Contribution of fundamental constants from atomic physics to the redefinition of kg

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"Redefinition of kg" : h or N_A

$$m_u = \frac{m(^{12}\mathrm{C})}{12}$$

$$M_u = 10^{-3} kg\ mol^{-1}$$

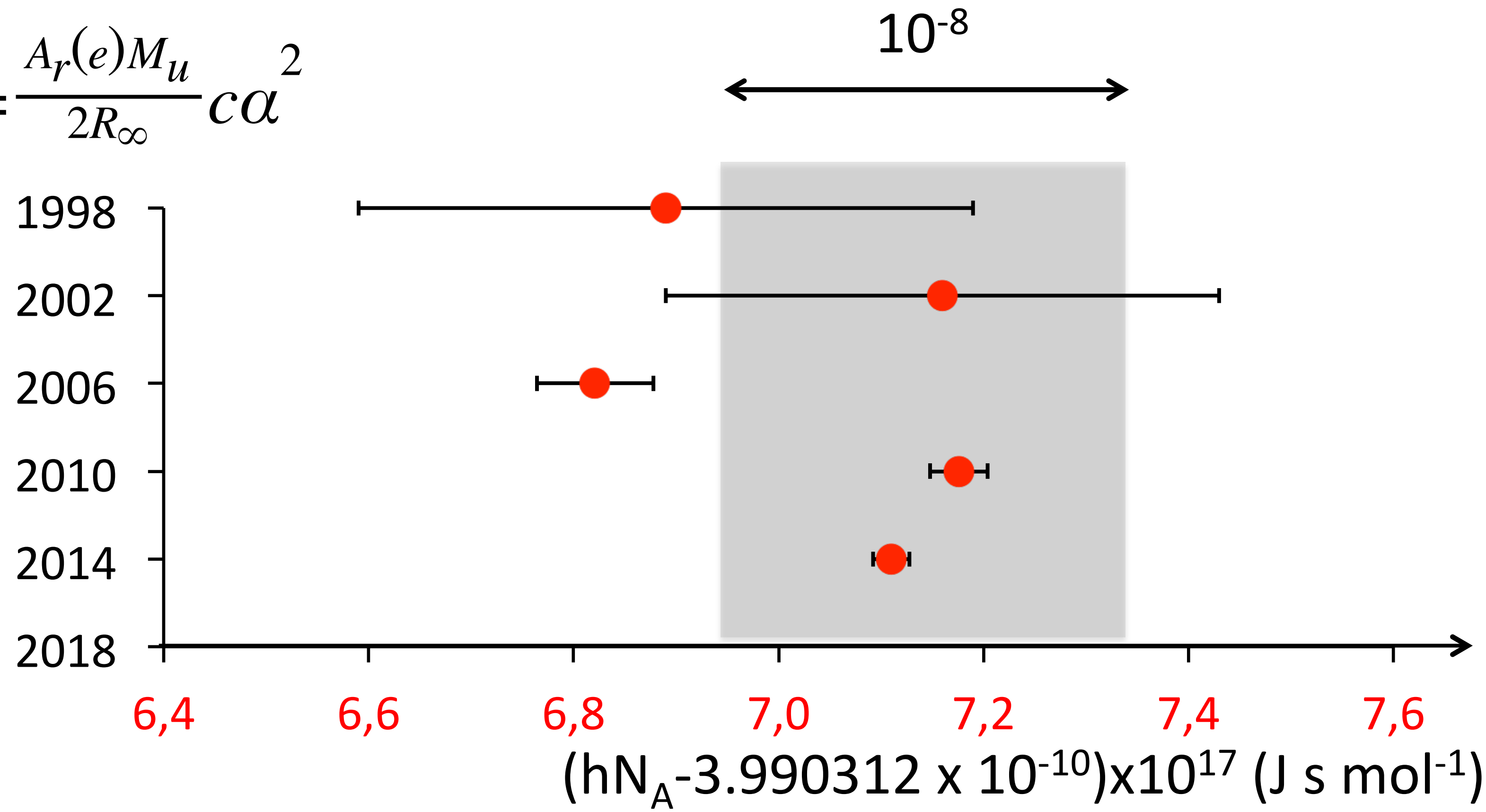
$$\left.\begin{aligned} R_\infty &= \frac{\alpha^2 m_e c}{2h} \\ A_r(e) &= \frac{m_e}{m_u} \\ N_A &= \frac{M_u}{m_u} \end{aligned}\right\} \quad hN_A = \frac{A_r(e) M_u}{2R_\infty} c\alpha^2$$

Comparison h ⇆ N_A

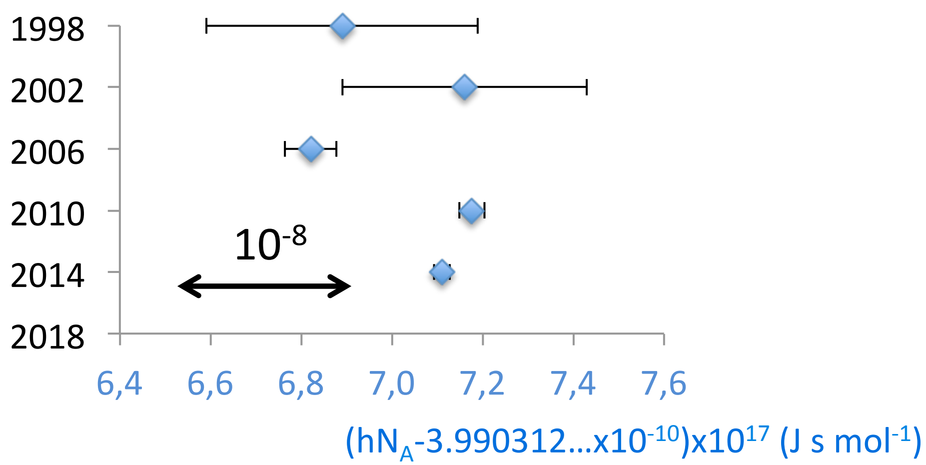
Consultative Committee for Mass and Related Quantities (CCM) Recommandation G1 2013

"… 1. At least three independent experiments, including work from watt balance and XRCD experiments, yield consistent values of the Planck constant with relative standard uncertainties not larger than 5 parts in 10^8 ,
2. At least one of these results should have a relative standard uncertainty not larger than 2 parts in 10^8…"

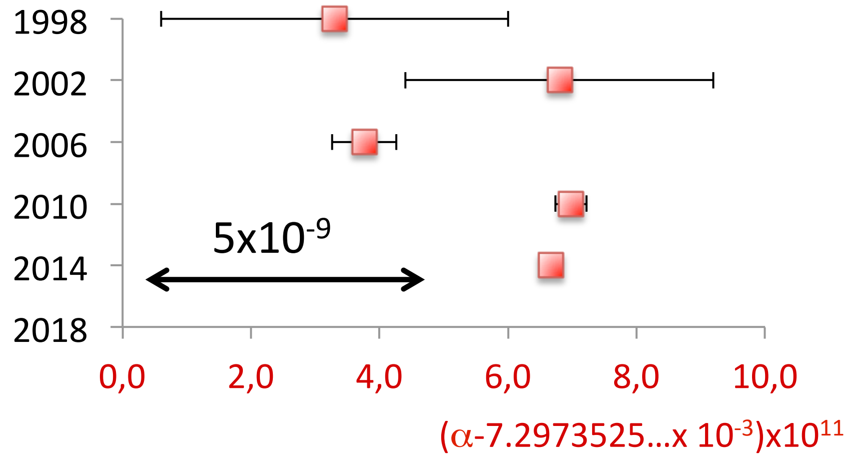
$$hN_A = \frac{A_r(e)M_u}{2R_\infty} c\alpha^2$$



Contribution to hN_A : α, $A_r(e)$, R_∞

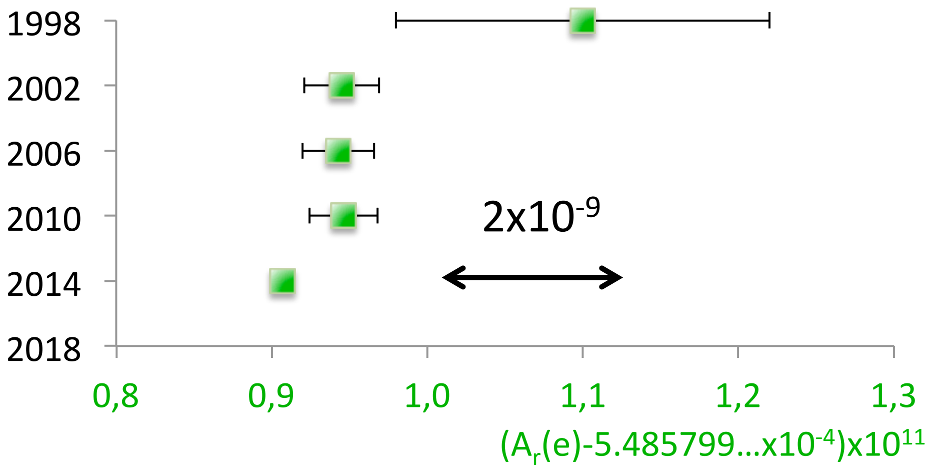


$r(hN_A,\alpha)=0.9979$

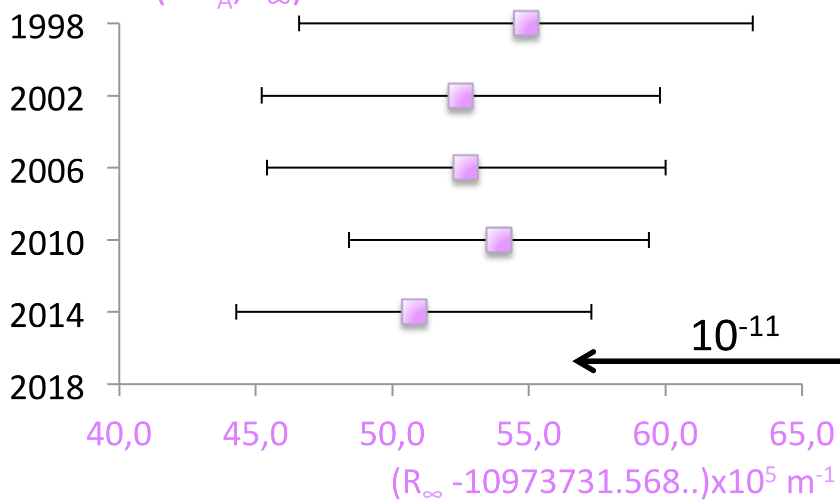


$$h N_A = \frac{A_r(e)\, M_u}{2R_\infty}\, c\, \alpha^2$$

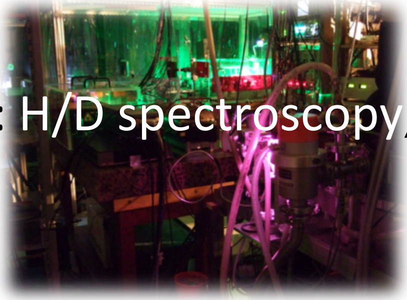
$r(hN_A,A_r(e))=0.0550$

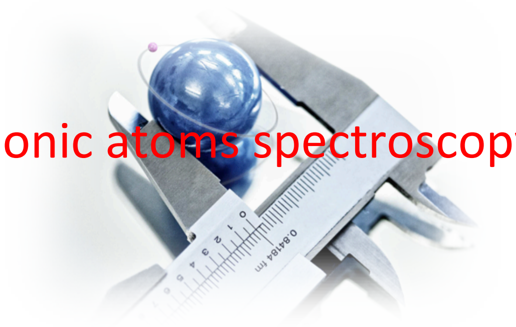


$r(hN_A,R_\infty)=-0.0123$

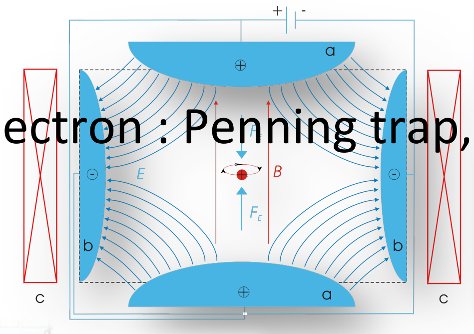


Outline

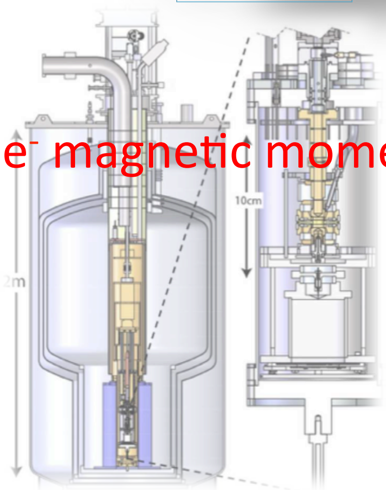


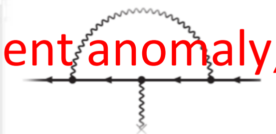


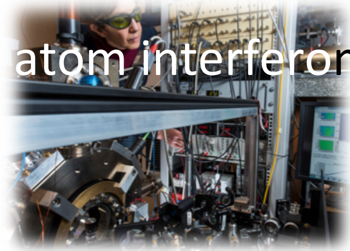
I. Rydberg constant R_∞ : H/D spectroscopy, muonic atoms spectroscopy



II. Relative atomic mass of the electron : Penning trap, $\bar{p}$He spectroscopy





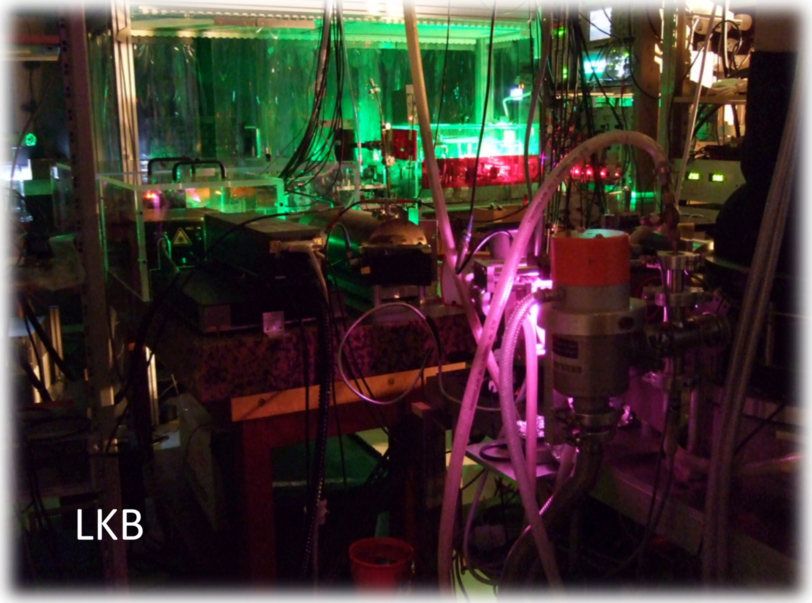


III. Fine structure constant : e^- magnetic moment anomaly, atom interferometry

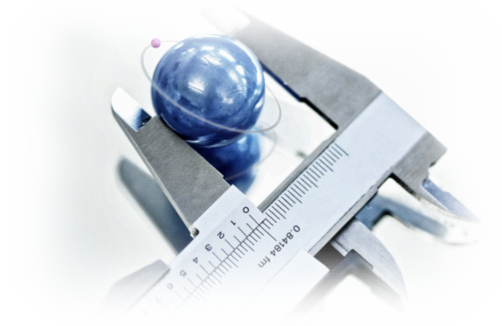
IV. Conclusion

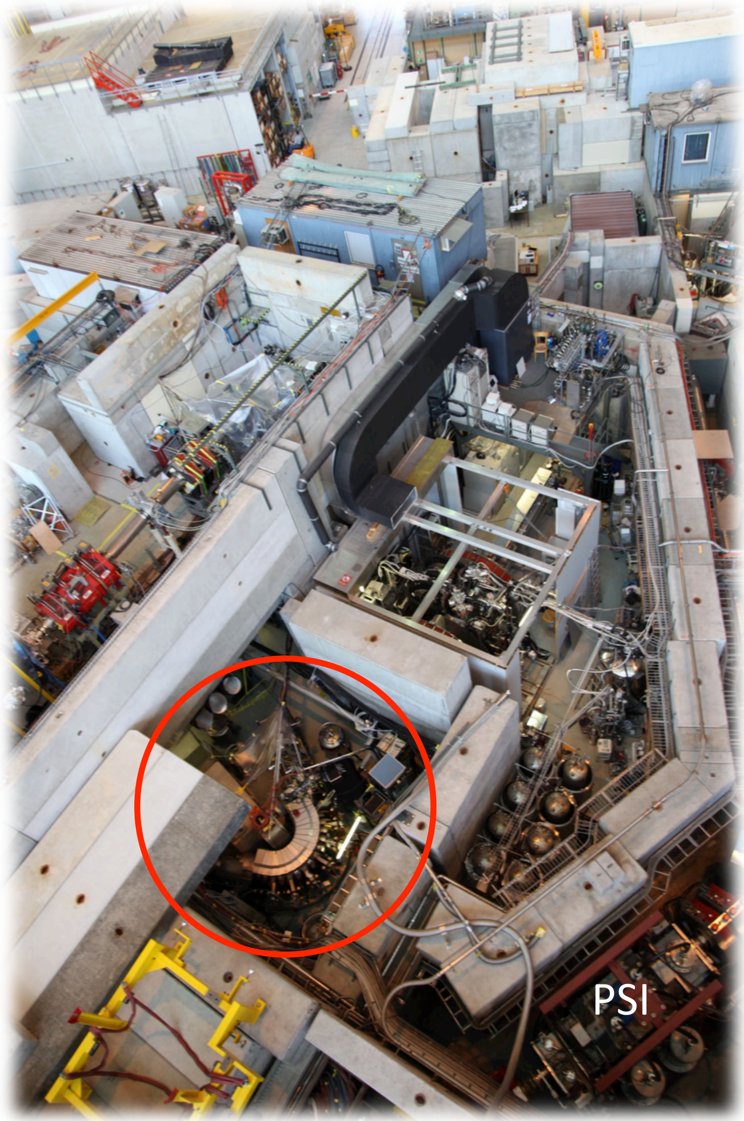
I. Rydberg constant :

- hydrogen/deuterium spectroscopy,

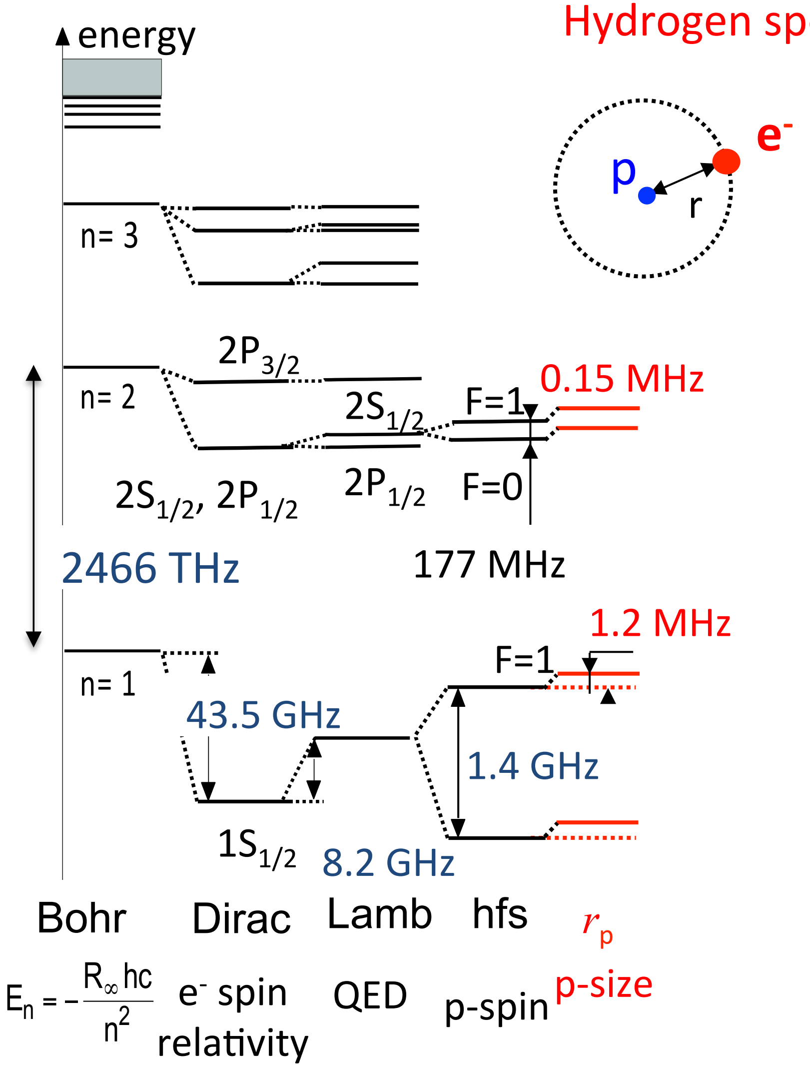


- muonic atoms spectroscopy





Hydrogen spectroscopy



$E(n,l,j)$ = Dirac + recoil + $LS(n,l,j,r_p) \approx \frac{R_\infty}{n^2} + LS(n,l,j,r_p)$

$hcR_\infty f(\alpha, m_e/m_p, n,l,j)$

exact

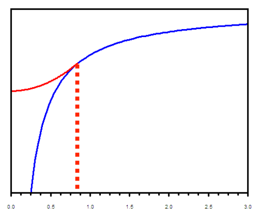
not exact

$LS(n,l,j)=hcR_\infty g(\alpha, m_e/m_p, n,l,j, r_p)$

g function includes :

- QED corrections ($1/n^3$)
- relativistic recoil
- charge radius of the proton ($m_r^3 r_p^2/n^3$)

Potential energy

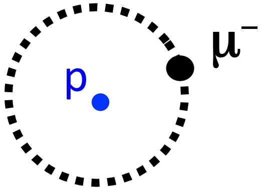


MPQ Garching	$\nu(1S-2S) = (1-\frac{1}{4})R_\infty + L(1S) - L(2S)$	
LKB Paris	$\nu(2S-8S) = (\frac{1}{4}-\frac{1}{64})R_\infty + L(2S) - L(8S)$	R_∞, $L^{exp}(1S)$ + QED
S. Karshenboim / K. Pachucki	$L(1S) - 8L(2S)$ = precisely calculated	

$\rightarrow r_p$ (1%)

Muonic hydrogen (PSI-CH) : proton radius

Exotic atom

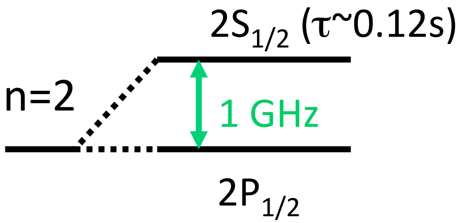


$m_\mu \sim 207\ m_e$

radius $\sim a_0/196$

$$E_{NS} = \frac{4}{3}\left(\frac{m_r}{m_e}\right)^3 \frac{R_\infty hc}{n^3}\left(\frac{r_p}{a_0}\right)^2 \delta_{l0}$$

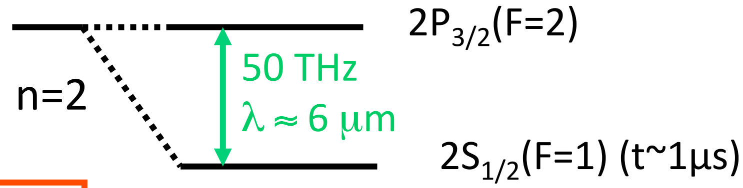
Electronic hydrogen : e-p



E_{NS}=146 kHz (0.014%)

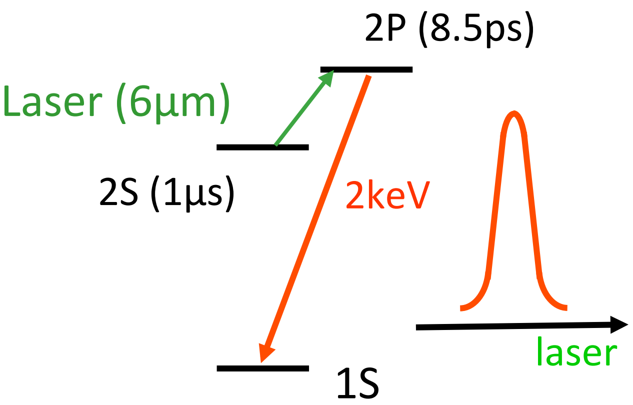
$$(f_{\mu p}/f_{ep}) \propto 1/(196)^3 \approx 10^{-7}$$

Muonic hydrogen : μ-p



E_{NS}=0.96 THz (2%)

Experiment: CREMA (Charge Radius Experiment with Muonic Atoms) international collaboration

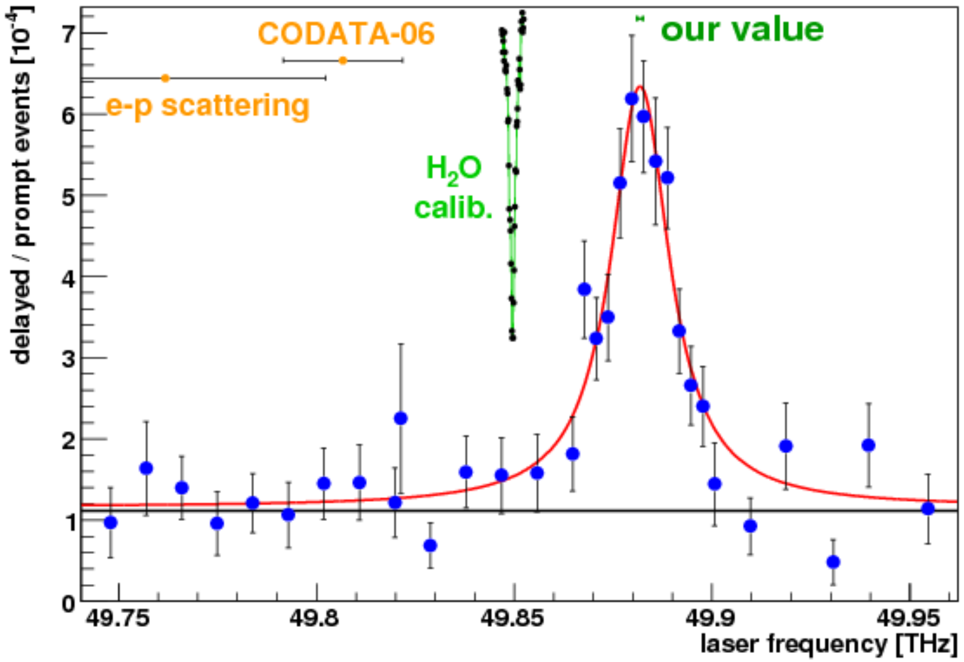


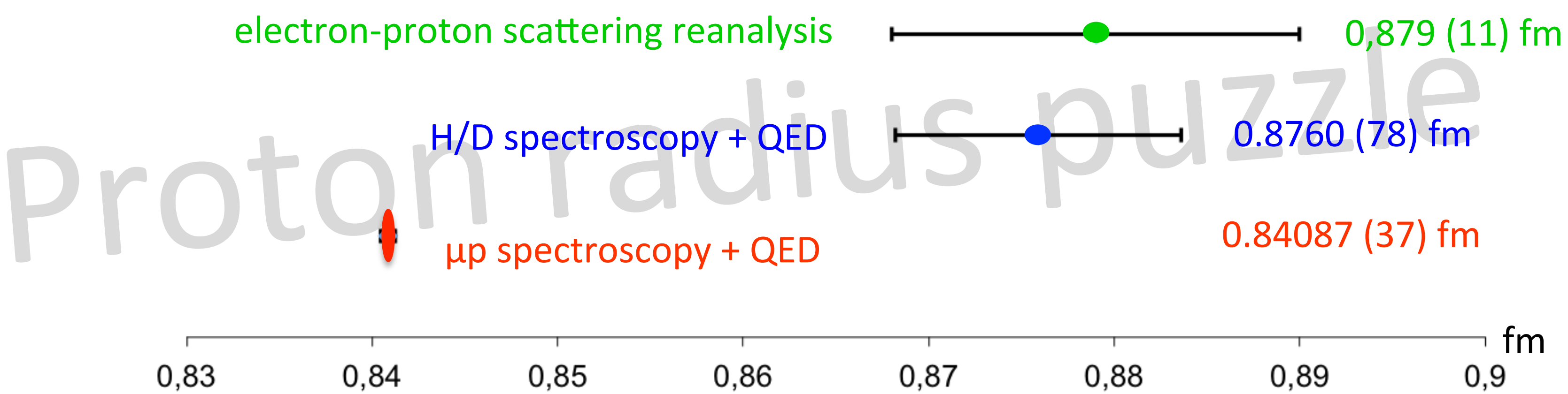
PAUL SCHERRER INSTITUT PSI

Challenges

- production of μp in 2S
- powerful trigger able 6μm laser
- small signal analysis

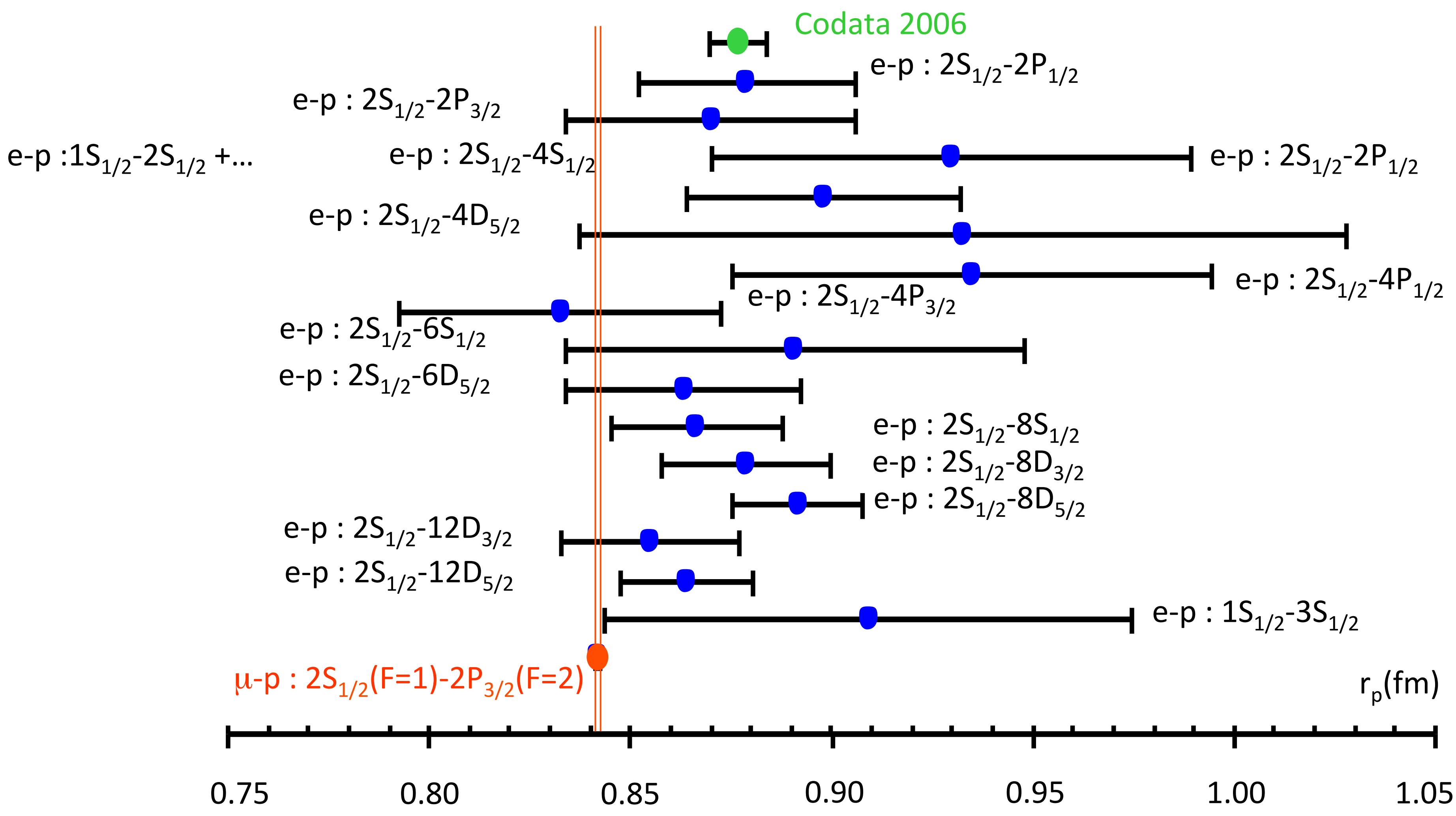
Proton radius puzzle hydrogen: $R_\infty \leftrightarrows r_p$





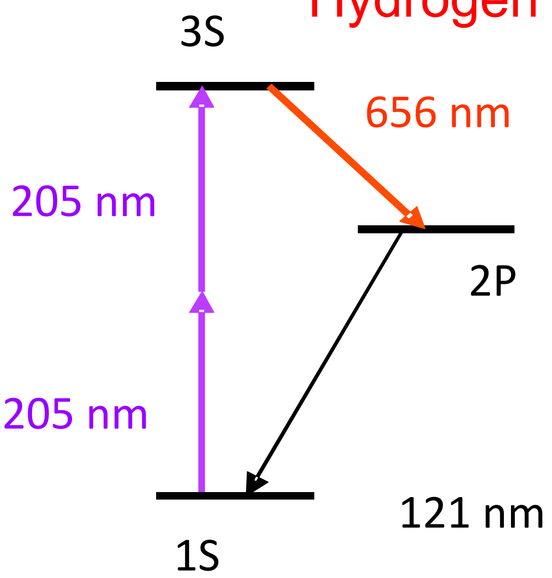
$$E(n,l,j) = \text{Dirac} + \text{recoil} + LS(n,l,j) = hcR_\infty\, f(\alpha, m_e/m_p, n,l,j) + hcR_\infty\, g(\alpha, m_e/m_p, n,l,j, r_p)$$

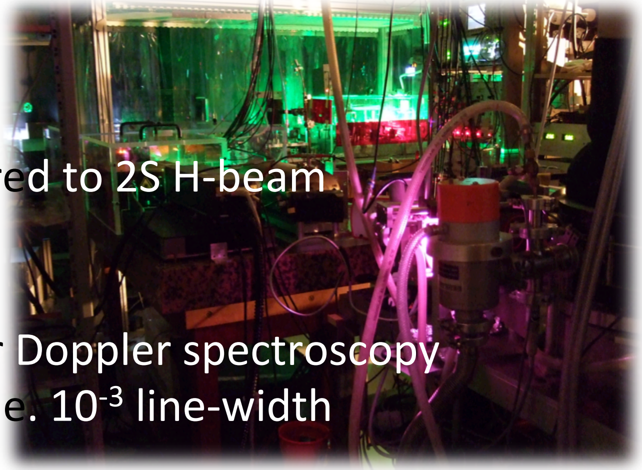
Proton radius puzzle hydrogen: $R_\infty \leftrightarrows r_p$



$E(n,l,j) = \text{Dirac + recoil} + LS(n,l,j) = hcR_\infty\, f(\alpha, m_e/m_p, n,l,j) + hcR_\infty\, g(\alpha, m_e/m_p, n,l,j, r_p)$

Hydrogen spectroscopy : 1S-3S LKB





Advantages

More atoms in 1S beam compared to 2S H-beam

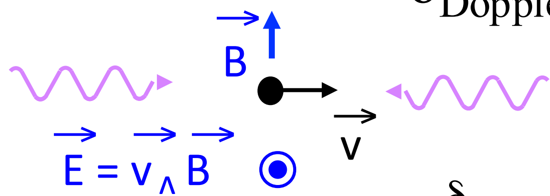
Difficulties

- Laser @ 205nm
- No "easy" optical transition for Doppler spectroscopy
- Aim for 1S-3S frequency 1kHz i.e. 10^{-3} line-width

CW laser source @ 205 nm 15mW

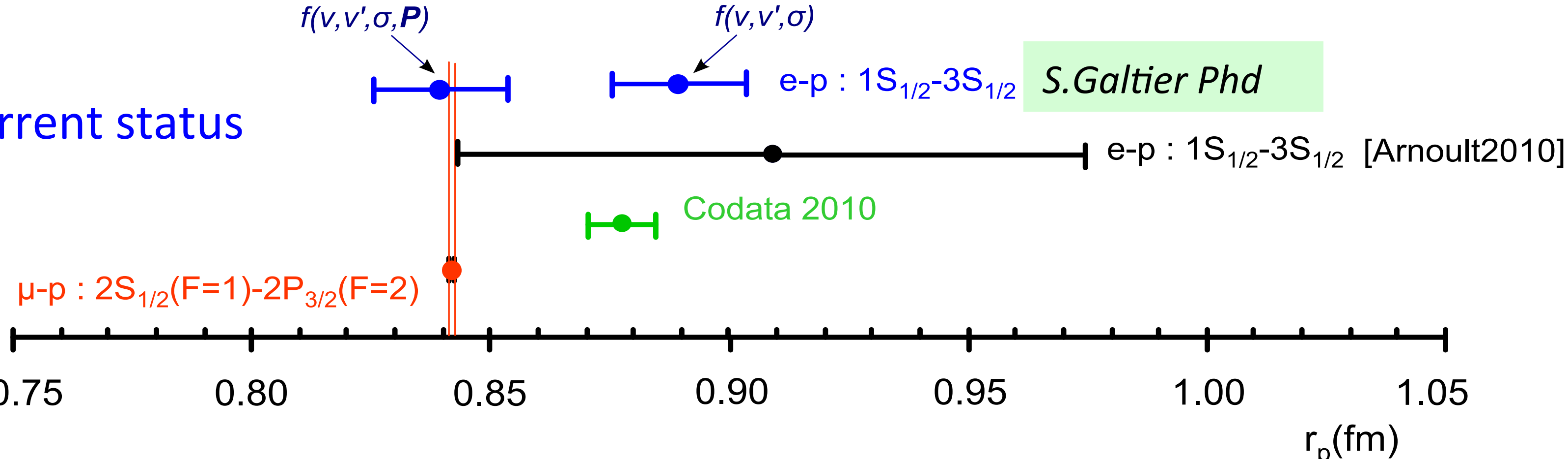
Compensation of 2nd order Doppler effect

$$\delta_{Doppler} = -\nu_{atomic} \frac{v^2}{2c^2}$$



$$\delta_{Stark} = \frac{E^2}{\Delta\nu_{SP}} = \frac{v^2 B^2}{\Delta\nu_{SP}}$$

Current status



Rydberg constant in 2017/2018 ?

2S-2P transition York university (E. Hessels) : “Ramsey method”

Measured @ 9kHz Lundeen and Pipkin PRL 72, 1172 (1994)

Γ(2S-2P)=100MHz proton radius : 11 kHz i.e. 10^{-4} of the linewidth

☺ : 2S-2P mainly QED weak dependence on the Rydberg constant, RF source well known

☹ : large line width 100MHz, lineshape controlled at 10^{-4} !

$^{20}Ne^{9+}$ Rydberg states NIST : U. D. Jentschura et al, PRL 100, 160404 (2008)

☺ : Rydberg states : high energy levels

- no contribution of the nucleus structure, QED well known ($1/n^3$)
- Direct measurement of the Rydberg constant

☹ : production of the ion $^{20}Ne^{9+}$

2S-4P transition MPQ Garching : Ann. Phys. (Berlin) 525 n°8-9 671-679 (2013)

Aim few kHz Γ(2S-4P)=13 MHz i.e. 10^{-3} of the linewidth

☺ : cold hydrogen source, one ph transition weak laser power needed

☹ : transverse excitation but OK now, controlled of the linewidth @ 10^{-3} quantum interference

$\nu(1S_{1/2}-2S_{1/2}) + r_p(\mu p)$: $R_\infty = R_\infty$(Codata)-110kHz ↔ 3.4×10^{-11}, ~5σ

(u=10Hz)

↕ u=19kHz, 5.9×10^{-12}

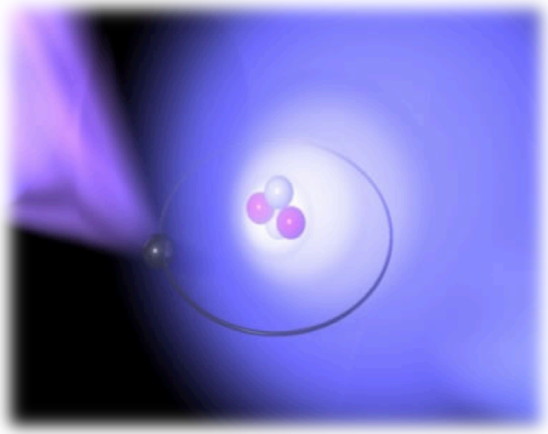
R_∞(2014) : hN_A = 3.9903127110(18)x10^{-10} J s mol^{-1}

R_∞(2018) ? : hN_A = 3.9903127111(18)x10^{-10} J s mol^{-1}

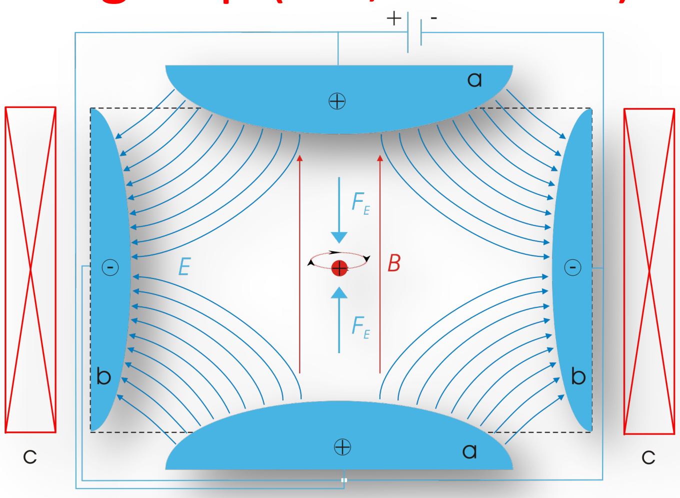
Rydberg constant and/or proton radius puzzle is not a limiting factor for hN_A

II. Relative atomic mass of the electron :

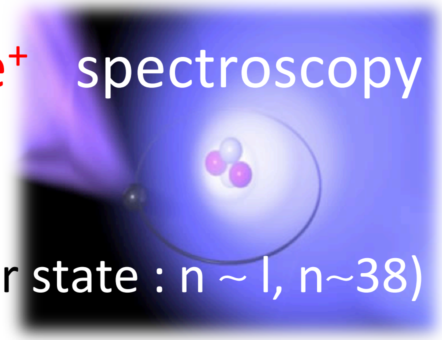
- $\bar{p}$He spectroscopy



- Penning trap (ion, electron)



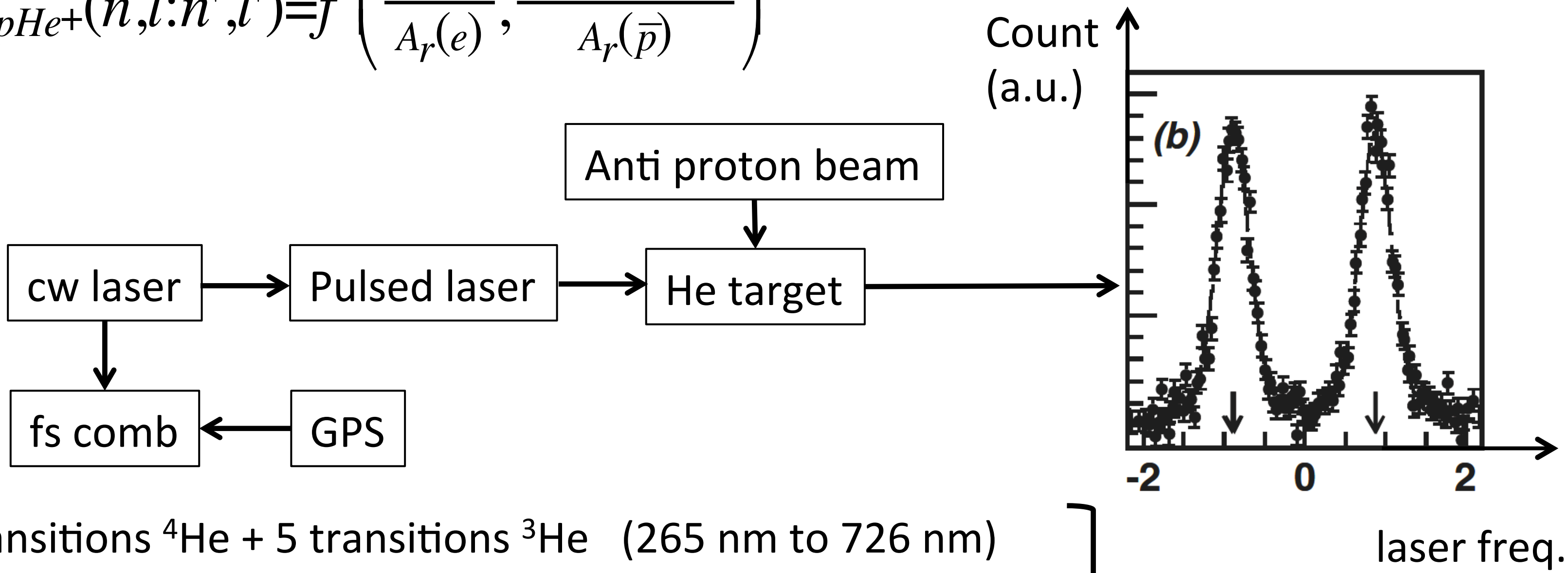
II. Relative atomic mass of the electron : $\bar{p}He^+$ spectroscopy



Anti proton ($\bar{p}$) facility at CERN

$\bar{p}He^+$ atom : $^4He^{2+}$ or $^3He^{2+}$ nucleus + e^- (1S state) + $\bar{p}$ (circular state : n ~ l, n~38)

$$\upsilon_{pHe^+}(n,l:n',l') = f\left(\frac{A_r(\bar{p})}{A_r(e)}, \frac{A_r(Nucleus)}{A_r(\bar{p})}\right)$$



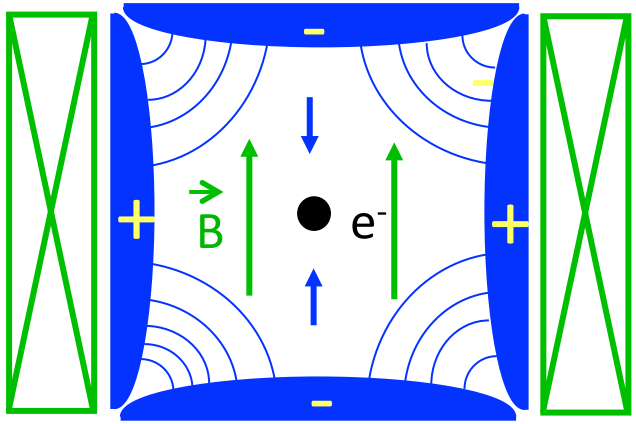
- 7 transitions ^{4}He + 5 transitions ^{3}He (265 nm to 726 nm)
- theory (V. Korobov)
- $\left|\frac{q}{m}\right|_p = \left|\frac{q}{m}\right|_{\bar{p}}$ at 9×10^{-11} $\mapsto$ $A_r(p)=A_r(\bar{p})$ (trap G. Gabrielse)
- A_r (Nucleus), $A_r(p)$: from Penning trap

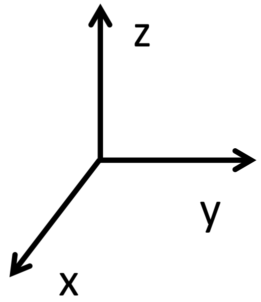
$A_r(e)$ @ 1.7×10^{-9}

M. Hori et al, PRL 96, 243401 (2006)

Penning trap

A quadrupolar electric potential is applied which confine the electron along the z axis



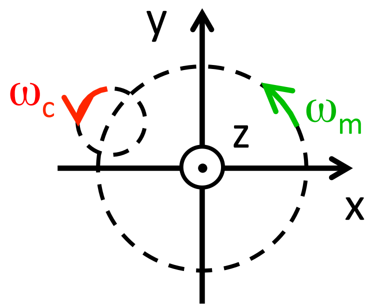


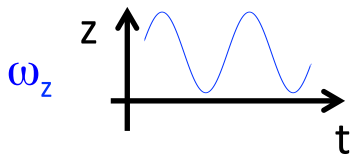
The transverse confinement is obtained by the application of the magnetic field

$$\omega_c = \frac{eB}{m_e}$$

The electron movement is the sum of :

- a cyclotron rotation at a slightly modified frequency
- an oscillation along the z axis
- a slow rotation at the magnetron frequency





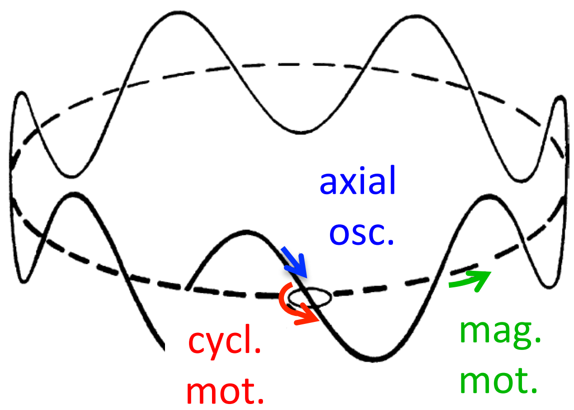
$$\omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2$$

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\left(\frac{\omega_c^2}{2}\right) - \frac{\omega_z^2}{2}}$$

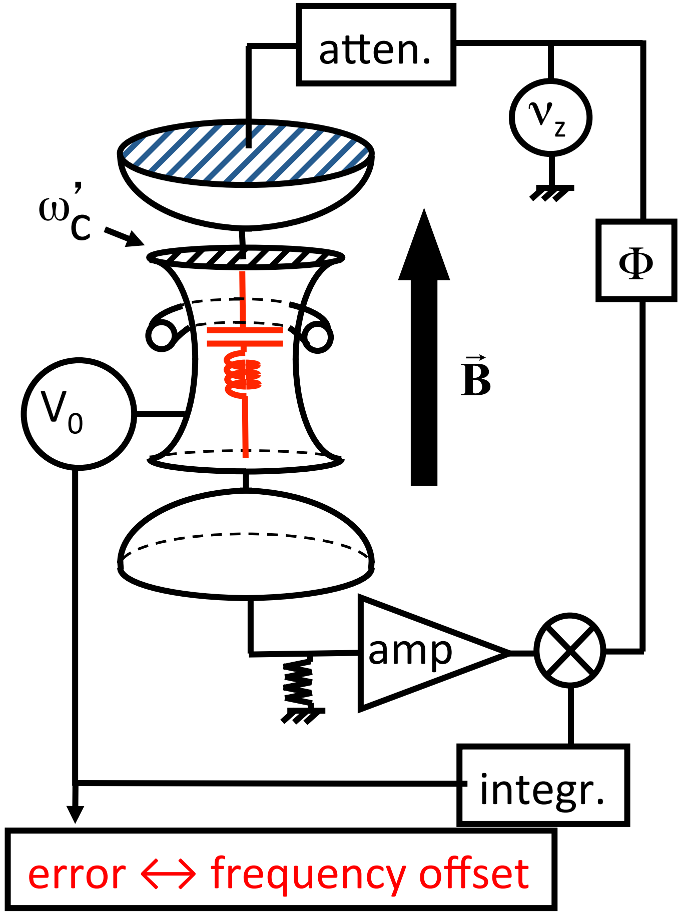
Modified cyclotron frequency

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\left(\frac{\omega_c^2}{2}\right) - \frac{\omega_z^2}{2}}$$

Modified magnetron frequency



Penning trap



Ni ring→ magnetic bootle

→~Stern-Gerlach : spin's info on z axis

$$\omega_c = \frac{eB}{m_e} \qquad \omega_L = g\frac{eB}{2m_e}$$

$$\nu_z \approx \nu_{z0} + \left(n + m + \frac{1}{2}\right)\delta$$

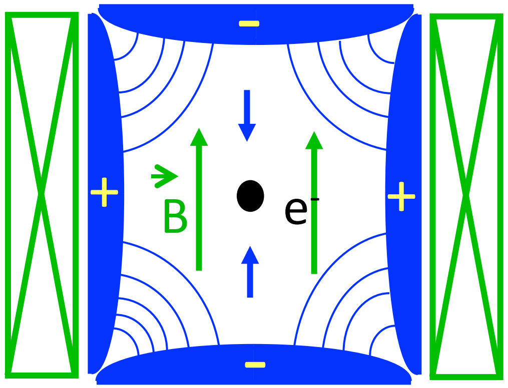
cyclotron (n)

spin (m)

depend on the trap (1,3Hz) (δ)

« Quantum electrodynamics » ed T. Kinoshita

Electron mass from the ratio of cyclotron frequencies of an electron and $^{12}C^{6+}$ ion



$$f_c(e^-) = \frac{eB}{2\pi\, m_e}$$

$$f_c(^{12}C^{\,6+}) = \frac{6eB}{2\pi\, m(^{12}C^{\,6+})}$$

$$\frac{f_c(^{12}C^{\,6+})}{f_c(e^-)} = \frac{6\, A_r(e)}{A_r(^{12}C^{\,6+})}$$

$$A_r(^{12}C^{\,6}) = 12 = A_r(^{12}C^{\,6+}) + 6\, A_r(e) - \frac{E_b^2(^{12}C)}{m_u c^2}$$

$\rightarrow A_r(e)$

also $A_r(p)$

Difficulties :
- Drift of B field
- Two different masses : ≠ running conditions of the trap (potential)
- Positive and negative charges

CODATA 98 : $f_c(^{12}C^{5+})$ and $f_c(e^-)$ → → $A_r(e)$ @ 2.1 x 10^{-9}

Van Dyck et al Phys. Rev. Lett. **75** (20) 3598 (1995)

$$m(X)\, c^2 = m(Nucleus)\, c^2 + Z\, m_e\, c^2 - E_b^2$$

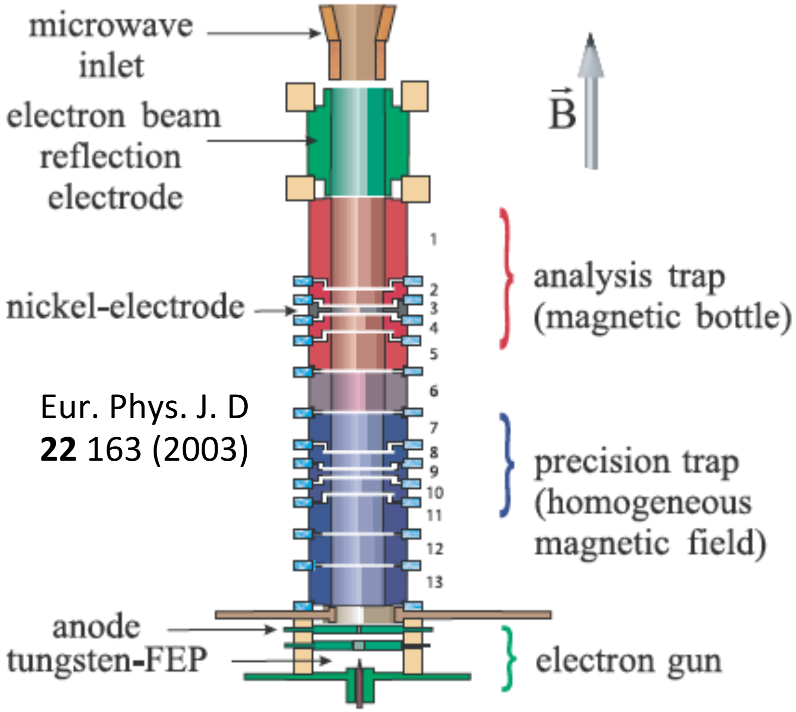
$$A_r(X) = A_r(Nucleus) + Z\, Ar(e) - \frac{E_b^2}{m_u c^2}$$

E_b binding energy (calculated precisely enough)

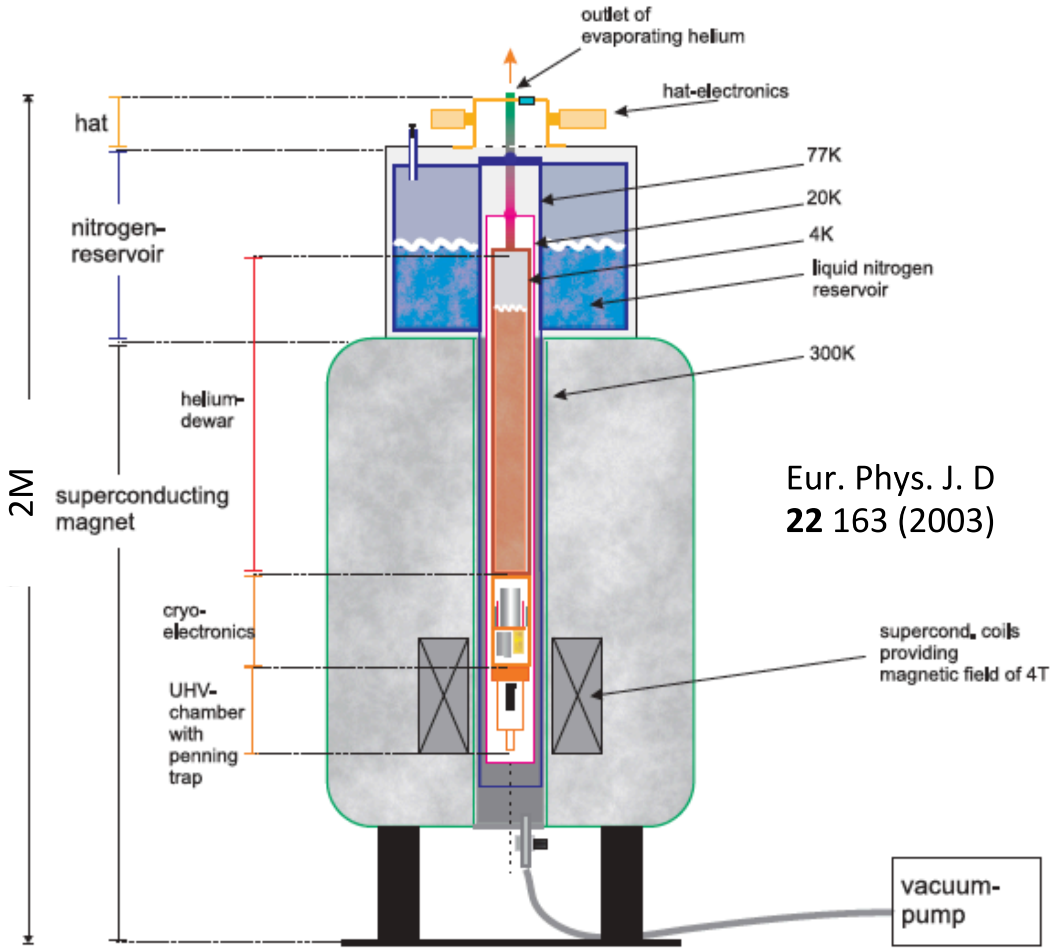
Electron mass from the ratio of cyclotron frequency to the spin-flip frequency of an ion

$$f_s\left({}^{A}X^{(Z\text{-}1)+}\right) = \frac{g_e\left({}^{A}X^{(Z\text{-}1)+}\right)}{h}\,\frac{eh}{4\pi\, m_e}B$$

$$f_c\left({}^{A}X^{(Z\text{-}1)+}\right) = \frac{(Z\text{-}1)e}{2\pi\, m\left({}^{A}X^{(Z\text{-}1)+}\right)}B$$



Eur. Phys. J. D **22** 163 (2003)



Eur. Phys. J. D **22** 163 (2003)

- Analysis trap : Ni ring → spin flip detection yes/no ?
- Frequency to induce spin flip : 104 MHz → shift of 0.7Hz on axial motion frequency (364kHz)
- Adiabatic transfer between the two trap 3cm in less than 1s
- Cyclotron frequency is measured simultaneously with the attempt of spin flip
  → drift B cancelled (1st order)

Electron mass from the ratio of cyclotron frequency to the spin-flip frequency of an ion

$$f_s\left({}^{A}X^{(Z-1)+}\right) = \frac{g_e\left({}^{A}X^{(Z-1)+}\right)}{h}\frac{eh}{4\pi m_e}B \qquad f_c\left({}^{A}X^{(Z-1)+}\right) = \frac{(Z-1)e}{2\pi m\left({}^{A}X^{(Z-1)+}\right)}B$$

Measured on the experiment

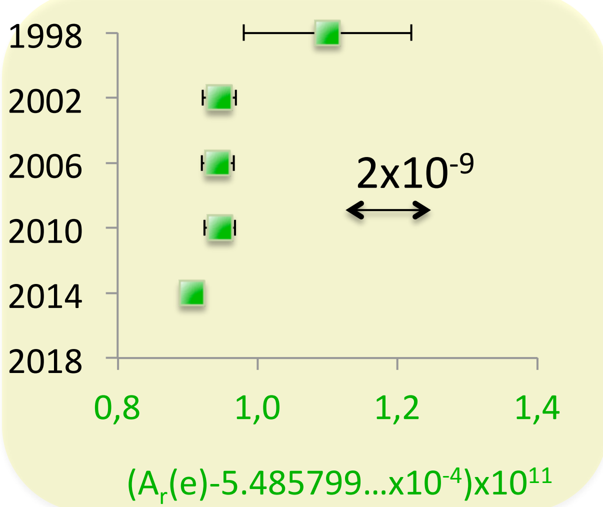
$$\frac{f_s\left({}^{A}X^{(Z-1)+}\right)}{f_c\left({}^{A}X^{(Z-1)+}\right)} = \frac{g_e\left({}^{A}X^{(Z-1)+}\right)}{2(Z-1)}\frac{A_r\left({}^{A}X^{(Z-1)+}\right)}{A_r(e)}$$

Measured with ions traps : pair of ions trapped $\Delta m/m \sim 10^{-3}$

QED :

$$g_{e^-}(X) = g_D + \Delta g_{rad} + \Delta g_{rec} + \Delta g_{Nucl.size} + \ldots$$

$$g_D = -\frac{2}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right)$$



CODATA 2002 : $f_c(^{12}C^{5+})$ and $f_c(e^-)$ → $A_r(e)$ @ 2.1 x 10^{-9}
$g_{e-}(^{12}C^{5+})$, $g_{e-}(^{16}O^{7+})$ → $A_r(e)$ @ 7.8 x 10^{-10} and 9.0 x 10^{-10}

CODATA 2006 : $f_c(^{12}C^{5+})$ and $f_c(e^-)$ → $A_r(e)$ @ 2.1 x 10^{-9}
$g_{e-}(^{12}C^{5+})$, $g_{e-}(^{16}O^{7+})$ → $A_r(e)$ @ 7.8 x 10^{-10} and 9.0 x 10^{-10}
$\bar{p}He^+$ spectroscopy → $A_r(e)$ @ 1.7 x 10^{-9}

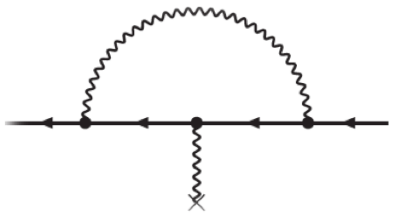
CODATA 2010 : $f_c(^{12}C^{5+})$ and $f_c(e^-)$ → $A_r(e)$ @ 2.1 x 10^{-9}
$g_{e-}(^{12}C^{5+})$, $g_{e-}(^{16}O^{7+})$ → $A_r(e)$ @ 5.2 x 10^{-10} and 7.6 x 10^{-10} (new QED)
$\bar{p}He^+$ spectroscopy → $A_r(e)$ @ 1.4 x 10^{-9}

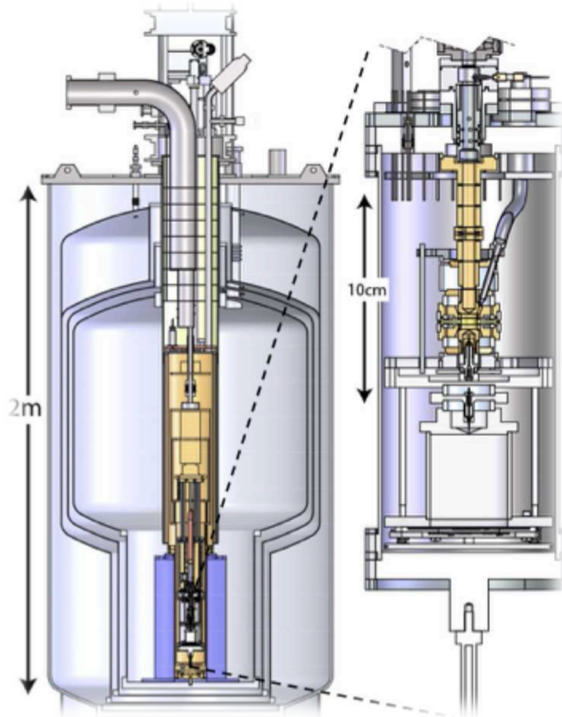
CODATA 2014 : $f_c(^{12}C^{5+})$ and $f_c(e^-)$ → $A_r(e)$ @ 2.1 x 10^{-9}
$g_{e-}(^{12}C^{5+})$, $g_{e-}(^{28}Si^{13+})$ → $A_r(e)$ @ 3.1 x 10^{-11} and 8.3 x 10^{-10} (new meas^t and new QED)
$\bar{p}He^+$ spectroscopy → $A_r(e)$ @ 1.4 x 10^{-9}

Electron mass determination in atomic mass unit for is not a limiting factor for hN_A and for α

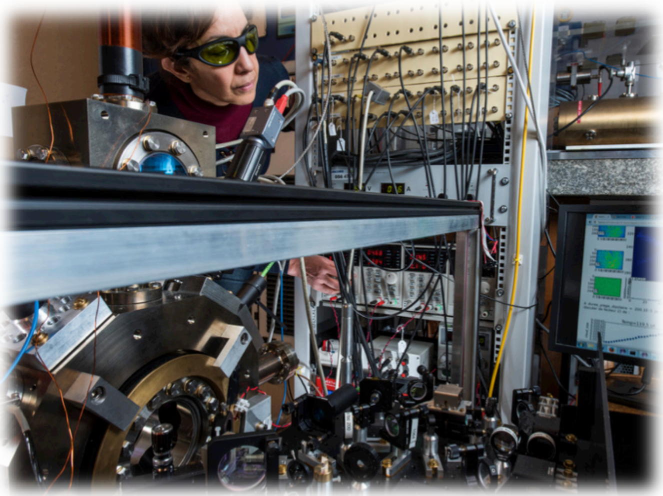
III. Fine structure constant :

- e^- magnetic moment anomaly,

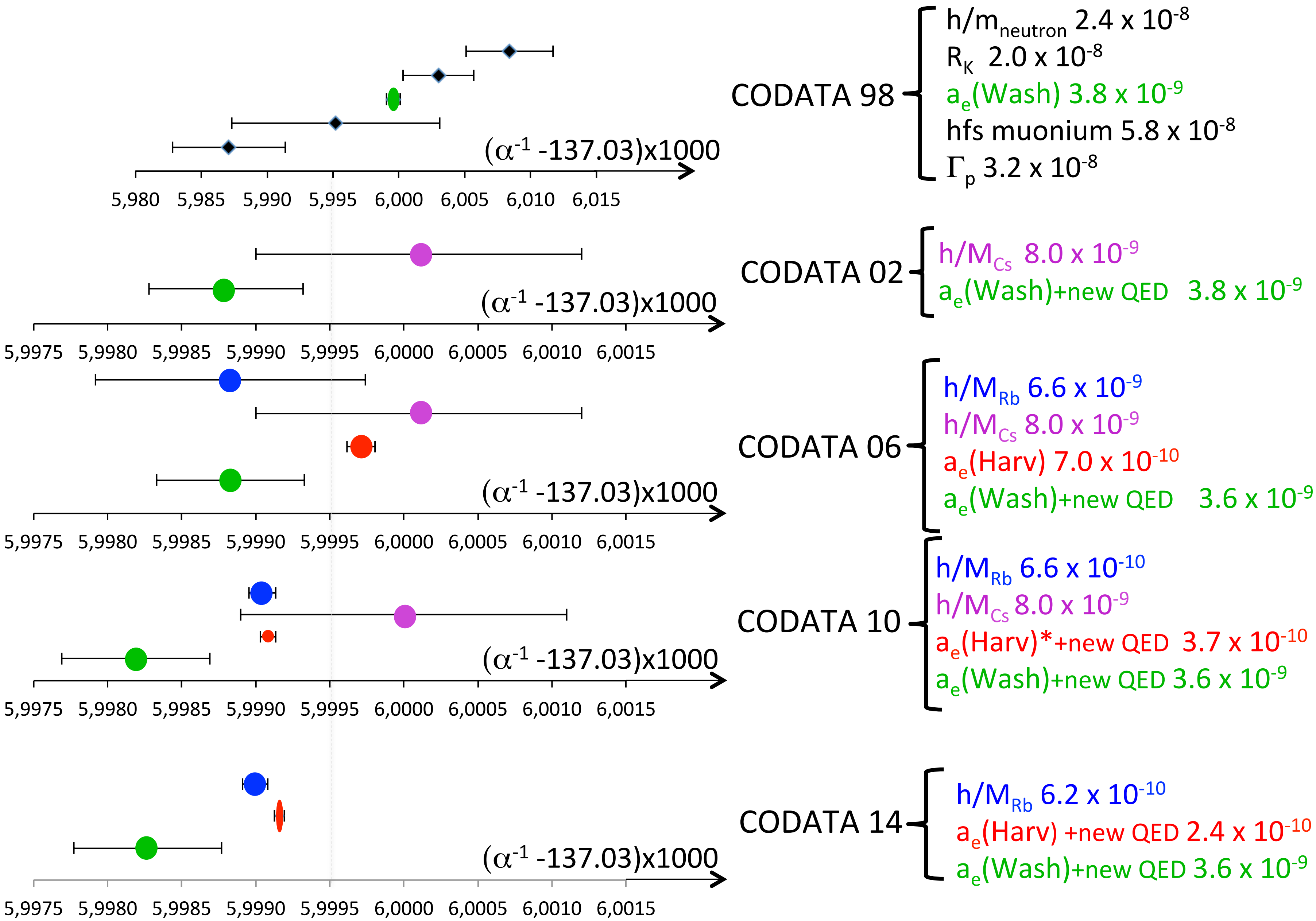




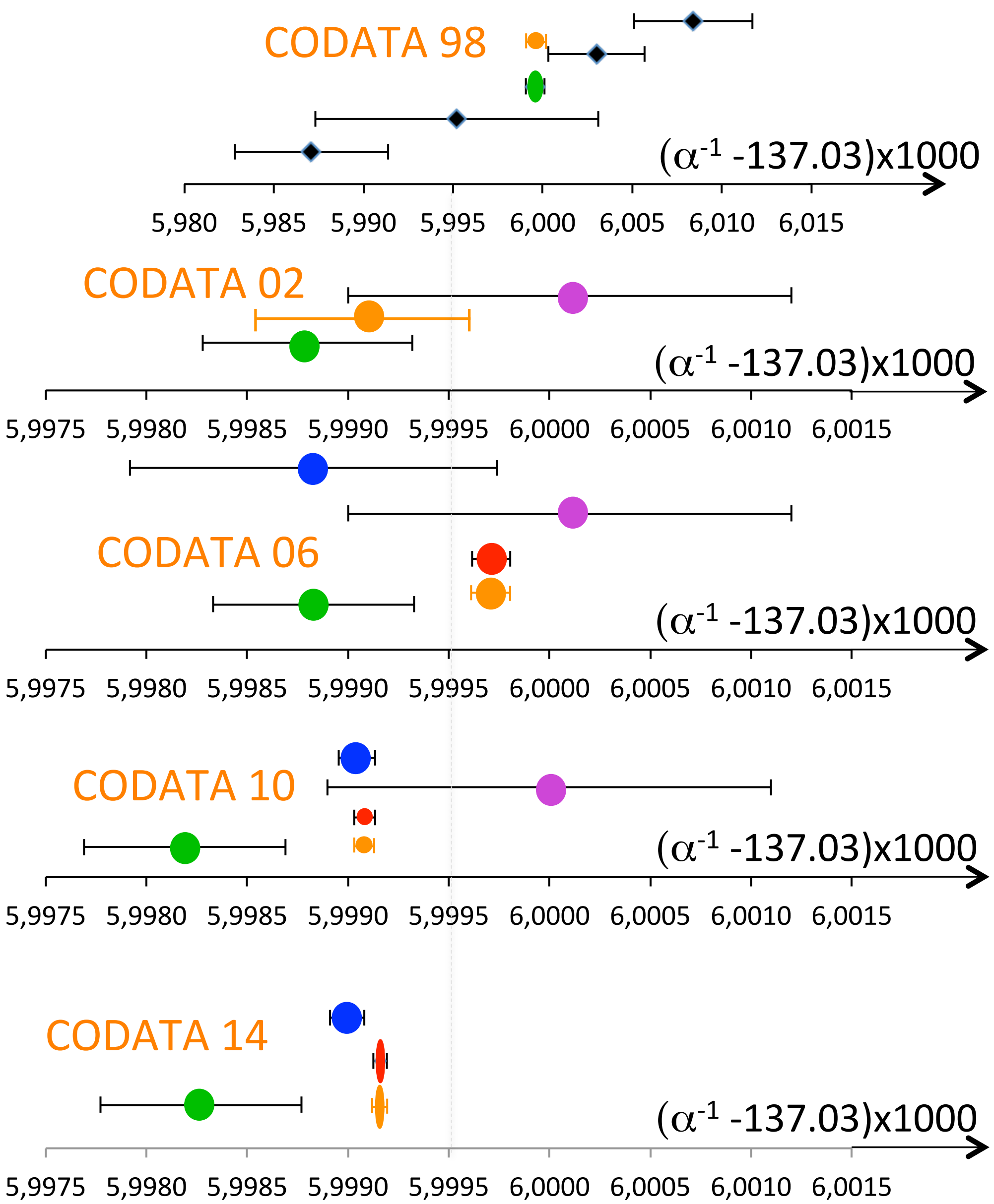
- atom interferometry.



Determinations of the fine structure constant



Determinations of the fine structure constant



CODATA 98
- $h/m_{neutron}$ 2.4×10^{-8}
- R_K 2.0×10^{-8}
- a_e(Wash) 3.8×10^{-9}
- hfs muonium 5.8×10^{-8}
- Γ_p 3.2×10^{-8}

CODATA 02
- h/M_{Cs} 8.0×10^{-9}
- a_e(Wash)+new QED 3.8×10^{-9}

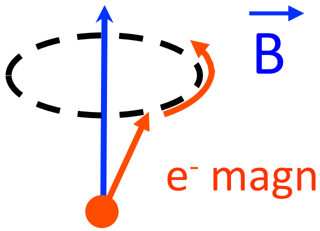
CODATA 06
- h/M_{Rb} 6.6×10^{-9}
- h/M_{Cs} 8.0×10^{-9}
- a_e(Harv) 7.0×10^{-10}
- a_e(Wash)+new QED 3.6×10^{-9}

CODATA 10
- h/M_{Rb} 6.6×10^{-10}
- h/M_{Cs} 8.0×10^{-9}
- a_e(Harv)*+new QED 3.7×10^{-10}
- a_e(Wash)+new QED 3.6×10^{-9}

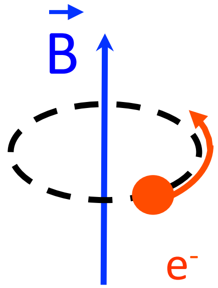
CODATA 14
- h/M_{Rb} 6.2×10^{-10}
- a_e(Harv) +new QED 2.4×10^{-10}
- a_e(Wash)+new QED 3.6×10^{-9}

Measurements of the electron g-factor

For a free electron the g-factor is simply deduced from $\frac{g}{2} = \frac{\omega_L}{\omega_C}$



Larmor frequency (spin) $$\omega_L = g\frac{eB}{2m_e}$$



and the cyclotron frequency (Lorentz) $$\omega_c = \frac{eB}{m_e}$$

and its anomaly is defined as $$a_e = \frac{g_e - 2}{2}$$ $a_e > 0$

In Nov. 1947, the first determination of a_e was performed by Kusch and Foley by Zeeman splitting in an atomic beam magnetic resonance experiment with Ga and then in Na and In (Apr. 1948)

Their result was $a_e = 0.00119\ (5)$

in agreement with the prediction of Schwinger (1948) $$a_e = \frac{\alpha}{2\pi} = 0.001162$$

Beginning of comparison theory-experiment of the g-2

Measurements of the electron g-factor

Nowadays experimental method : Study of transitions induced by a RF field in a Penning trap in a given magnetic field (Washington, Mainz, Stanford, Harvard)

The energy levels of one electron in a magnetic field are given by :

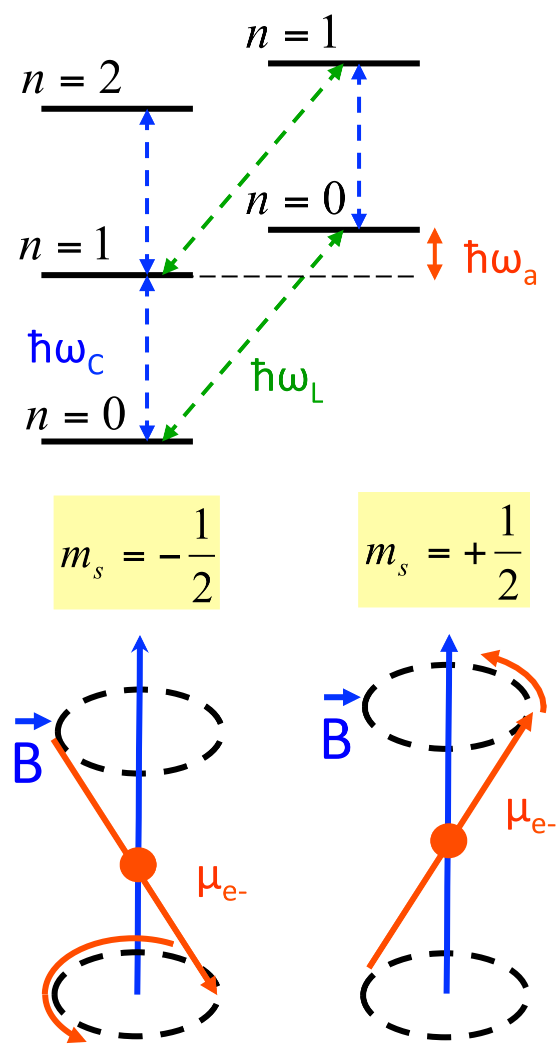
$$E(n,m_s) = \left(n + \frac{1}{2}\right)\hbar\omega_c + m_s\,\hbar\omega_L$$

where $$\frac{\omega_L}{\omega_c} = \frac{g}{2} = 1 + \frac{\omega_a}{\omega_c}$$

and ω_a is the anomaly frequency $\omega_a = \omega_L - \omega_c$

directly related to a_e $$a_e = \frac{\omega_a}{\omega_c}$$

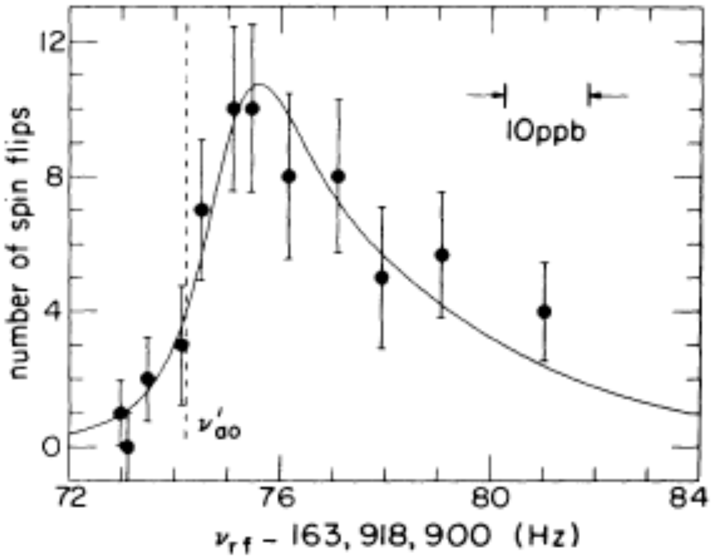
Rabi-Landau levels

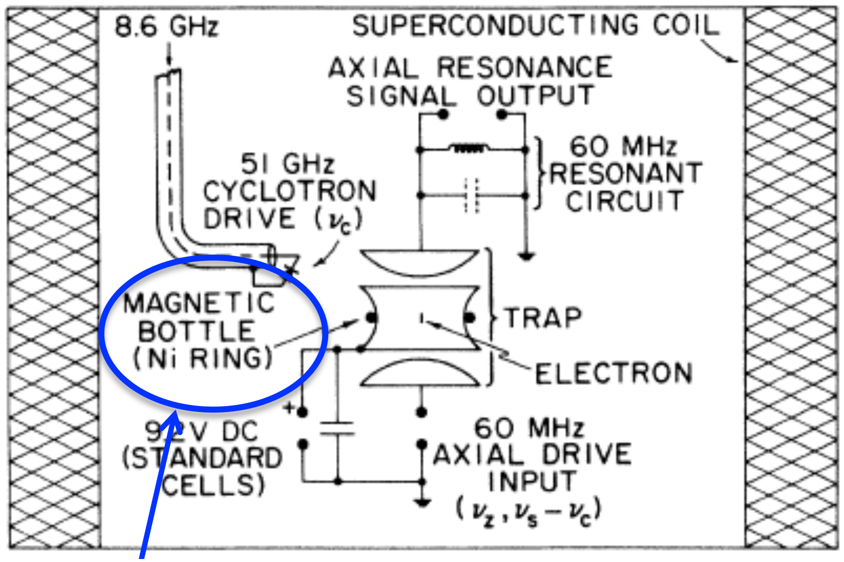


Measurements of the electron g-factor : Pioneer work in Washington

- Penning trap at 4K
- single electron stored

Measurement of the cyclotron frequency and of the anomaly frequency





Detection of spin-flip through the induced shift of the axial frequency

$$\nu_z \approx \nu_{z0} + \left(n + m + \frac{1}{2}\right)\delta$$

depend on the trap (here 1.3Hz)

Results :

$$g_{e^-}/2 = 1.001\,159\,652\,200\,(40)$$

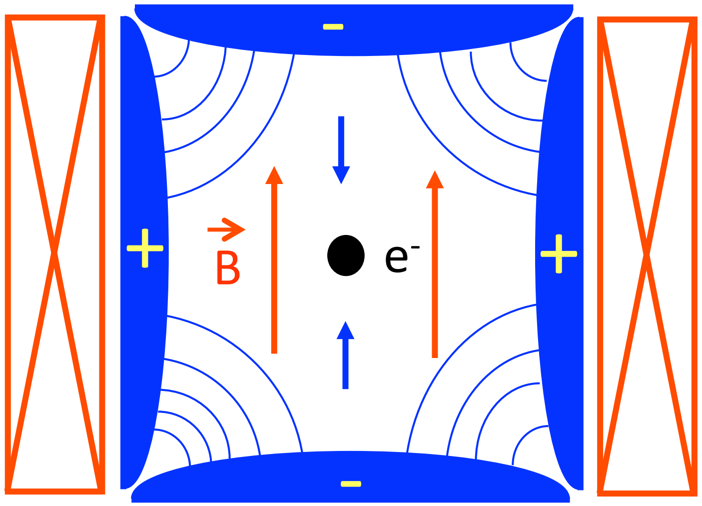
4 x 10^{-11}

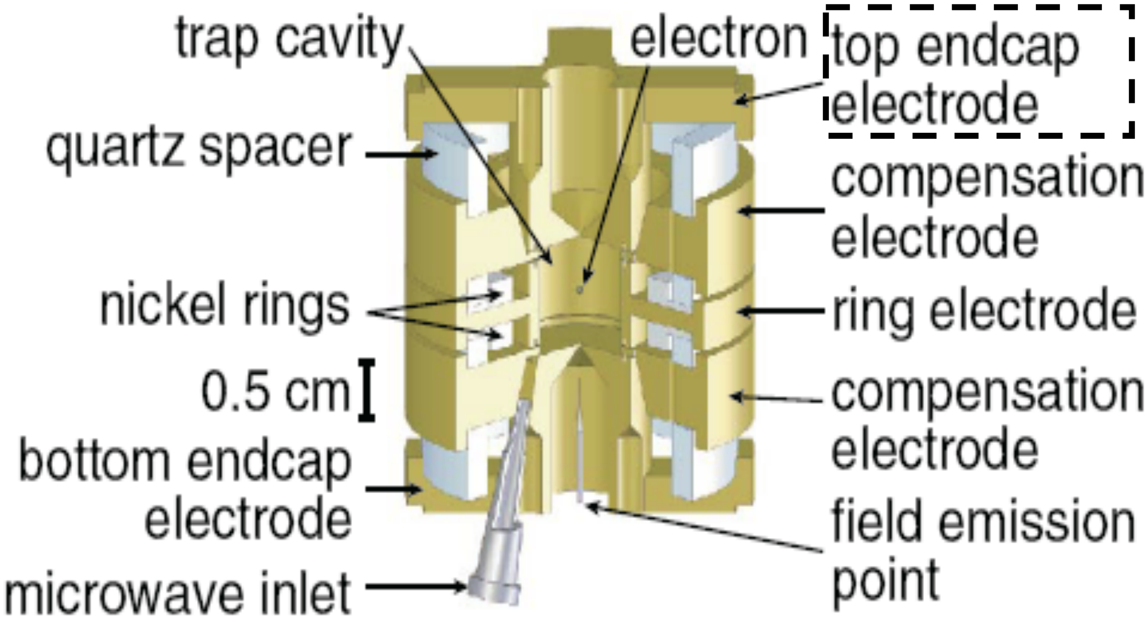
and

$$g_{e^-}/g_{e^+} = 1 + (0.5 \pm 2.1) \times 10^{-12}$$

R.S. Van Dyck Jr, P.B. Schwinger and H.G. Dehmelt, Phys. Rev. D 34, 722 (1986) and Phys. Rev. Lett. 59, 26 (1987)

Measurements of the electron g-factor : the Harvard experiment
the most precise determination of the electron g-factor





- Cylindrical Penning trap invented to form a microwave cavity that could inhibit spontaneous emission (by a factor of up to 250)
→ narrowed line width

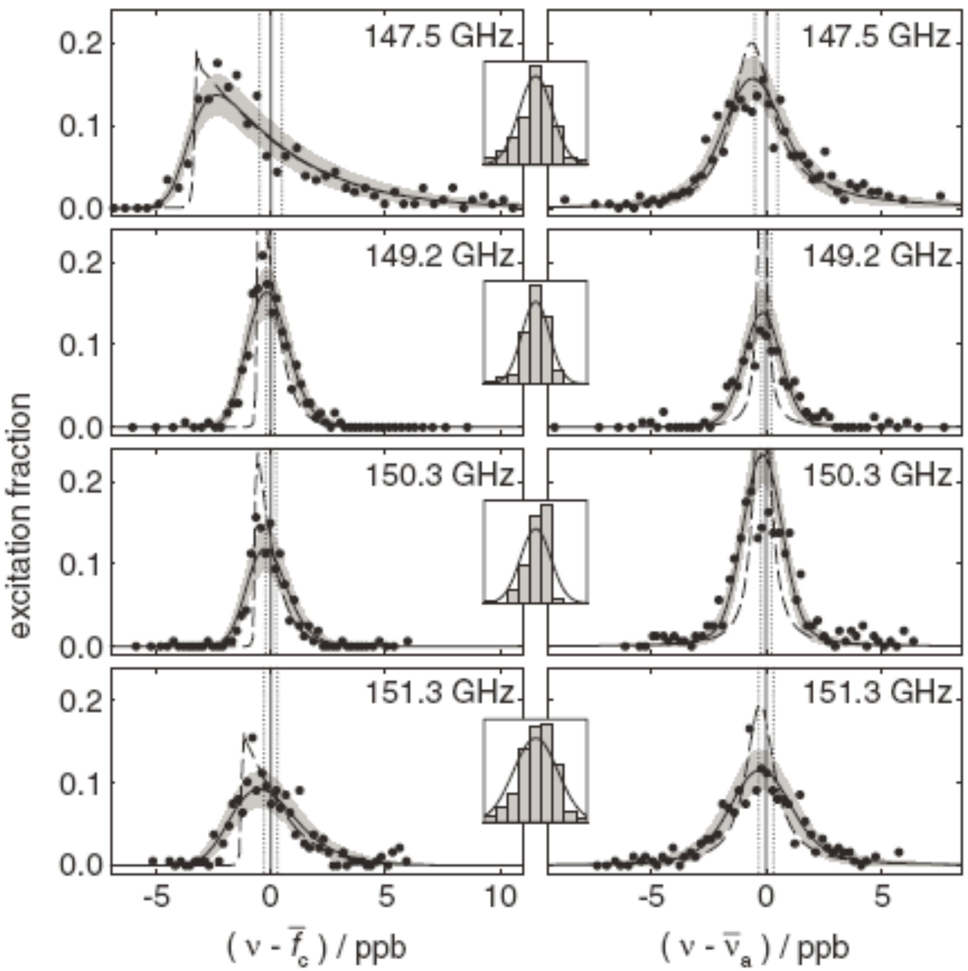
- Trap cavity cooled to 100 mK
→ the electron cyclotron motion is its ground state

- "Calculable" trap
→ careful control and probe of radiation field and magnetic field in the trap cavity

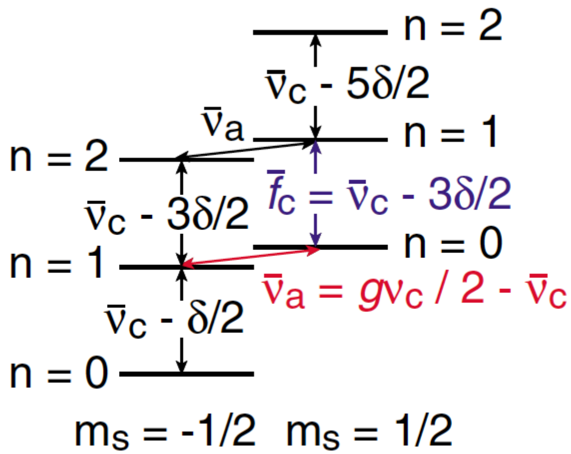
The one quantum change in cyclotron motion is resolved

Measurements of the electron g-factor : the Harvard experiment
the most precise determination of the electron g-factor

Quantum-jump spectroscopy : measuring the quantum jumps per attempt to drive them as a function of drive frequency (different modes of the trap cavity)



cyclotron transitions anomaly transitions



Result :

in 2006

$$g/2 = 1.001\ 159\ 652\ 180\ 85\ (76)$$

7.6×10^{-13}

in 2008

$$g/2 = 1.001\ 159\ 652\ 180\ 73\ (28)$$

2.8×10^{-13}

B. Odom *et al.*, Phys. Rev. Lett. 97, 030801 (2006)
D. Hanneke *et al*, Phys. Rev. Lett. 100, 120801 (2008) and Phys. Rev. A 83, 052122 (2011)

electron anomaly : discussion

The last result obtained in Harvard is :

$$a_e = 1\ 159\ 652\ 180.73\ (0.28) \times 10^{-12}$$ 2.4 x 10^{-10}

- Taking into account the presence of the muon and tau particles, the QED contribution to the electron g - 2 can be written :

$$a_e = A_1 + A_2(m_e/m_\mu) + A_2(m_e/m_\tau) + A_3(m_e/m_\mu, m_e/m_\tau)$$

where $A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + .A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 ..$ and $A_1^{(2)} = \frac{1}{2}$

- Since the experimental uncertainty is less than 1% of $\left(\frac{\alpha}{\pi}\right)^4 \approx 29 \times 10^{-12}$ the coefficient $A_1^{(8)}$ is needed to match the precision of theory with experiment
- In addition, the total non QED (hadronic) contribution to a_e is 1.72(2) x 10^{-12}

see :T. Kinoshita in *Lepton dipole moments*, Ed. World Scientific (2010)

But the comparison of theory with measured electron anomaly needs also a value of α obtained by an independent measurement

electron anomaly : discussion

Complexity of QED calculations

$$a_e = A_1 + A_2\left(m_e/m_\mu\right) + A_2\left(m_e/m_\tau\right) + A_3\left(m_e/m_\mu, m_e/m_\tau\right)$$

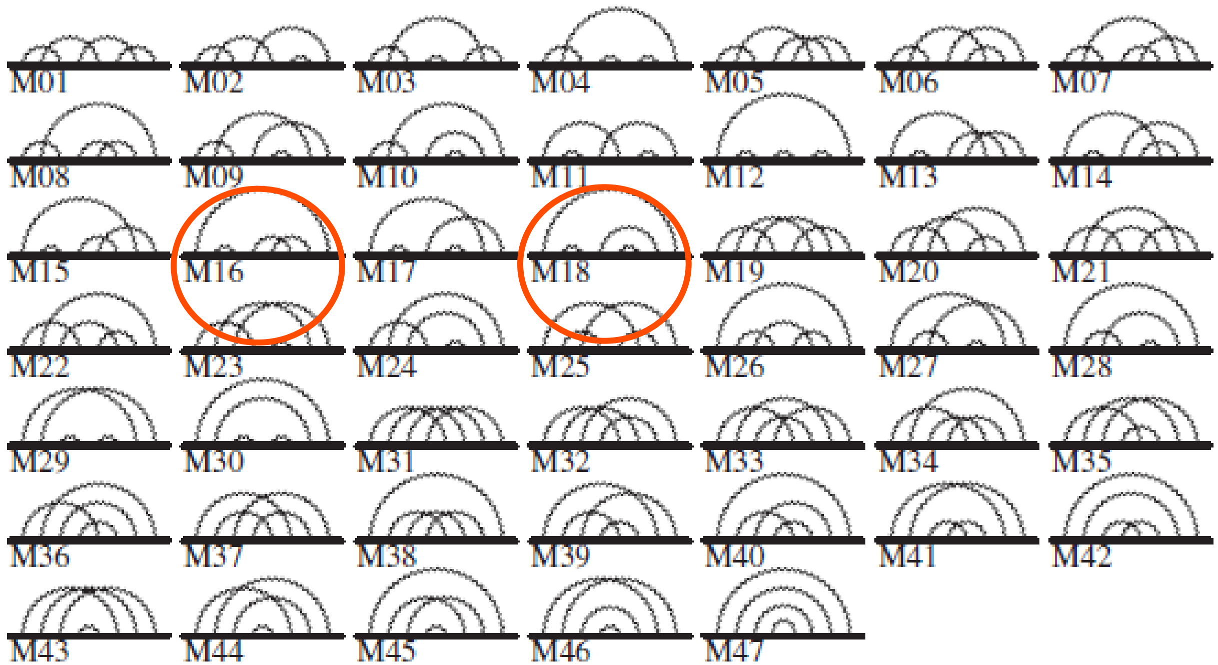
where $$A_i = A_i^{(2)}\left(\frac{\alpha}{\pi}\right) + A_i^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_i^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + .A_i^{(8)}\left(\frac{\alpha}{\pi}\right)^4 ..$$

$$A_1^{(2)} = \frac{1}{2}$$

$$A_1^{(8)} = -1.9144(35)$$

891 Feynman diagrams !
(mostly numerical calculations)

- 373 calculated by 2 independent methods
- 518 "vertex" diagrams amalgamated in 47 diagrams



see : T. Kinoshita in *Lepton dipole moments*, Ed. World Scientific (2010) and ref. therein

electron anomaly and fine structure constant

On another hand, the last measurement of the electron g-factor, combined with recent calculations of $A_1^{(8)}$ and $A_1^{(10)}$ coefficients gives the most precise determination of the fine structure constant

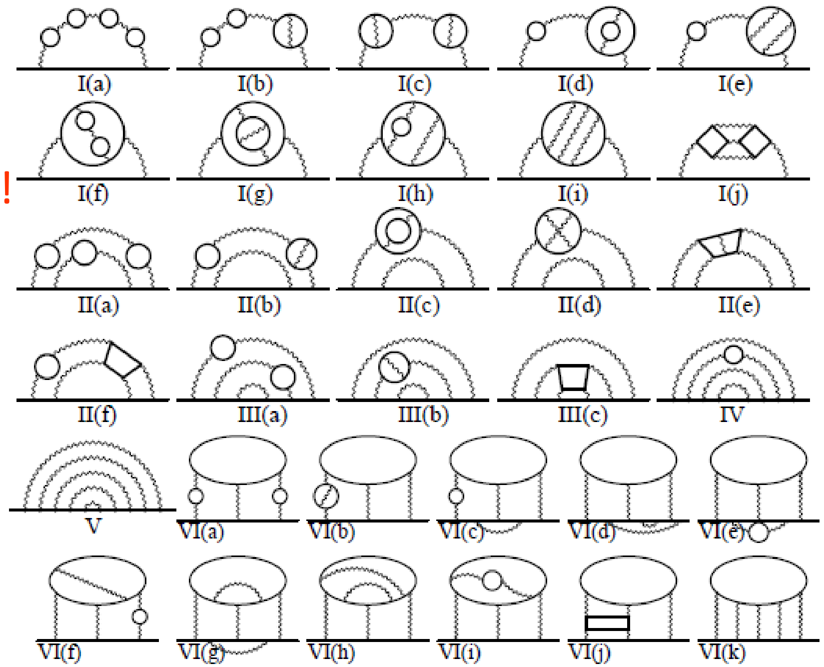
$$\alpha^{-1} = 137.035\,999\,1570\,(334)$$

2.4×10^{-10}

B. Odom *et al.*, Phys. Rev. Lett. 97, 030802 (2006) and 99, 039902 (2007)
D. Hanneke, S. Fogwell and G. Gabrielse, Phys. Rev. Lett. 100, 120801 (2008)
$A_1^{(10)}$: T. Aoyama, M.Hayakawa, T. Kinoshita and M. Nio, Phys. Rev. D **91**(3) 033006 (2015)

12672 diagrams !

$A_1^{(10)} = 9.16(58)$



Determination of the fine structure constant α from h/m

Rydberg constant in terms of energy :

$$h c R_\infty = \frac{1}{2} m_e c^2 \alpha^2$$

$$\alpha^2 = \frac{2R_\infty}{c} \times \frac{m\left(^{87}Rb\right)}{M_P} \times \frac{M_P}{m_e} \times \frac{h}{m\left(^{87}Rb\right)}$$

Bound systems (with hydrogen) back in the α competition

- Rydberg constant : 5×10^{-12} (hydrogen spectroscopy) (CODATA 2010)
- atom-to-proton mass ratio :1.4×10^{-10} (ion trap)
- electron-to-proton mass ratio : 4.2×10^{-10} (ion trap)

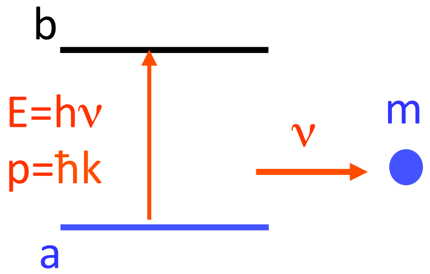
$$\alpha^2 = \frac{2R_\infty}{c} \times \frac{A_r\left(^{87}Rb\right)}{A_r(e)} \times \frac{h}{m\left(^{87}Rb\right)}$$

$Ar(^{87}Rb)$ is the mass of ^{87}Rb in atomic mass unit (ref ^{12}C)
$Ar(e)$ is the electron mass in atomic mass unit (ref ^{12}C)

Determination of the fine structure constant α from h/m

Recoil effect → h/m

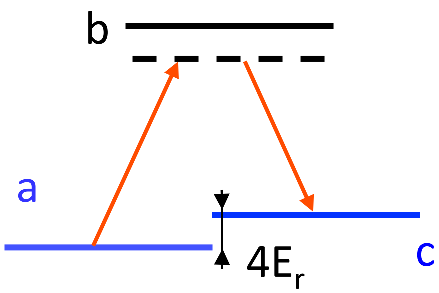
The recoil velocity is directly related to the h/M ratio — J.L. Hall et al, : PRL **37**,1339 (1976)



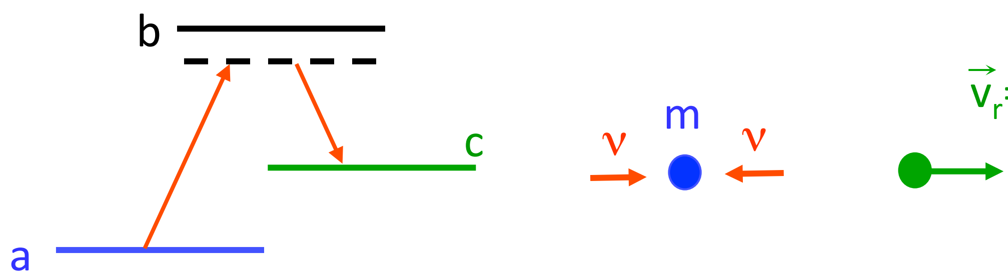
$$\vec{v}_r = \frac{\hbar \vec{k}}{m}$$

and can be measured very precisely in terms of frequency (Doppler effect)

Spontaneous emission → Raman two photon transition



Same internal state



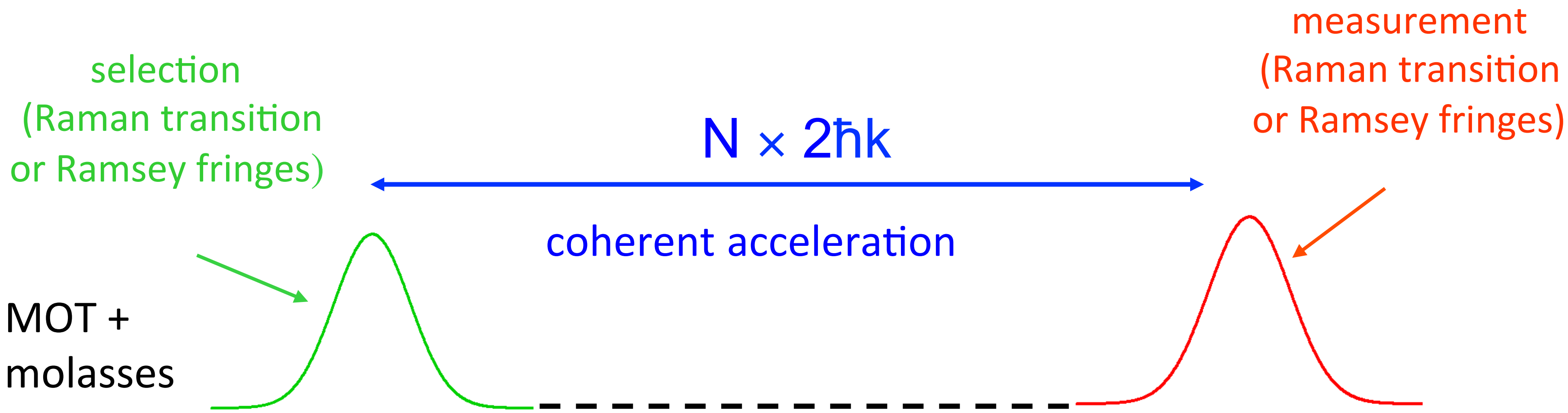
$$\vec{v}_r = 2\text{x}\, \frac{\hbar \vec{k}}{m}$$

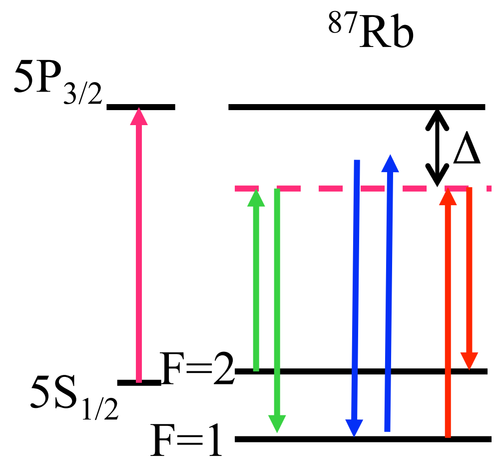
Two different internal states

- Momentum transfer almost perfectly defined
- 2 photon transition → light shift

^{87}Rb v_r~6mm/s @300°K v~300m/s → need to cool atom sample

Principle of our experiment



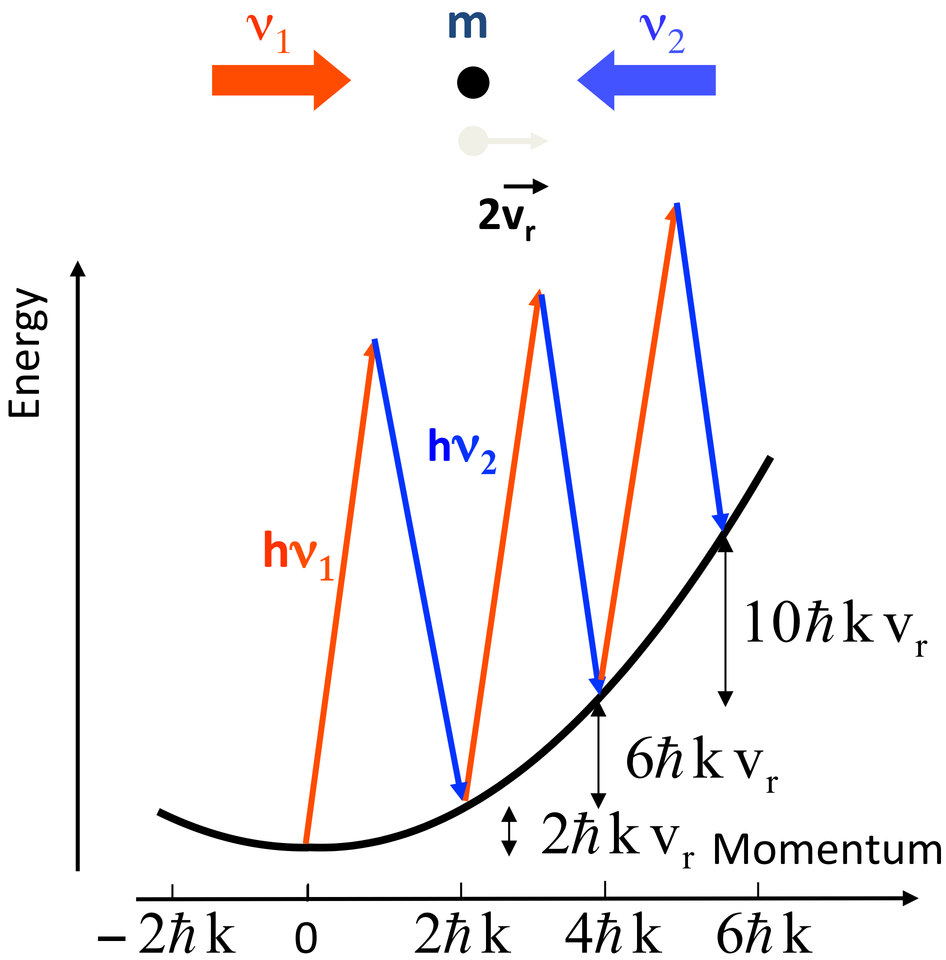


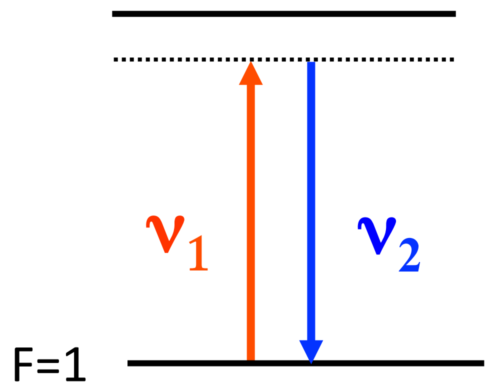
- selection of an initial sub-recoil velocity class
- coherent acceleration : *N* Bloch oscillations, momentum transfer 2Nħk
- measurement of the final velocity class

$$\sigma_{vr} = \sigma_v \ / \ (2N)$$

Coherent acceleration of atoms : simple approach

Succession of stimulated Raman transitions
(same hyperfine level)





$2\hbar k$ per cycle

$$\delta = \nu_1 - \nu_2 \propto t$$

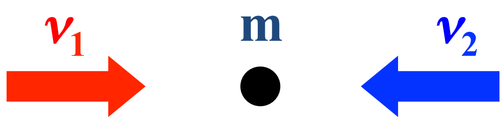
Addiabatic passage : acceleration of the atoms

The atom is placed in an accelerated standing wave: in its frame, the atom is submitted to an inertial force

→ Bloch oscillations in a periodic potential

LKB (1996)

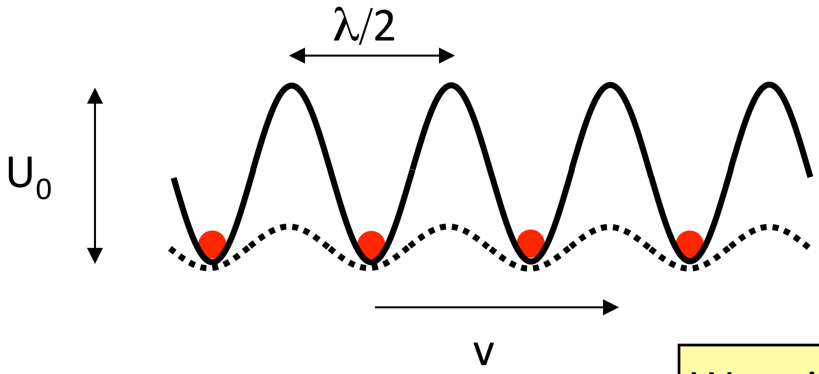
Atom in an accelerated lattice

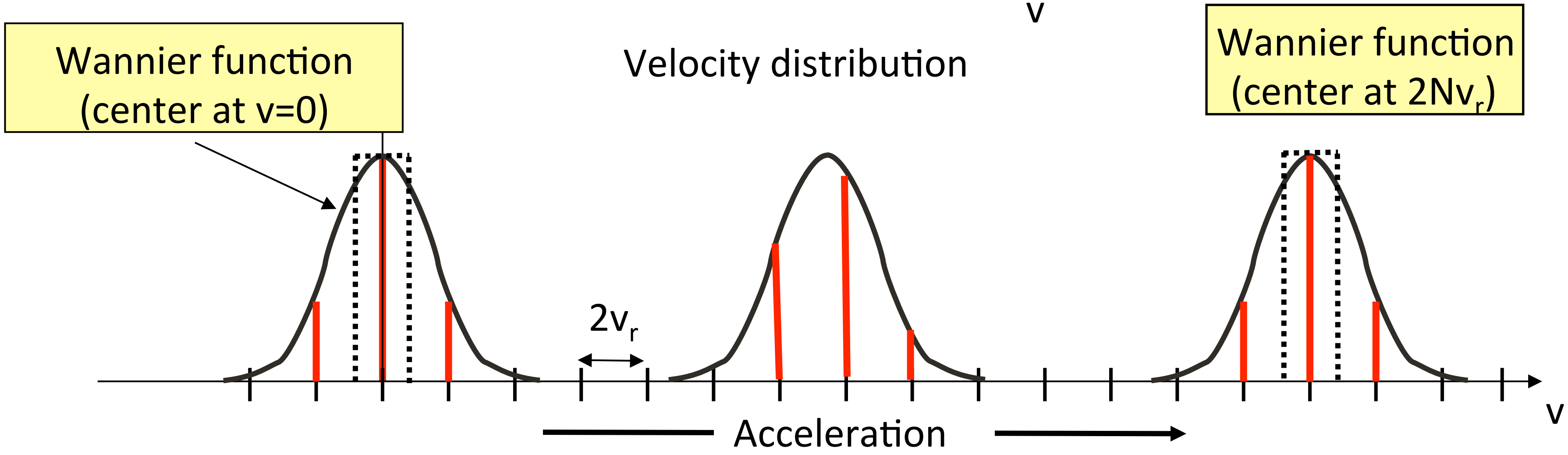


ν_1 & ν_2 $\rightarrow$ Velocity of the lattice $v=(\nu_1 - \nu_2)/2k$

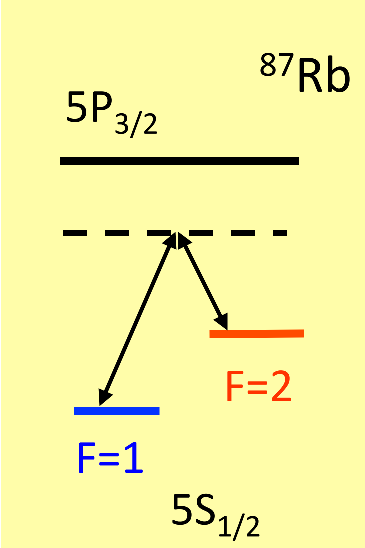
Light shifts : Periodic potential

$$U(x,t) = \frac{U_0}{2}\cos(2k - vt)$$





See also Course 188 - **Atom Interferometry** **P.Cladé talk (July 2013)**



Velocity measurement : π pulses

Raman transition ↔ Doppler sensitive

In our earlier experiment (π−π configuration), two π Raman pulses were used

- to select a subrecoil velocity distribution
- and to measure the final velocity distribution

Detection pop. in atomic state (F=1 and F=2) : fluorescence resonance in a laser beam

Selection: F=2 → F=1 δ_{sel}

Blow away beam

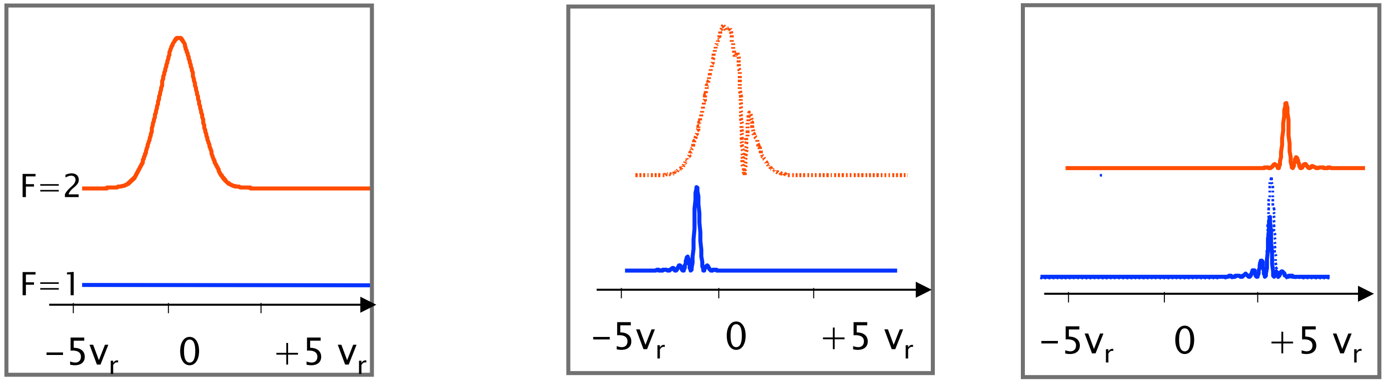
Measurement: F=1 → F=2

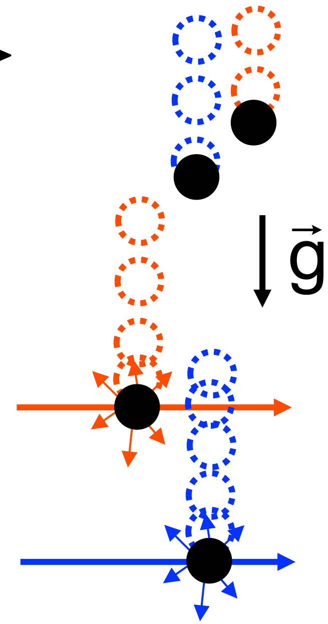
Tuned Frequency δ_{meas}

MOT + optical molasses

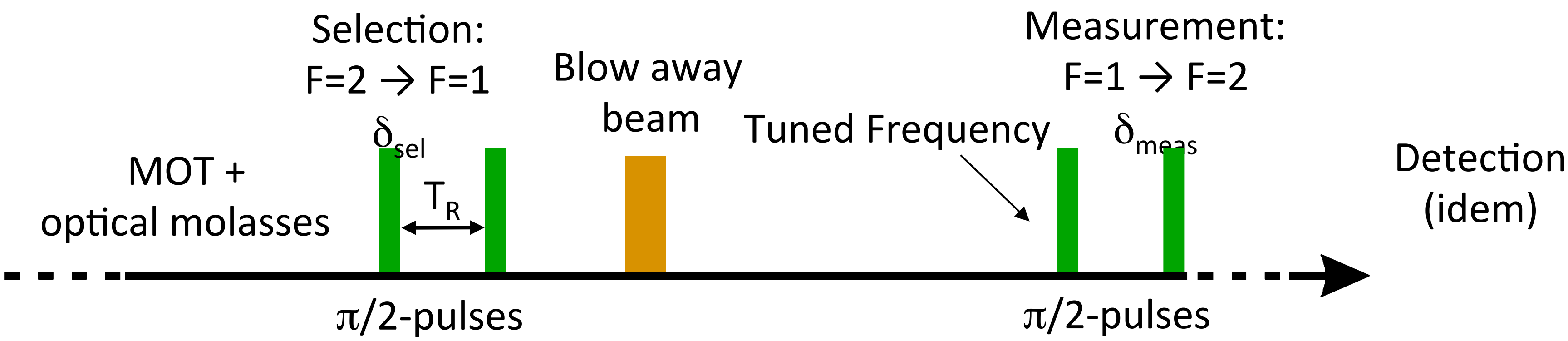
π-pulse

π-pulse



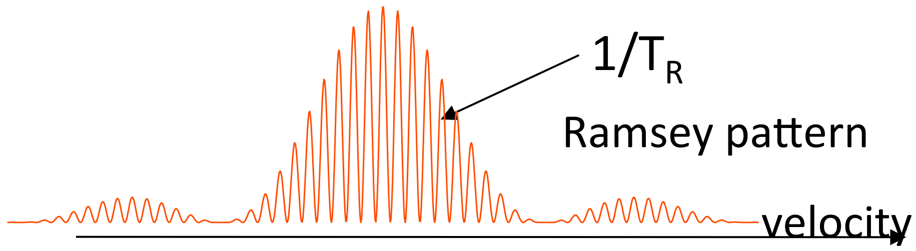


Improvement of the velocity selection



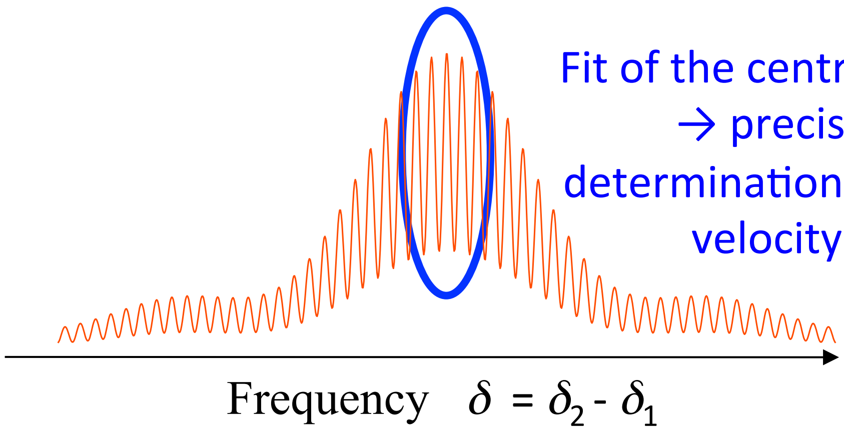
In our present experiment ($\{\pi/2, \pi/2\} - \{\pi/2, \pi/2\}$ configuration),

- the first pair of $\pi/2$ pulses (frequency δ_1) selects a velocity pattern



- the second pair of $\pi/2$ pulses (frequency δ_2) selects another velocity pattern

When the detection frequency δ_2 is swept, the signal obtained is the convolution of two Ramsey patterns



Fit of the central part → precise determination of the velocity

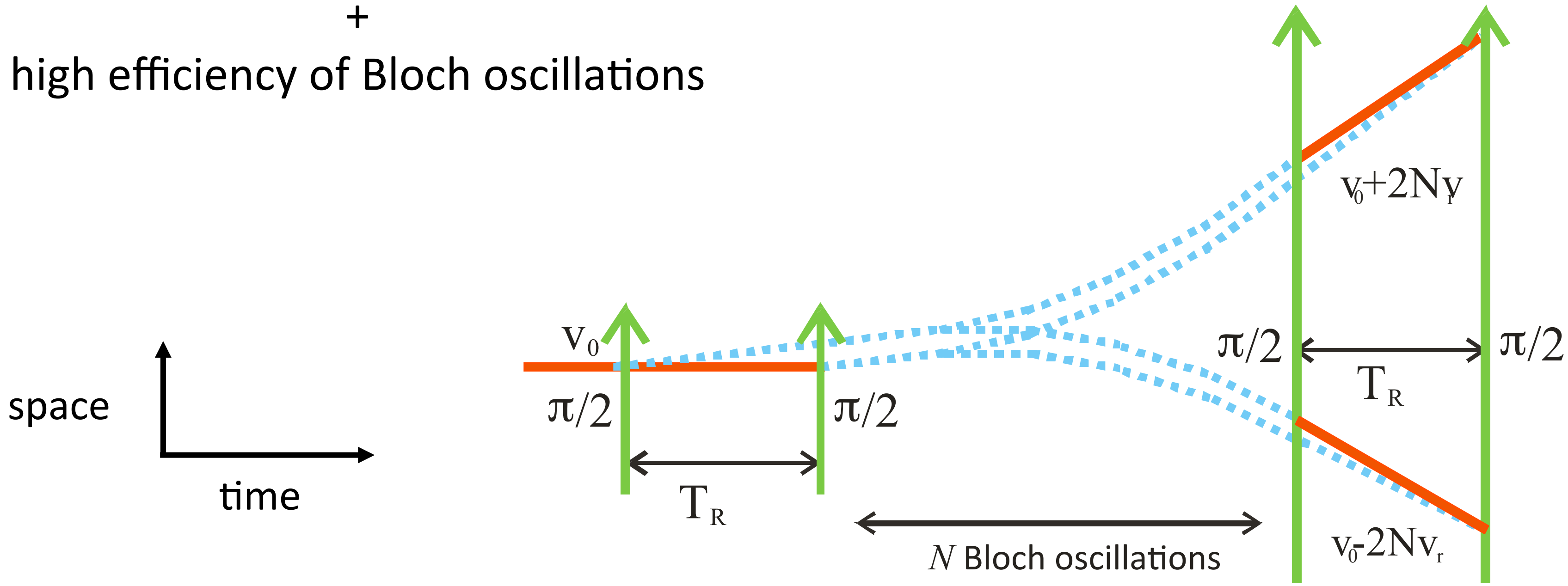
Interferometric method

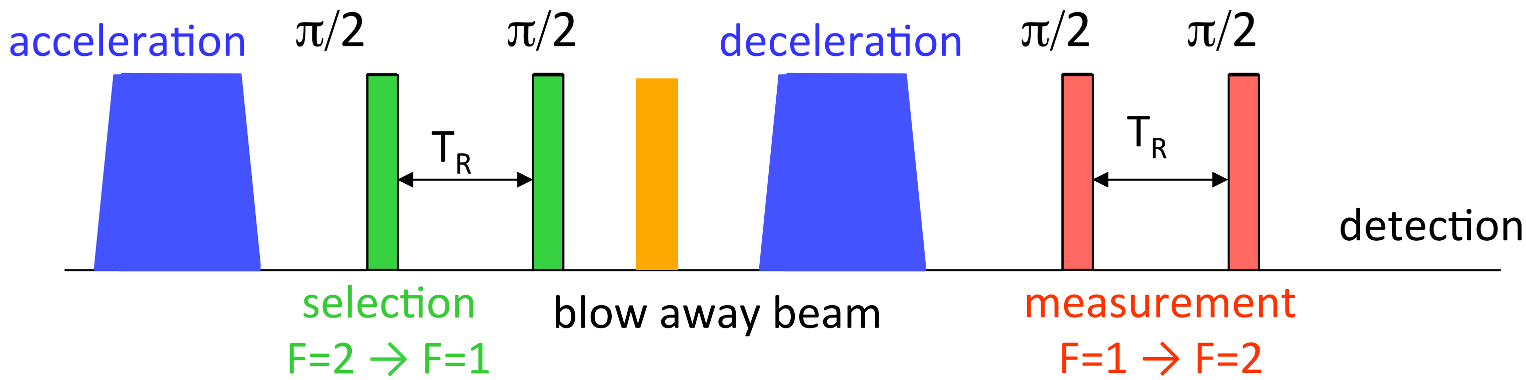
Bloch oscillations and atomic interferometry

high sensitivity of atomic interferometry

+

high efficiency of Bloch oscillations

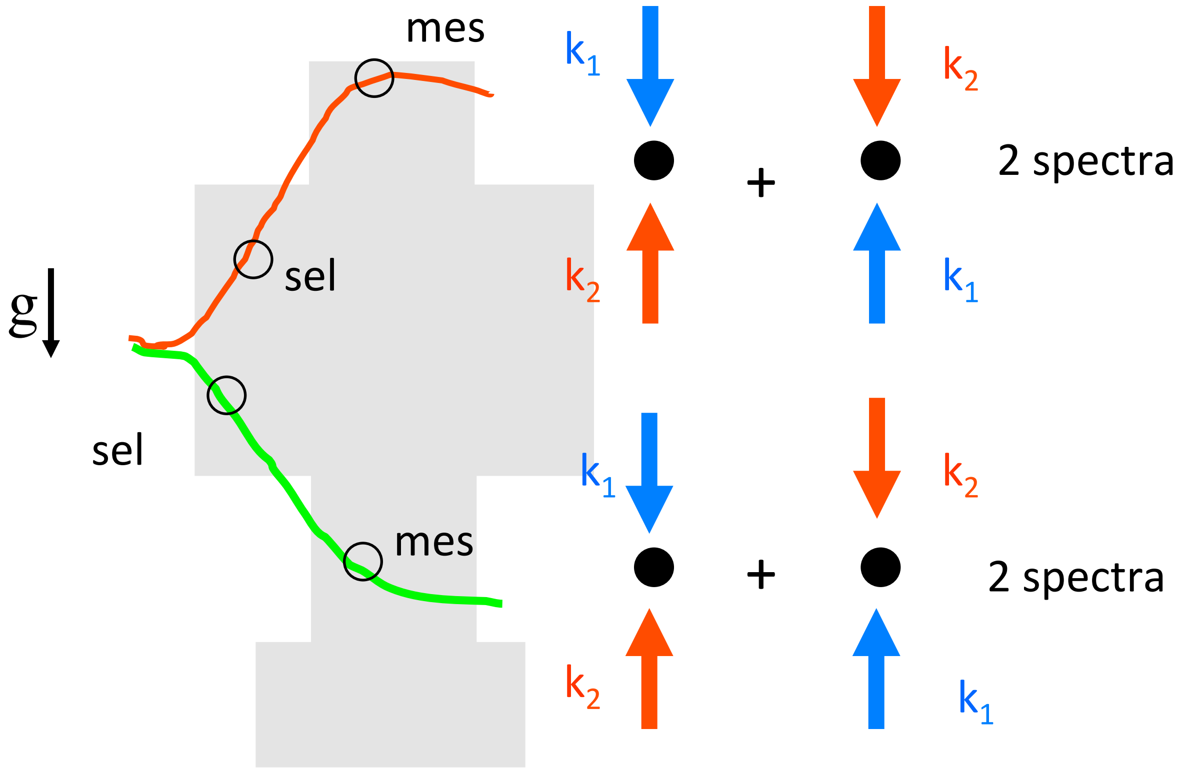




Measurement of the recoil velocity

upwards acceleration

downwards acceleration

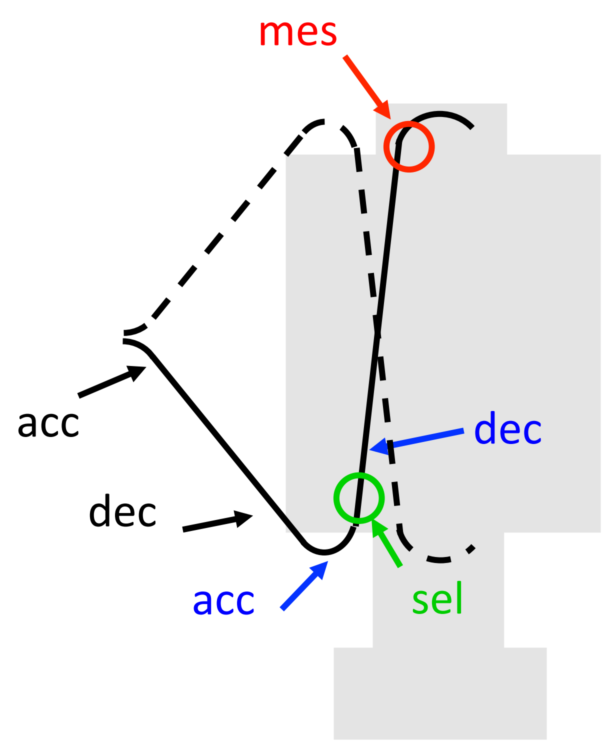


We measure (Doppler effect) : $\Delta V = \frac{\hbar(\delta_{sel} - \delta_{meas})}{(k_1 + k_2)}$ with $\Delta V = \mathrm{Avg}(\Delta V_{1,2}, \Delta V_{2,1})$

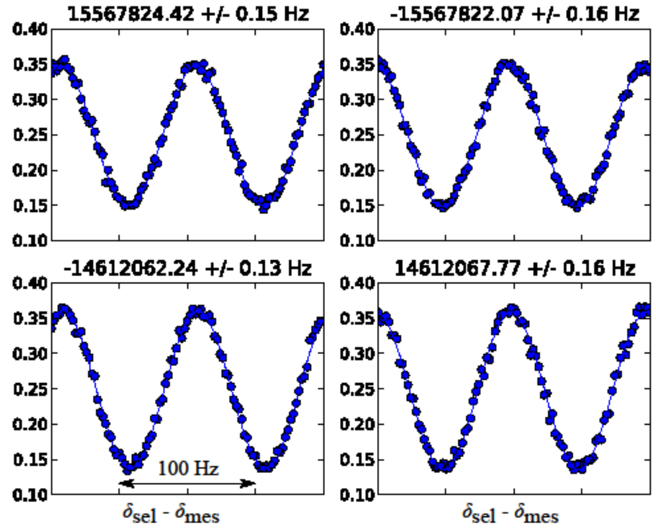
Acceleration in both opposite directions : $v_r = \frac{\Delta V^{up} - \Delta V^{down}}{2(N^{up} + N^{down})}$ (no contribution of g)

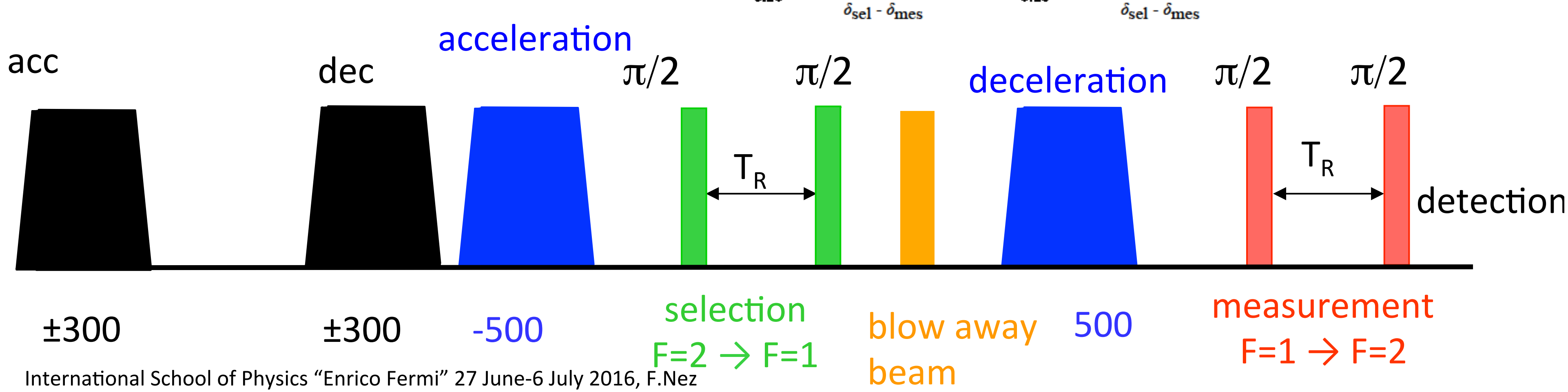
$$v_r = \frac{\hbar k_B}{m} \Rightarrow \boxed{\frac{\hbar}{m} = \frac{(\delta_{sel} - \delta_{meas})^{up} - (\delta_{sel} - \delta_{meas})^{down}}{2(N^{up} + N^{down})(k_1 + k_2)k_B}}$$

« Atom elevator »

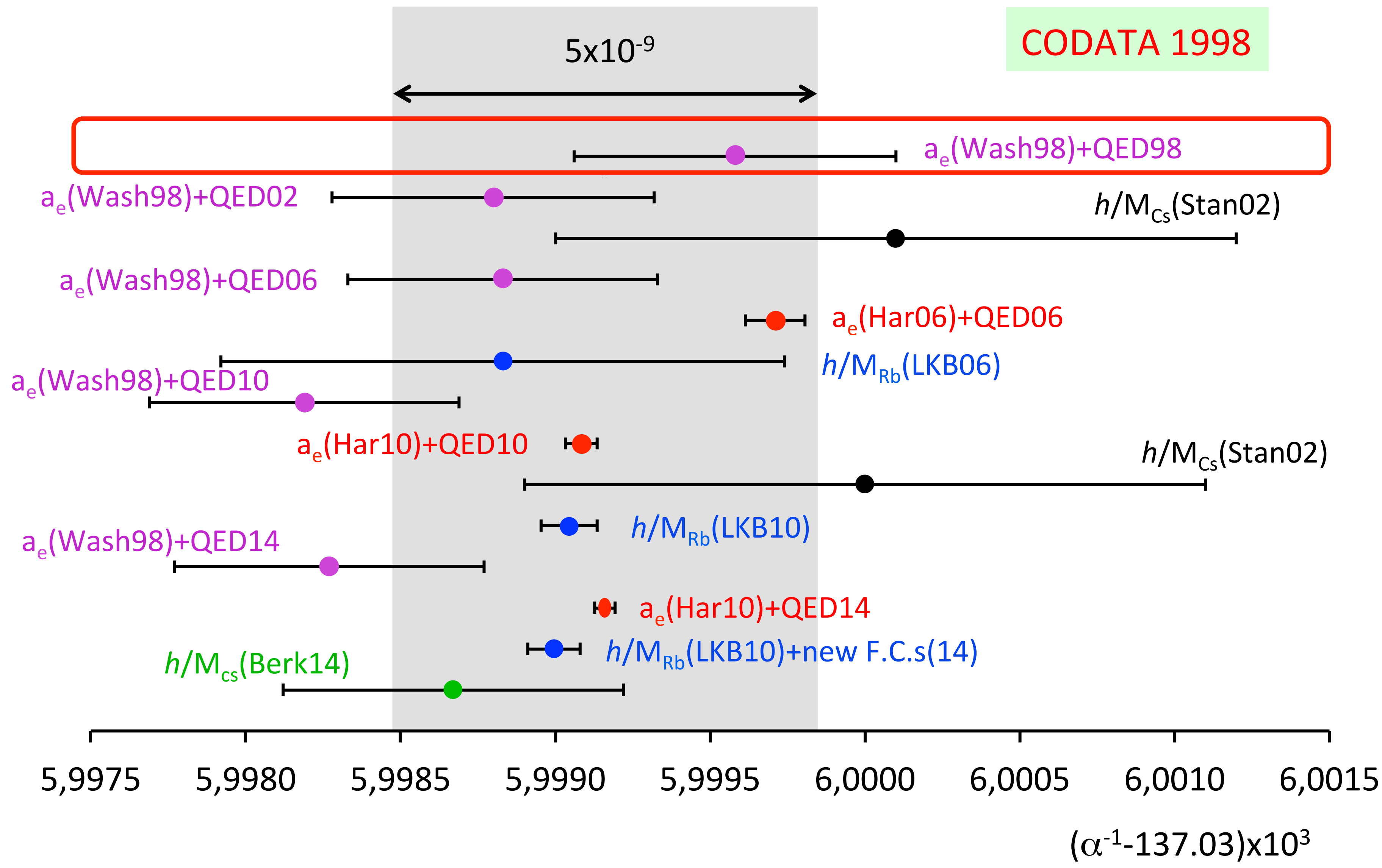


Bloch oscillations = high efficiency (99.95% per recoil)
⇒ “increase” the size of the vacuum chamber
more recoils transferred to the atoms ⇒ higher accuracy on recoil determination

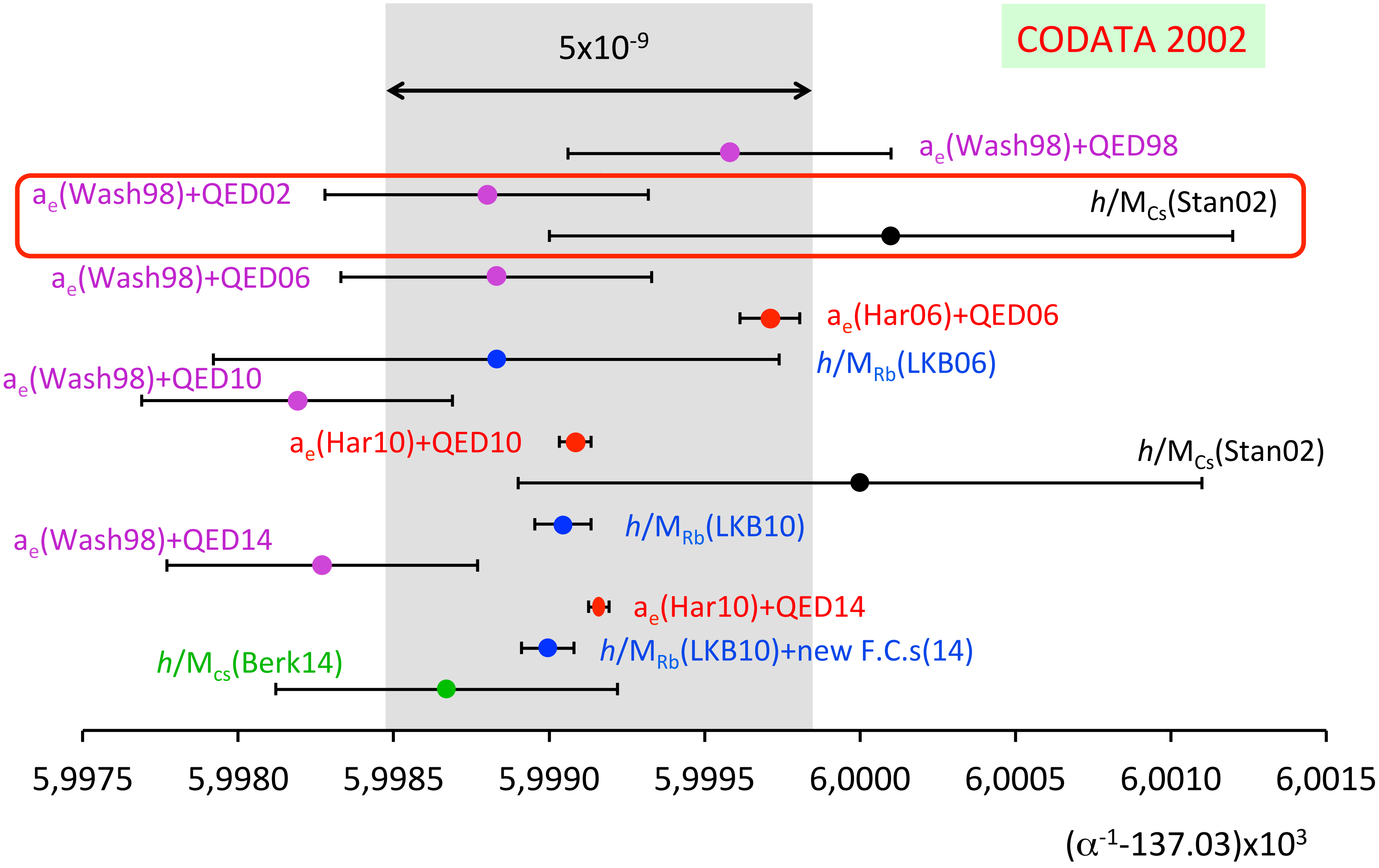




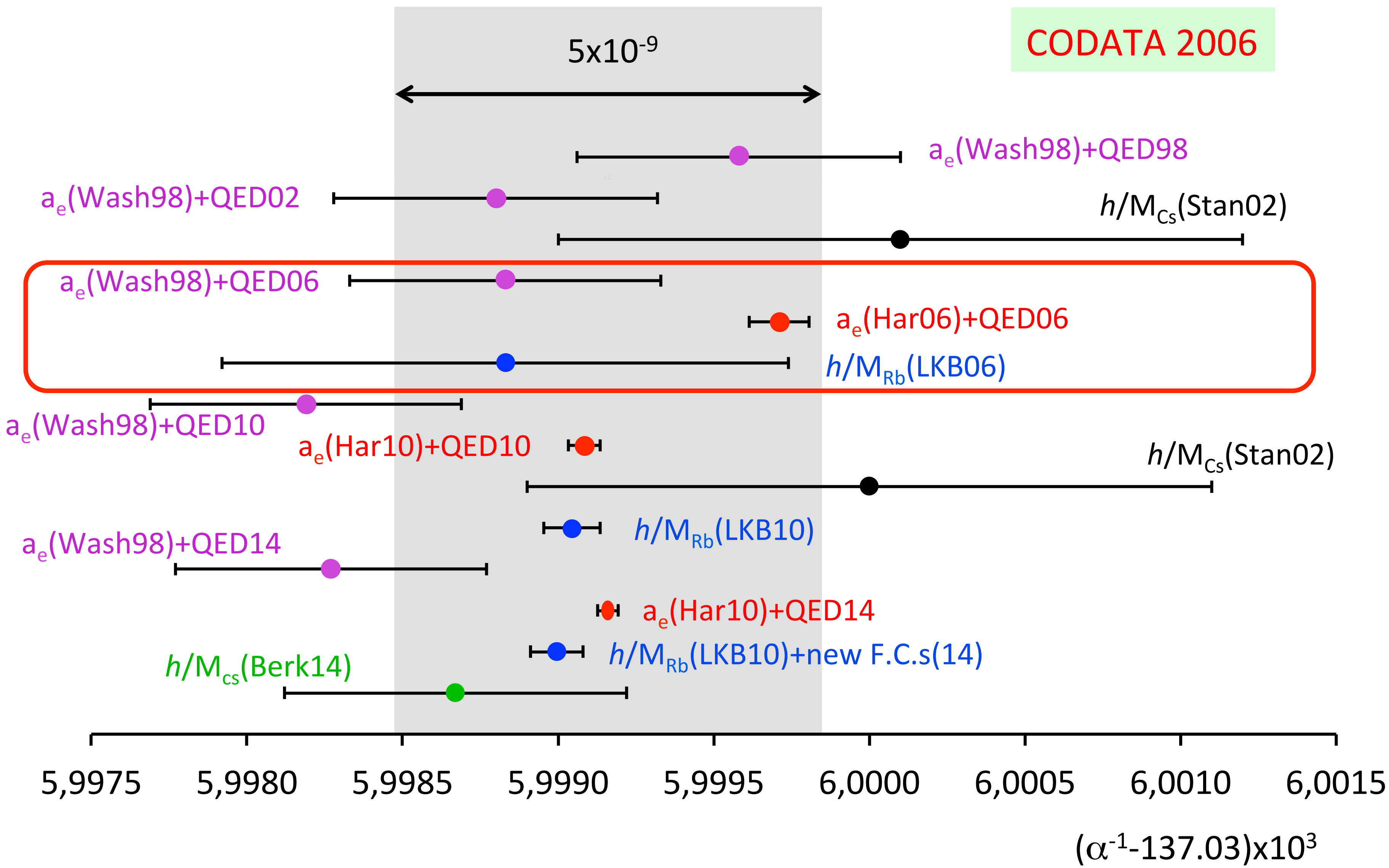
Most precise determinations of α used by CODATA since 1998



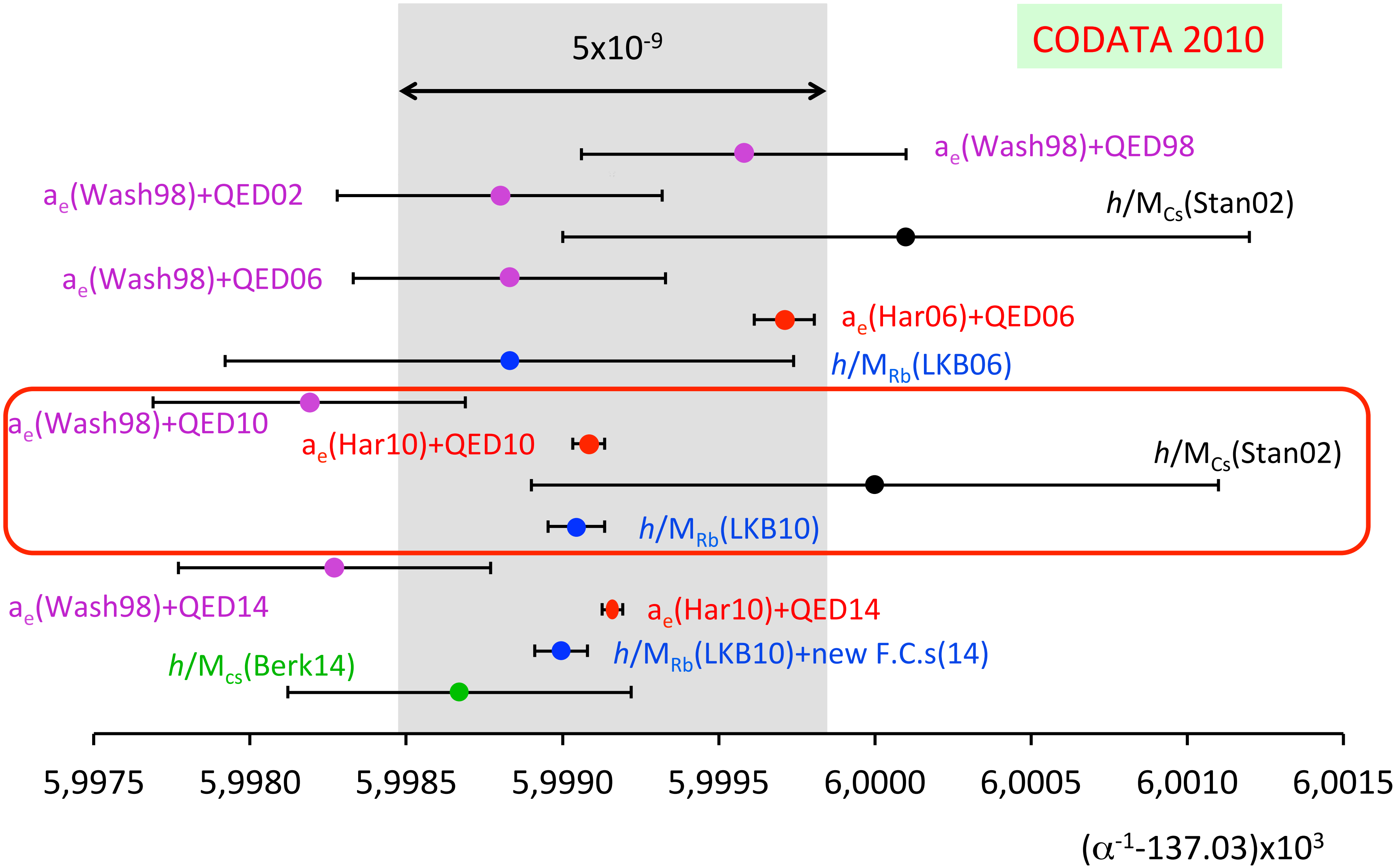
Most precise determinations of α used by CODATA since 1998



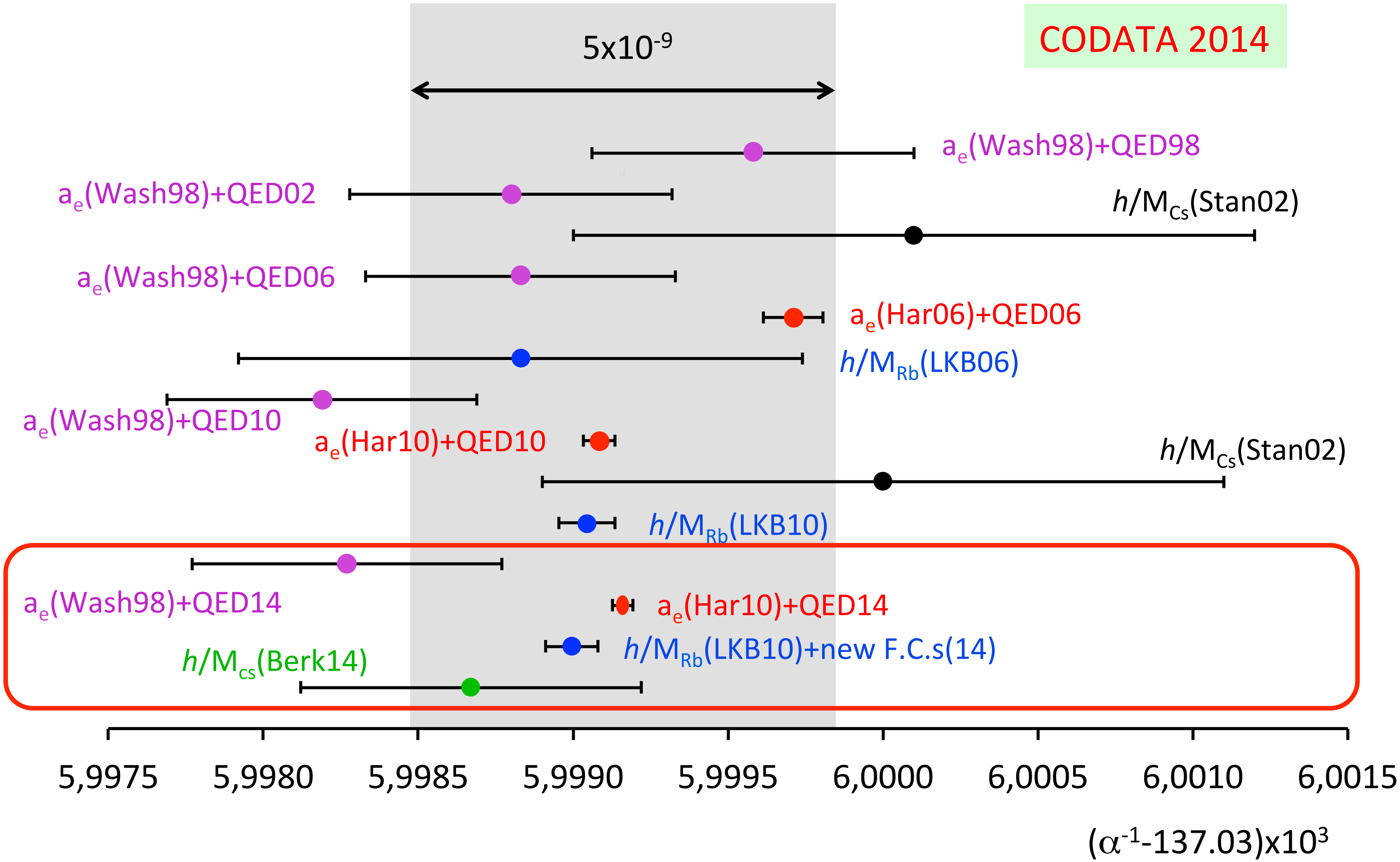
Most precise determinations of α used by CODATA since 1998



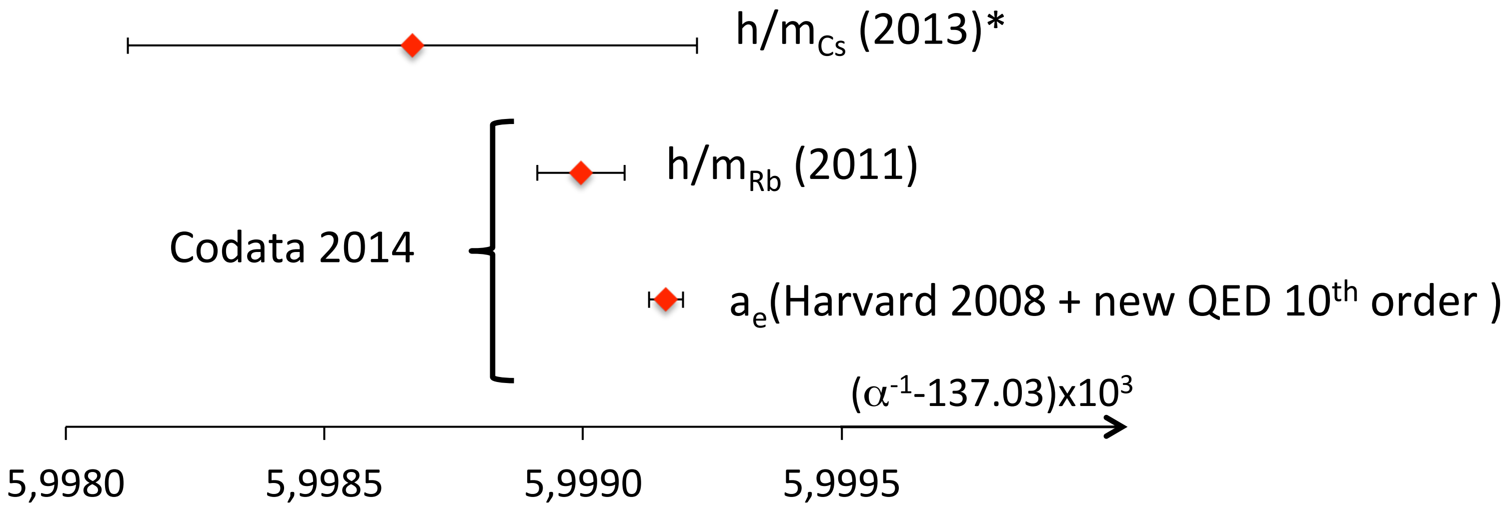
Most precise determinations of α used by CODATA since 1998



Most precise determinations of α used by CODATA since 1998



Fine structure determinations 2014



Cs : S. Y. Lan et al, Science **339** 554-557 (2013)

Rb: R. Bouchendira et al, Phys. Rev. Lett. **106**(8) 080801 (2011)

a_e : D Hanneke et al, Phys. Rev. Lett. **100**(12) 120801 (2008)
T Aoyama et al, Phys. Rev. D **91**(3) 033006 (2015)

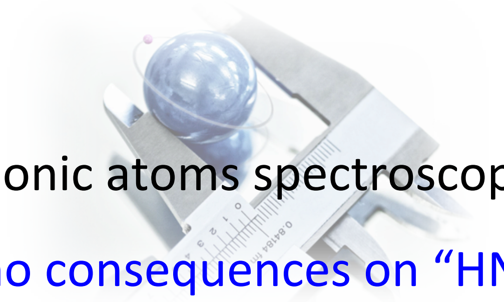
Systematics in these determinations ?

Active researches in progress (Rb, Cs, a_e, QED)

Long term prospect : new determination of α from g factor of H- and Li-like

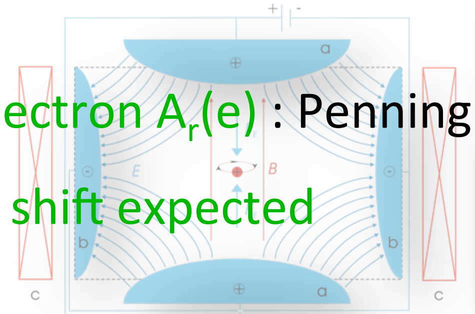
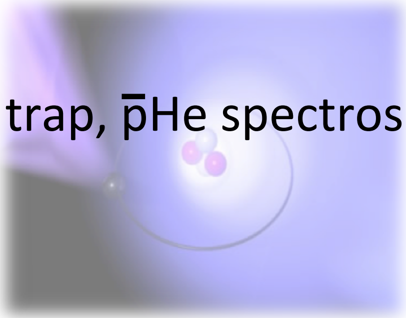
Conclusion

$$hN_A = \frac{A_r(e) M_u}{2R_\infty} c\alpha^2$$

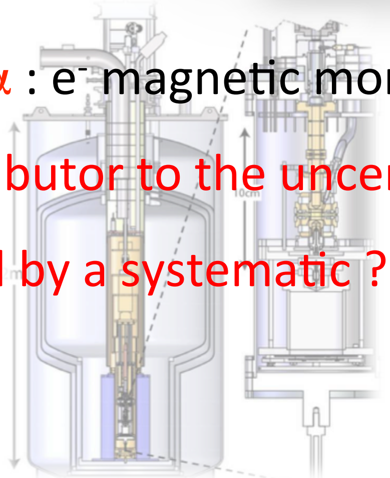
Rydberg constant R_∞ : H/D spectroscopy, muonic atoms spectroscopy

Possible shift before 2017 but no consequences on "HN_A"

Relative atomic mass of the electron $A_r(e)$: Penning trap, $\bar{p}$He spectroscopy

Well known, not shift expected



Fine structure constant α : e^- magnetic moment anomaly, atom interferometry

Most contributor to the uncertainty of "hN_A"

h/M shifted by a systematic ?


Thank you for your attention