The Newtonian constant of gravitation G – a constant too difficult to measure?

Terry Quinn

Varenna July 2016

The Newtonian constant of gravitation G

$$F = M_1\, M_2\, G/r^2$$

The 2014 CODATA value for G is

G = 6.67408 (31) x 10^{-11} kg^{-1} m^3 s^{-2}

The uncertainty represents 47 ppm

Current situation in the measurement of G from CODATA 2014

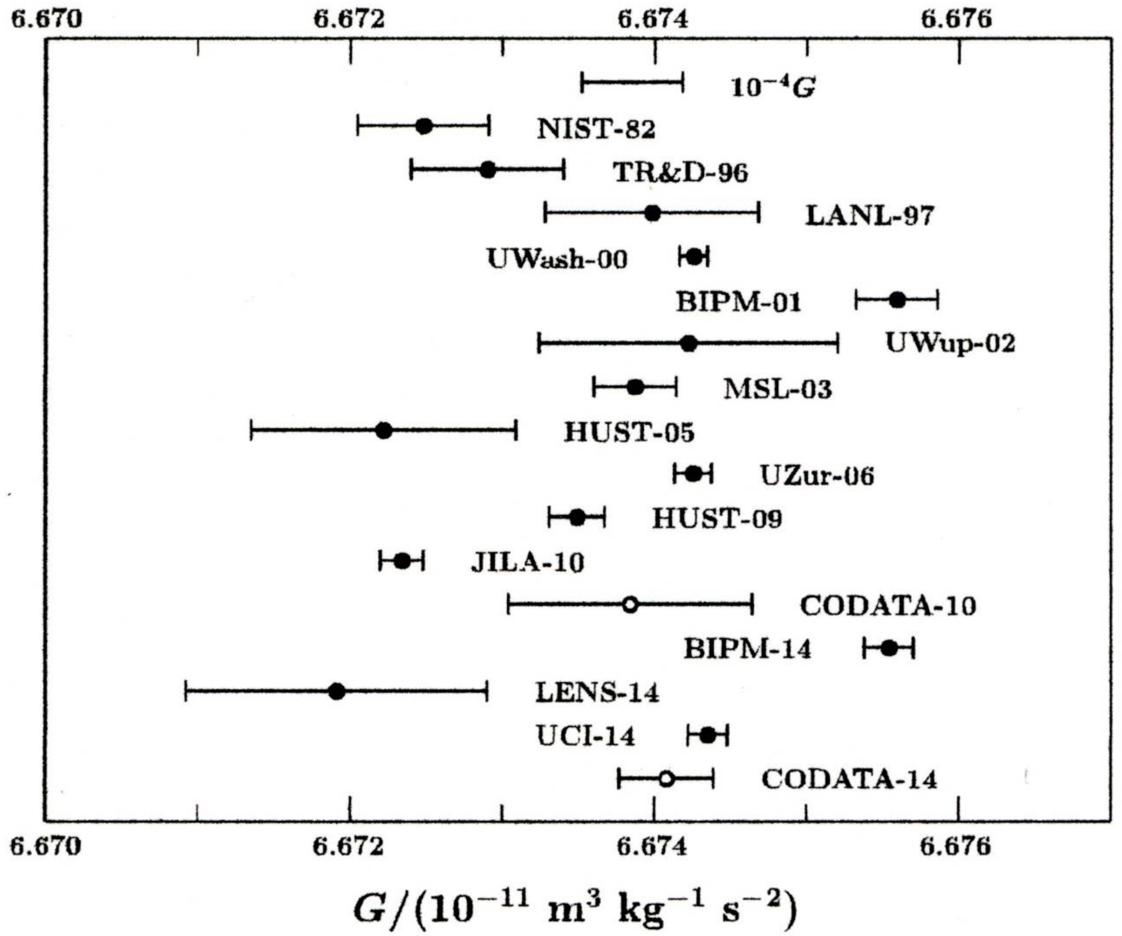
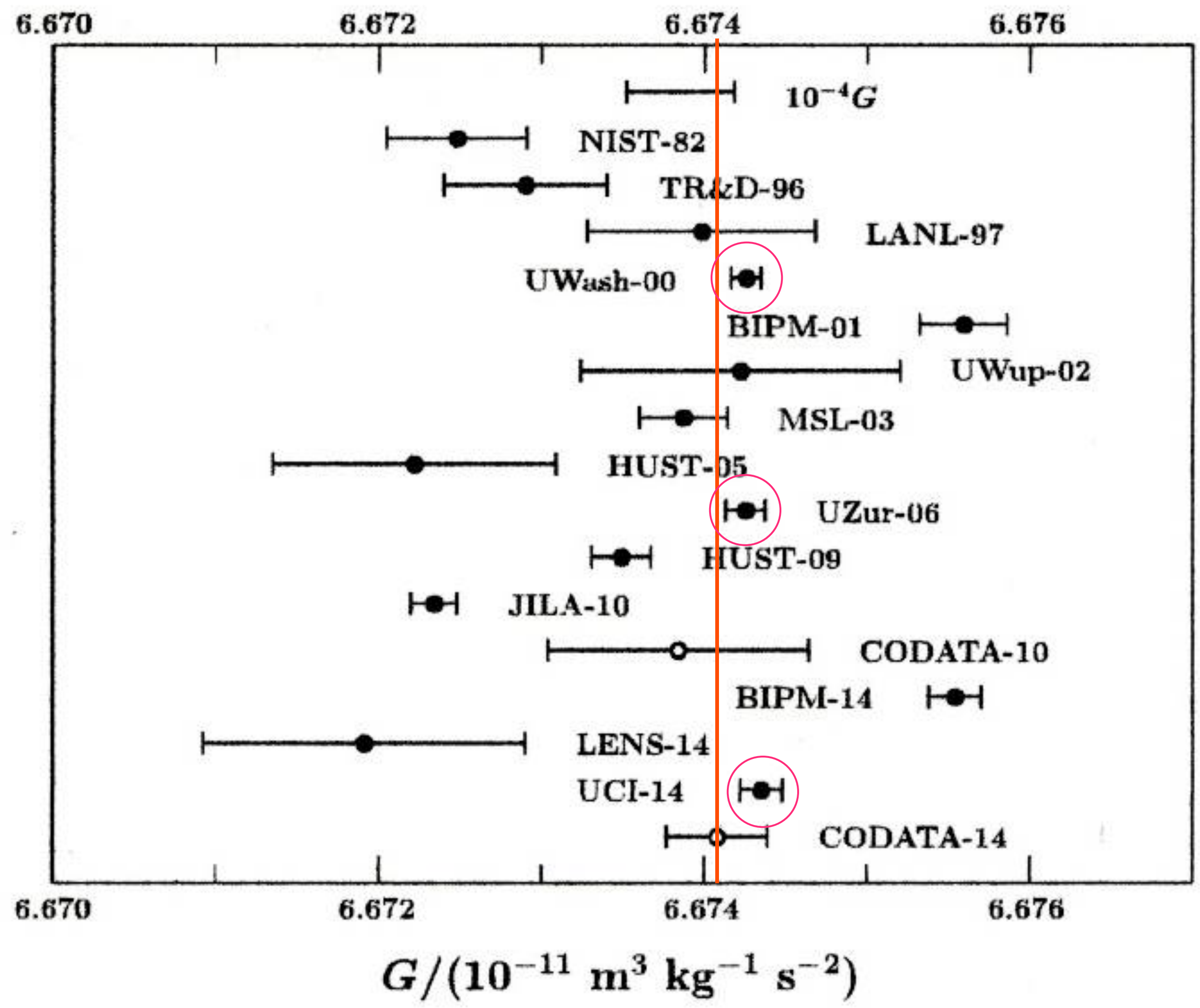


FIG. 6 Values of the Newtonian constant of gravitation G in Table XXVII and the 2010 and 2014 CODATA recommended values in chronological order from top to bottom.



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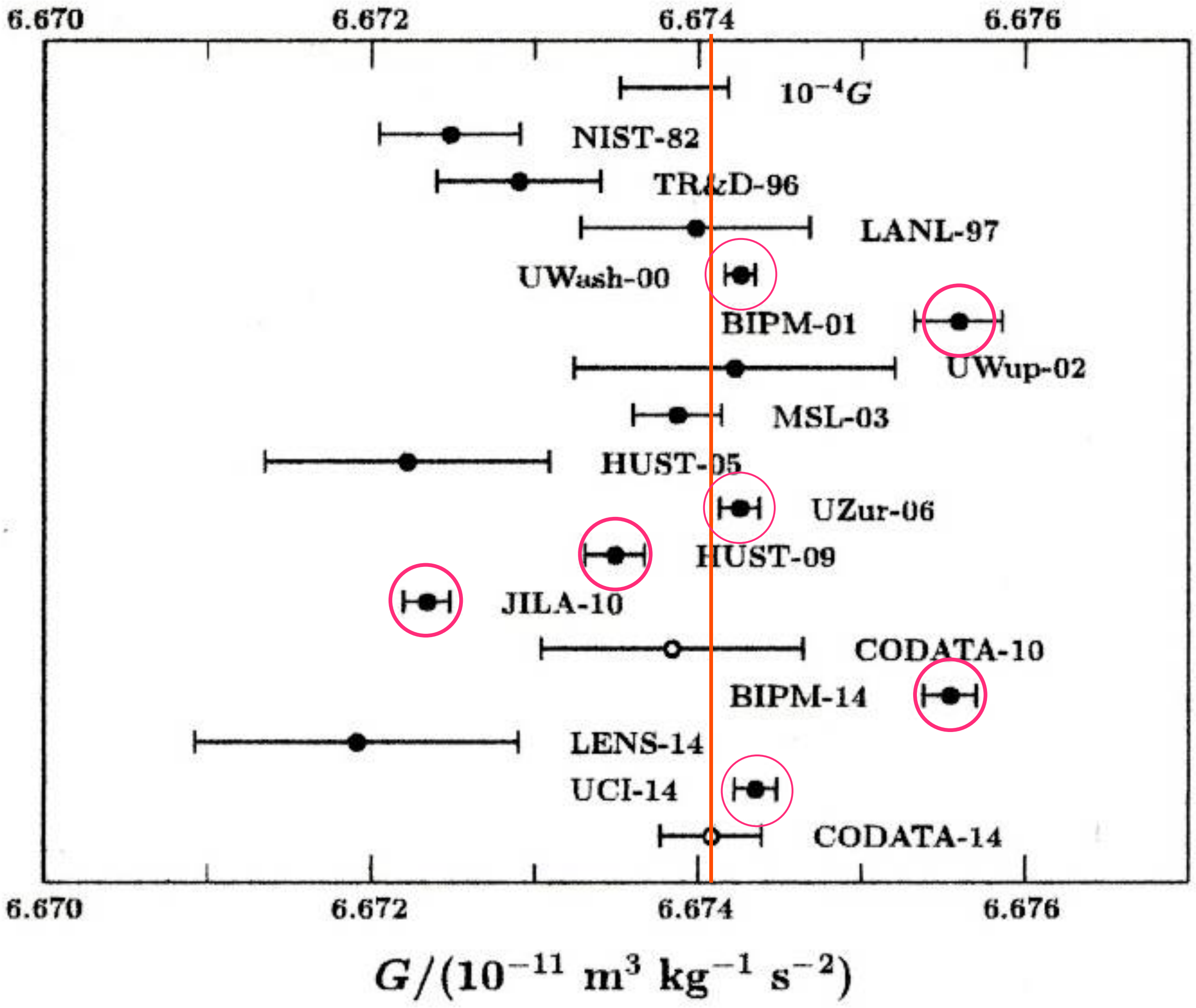
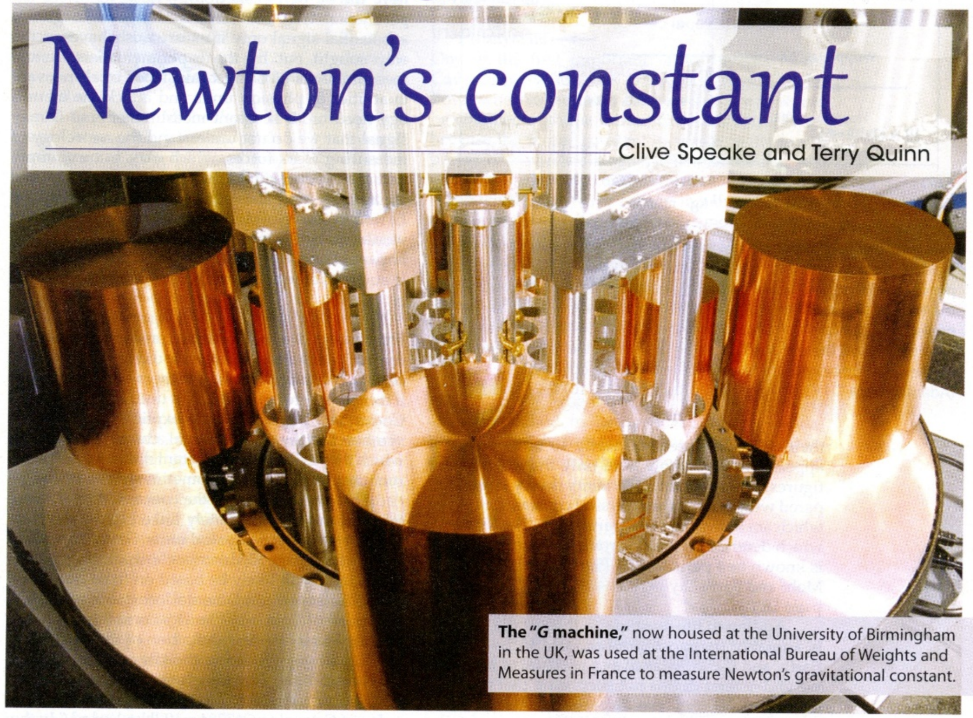


FIG. 6 Values of the Newtonian constant of gravitation G in Table XXVII and the 2010 and 2014 CODATA recommended values in chronological order from top to bottom.

The search for Newton's constant

Clive Speake and Terry Quinn



The "*G* machine," now housed at the University of Birmingham in the UK, was used at the International Bureau of Weights and Measures in France to measure Newton's gravitational constant.

Three decades of careful experimentation have painted a surprisingly hazy picture of the constant governing the most familiar force on Earth.

Gravity has a special place in physics. For starters, it is the only fundamental interaction that cannot be described by a quantum theory. Whereas the prevailing theories of gravity—Newton's law and Einstein's general relativity—consider space and time to be continuous classical quantities, the theories that describe electromagnetism and the nuclear forces are based on conserved quanta.

Gravity is also by far the weakest of the fundamental forces; its strength becomes comparable to that of the others only at energies near the Planck scale, 1.22×10^{19} GeV, some 15 orders of magnitude higher than the energies currently being explored by the Large Hadron Collider. The mismatch calls into question the validity of the standard model of particle physics, which is thought to be incompatible with such an immense fundamental energy scale.

It is fitting, then, that gravity, more than any other force, stubbornly eludes precise measurement. Newton's law, which approximates general relativity in the limit of small gravitational fields and nonrelativistic speeds, states that the magnitude F of the force attracting two spherical bodies of mass M_1 and M_2, separated by a distance r, is given by $F = GM_1M_2/r^2$. The constant G is known, unsurprisingly, as Newton's constant of gravitation. It is considered to be a fundamental constant of nature. But more than three centuries after Newton's law was proposed, experiments have yet to yield a consensus on the constant's value.

According to the Committee on Data for Science and Technology (CODATA), which issues recommended values of fundamental constants once every four years, $G = 6.673\,84(80) \times 10^{-11}$ kg^{-1} m^3 s^{-2}. That value, from 2010, reflects the results of nearly a dozen experimental measurements made during the past three decades (see figure 1).[1] Although many of the individual measurements have an uncertainty of less than 50 parts per million (ppm),

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bers, not the wooden boxes used in the Cavendish experiment, the basic principle of separating the minute gravitational force between laboratory-scale masses from Earth's large, downward pull remains the same.

Cavendish would have been surprised, however, to find that after so many years, measurement accuracy has improved only modestly—not nearly as much as it has for almost every other physical quantity. We now estimate the accuracy of Cavendish's measurements to be something like 1%, which is not much worse than the spread of measurements that figure into the current CODATA value. To understand how we've arrived at this situation, let's first

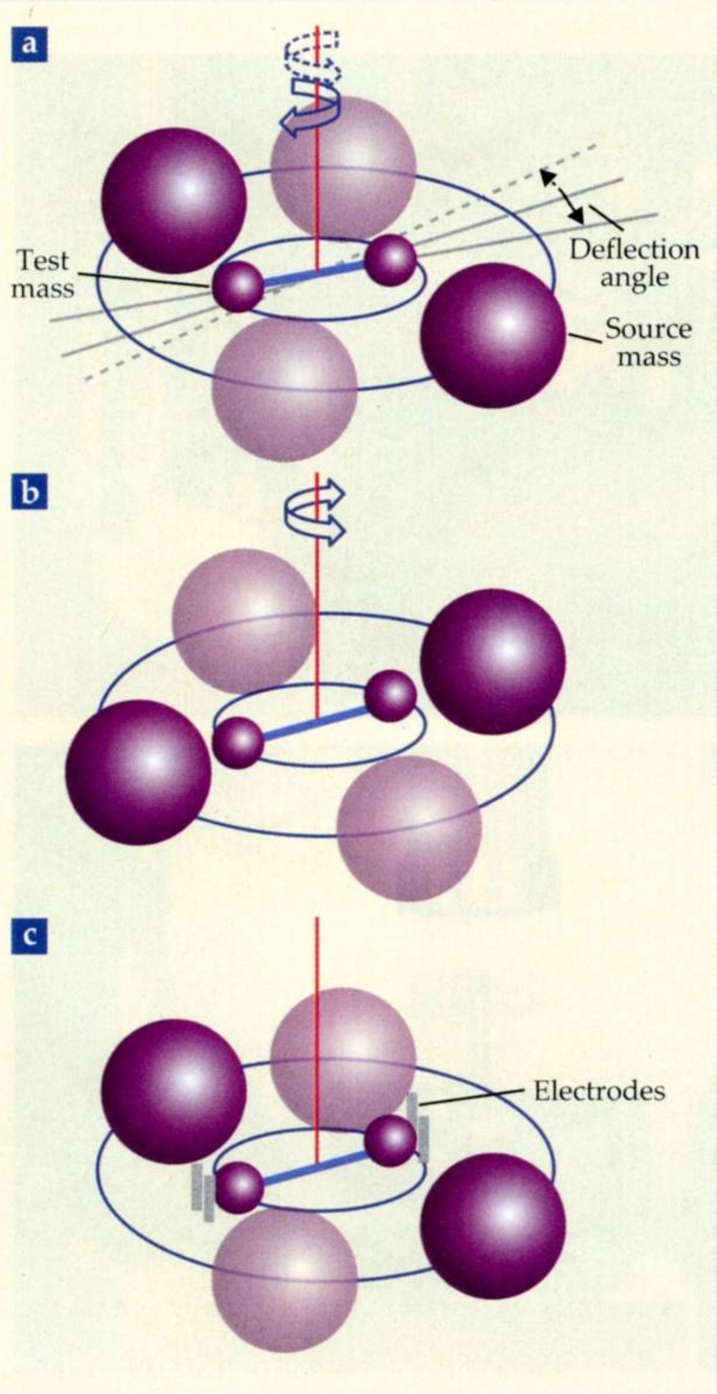
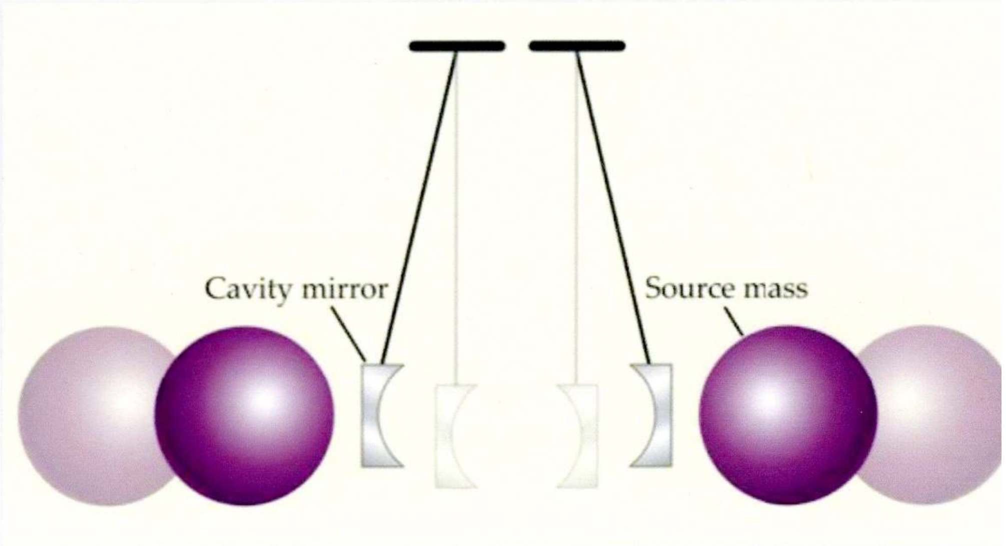


Figure 2. A torsion-balance experiment has, as its central element, two test masses balanced on a beam suspended by a thin metal wire. **(a)** In the original setup conceived by John Michell and later used by Henry Cavendish, two large source masses are positioned to exert a gravitational force that causes the torsion balance to turn through a small angle. The arrangements indicated by the dark and light source masses would yield clockwise and counterclockwise displacements, respectively. **(b)** In so-called time-of-swing experiments, *G* is calculated from the change in oscillation period when source masses are repositioned between arrangements lying along (dark spheres) and orthogonal to (light spheres) the resting test-mass axis. **(c)** In a third approach, the electrostatic servo-control technique, the gravitational force is calculated from the voltage that must be applied to nearby electrodes to hold the test assembly in place. In all three configurations, the gravitational coupling between the source masses and the whole of the torsion-balance assembly has to be calculated.

Figure 4. A simple pendulum gravity gradiometer consists of a microwave or optical cavity formed by two hanging mirrors. When source masses are moved toward the cavity mirrors, the varying gravitational pull leads to a change in the cavity's optical length and, hence, a change in its resonant frequency. In a Fabry–Perot experiment performed at JILA, the change in the optical length was on the order of tens of nanometers.



result by Winfried Michaelis and coworkers at Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, Germany.[10] Michaelis and his colleagues used a novel torsion balance in which the

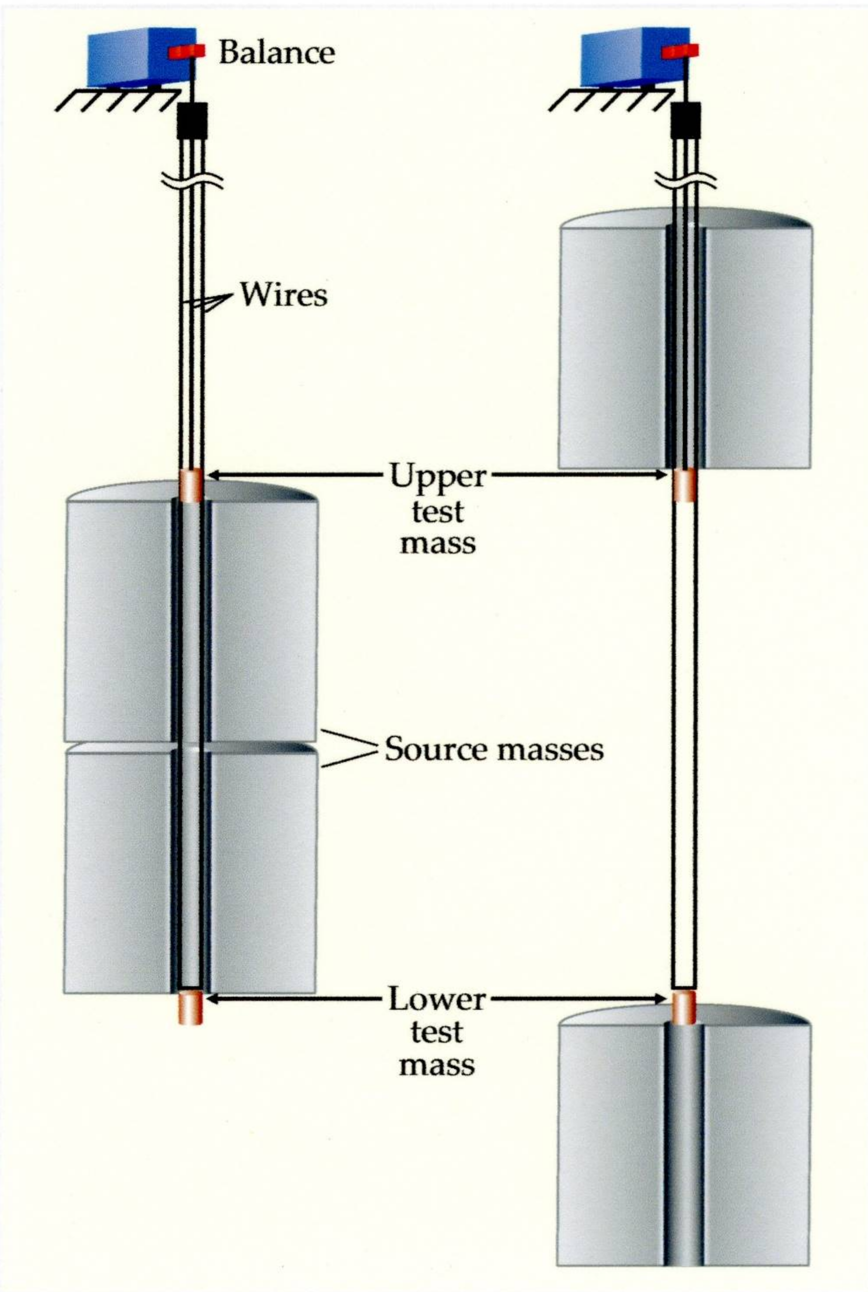


Figure 5. In a beam-balance experiment, a Zürich team compared the weights of two 1.1-kg test masses suspended just above and just below 6.5-ton source masses. In switching between the left and right configurations, the test masses' differential weight changes by an amount equivalent to the weight of a millimeter-sized drop of water. (Adapted from ref. 18.)

for biases through a number of experimental configurations housed in the same laboratory and publishing a final result only when the measurements agree should lead to more reliable values of G.

Beyond the torsion balance

Since the 1990s a few groups have developed successful alternatives to the torsion balance. Among the firsts, researchers at the University of Wuppertal in Germany devised a simple pendulum gravity gradiometer, which consisted of two metal mirrors suspended by thin wires to form a hanging microwave cavity, as illustrated in figure 4. When 125-kg source masses were positioned behind each mirror, they induced a slight displacement of the mirrors, detectable as a change in the cavity resonance frequency.

By 2002 the Wuppertal group had refined the technique sufficiently to measure G with a reported uncertainty of 100 ppm.[16] Soon after, Harold Parks and James Faller of JILA adopted a similar ap-

The effect is evident at periods ranging from less than a second to more than 10 minutes. We were able to relate the anelastic aftereffect to the presence of so-called $1/f$ noise arising from the movement of dislocations in the metal wire.

Kazuaki Kuroda then deduced that anelastic behavior would subject time-of-swing measurements to an error inversely proportional to the quality factor Q, a quantity indicating how closely the balance approximates a lossless elastic spring.[9] He calculated corrections for many of the classic torsion-balance measurements; he revised all of them downward, in most cases by a few tenths of a percent. The NBS measurements on which the 1986 CODATA value was based were revised downward by about 50 ppm following confirmatory experiments by Bagley and Luther, who used two wires of widely different Q.

In 1996 a second development shook confidence in the CODATA value: the publication of a

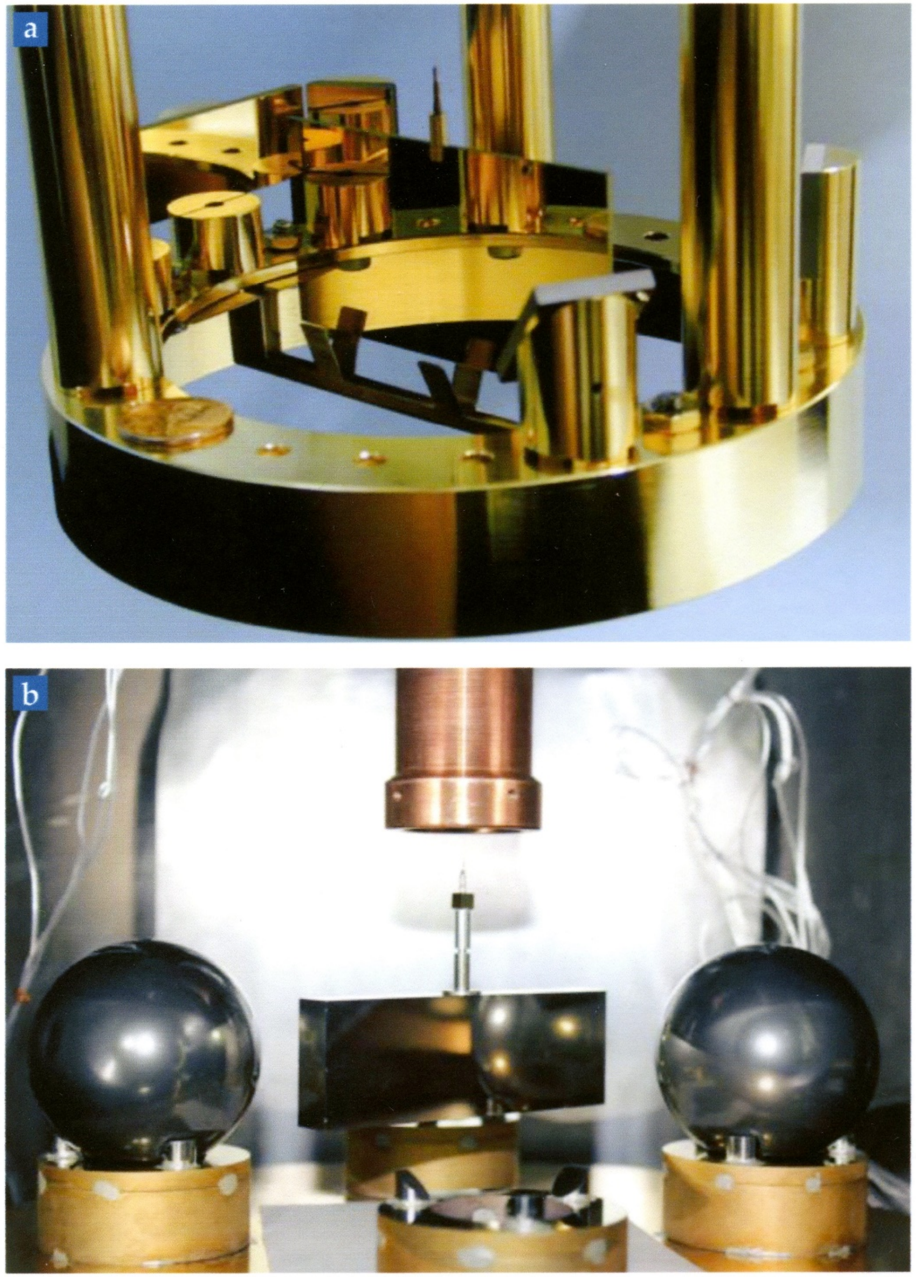


Figure 3. Two twists on the torsion balance. **(a)** A group at the University of Washington used the flat plate visible at center, rather than the traditional dumbbell arrangement, as the test mass in a torsion-balance measurement of the gravitational constant G. (A penny at the bottom left conveys the scale.) In such a geometry, the derived value of G is almost completely independent of the mass distribution of the test masses. (Image courtesy of Jens Gundlach.) **(b)** Researchers at Huazhong University of Science and Technology in Wuhan, China, used a quartz slab as the test mass, which offers similar metrology advantages. The source masses are arranged in the so-called time-of-swing configuration, detailed in figure 2b. (Image courtesy of Jun Luo.)

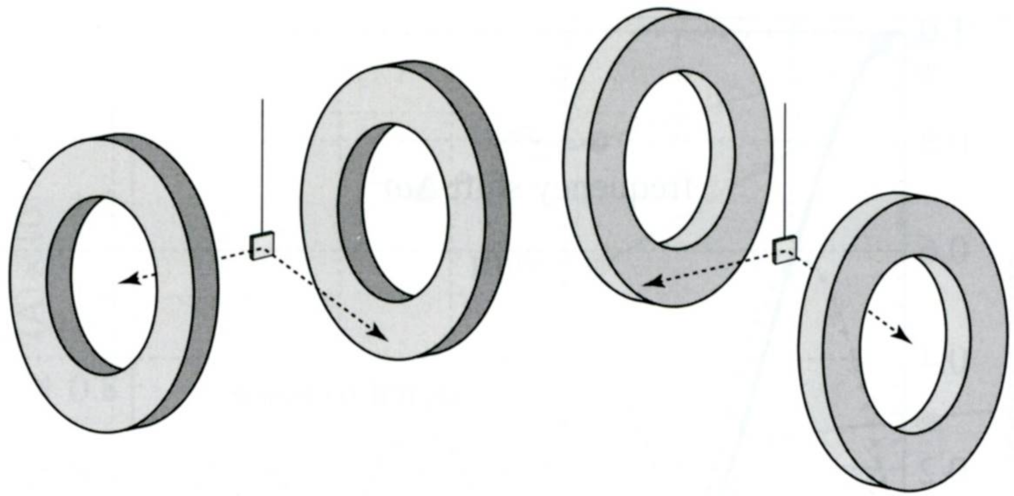


Figure 1. Diagram of source mass rings and the torsion pendulum in the two measurement positions 90° apart.

The cryogenic torsion balance of University of California

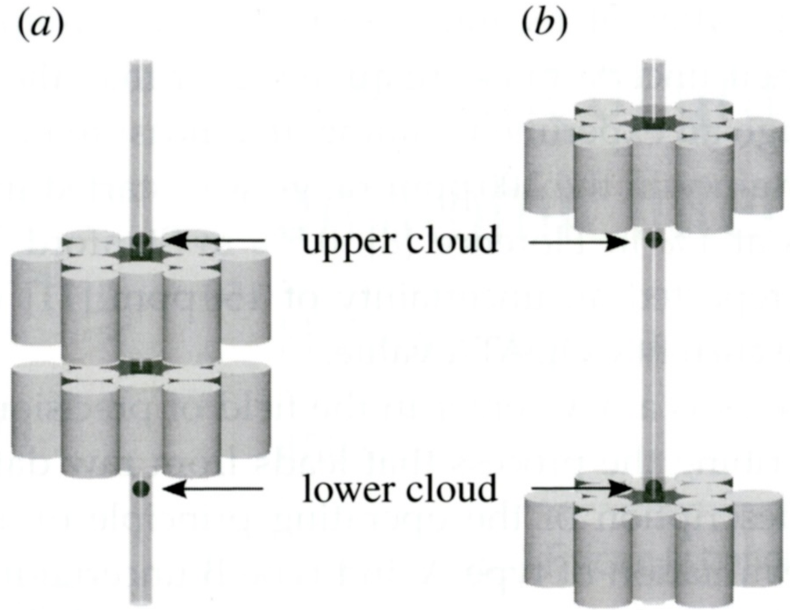
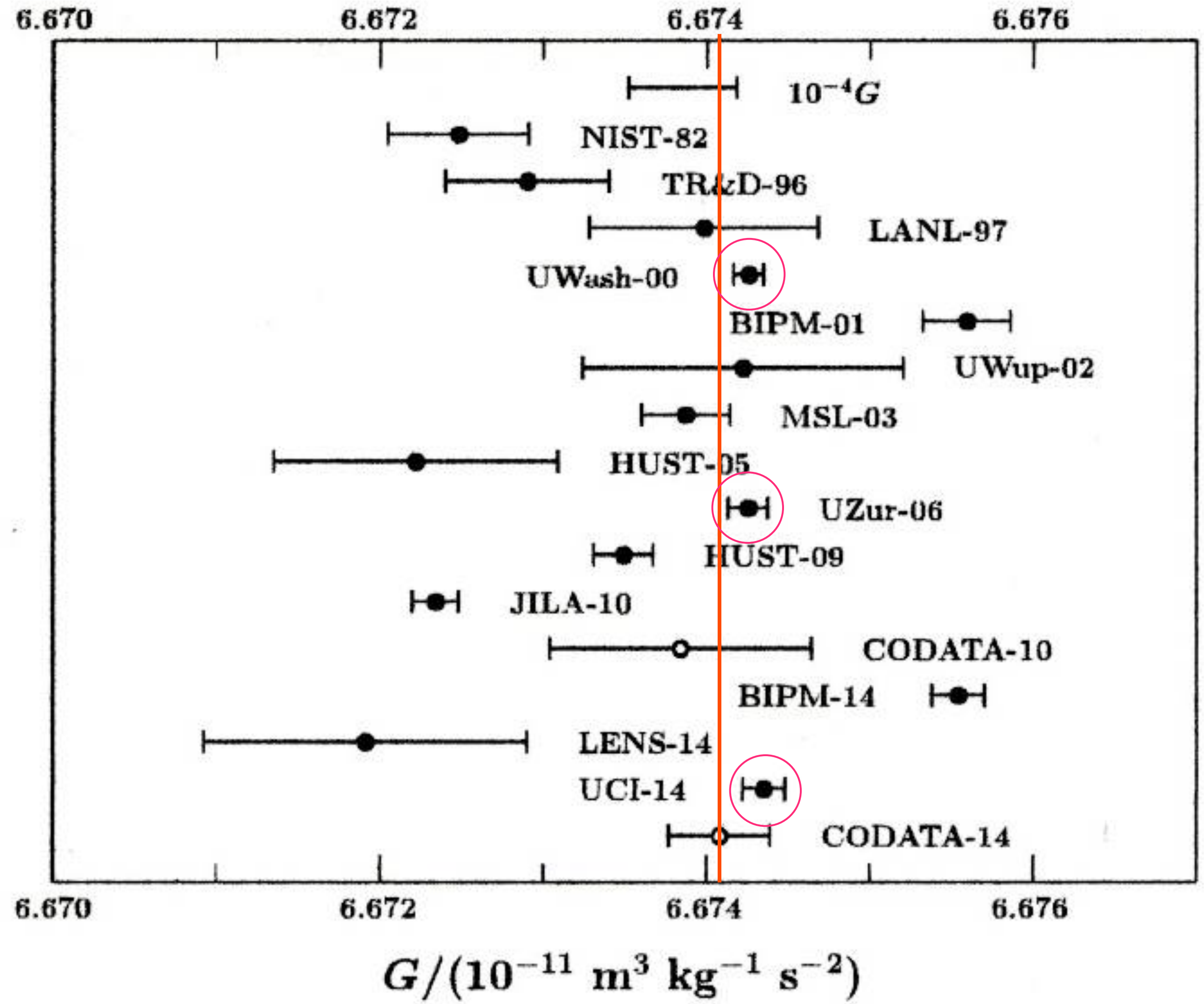


Figure 1. Schematic drawing of our apparatus. The clouds of cold atoms are represented at their apogees inside the long vertical vacuum tube. In (*a*), the source masses are pulling the clouds together while in (*b*) they are tearing them apart. Each source mass is a group of 12 cylinders arranged in hexagonal symmetry. The structures supporting the cylinders are not shown. (Online version in colour.)

Atom interferometer determination of G, university of Bologna



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The BIPM *G* experiment has passed through three phases:

1 A preliminary small-scale version(1996) to explore the behaviour of torsion strips (Metrologia, 1997, 34, 245-249). We obtained a value of *G* with a relative uncertainty of 1.7 10^{-3}

2 The first full scale version produced a value for *G* with an uncertainty of 41 ppm in 2001 (PRL, 2001, 87, 111101). In this version we used two methods of measurement.

3 A second full scale version, using the same two methods of measurement but completely rebuilt, produced a value of *G* in 2013 with an uncertainty of 27 ppm, statistically consiste with the first, (PRL, 2013, 111, 101102)

In every experiment to measure *G* except ours, each experimenter has used only one method of measurement.

Different experimenters have used different methods, but this is not the same because the errors in one experiment are not directly constrained by the results of different methods in other experiments.

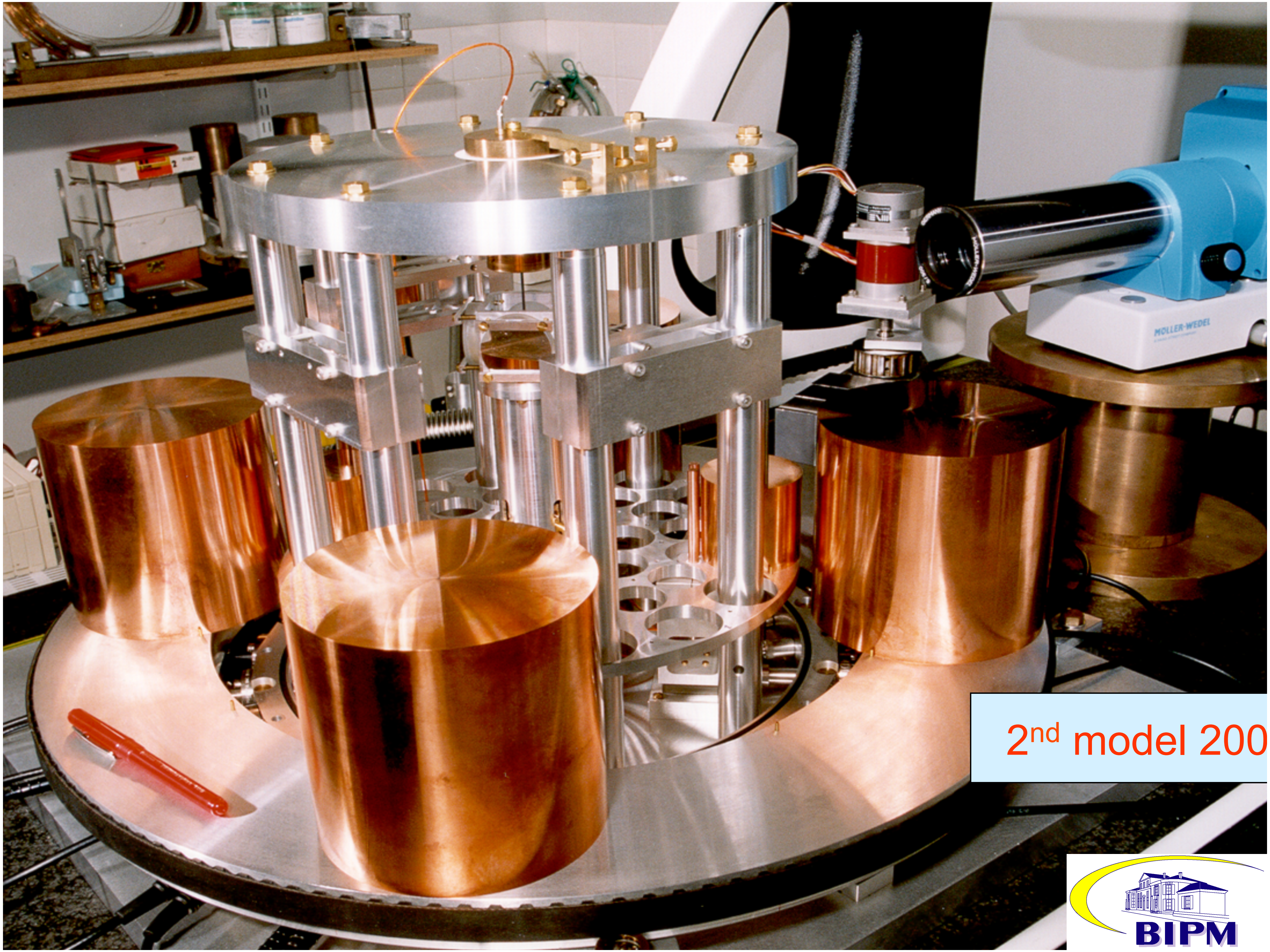
In an experiment in which there are two or more independent methods, one has first to look for errors in each until they all agree. When this is the case, the only errors that can remain are those in the much more limited set common to all.

This is the principal feature of our *G* experiment.
We have used two methods with potential for a third and we have done the whole experiment twice

BIPM

The principal characteristics of the BIPM torsion balance experiment used to measure *G* are the following:

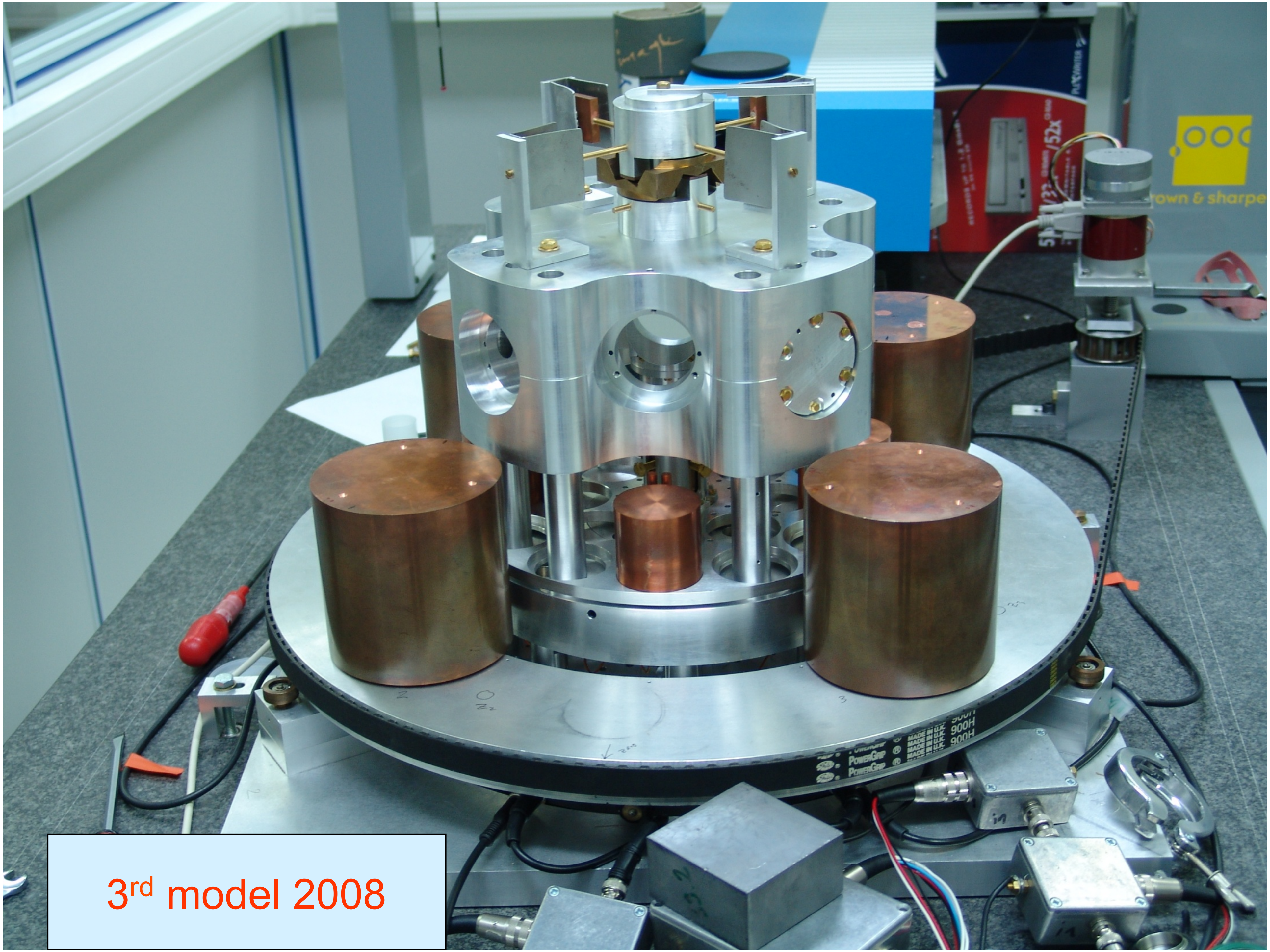
- a large mass of the torsion balance (some 6 kg) leading to a *G* signal of 3×10^{-8} Nm
- a hexadecupole test-mass distribution leading to insensitivity to local external gravity fields
- the whole placed on the platform of a coordinate measuring machine to give the best chance of accurate metrology
- a heavily loaded (some 6 kg) torsion strip as balance suspension having a $Q \geq 10^5$
- two complete experiments with the apparatus almost wholly rebuilt so that we have two statistically independent and statistically consistent results.
- two modes of operation, Cavendish and servo, giving two largely independent results for each experiment



2^{nd} model 200

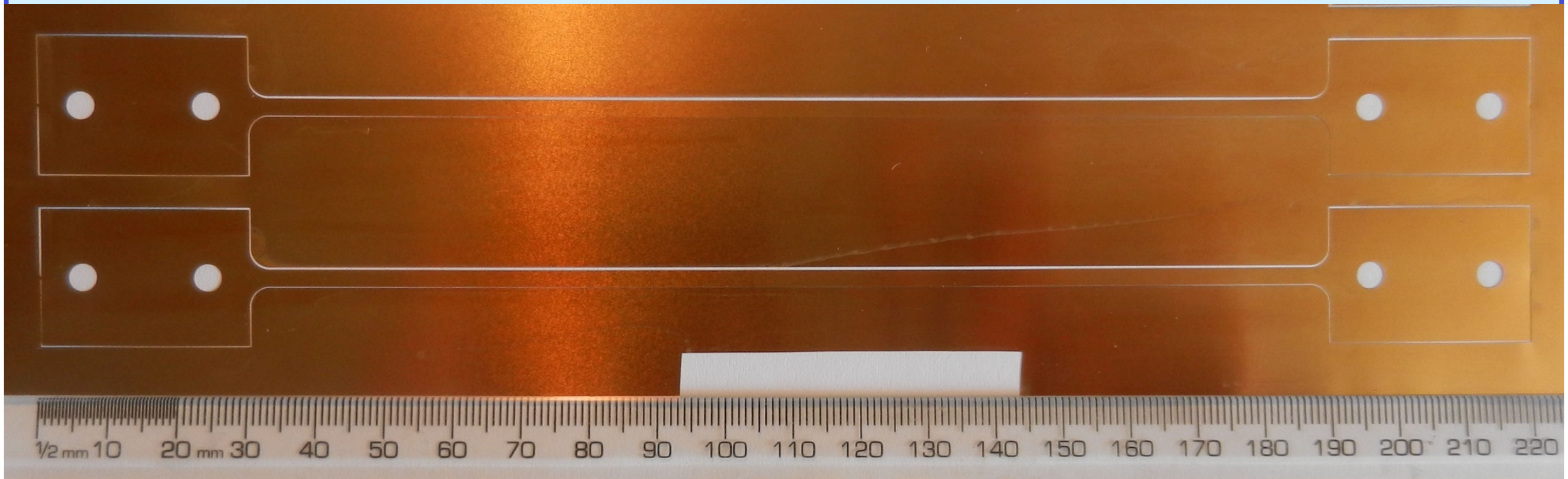
BIPM

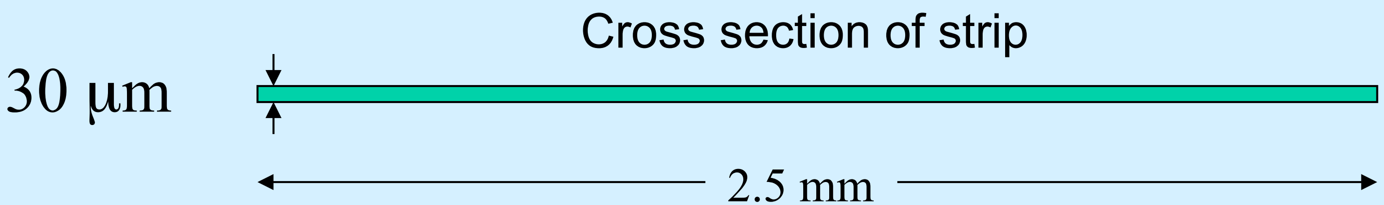

DEA
XYZ



3rd model 2008

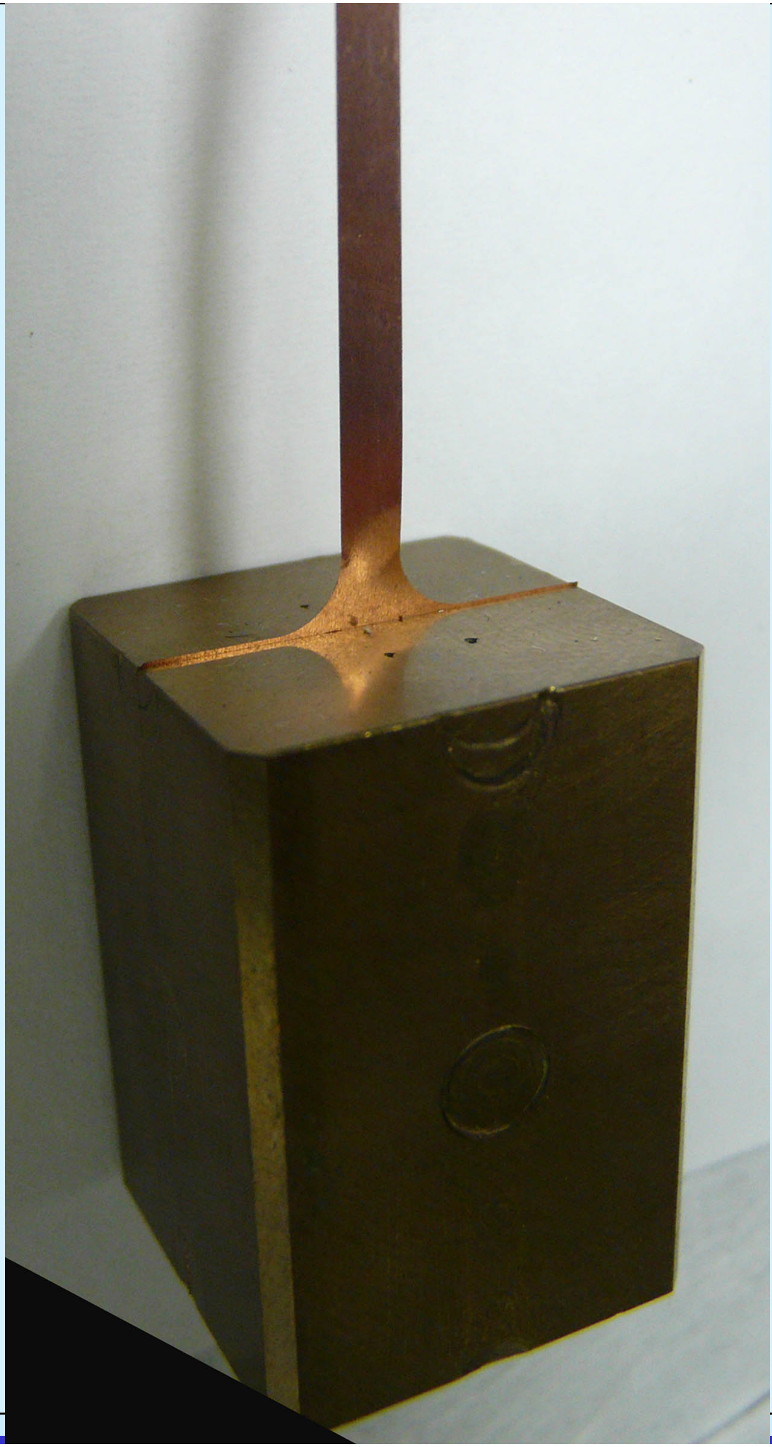
The Cu-Be torsion strip, 160 mm long, 2.5 mm wide and 30 μm thick

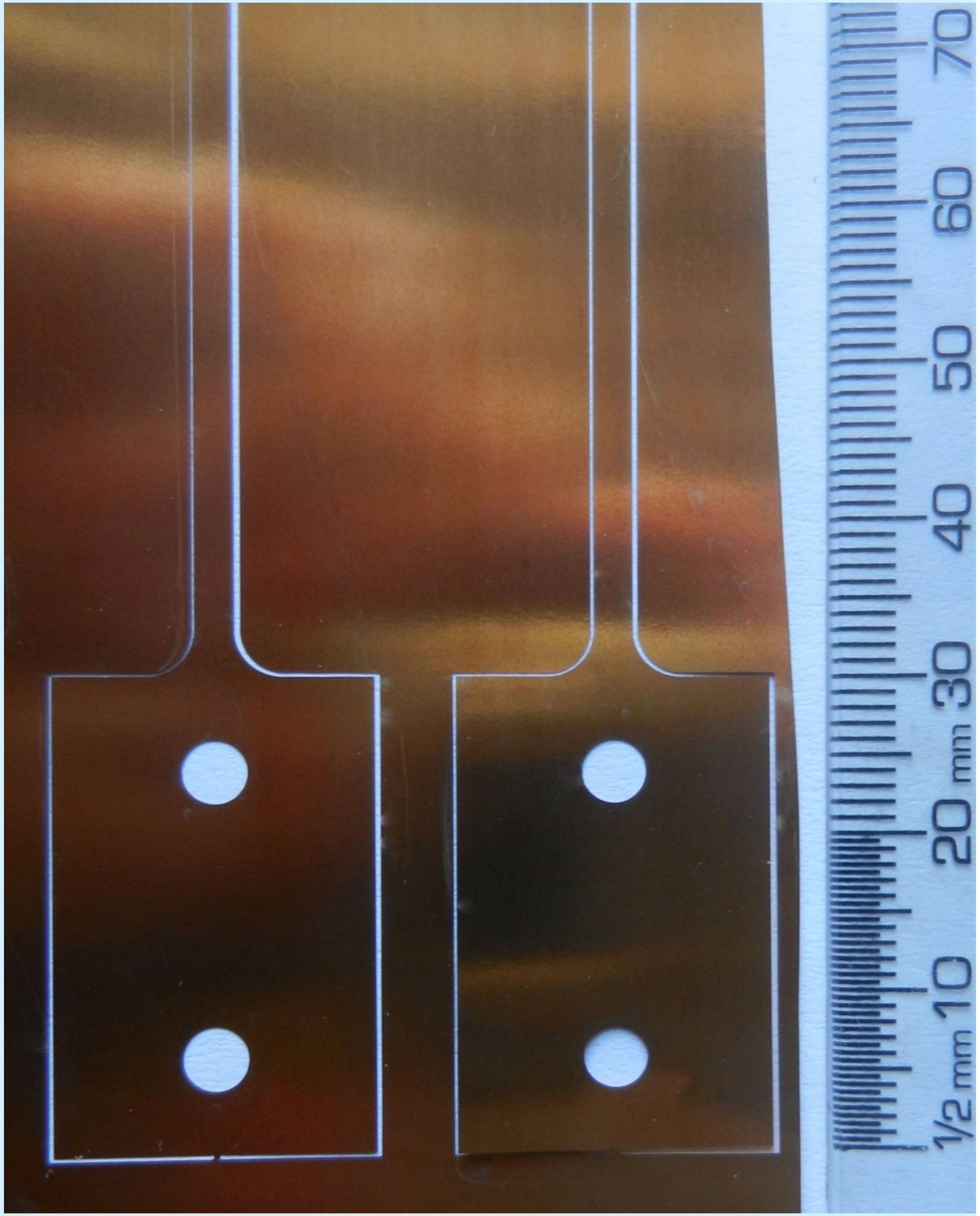




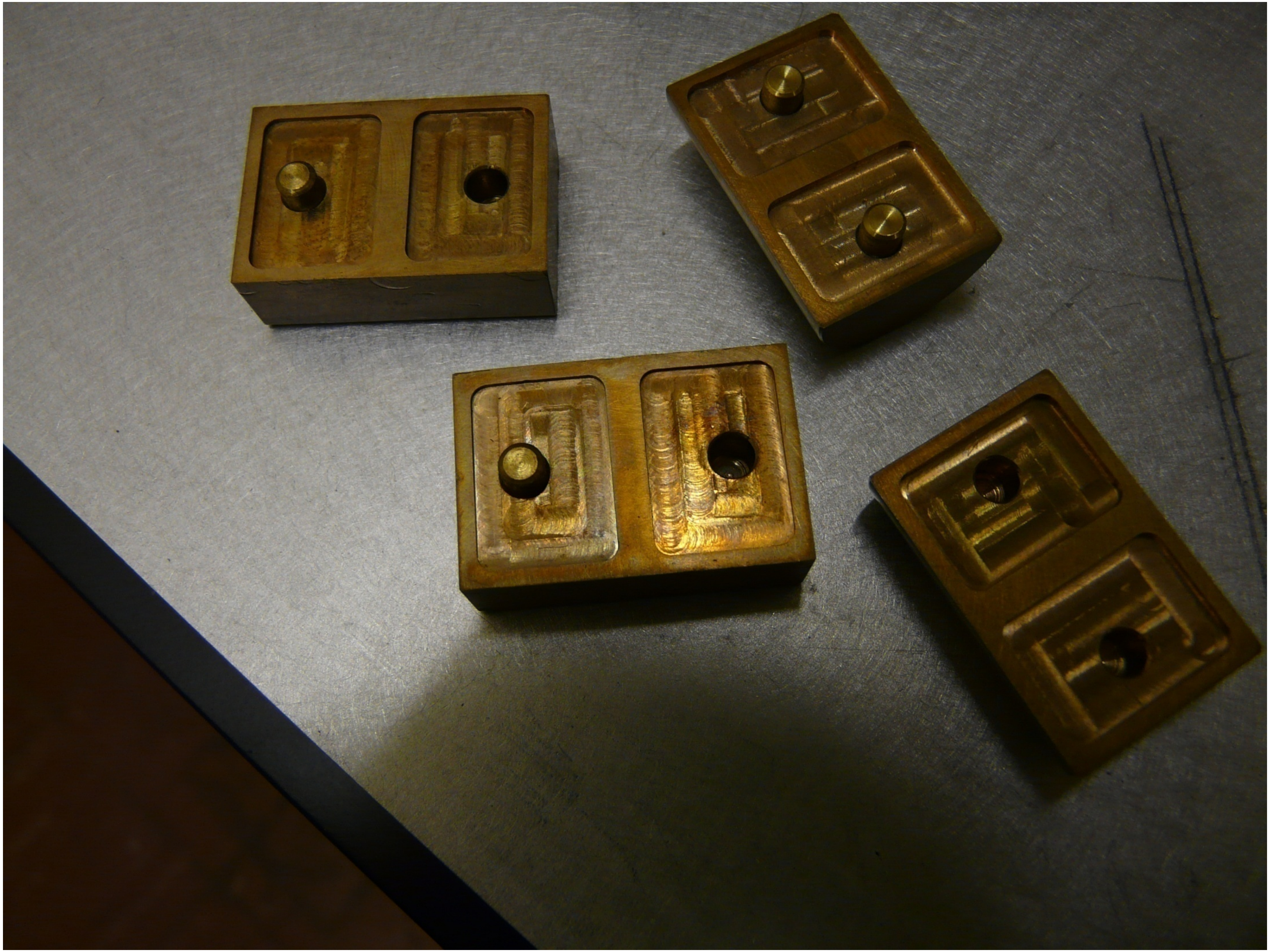
The strip loaded to about 2/3 of its yield stress and stretches by nearly 1 mm as the load is applied.

BIPM





70
60
50
40
30
20 mm
10
1/2 mm



The restoring torque of a strip of thickness *t,* width *b,* length *L* under a load *Mg* is given by:

$$c = bt^3 F/3L + Mgb^2/12L$$

In this expression only the term in red contains the shear modulus of elasticity, i.e., only the red term is elastic and subject to anelasticity. The second term is purely gravitational and represents the gravitational potential energy as the end of the strip rises and falls as it twists. This term is thus lossless and in our strip accounts for 97% of the restoring torque.

Provided that there are no losses at the ends of the strip where it is held, the whole thing should have a very high Q

This is indeed the case, the Q was 3×10^5 in the 2001 experiment and 1.2×10^5 in the 2013 experiment. The torsion balance was very stable as one would expect of a system having a $Q \geq 10^5$.

For example during ten days of the Cavendish runs the zero angle, measured at the beginning and end of each run, drifted by a total of 0.08 μrad (with a std dev of 0.07 μrad) equivalent to 10 nm at the periphery of the disk and 1 nm at the edge of the strip.

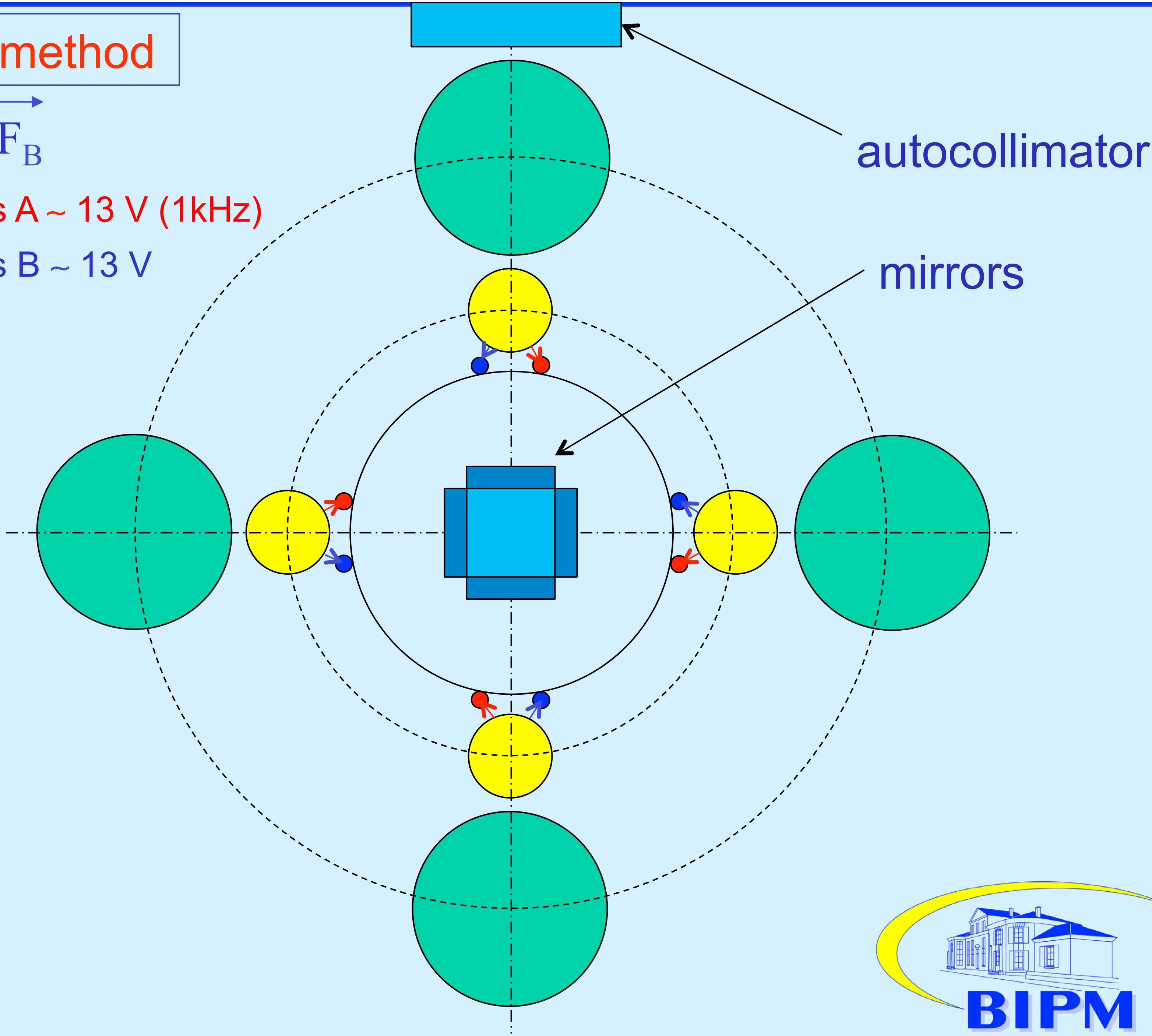
The only significant perturbing factor is the temperature.

Servo method

$\vec{F}_A = \vec{F}_B$

Electrodes A ~ 13 V (1kHz)

Electrodes B ~ 13 V



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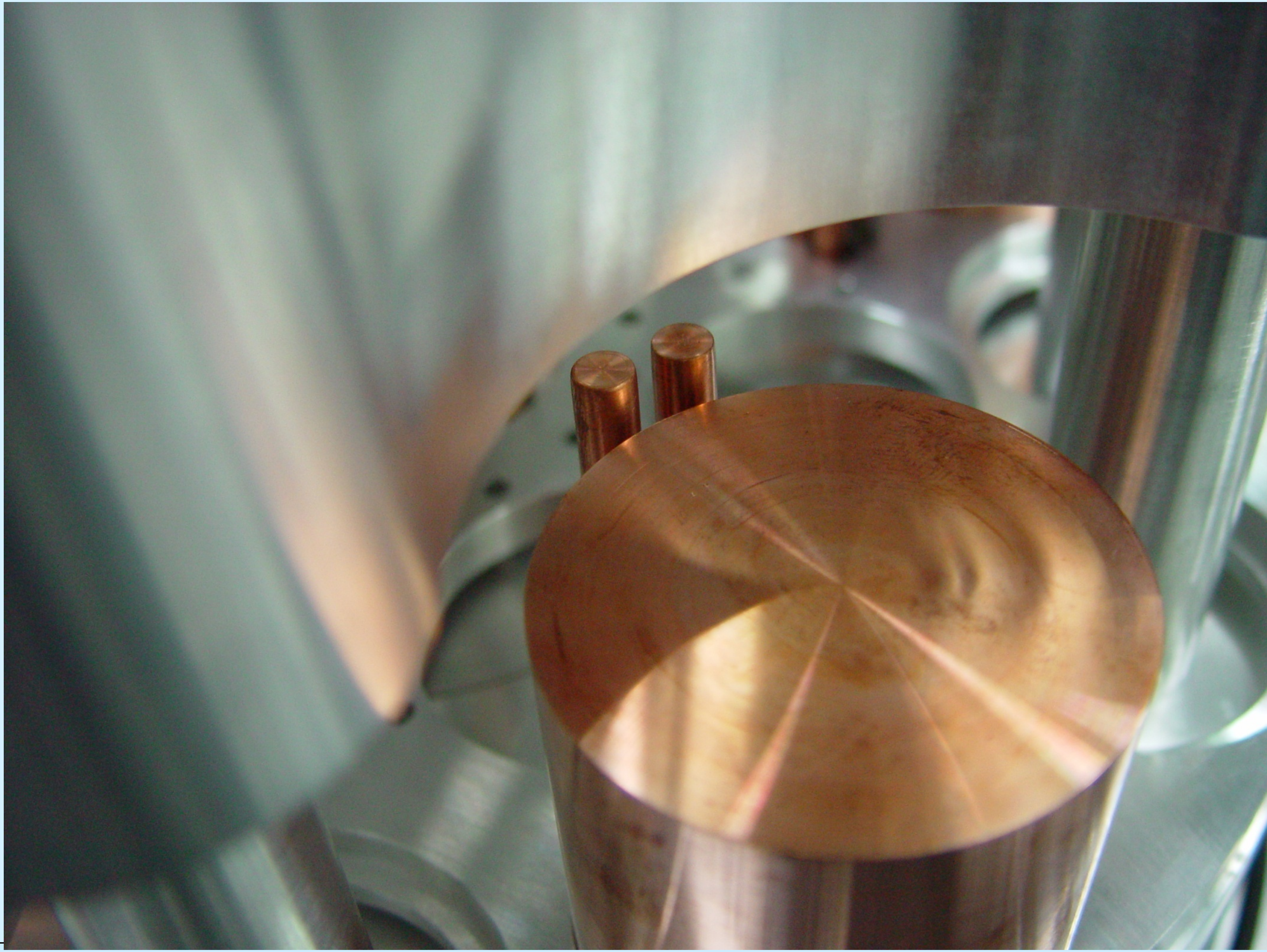
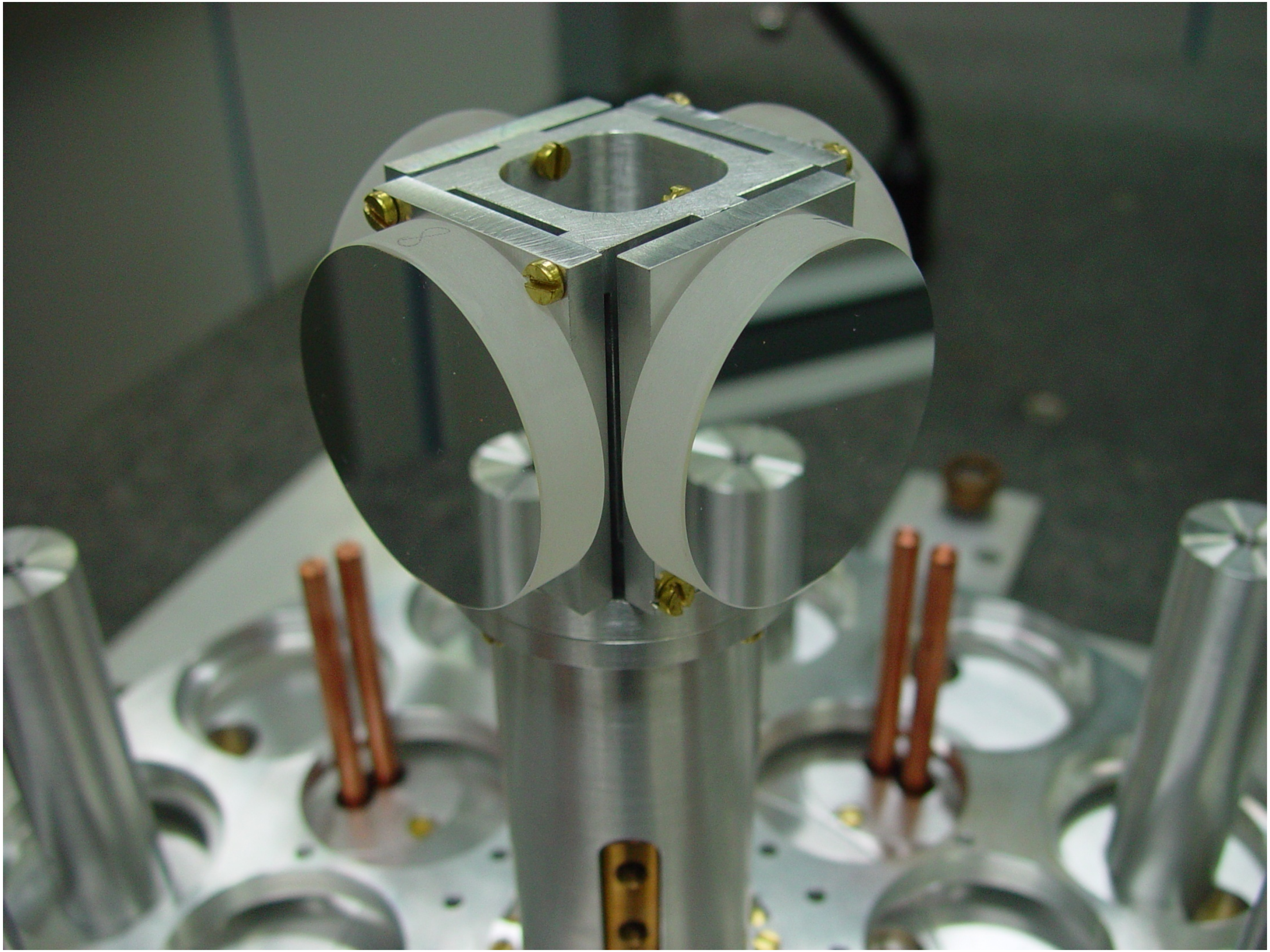
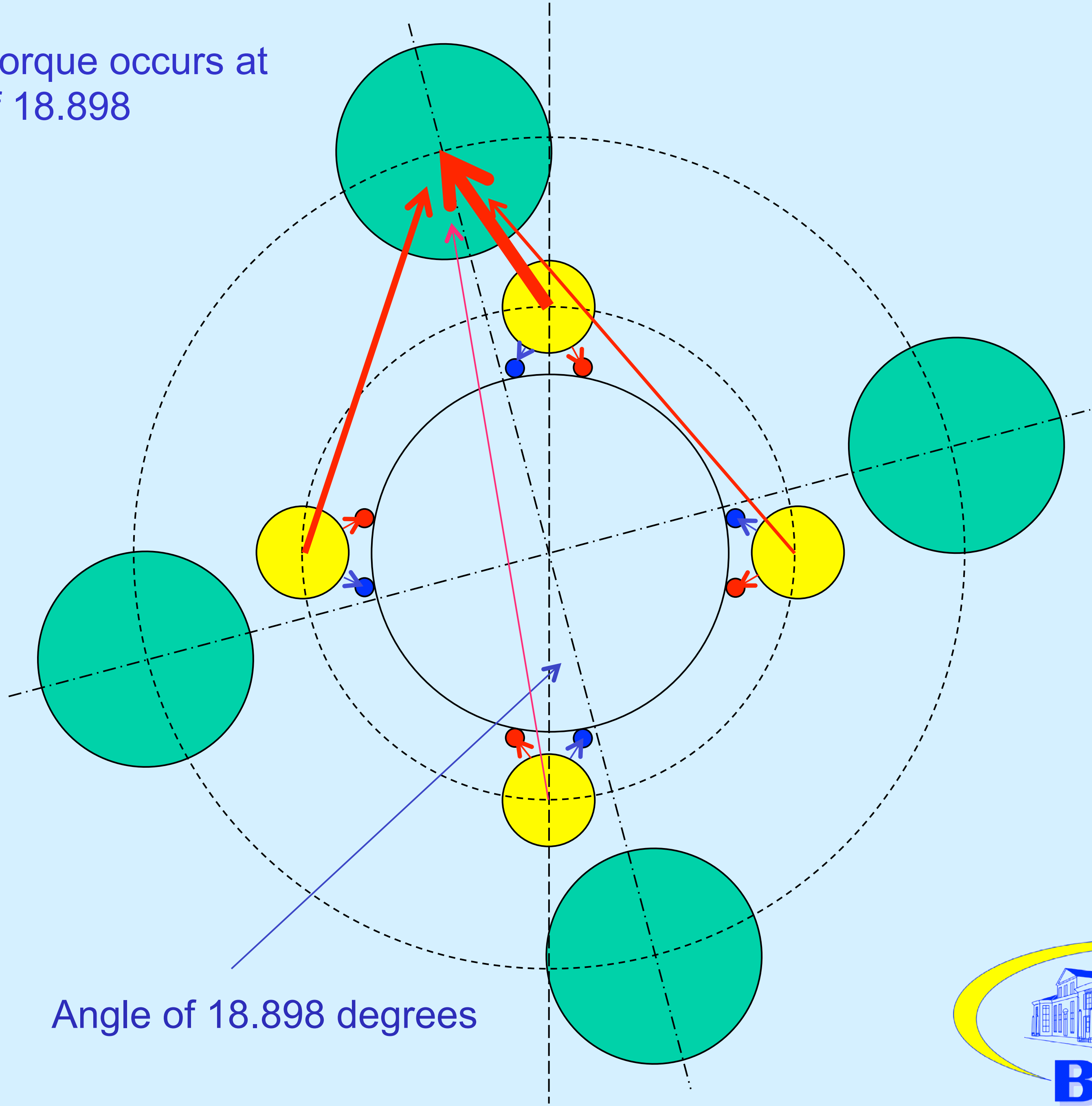


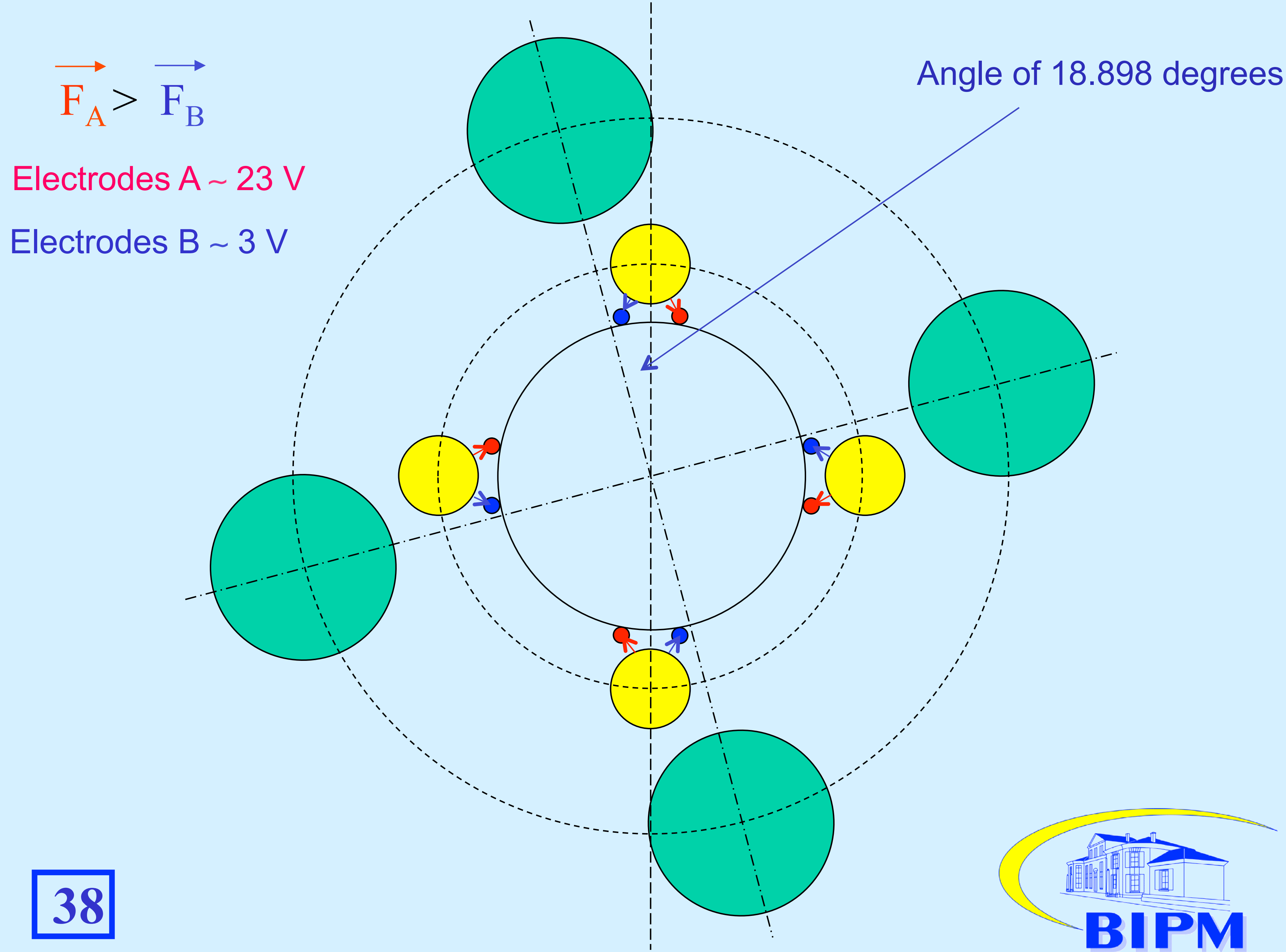
The design of the electrodes was modelled using a 2d finite element method. The radii of the electrodes and their distance from the test masses were designed such that the capacitance versus angle was linear with $dC/d\theta$ a maximum.

This means that $d^2C/d\theta^2$ is nominally zero. This enables large voltages to be applied to the electrodes (100s of volts) without instabilities.



Maximum torque occurs at an angle of 18.898 degrees.





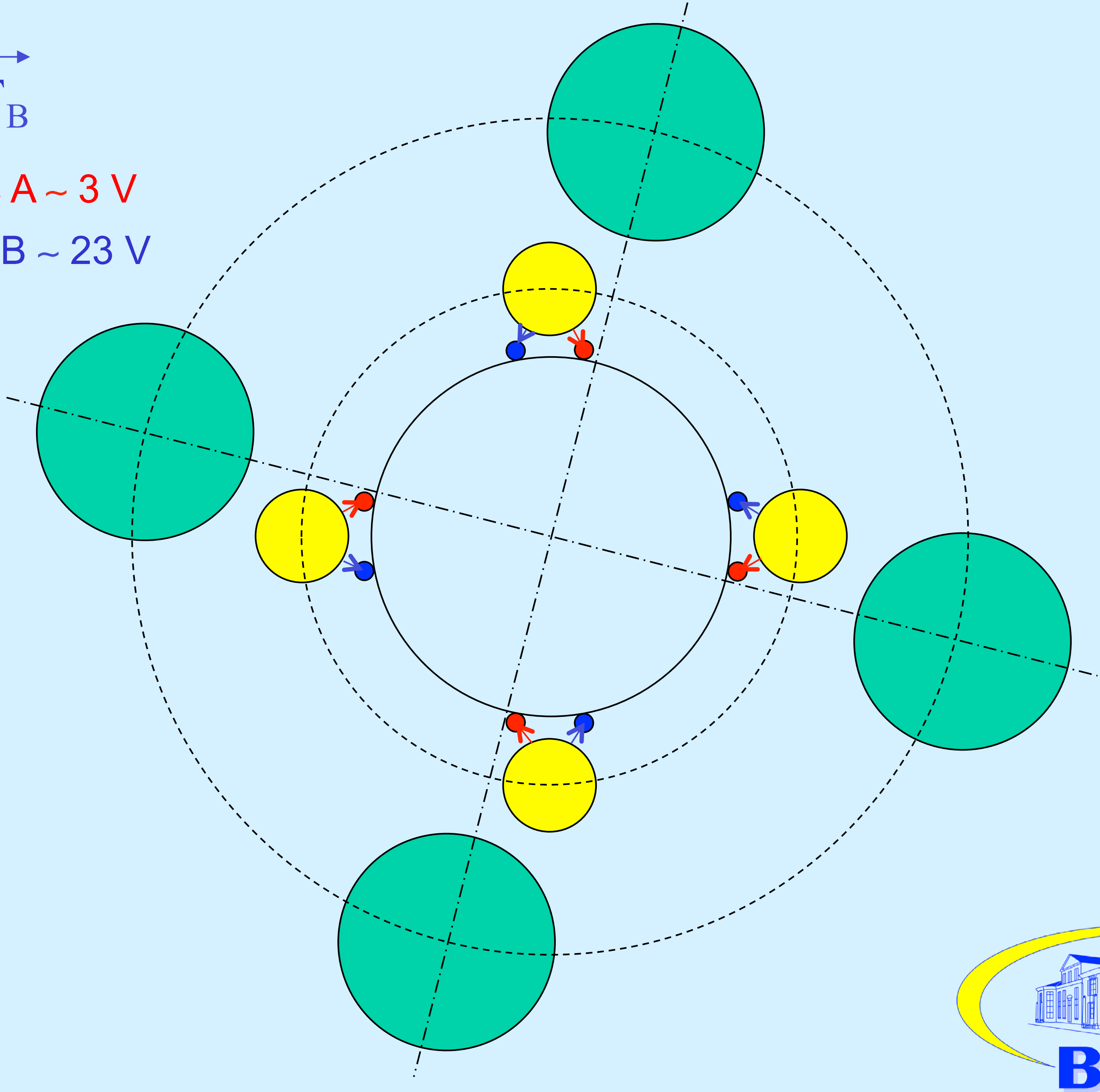
$\vec{F}_A > \vec{F}_B$
Electrodes A ~ 23 V
Electrodes B ~ 3 V
Angle of 18.898 degrees

BIPM

$\vec{F}_A < \vec{F}_B$

Electrodes A ~ 3 V

Electrodes B ~ 23 V



BIPM

The principle of the servo method:

Impossibile visualizzare l'immagine. La memoria del computer potrebbe essere insufficiente per aprire l'immagine oppure l'immagine potrebbe essere danneggiata. Riavviare il computer e aprire di nuovo il file. Se viene visualizzata di nuovo la x rossa, potrebbe essere necessario eliminare l'immagine e inserirla di nuovo.

- The electrostatic restoring torque that balances the gravitational torque $G\Gamma$ is given by:

$$G\Gamma = \frac{1}{2}\left[\frac{dC_A}{d\theta} V_A^{\,2} + \frac{dC_B}{d\theta} V_B^{\,2} + \frac{dC_{AB}}{d\theta} \left(V_A - V_B\right)^2 \right]$$

- where Γ is the gravitational coupling constant between the source masses and torsion balance; A and B signify the electrostatic servo electrodes

BIPM

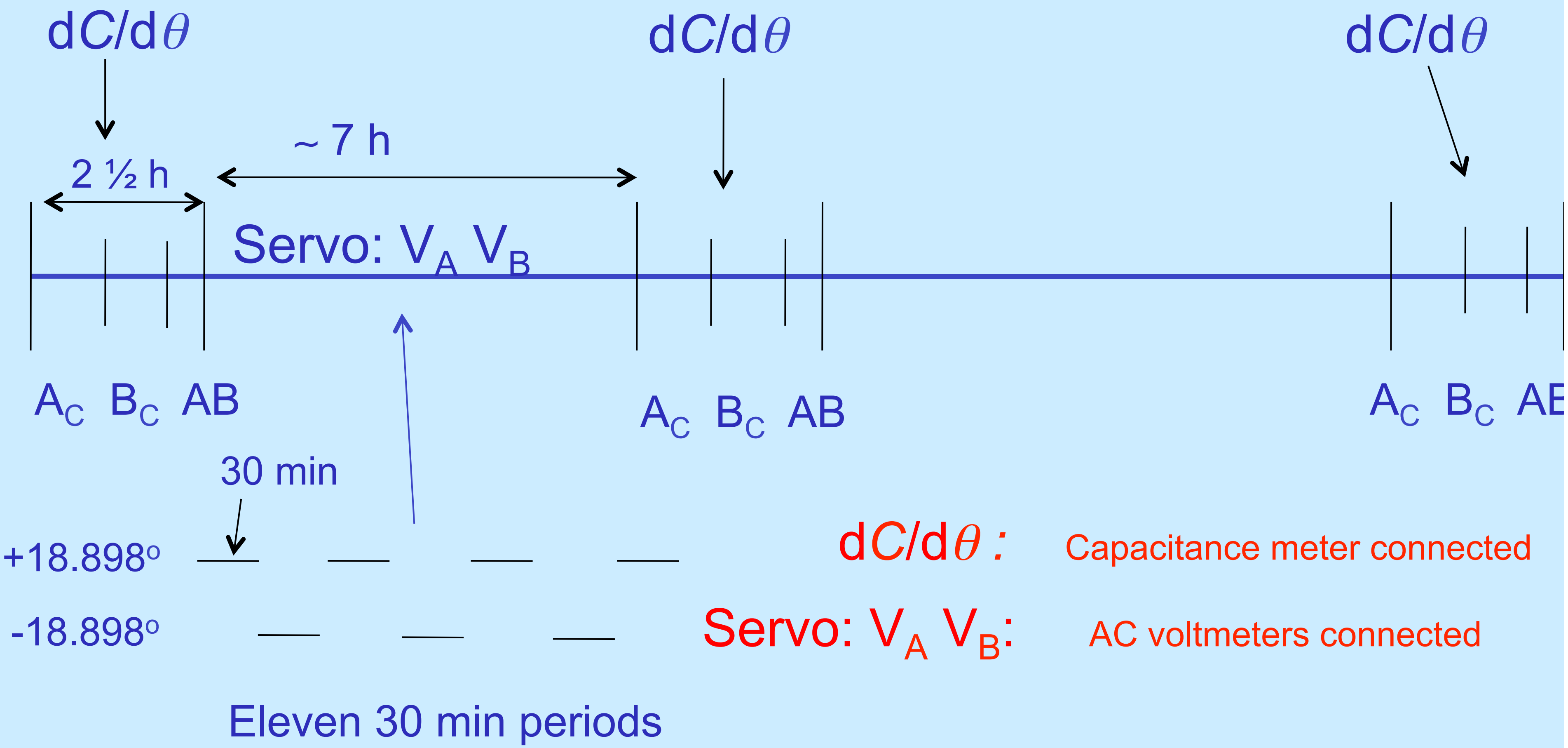
- We have to measure:

$dC/d\theta$ for both electrodes where C_A and C_B are the capacitances of each electrode to all of their surroundings and C_{AB} is the capacitance between the electrodes

AC voltages (≈ 25 volts) at 1 kHz that are applied to the electrodes

$$G\Gamma = \frac{1}{2}\left[\frac{dC_{A}}{d\theta}V_{A}^{2} + \frac{dC_{B}}{d\theta}V_{B}^{2} + \frac{dC_{AB}}{d\theta}\left(V_{A} - V_{B}\right)^{2}\right.$$

Sequence of measurements for a servo run

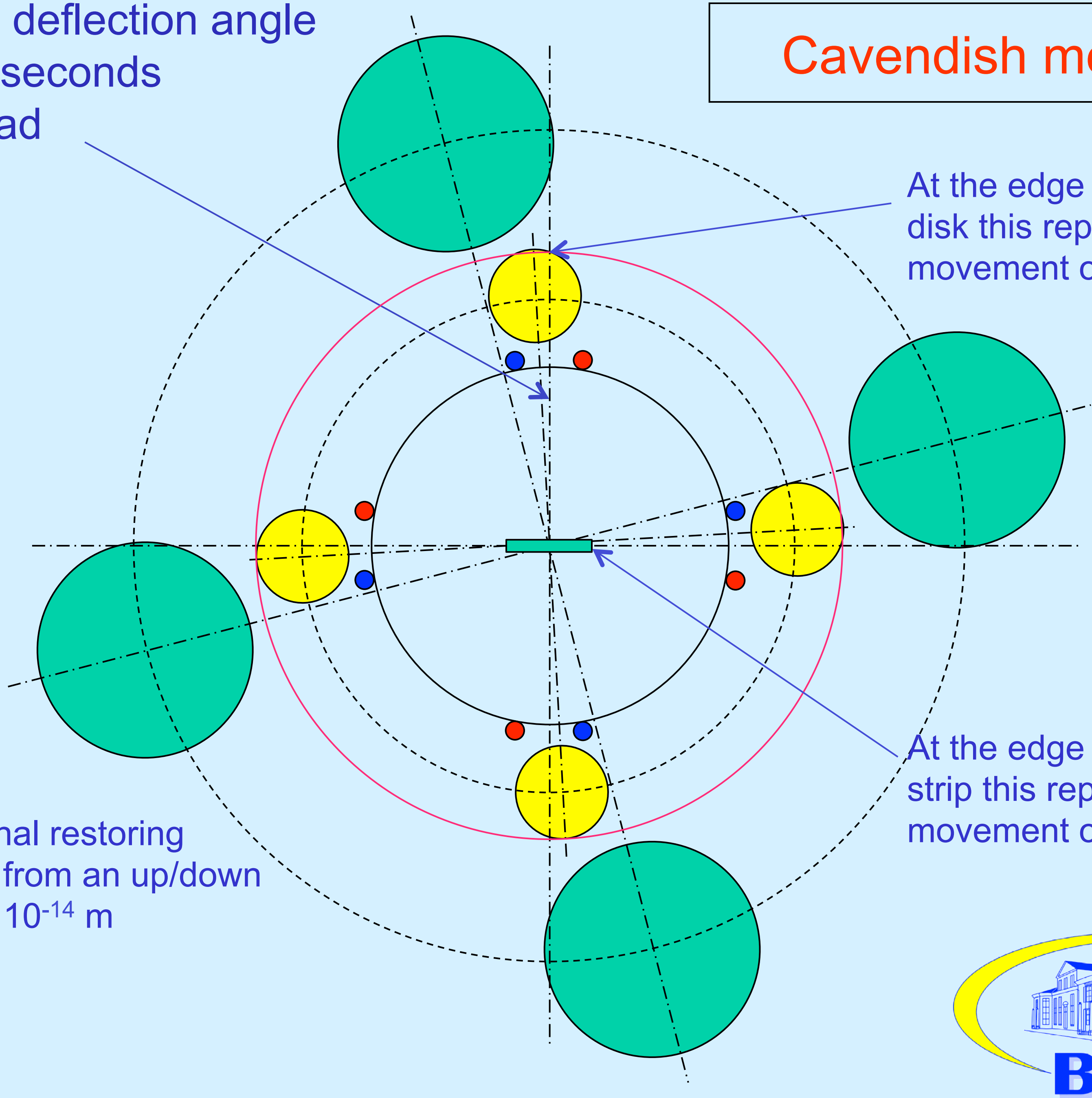


Total measurement time about 23 h

BIPM

Cavendish method

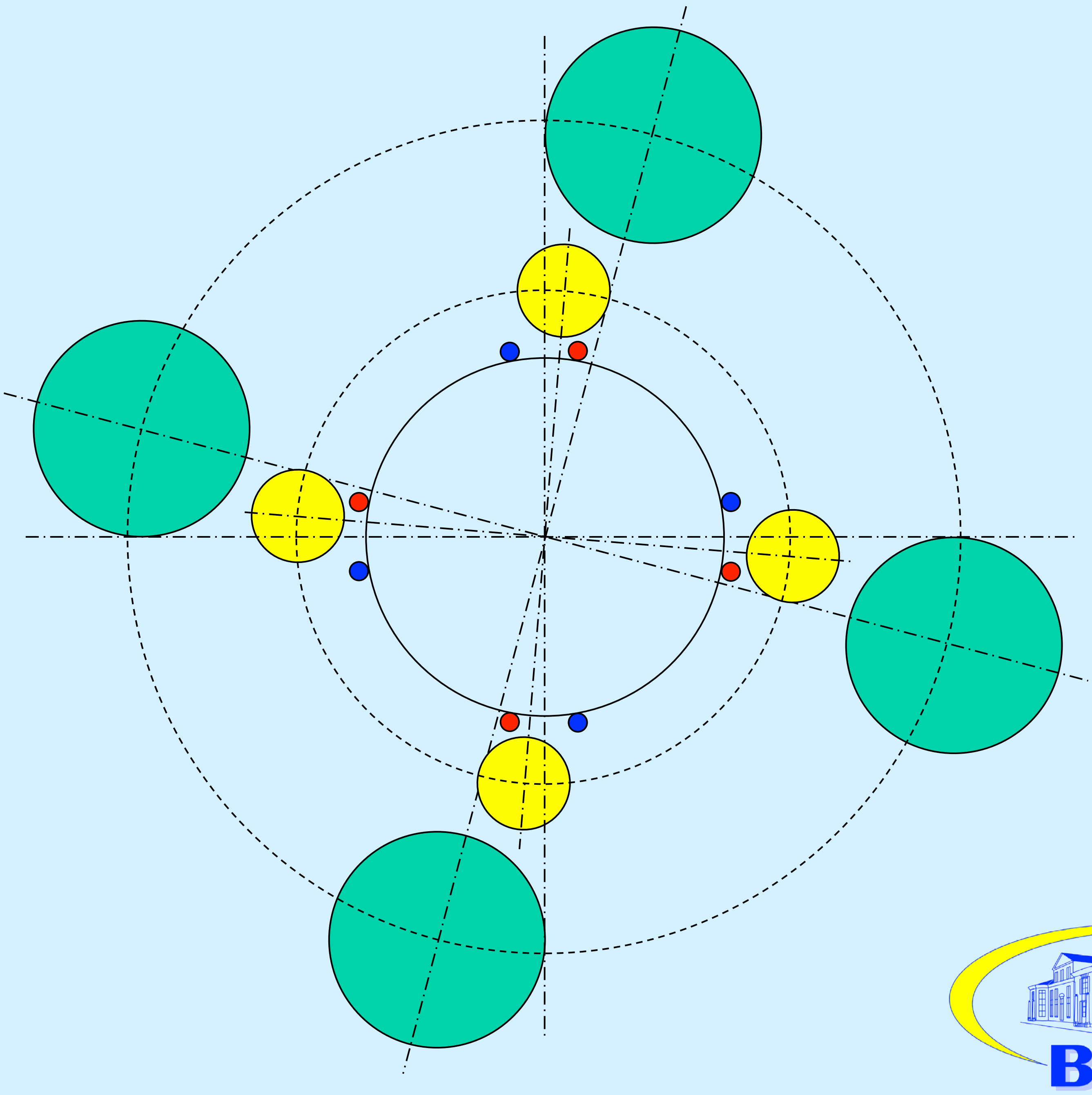
Measured deflection angle
≈15.8 arc seconds
or 76.5 μrad



At the edge of the torsion disk this represents a movement of 10 μm

At the edge of the torsion strip this represents a movement of 1 μm

The gravitational restoring torque comes from an up/down movement of 10^{-14} m



BIPM

Principle of the Cavendish (free-deflection) method:

- The angular deflection, θ, is related to the gravitational torque $G\Gamma$ by the relation $G\Gamma = c\,\theta$, where c is the torque constant of the balance given by $c = I\omega^2$ so that:

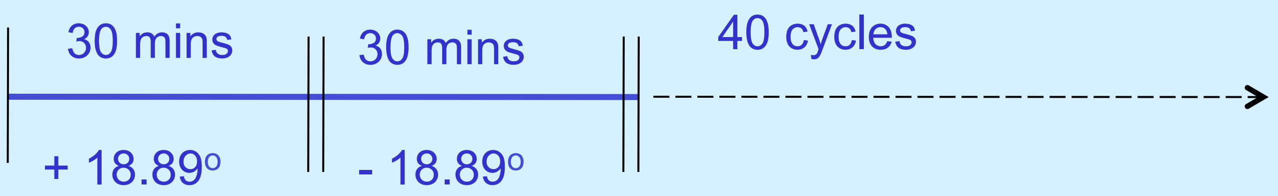
$$G\Gamma = I\omega^2\,\theta$$

- c is determined from measurements of period and calculation and measurement of I, the moment of inertia of the torsion balance.

- since the angle is observed by the autocollimator in air but actually the rotation is in vacuum we have to multiply the observed angle by the refractive index of air 1.000271

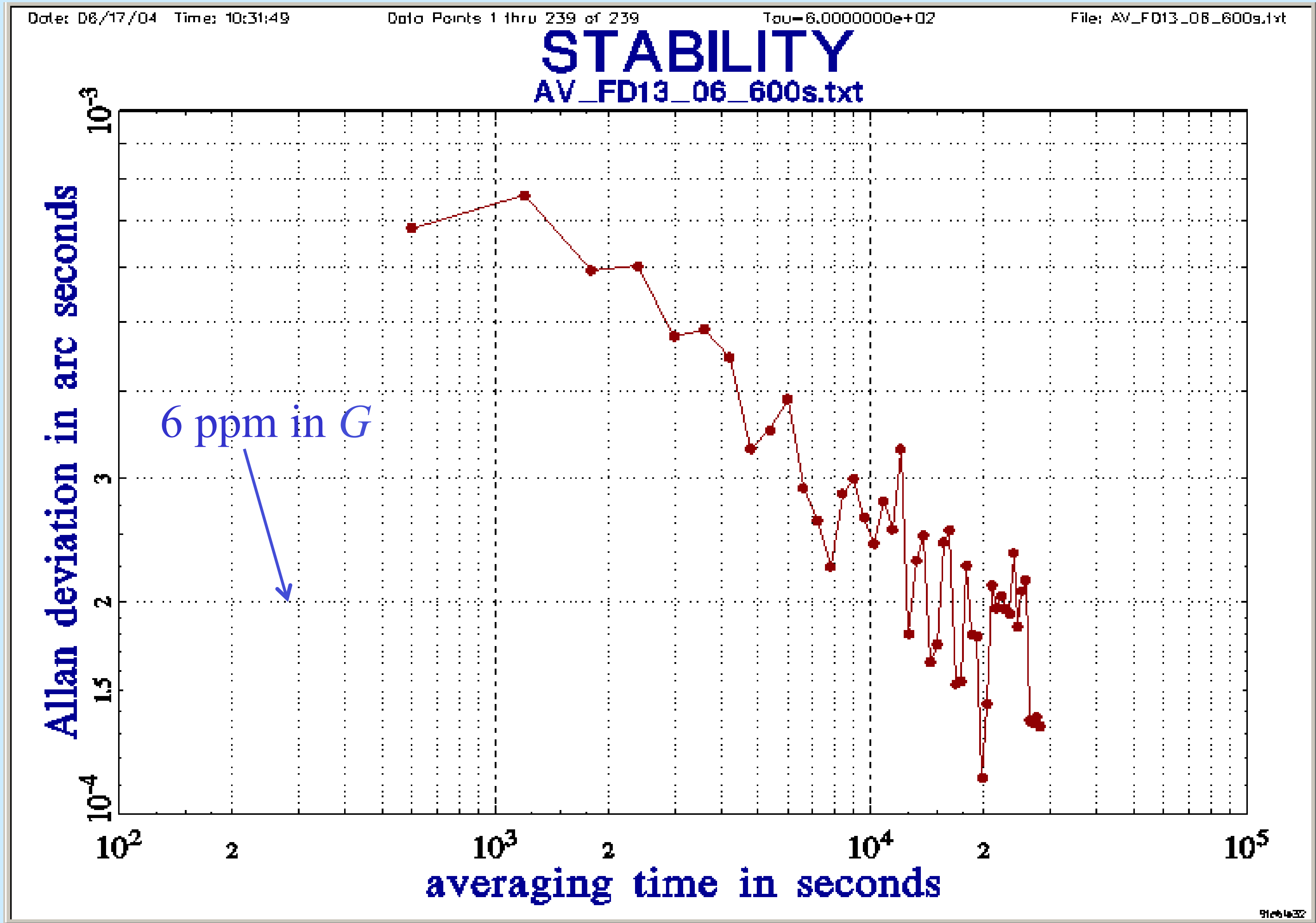
BIPM

Sequence of operations for the Cavendish method



Total run time for one data point ≈ 22 hours

Allan deviation for free-deflection measurements



Comparing the expressions for servo and Cavendish:

Impossibile visualizzare l'immagine. La memoria del computer potrebbe essere insufficiente per aprire l'immagine oppure l'immagine potrebbe essere danneggiata. Riavviare il computer e aprire di nuovo il file. Se viene visualizzata di nuovo la x rossa, potrebbe essere necessario eliminare l'immagine e inserirla di nuovo.

$$G\Gamma = \frac{1}{2}\left[\frac{dC_{\mathrm{A}}}{d\theta} V_{\mathrm{A}}^{2} + \frac{dC_{\mathrm{B}}}{d\theta} V_{\mathrm{B}}^{2} + \frac{dC_{\mathrm{AB}}}{d\theta} \left(V_{\mathrm{A}} - V_{\mathrm{B}}\right)^{2} \right]$$

$$G\Gamma = I\omega^2 \theta$$

Note:

(a) $I \approx 4\, M(\text{test})\, R^2$ and so M(test) appears in both Γ and I and is thus eliminated in the Cavendish method and

(b) θ appears in the denominator in the servo and numerator in the Cavendish method. Thus the average of the servo and Cavendish methods eliminates a <u>common</u> angle error.

Measured quantities specific to the servo method are:

Capacitance, angle ($dC/d\theta$) and AC volts at 1 kHz

For the Cavendish method:

Angle θ, period T and moment of inertia I

Common to both are:

(a) values of the (test) and source masses and their density inhomogeneities

(b) Relative positions of source masses and test masses and torsion balance with respect to source masses (dimensional metrology)

(c) the gravitational coupling between source and test masses and all the other components of the torsion balance.

BIPM

- Density inhomogeneities

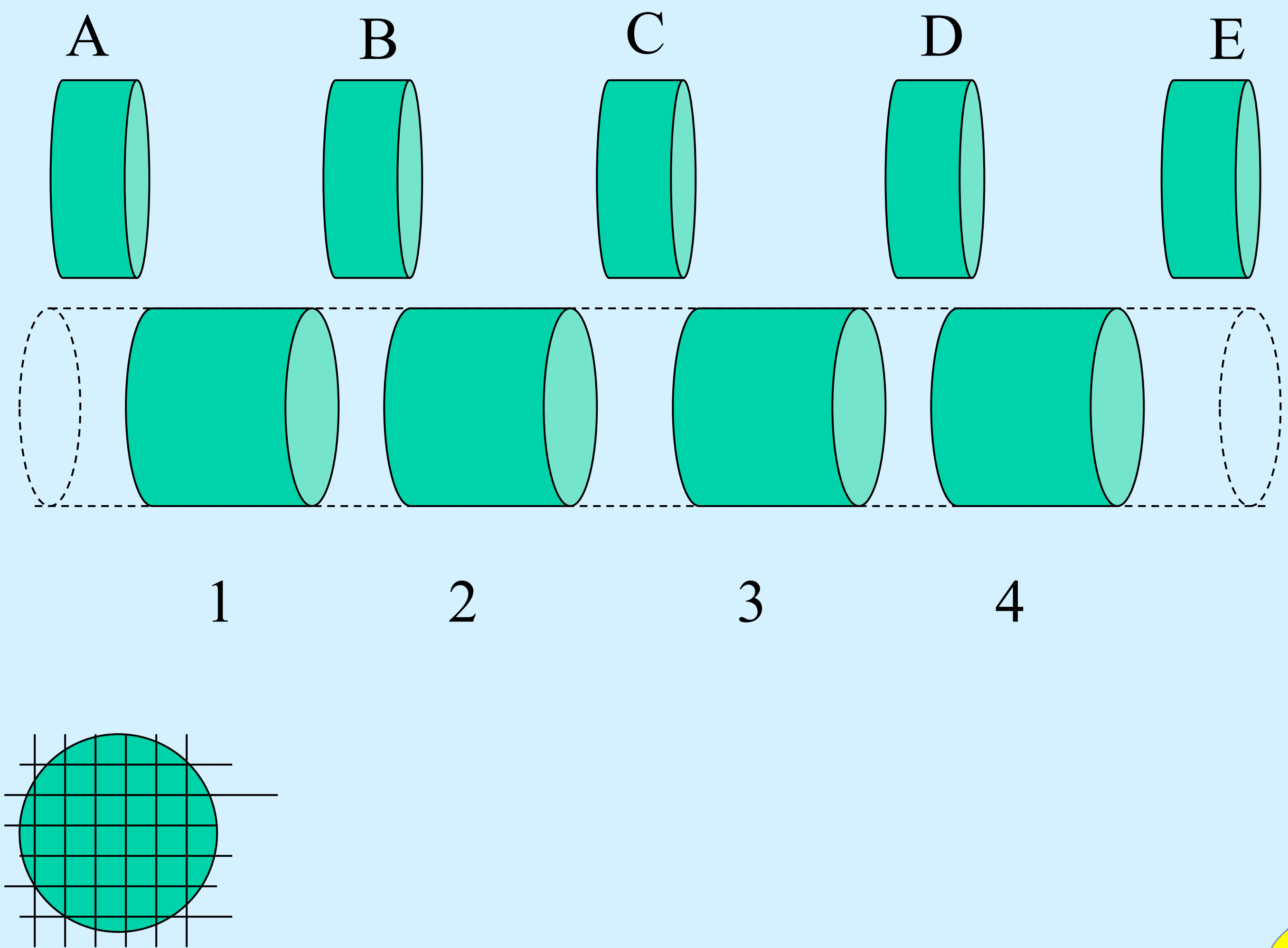
  (a) hydrostatic weighing of samples cut from the original ingots
  (b) centre of gravity of source masses by air bearing

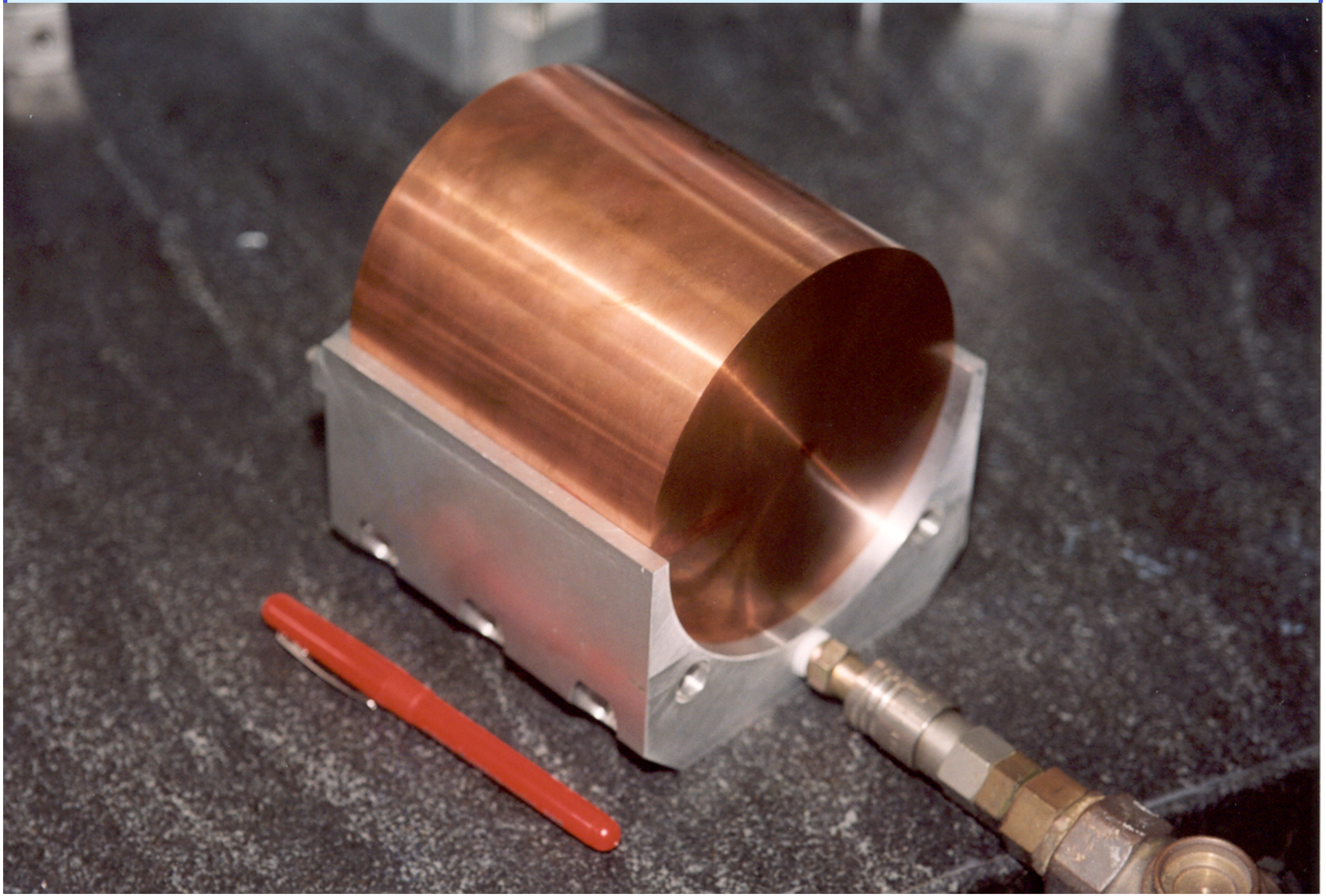
- During the measurements of *G*, we turned the source masses through successive angles of 120^{o} and took the average of the results.

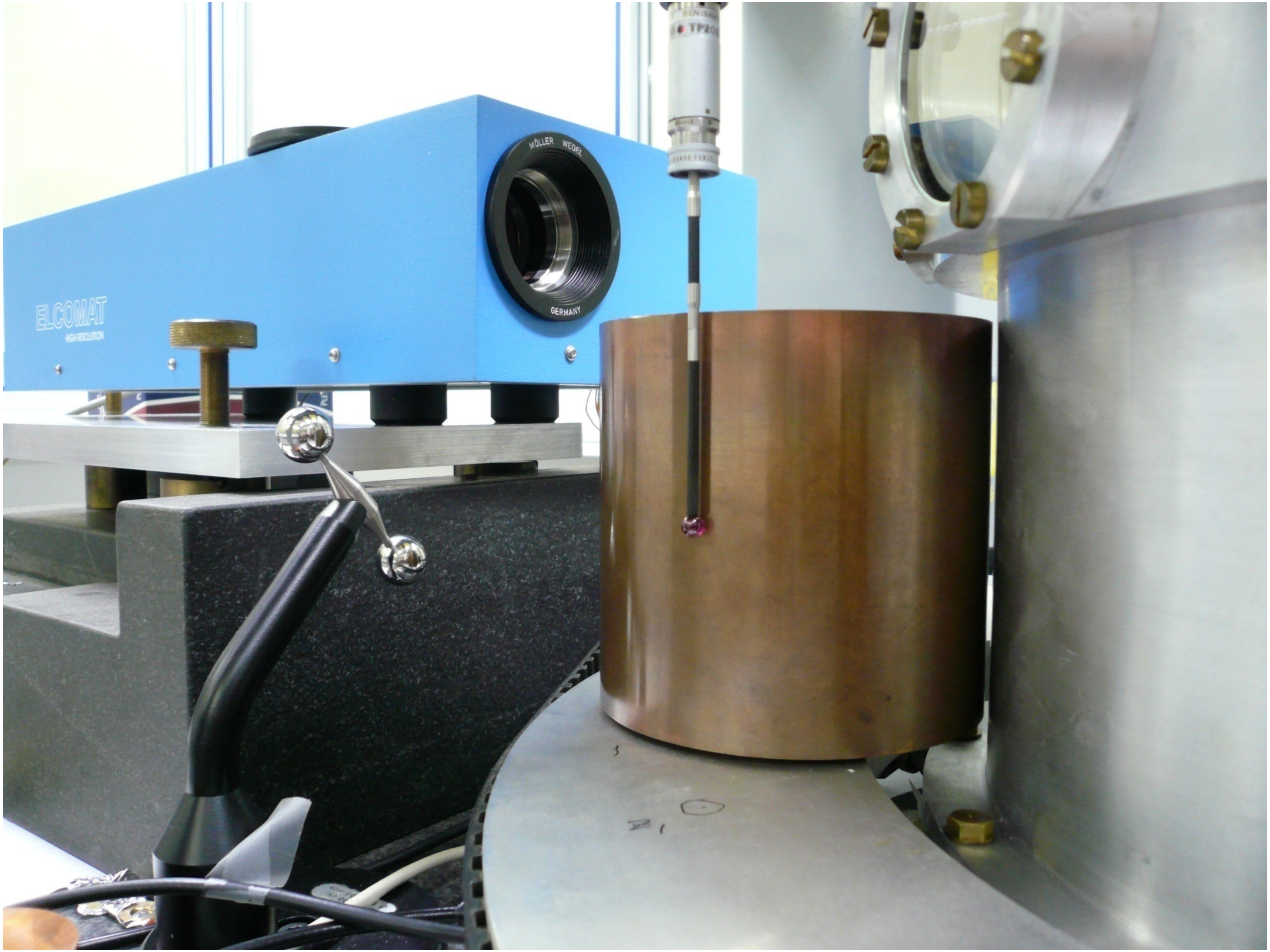
BIPM

The BIPM measurement of *G*

- Measurement of density inhomogeneities in source masses



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GERMANY
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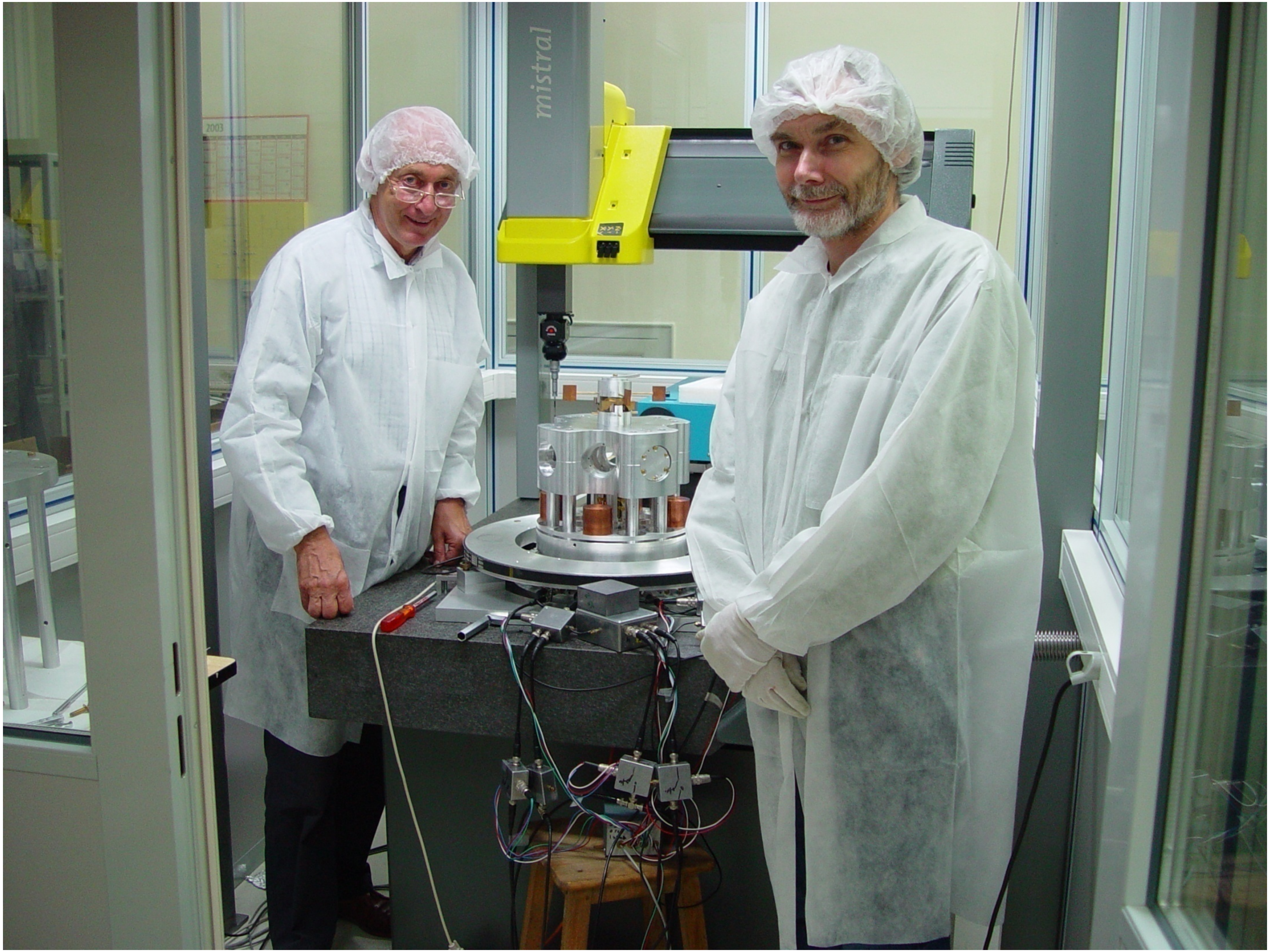
HIGH RESOLUTION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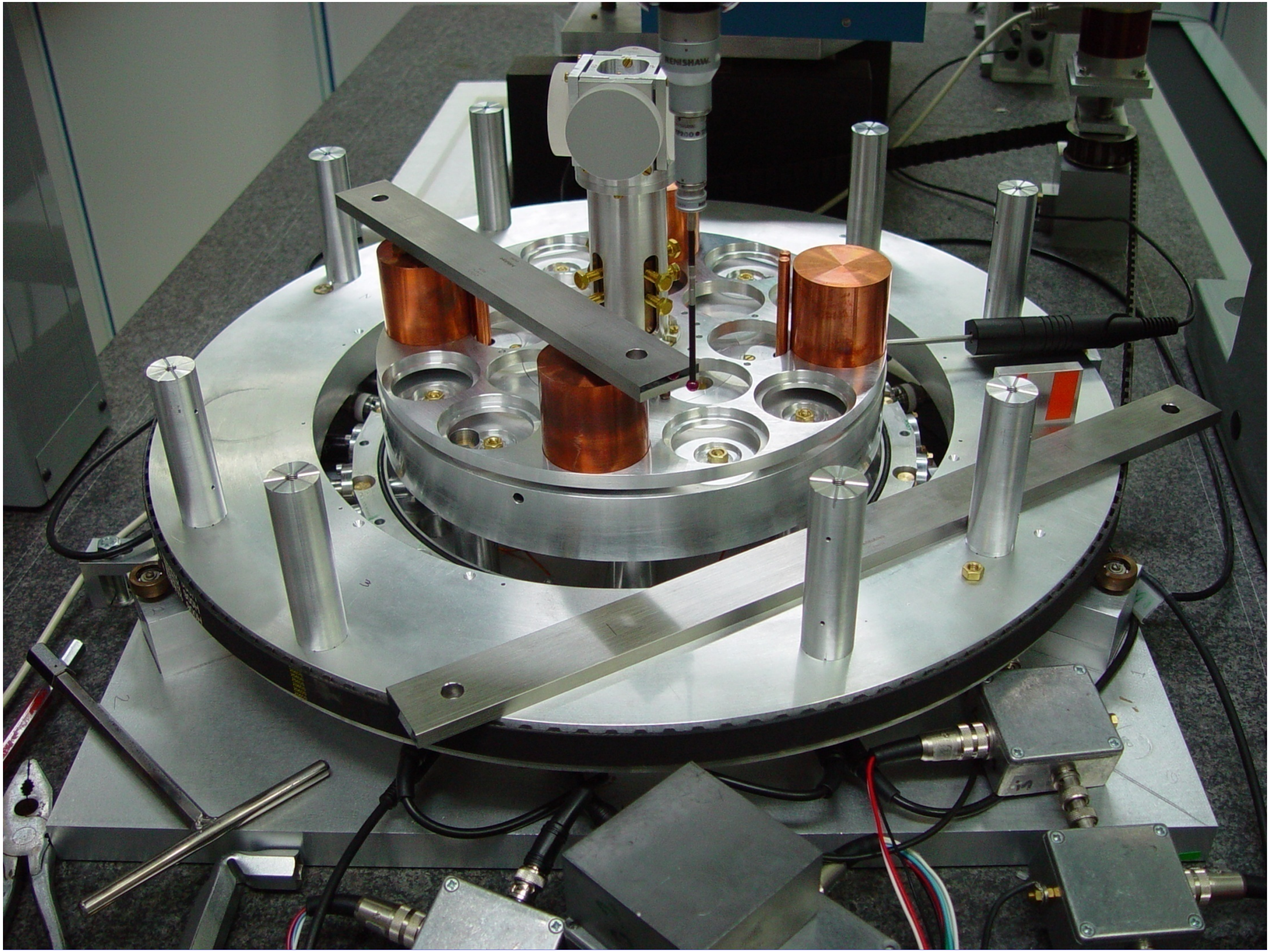
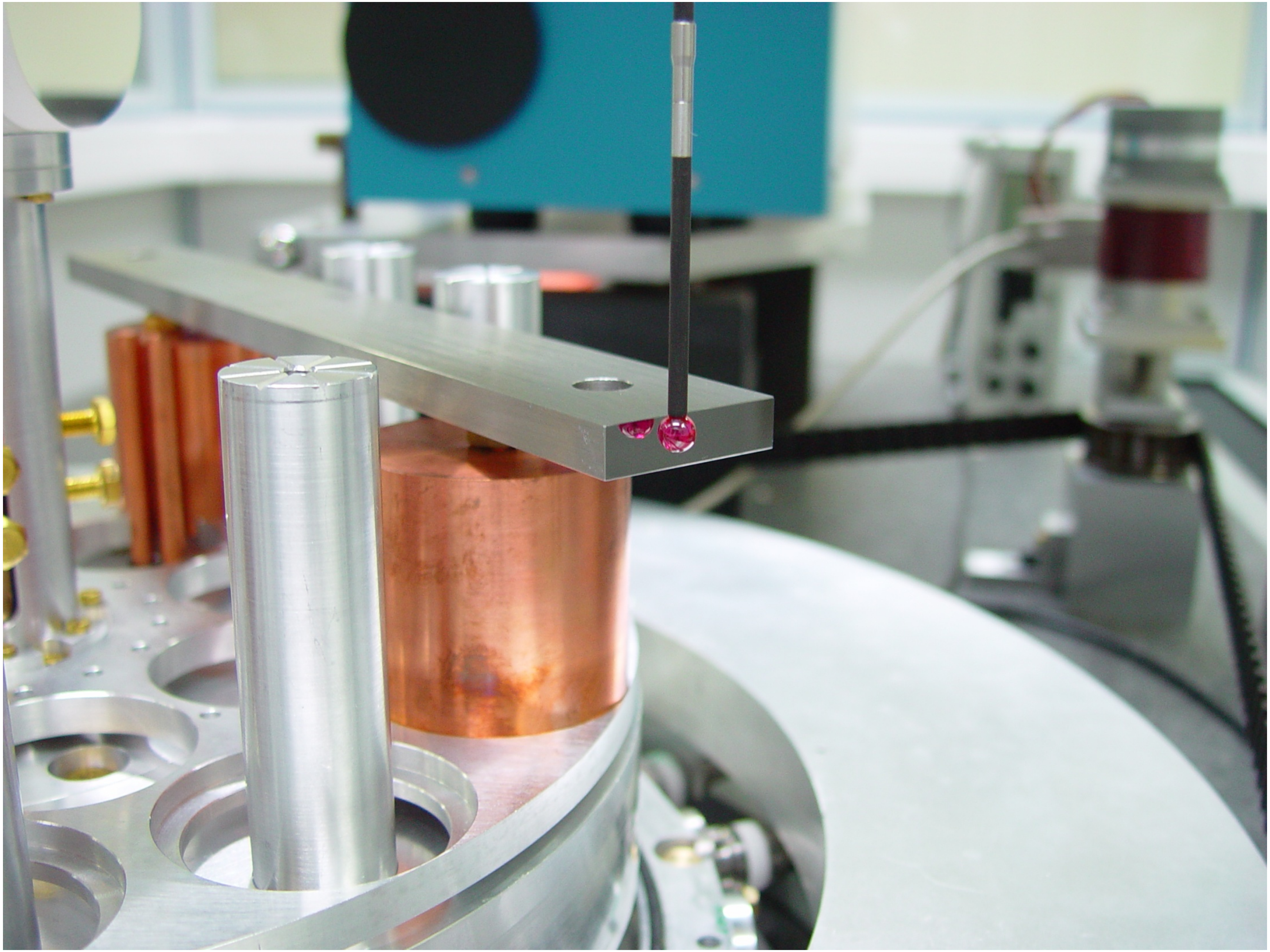
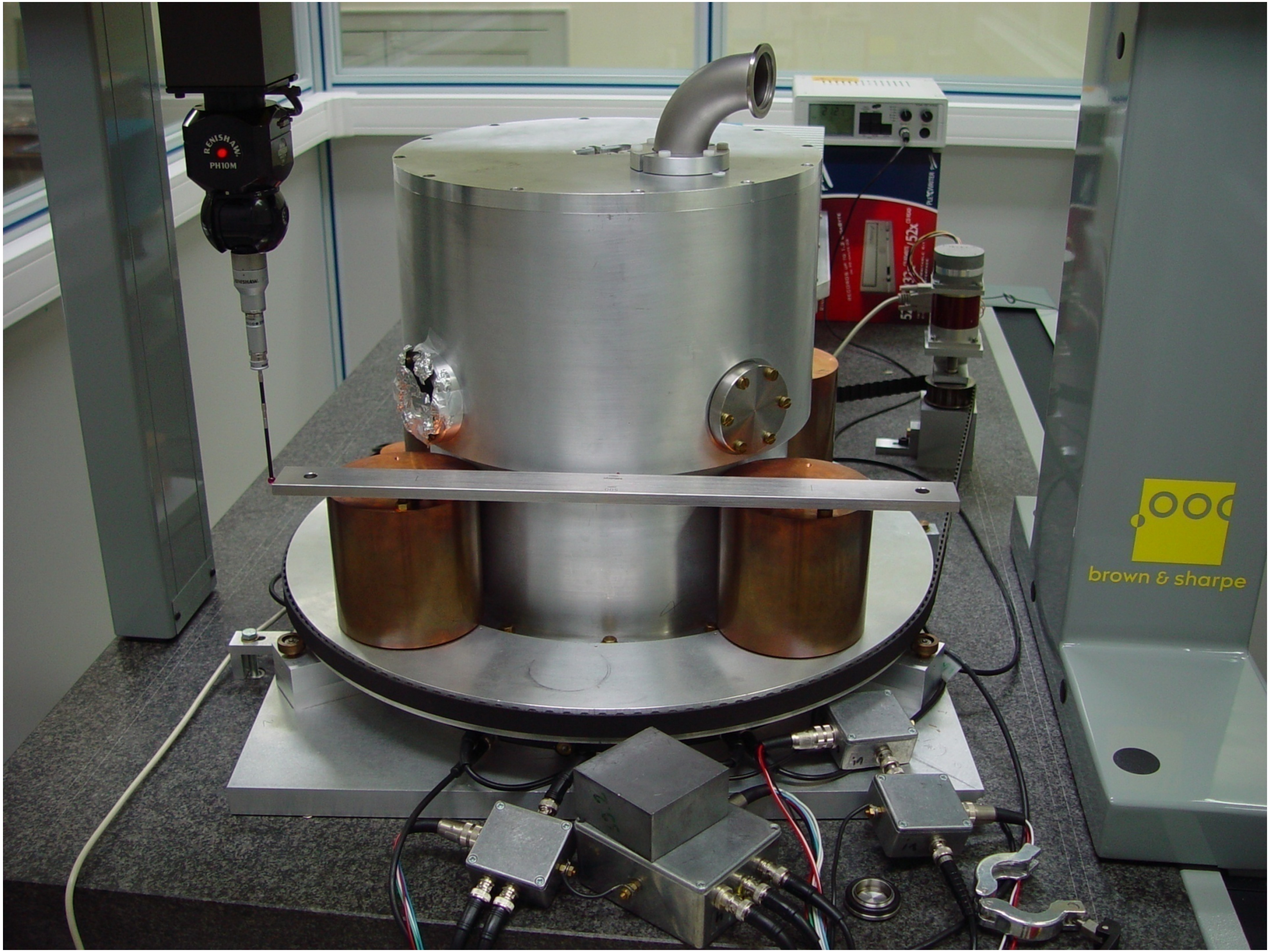
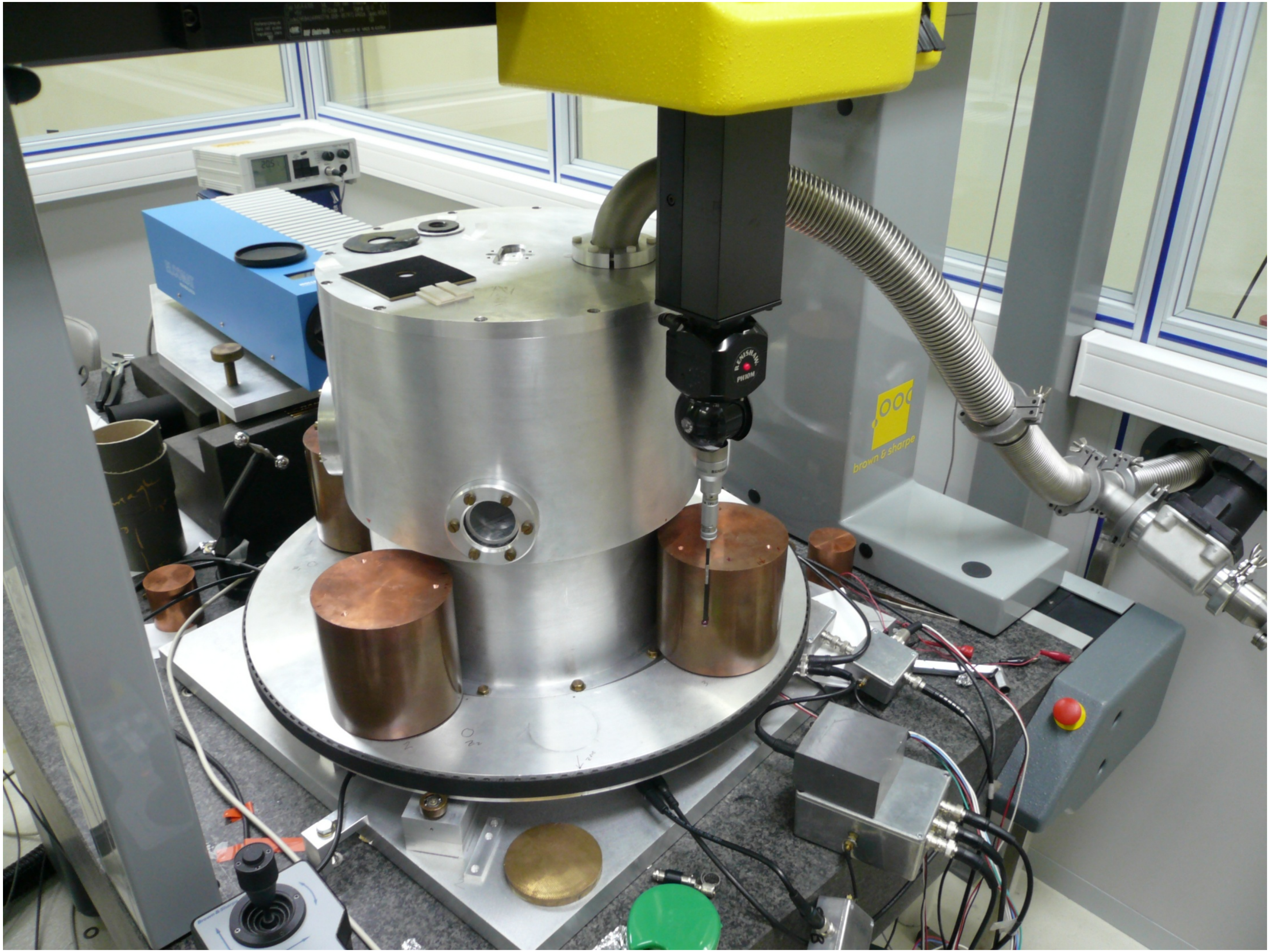
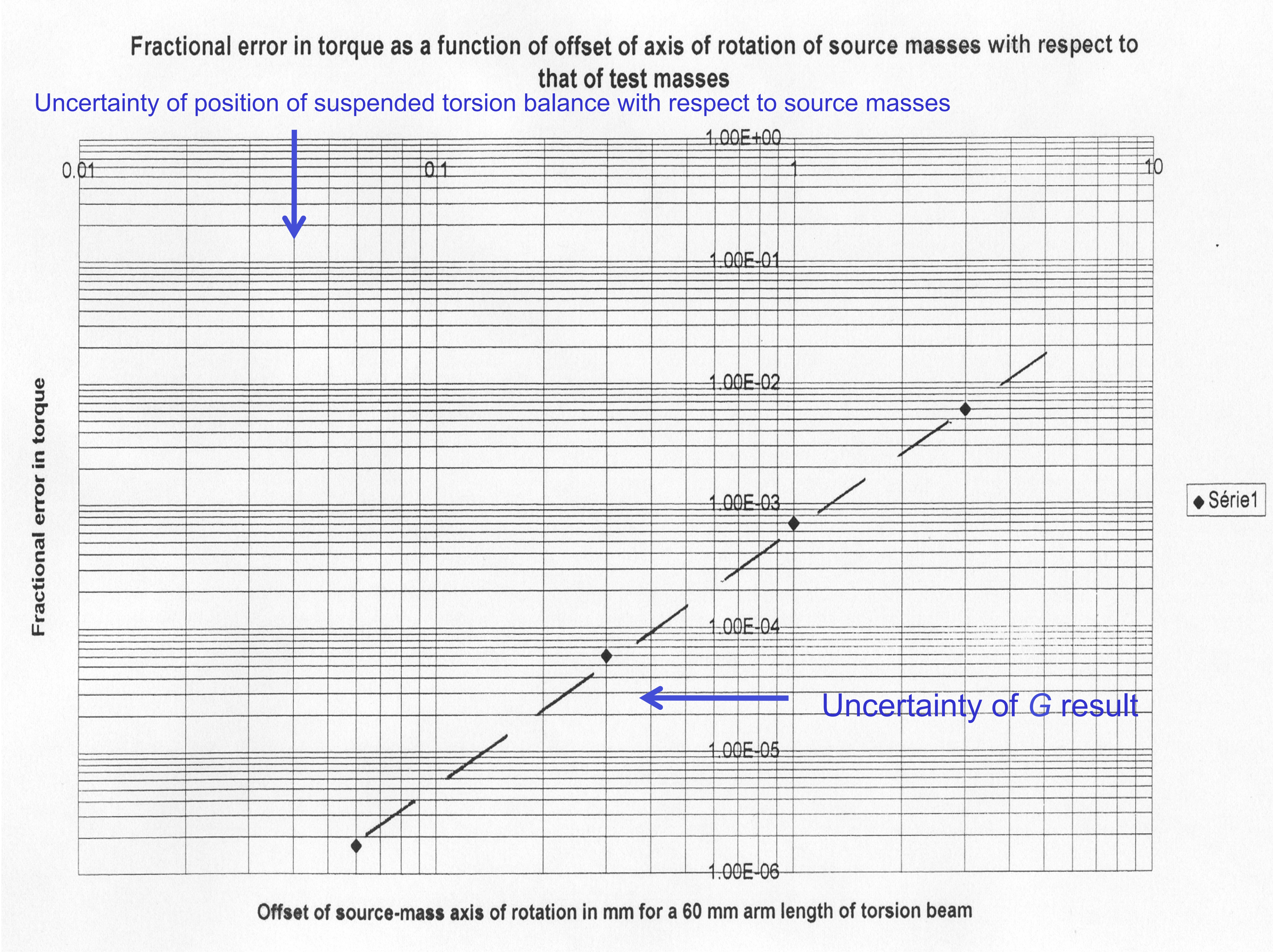





RENISHAW
PH10M
brown & sharpe


RENISHAW
PH10M
brown & sharpe


Fractional error in torque as a function of offset of axis of rotation of source masses with respect to that of test masses
Uncertainty of position of suspended torsion balance with respect to source masses
Uncertainty of G result
Fractional error in torque
Offset of source-mass axis of rotation in mm for a 60 mm arm length of torsion beam
0.01
0.1
1
10
1.00E+00
1.00E-01
1.00E-02
1.00E-03
1.00E-04
1.00E-05
1.00E-06
Série1

Source mass positions (millimetres) October 2007

Estimated accuracy of cmm calibration: 0.4µm
and of test and source mass positions: 0.5 µm

mm

T = 20.5 at z = -113.5 (halfway up source masses)

mass 1		x	439.8204			
		y	194.6098		213.9361	
mass 2		x	710.3151			
		y	58.9622		214.0259	
mass 3		x	845.8945			
		y	329.5344		213.9667	
mass 4		x	575.3332			
		y	465.1887		214.0396	
	centre	x	642.8408	average	**213.9921**	centre
		y	262.0738			

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Source mass coordinates March 2008, position B

Source mass positions 3 March 2008 **measurements at + 18.898 degrees**

T=20.6 z=-113.5

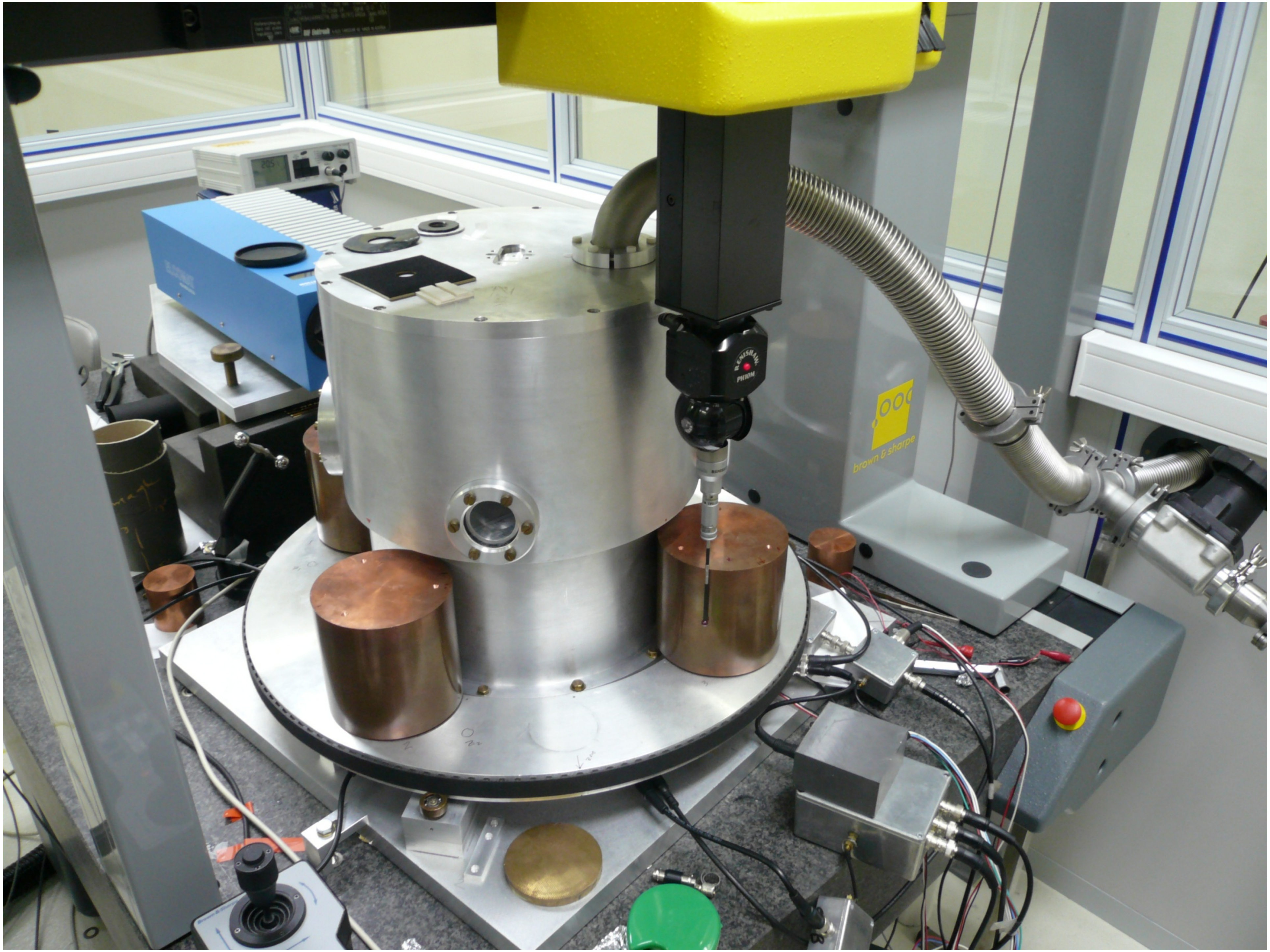
mass 1	x	439.9185	
	y	194.336	213.9369
mass 2	x	710.6025	
	y	59.0676	214.0263
mass 3	x	845.8037	
	y	329.8305	213.9668
mass 4	x	575.0539	
	y	465.1066	214.0404
		642.8447 average	**213.9926**
		262.0852	

213.9921
at 20.5 °C

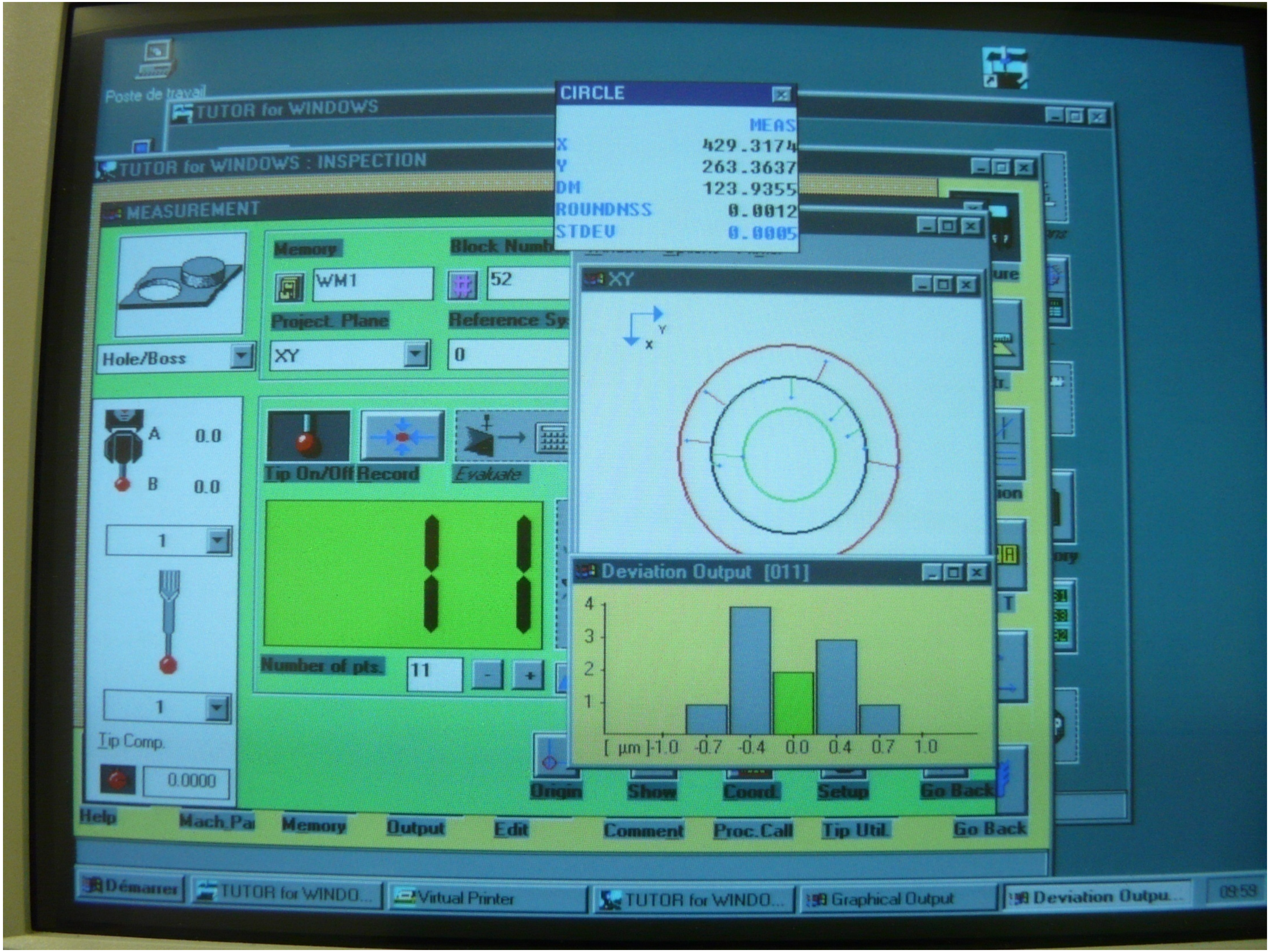
Source masses in position A

213.9880 at 20.6 °C → 213.9876 at 20.5 °C January 2006
213.9885 at 20.65 °C → 213.9880 at 20. 5 °C June 2007
213.9876 at 20.4 °C → 213.9880 at 20.5 °C Sept 2007

BIPM



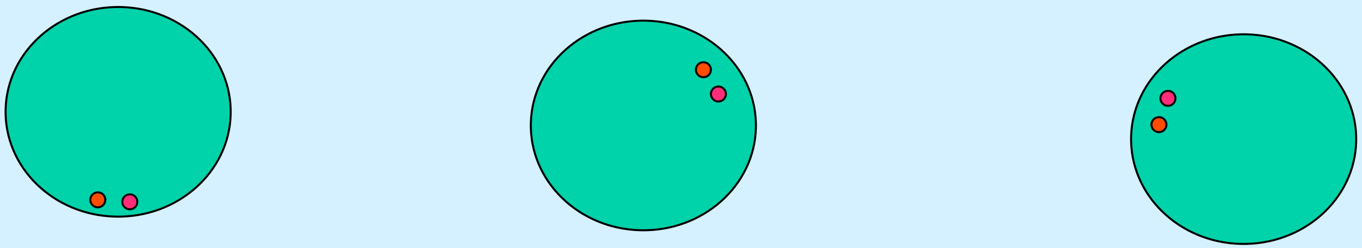
brown & sharpe



Poste de travail
TUTOR for WINDOWS
TUTOR for WINDOWS : INSPECTION
MEASUREMENT
CIRCLE
MEAS
X 429.3174
Y 263.3637
DM 123.9355
ROUNDNSS 0.0012
STDEV 0.0005
Memory
WM1
Block Numb
52
Project. Plane
XY
Reference Sy
0
Hole/Boss
A 0.0
B 0.0
1
1
Tip Comp.
0.0000
Tip On/Off
Record
Evaluate
Number of pts.
11
XY
Deviation Output [011]
[µm] -1.0 -0.7 -0.4 0.0 0.4 0.7 1.0
Origin
Show
Coord.
Setup
Go Back
Help
Mach.Pa
Memory
Output
Edit
Comment
Proc.Call
Tip Util.
Go Back
Démarrer
TUTOR for WINDO...
Virtual Printer
TUTOR for WINDO...
Graphical Output
Deviation Outpu...
09:59

Radii of source masses, 1, 2, 3 and 4 (millimetres) at all three orientations

	23 Jan	29 Jan	13 Feb	20 Feb	21 Feb	28 Feb	12 Mar	average	σ μ
1	58.9766	.9780	.9765	.9762	.9773	.9774	.9764	58.9769	0.6
2	58.9866	.9863	.9871	.9872	.9879	.9871	.9873	58.9871	0.5
3	58.9853	.9852	.9839	.9840	.9855	.9848	.9855	58.9849	0.6
4	59.0015	.0015	.0015	.0016	.0018	.0012	.0015	59.0015	0.2
	58.9876		58.9876			58.9876		58.9876	



0^o 120^o 240^o

The measured radii of source mass is independent of their orientation

A third method of measurement, in addition to the servo and Cavendish, is the timing method.

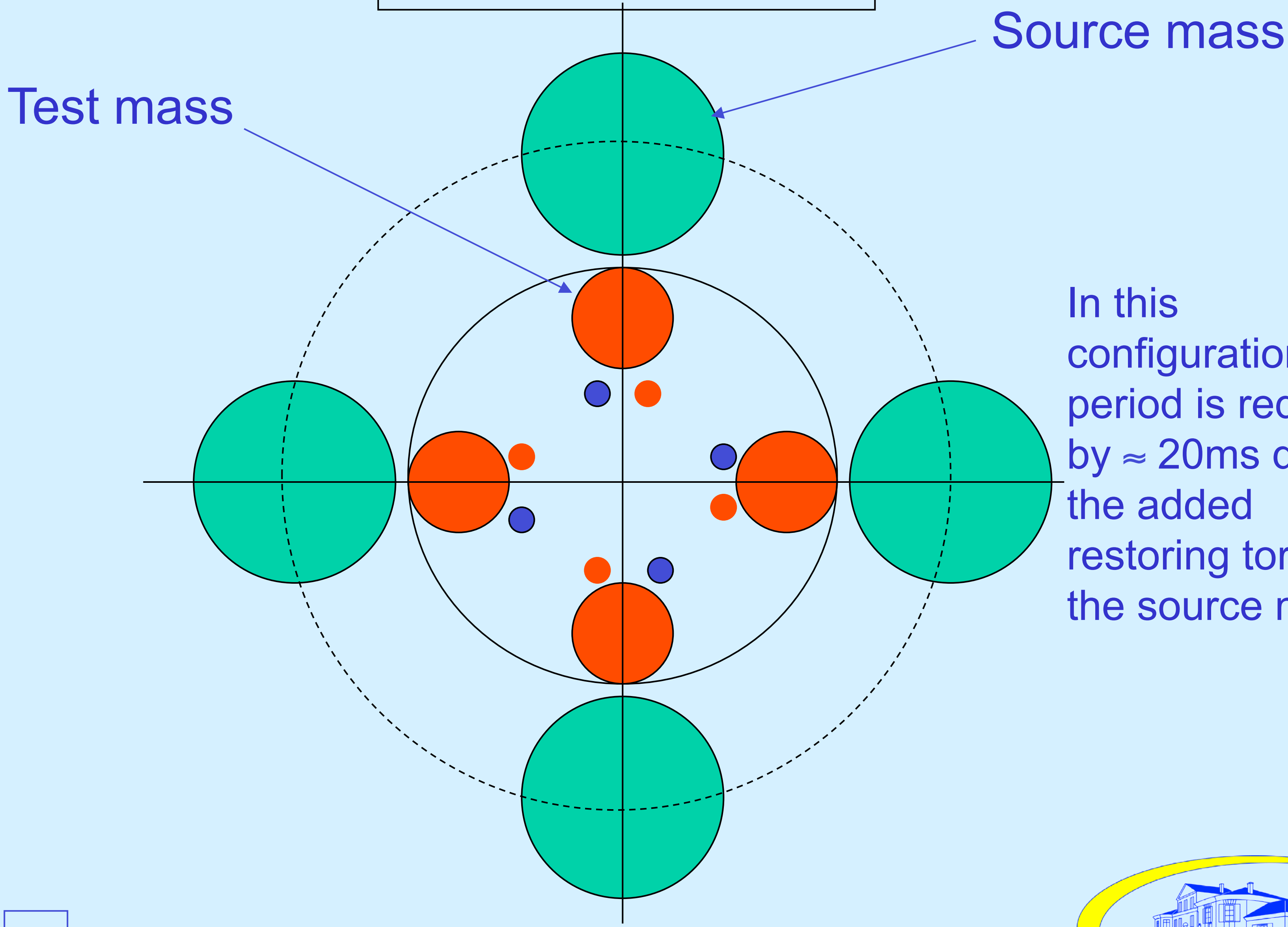
While we explored this method, the temperature of the laboratory could not be maintained sufficiently stable to give useful results.

The method requires the measurement of the small change in period, ≈ 40 ms, in the natural period of the torsion balance of 120 s.

1

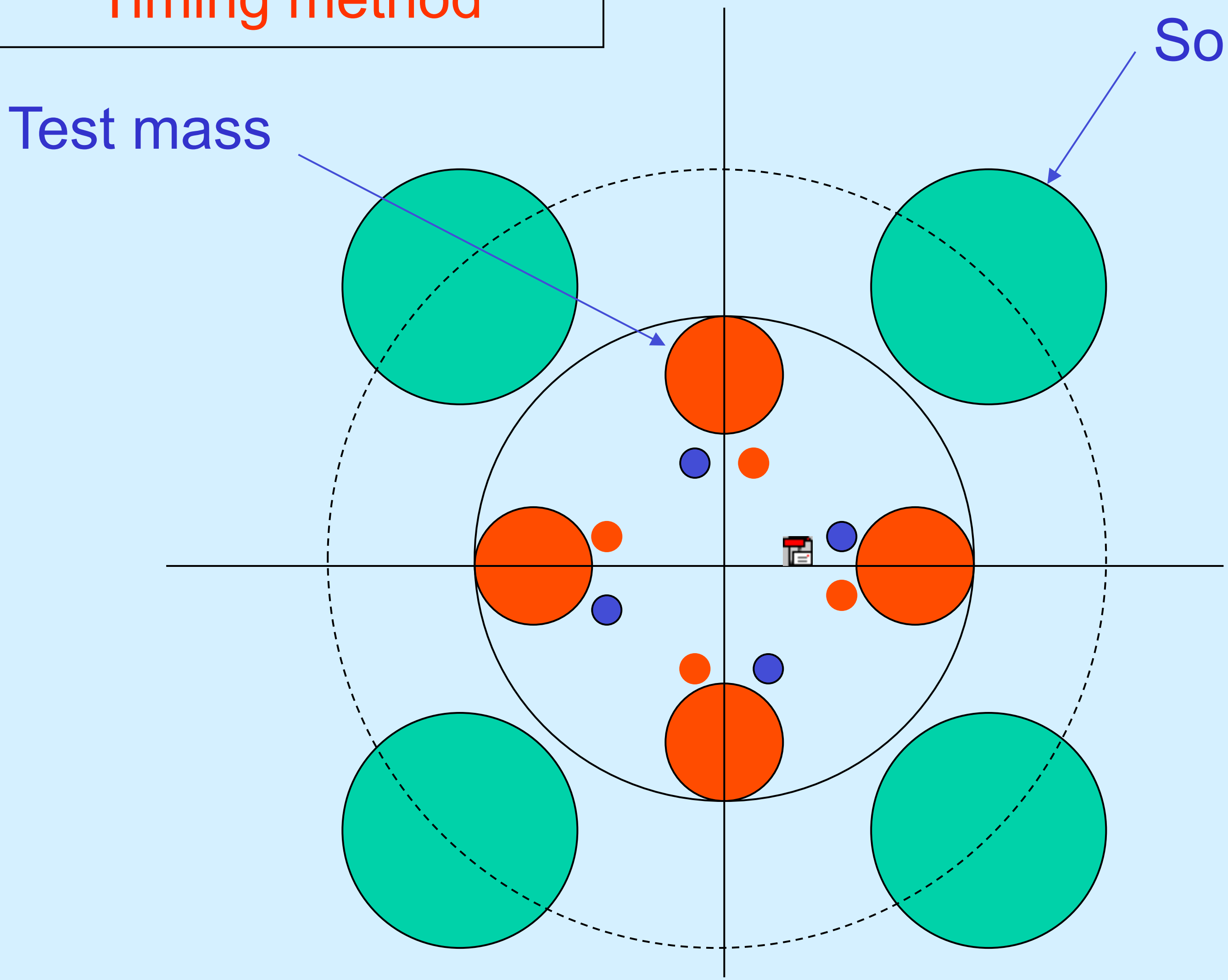
BIPM

Timing method



In this configuration the period is reduced by ≈ 20ms due to the added restoring torque of the source masses

Timing method



In this configuration the period is increased by ≈ 20ms due to the attraction of the source masses

The relation between measured quantities and G for the timing method is:

$$I(\omega^2_0 - \omega^2_{45}) = (\Gamma_{45} - \Gamma_0)\, G$$

Where ω_0 and ω_{45} are the angular frequencies of free oscillation of the torsion balance for source mass positions of 0 and 45 degrees and Γ_{45} and Γ_0 are the corresponding gravitational coupling coefficients and I is the moment of inertia of the torsion balance. This can be written in terms of the measured periods:

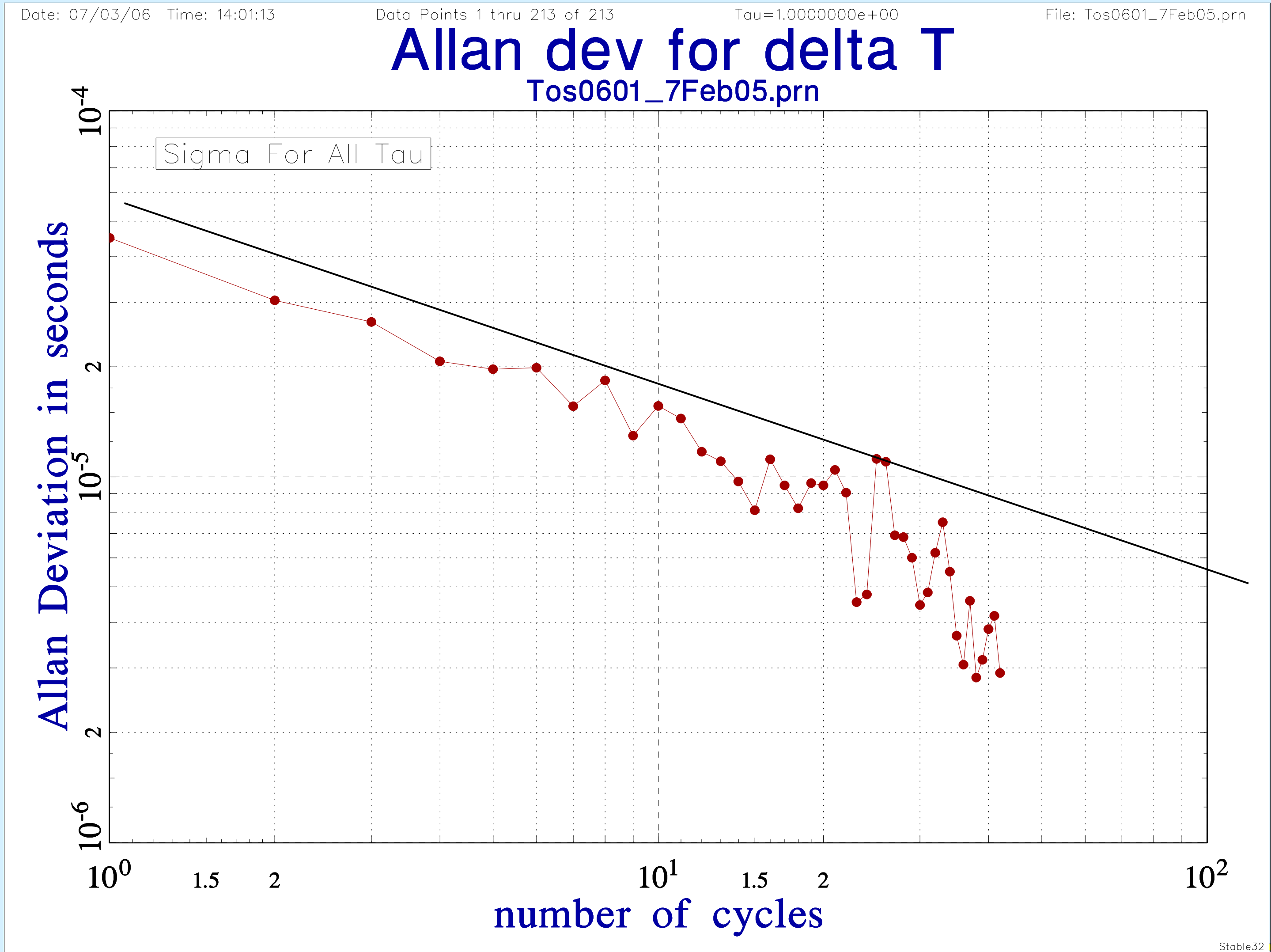
$$(\Gamma_{45} - \Gamma_0)\, G = 8\pi^2 I\, \Delta T/(T_{av}^3)$$

$T_{av} \approx 120$ s and $\Delta T \approx 40\ ms$

In order to reach an uncertainty in G of 30 ppm we need an uncertainty in ΔT of 1 microsecond or 1 part in 120 million in T which would need a temperature stability of the strip of about 1 mK.

BIPM

Allan deviation for time-of-swing measurements



the black line of slope -1/2 corresponds to the Allan deviation expected for a white noise process

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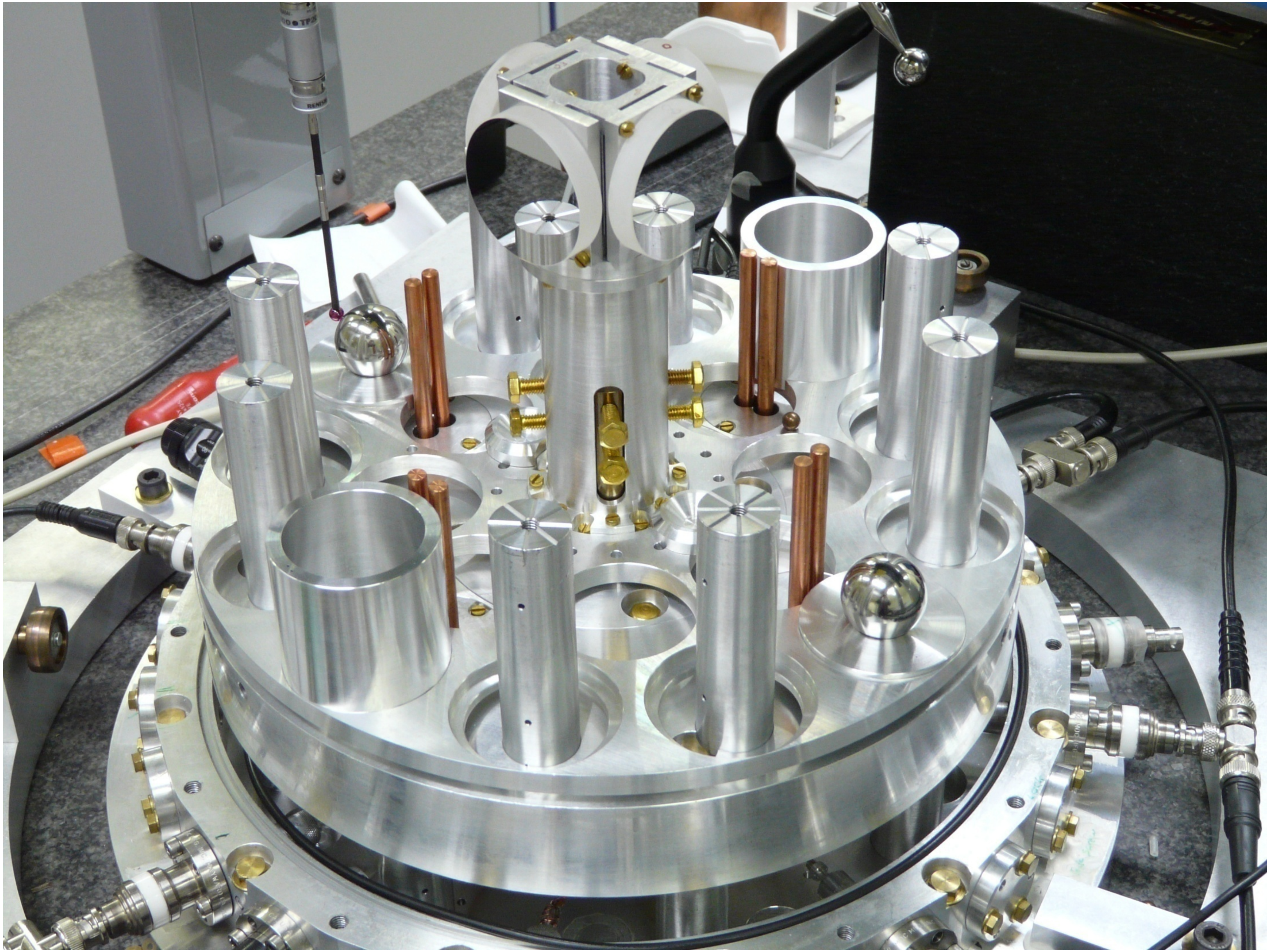

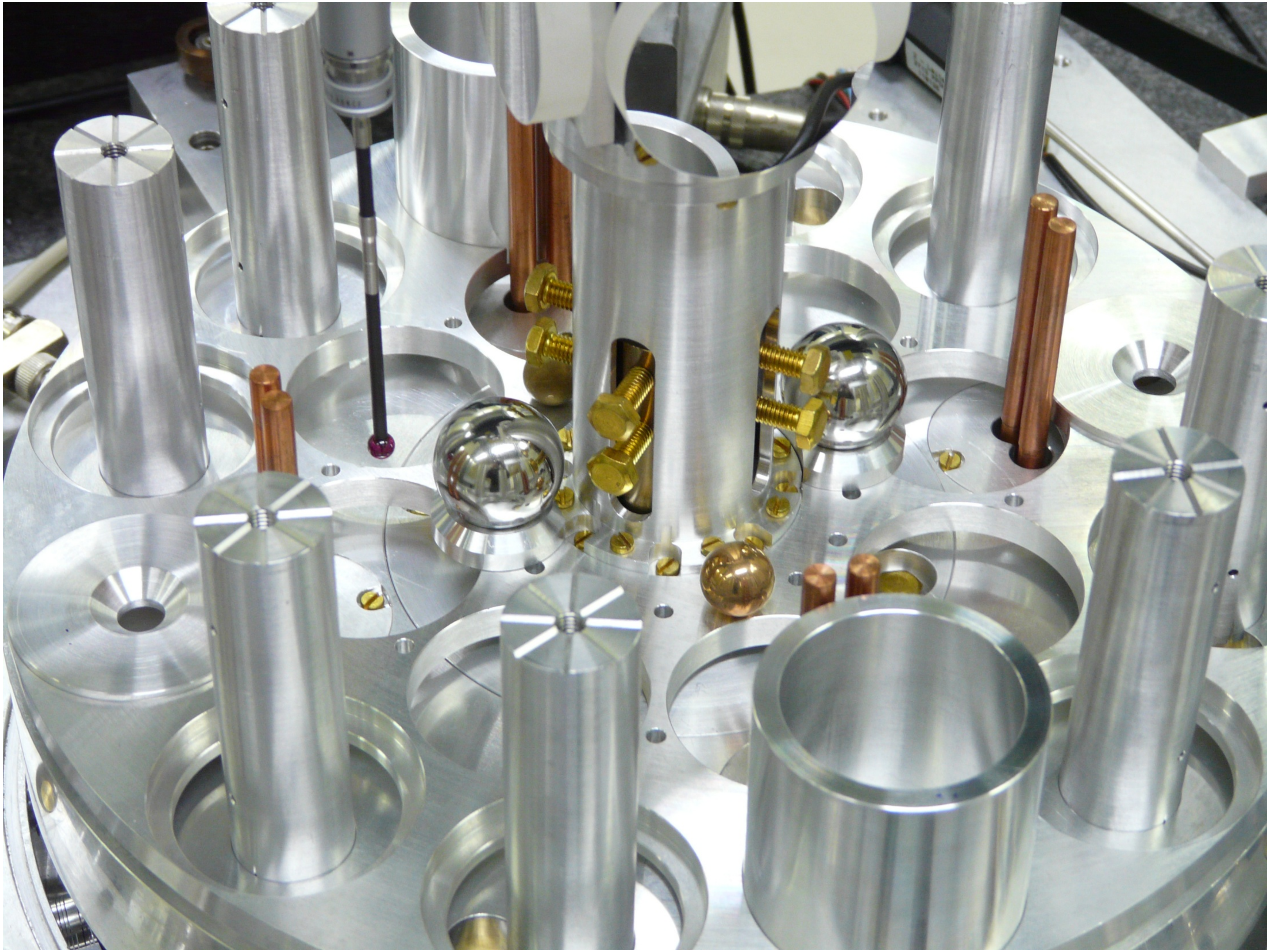


MÖLLER WEDEL
GERMANY

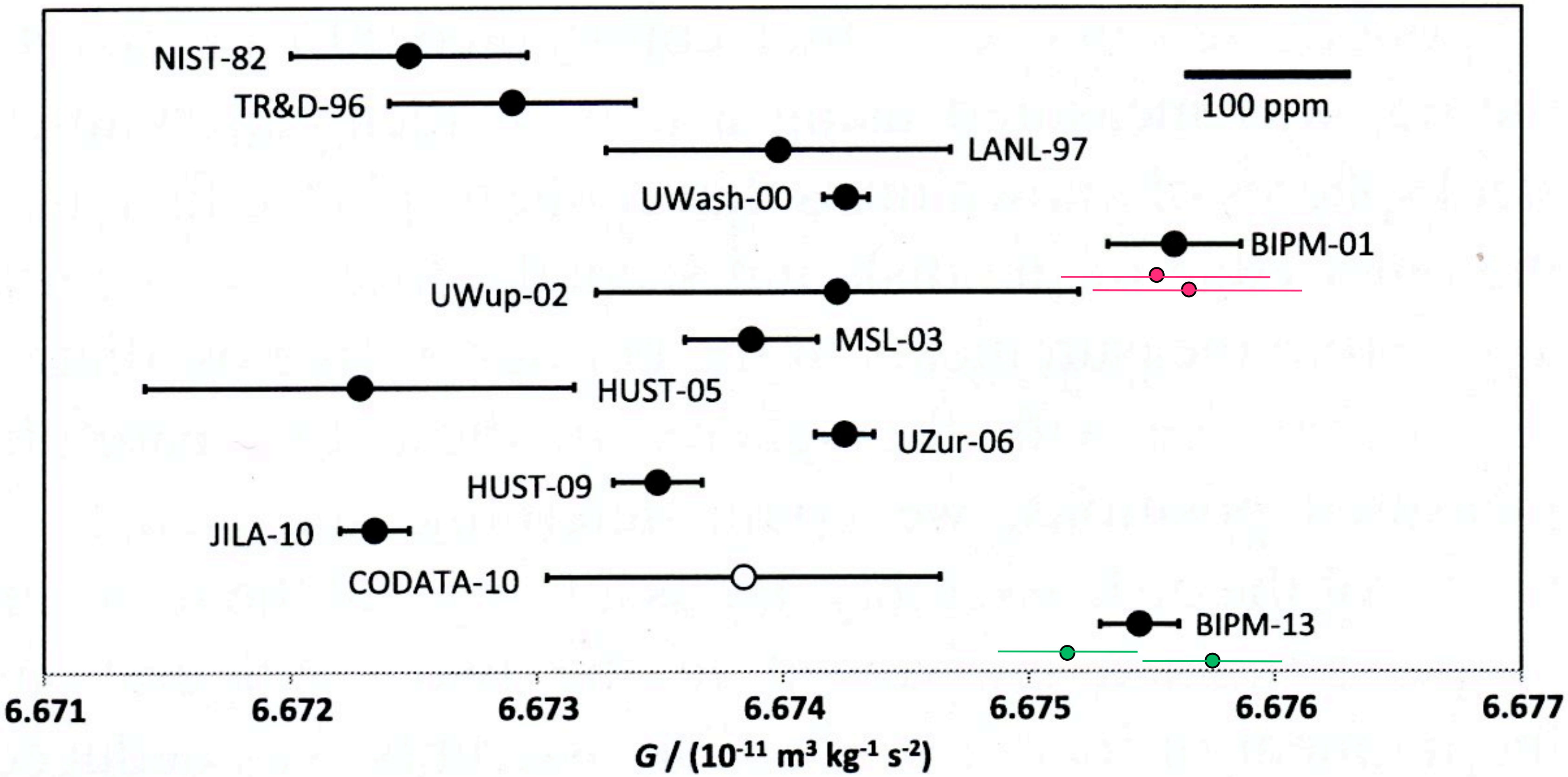

FIG. 3. The present result (BIPM-13) compared with recent measurements of G [6].

BIPM

What of the future? I would like to see the following:

The BIPM apparatus with all three methods:
Servo, which needs electrical and angle measurements
Cavendish, which needs angle, timing and moment of inertia measurements
Timing, which needs timing measurements

With all three needing:
Dimensional metrology
Well characterized source and test masses
Gravitational coupling calculations

General improvements would include an angle interferometer in the vacuum chamber, low thermal expansion disk material.

The aim: sub 10 ppm in all three methods

BIPM

HENRY CAVENDISH

BIPM

BIPM

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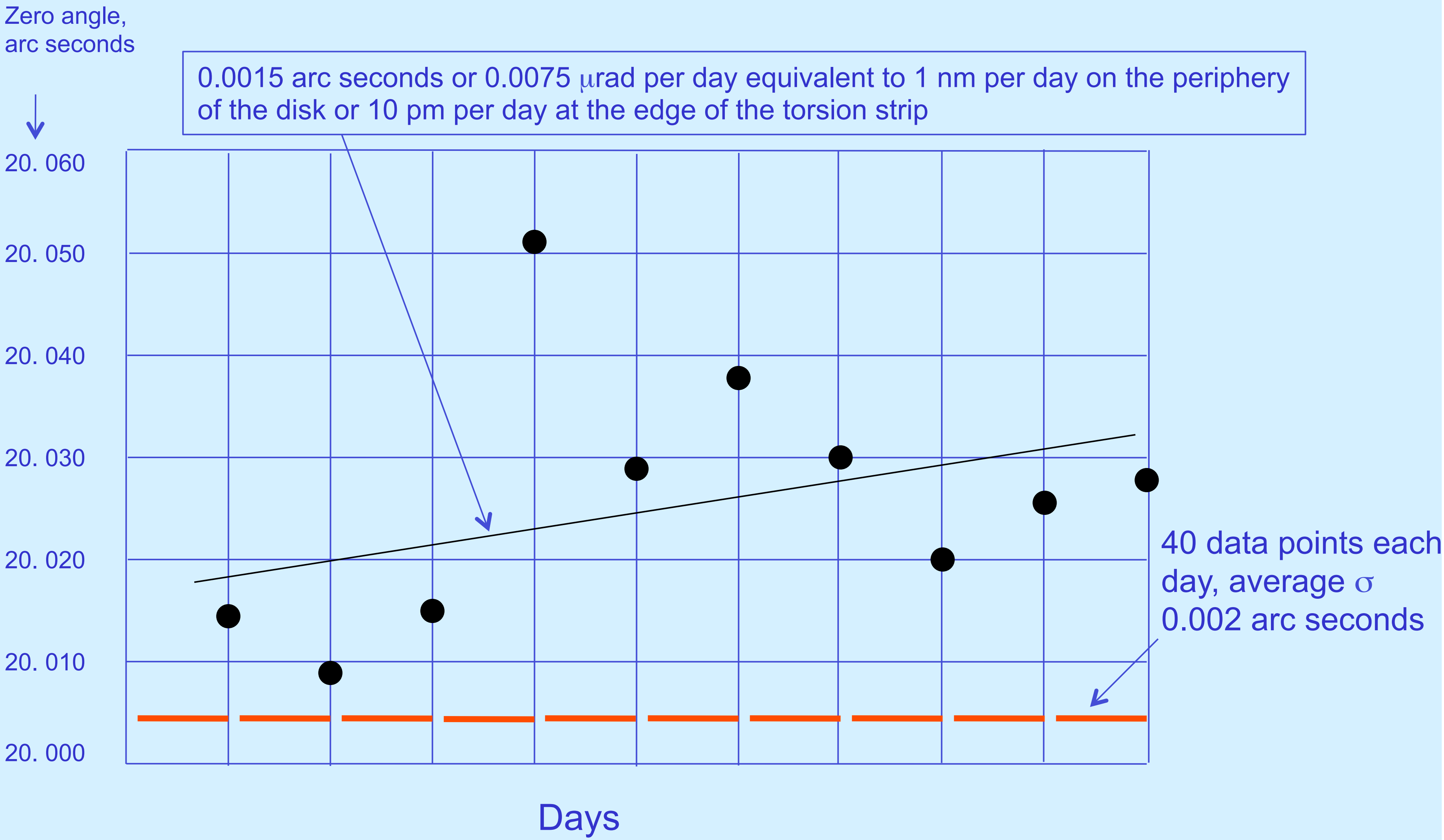
BIPM

Zero drift of the torsion balance during the Cavendish runs



Note: total *G* signal 32 arc seconds

Note: during the whole 10 days the temperature was 20.4 °C within 0.1 °C