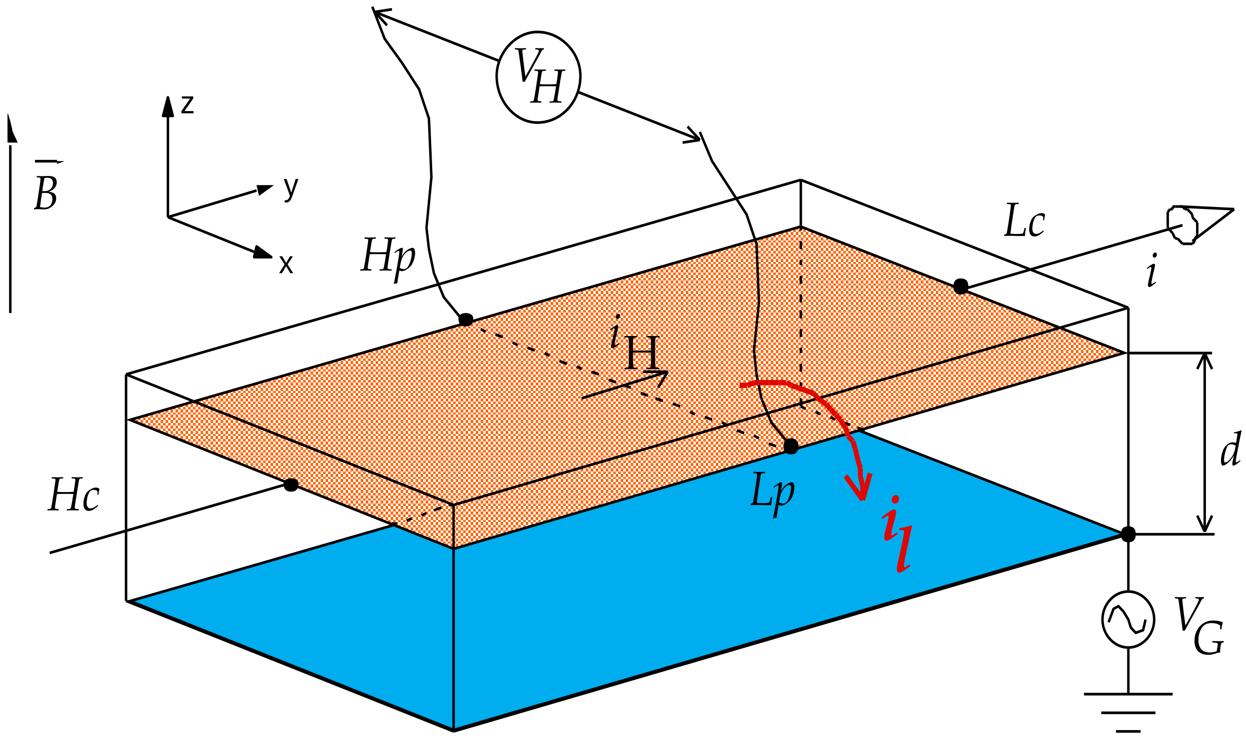
$\delta = 0.0129\ 10^{-6}/\mathrm{kHz}^2$

# AC measurements of the QHE
## (with grounded back gate)

**Linear frequency dependence of the Hall resistance observed**



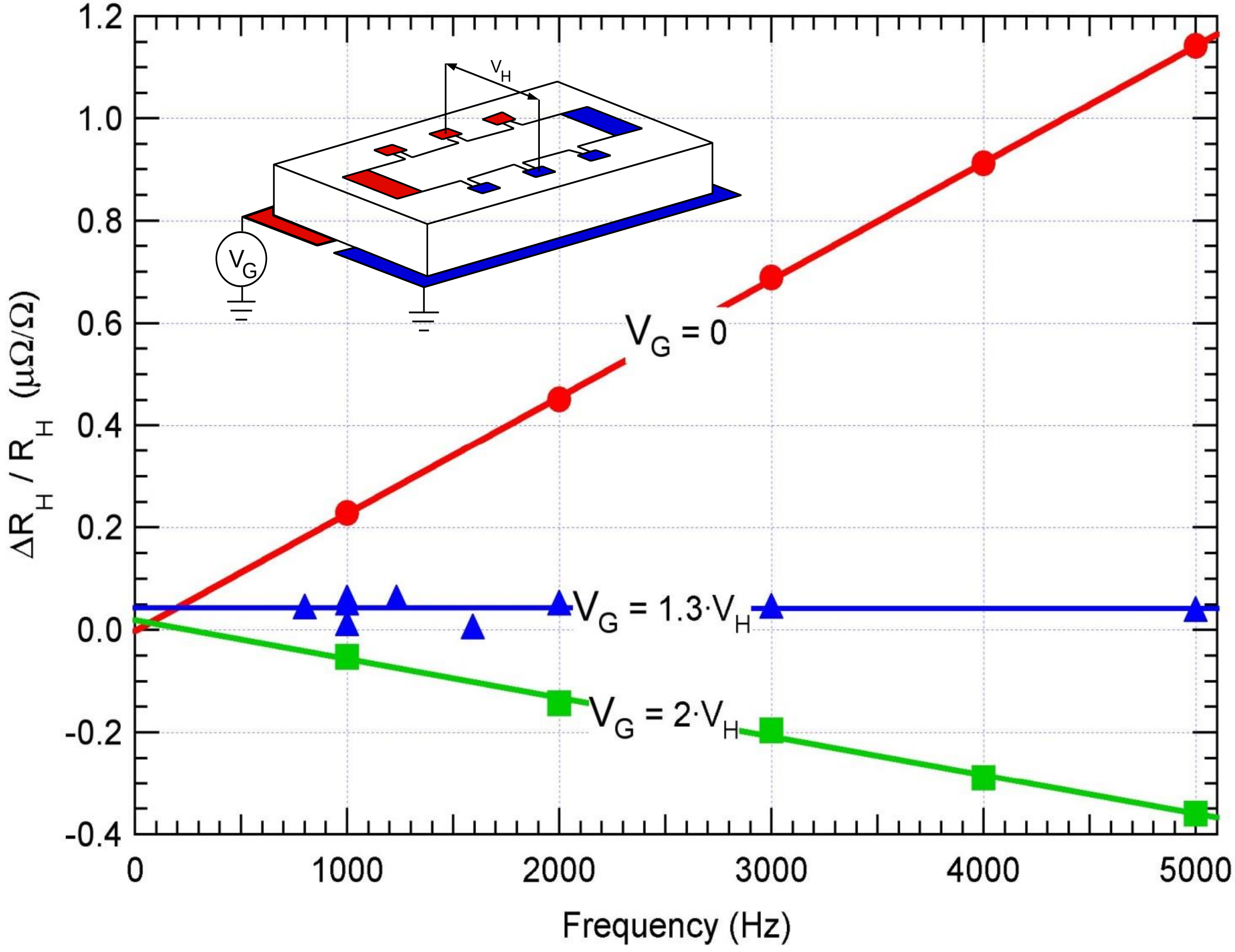
# Model for ac Losses



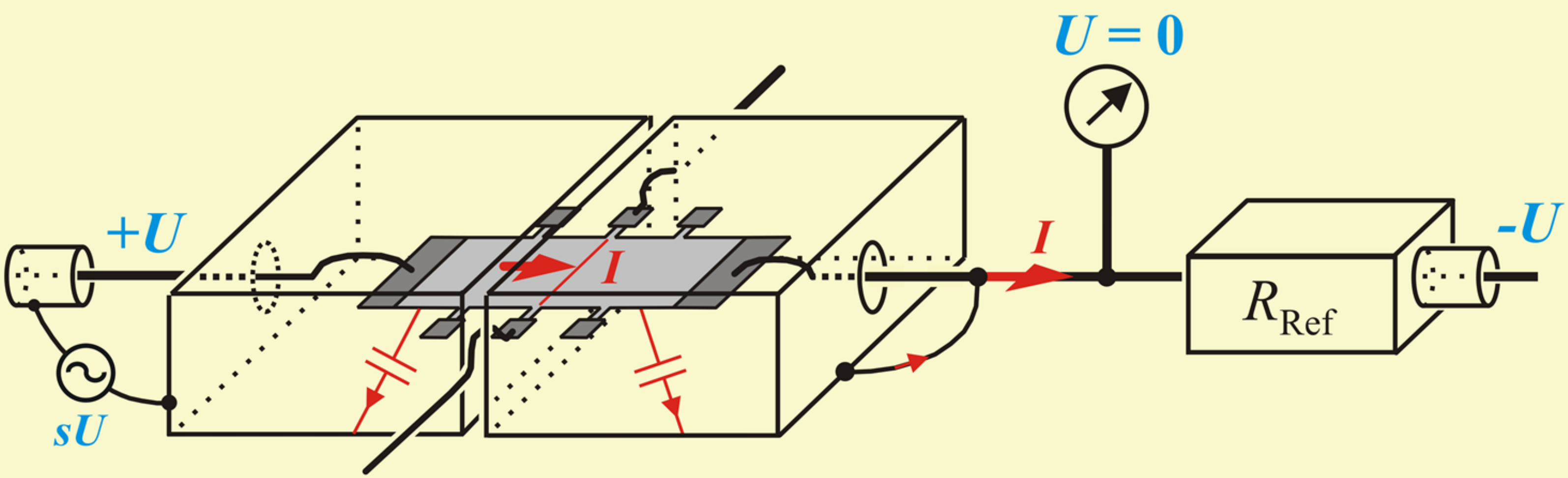

$$Z_H = \frac{V_H}{i} = \frac{V_H}{i_H - i_l} \approx R_H\left(1 + \frac{R_H}{V_H} i_l\right) = R_H(1+\Delta)$$

**Overney et al., 2003**

# Adjusting capacitive losses
(2 back gates)



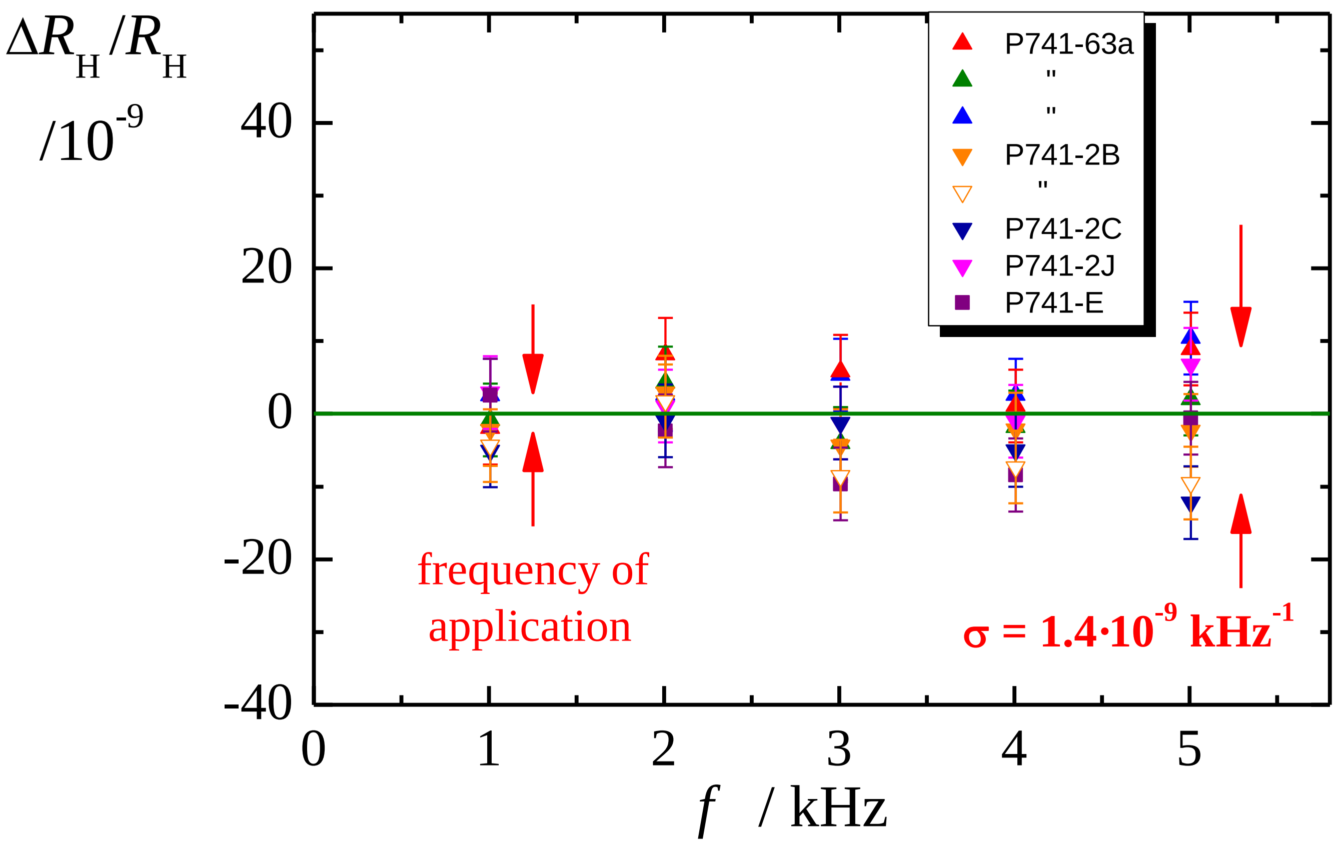
# Double shielding technique




- Meet the defining condition: ALL currents which have passed the Hall-potential line are collected and measured.
- Adjust the high-shield potential $sU$ so that $dR_H/dI = 0$.

**B.P. Kibble, J. Schurr, *Metrologia* 45, L25-L27 (2008).**

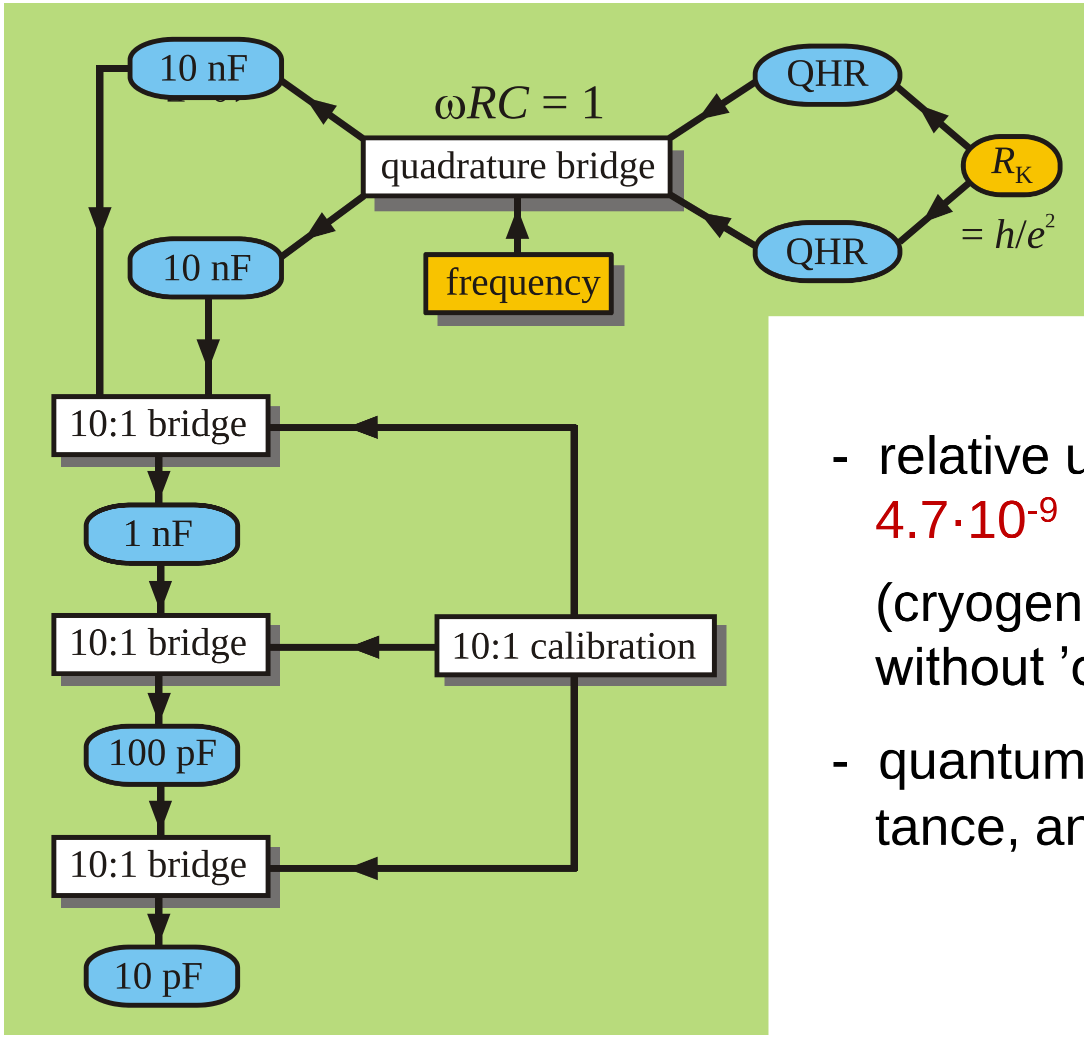
# QHR as impedance standard




⇒ capacitive effects < $1.4 \cdot 10^{-9}$ $kHz^{-1}$ for different devices
⇒ better than artefacts

J. Schurr, J. Kucera, K. Pierz, and B.P. Kibble, *Metrologia* 48, 47-57 (2011).

# Realization of the Farad




- relative uncertainty of 10 pF: 4.7·10$^{-9}$ ($k$ = 1)

  (cryogenic quantum effect without ’calculable‘ artefacts)

- quantum standard of capaci-tance, analogous to $R_{DC}$

J. Schurr, V. Bürkel, B. P. Kibble, *Metrologia* 46, 619-628, 2009

# Conclusions

- $R_H$ is a universal quantity
- QHR improved electrical calibrations in National Metrology Institutes considerably
- QHR plays an important role in the determination of the fine structure constant
- QHR can be used as quantum standard for impedance

# Thank you very much for your attention