Inversion of Earthquake Rupture Process： Theory and Applications

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1. An Overview

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1.2 Retrieval of the source time function
1.3 Construction of the slip distribution based on the ASTFs
1.4 Construction of the slip distribution based on the waveform data

The pioneering works of rupture process inversion can be referred to Olson and Apsel (1982), Kikuchi and Kanamori (1982), and Hartzell and Heaton (1983), Das and Kostrov (1990).

The techniques were further developed by following researchers (*e.g.*, Ji *et al.*, 2002). An alternative method is to invert the apparent source time functions (ASTFs), which should have been obtained by deconvolving the Green's function (sometimes empirical Green's function of a small shock) from the mainshock seismograms (Mori and Hartzell, 1990; Chen and Xu, 2000). In principle, the seismogram inversion and the ASTF inversion are equally effective to estimate the rupture model (Zhang *et al.*, 2010).

The fault slip model can be also determined by inverting geodetic deformation data. Compared with the seismic data inversion, geodetic inversion of the co-seismic deformation data has fewer unknown parameters and thus has a higher stability. Meanwhile, the disadvantage is that it cannot constrain the temporal evolution of slip accumulation. Combination of the seismic and geodetic data can synthesize their advantages and results in a better resolution power at different depths.

In the past 20 years, joint inversion of seismic and geodetic data has been the powerful tool to study the earthquake source process and its physics. All seismic and geodetic datasets, *e.g.*, strong motion data, broad band data at local, regional and teleseismic distances, high-rate/campaign GNSS (Global Navigation Satellite System) data, InSAR (Interferometric Synthetic Aperture Radar) data, and leveling data, have been used and inverted for that purpose (*e.g.*, Delouis *et al.*, 2002; Zhang *et al.*, 2012).

The joint inversion much expands the frequency band of source inversions, and provides better estimates of source parameters, such as magnitude, source dimension and duration, rupture velocity and rupture direction, *etc*.

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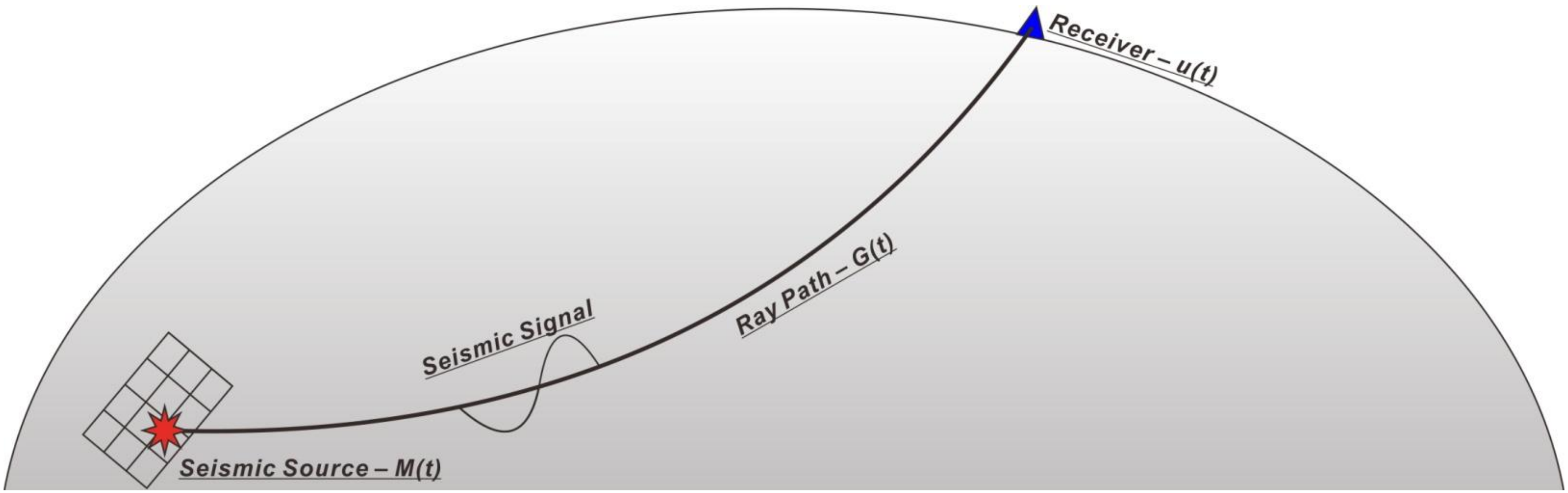
As the linear dimension of an earthquake source is small compared with the wavelength of interest, the displacement at the field point r caused by the earthquake at the origin of the coordinate system can be written as (Aki and Richards, 1980)

$$u_i(\mathbf{r},t) = G_{ij,k}(\mathbf{r},t) * M_{jk}(t)$$

$u_i(\mathbf{r},t)$	observed displacement
$M_{jk}(t)$	moment tensor function
$G_{ij,k}(\mathbf{r},t)$	Green's function

Green’s function



By means of the transformation to frequency domain, the above displacement can be transformed into

$$u_i\left(\mathbf{r},\omega\right)=G_{ij,k}\left(\mathbf{r},\omega\right)M_{jk}\left(\omega\right)$$

ω angular frequency

Inversion of the moment tensor can be conducted in time domain as well as in frequency domain. In any of the domains, the inversion equation can be written as

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In general, the instrument-recorded displacement of an earthquake can be expressed as

$$u(t) = M_0 S(t) * P(t) * I(t)$$

M_0 **scalar seismic moment**

$S(t)$ **normalized far-field STF**

$P(t)$ **response of the path**

$I(t)$ **response of the instrument**

By Fourier transformation, we have

$$u(\omega) = M_0 S(\omega) P(\omega) I(\omega)$$

By analog to the above expression, we have the following one for a second event with the same focal mechanism, which may be real event or synthetic one

$$u'(\omega) = M_0' S'(\omega) P'(\omega) I'(\omega)$$

If the second event has the same source location and is recorded at the same station by the same instrument, and if the second is so small that its source time function can be approximated with a Dirac δ-function, then we have the following

$$\frac{u(\omega)}{u'(\omega)} = \frac{M_0}{M_0'} S(\omega)$$

Since

$$P'(\omega) = P(\omega)$$

$$I'(\omega) = I(\omega)$$

$$S'(\omega) = 1$$

The above equation indicates that the source time function of a larger earthquake can be retrieved by means of a smaller one.

- **The smaller one can often be found, which may be an aftershock, or a pre-shock.**

- **The synthetic earthquake can be adopted as the real earthquake can not be found.**

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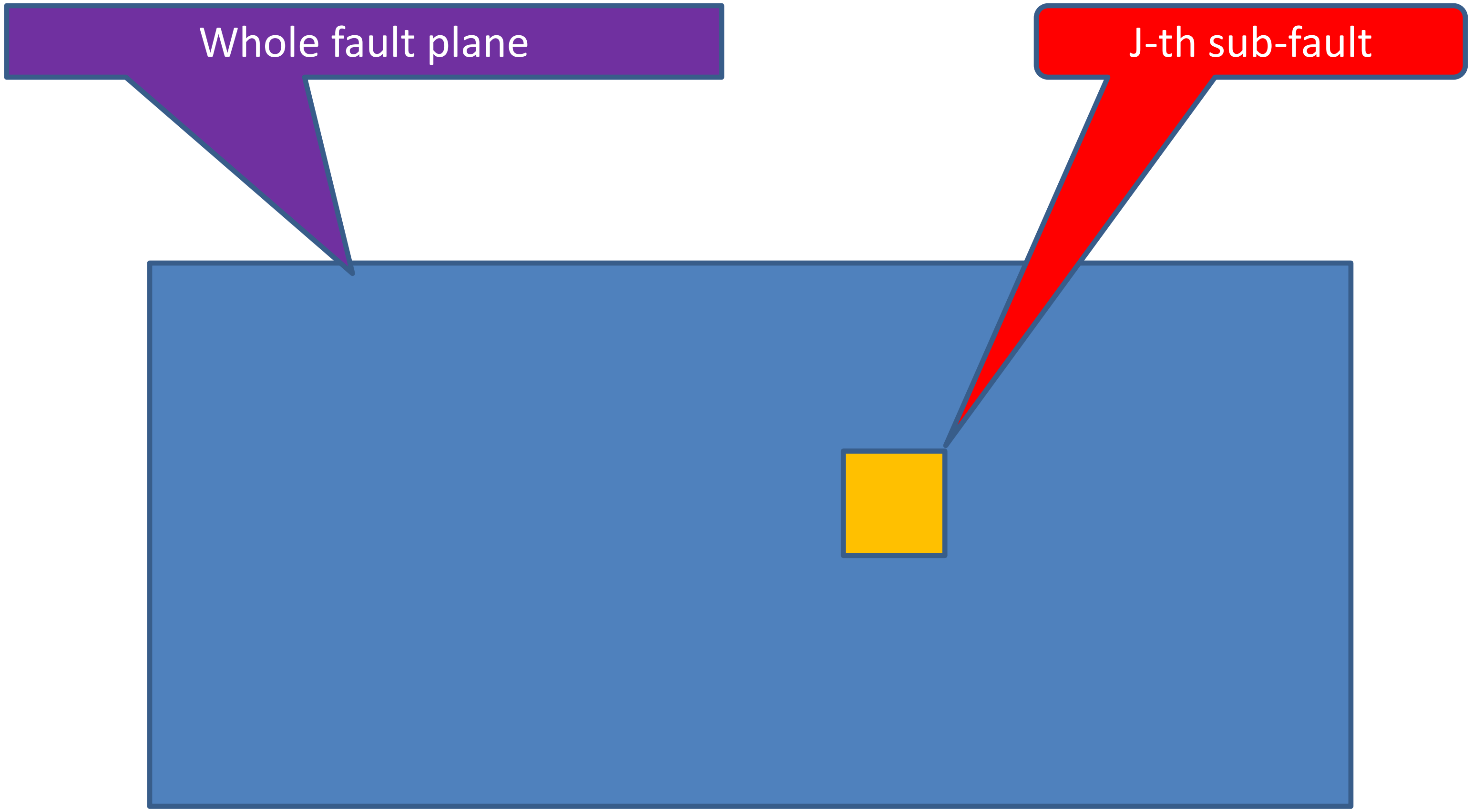


A fault can be divided into many sub-faults

In general, the slip on a fault plane is a function of time. If the area of the fault plane is assumed constant, the slip will only depend on the scalar moment as

$$D(t) = \frac{M_0(t)}{\mu A}$$

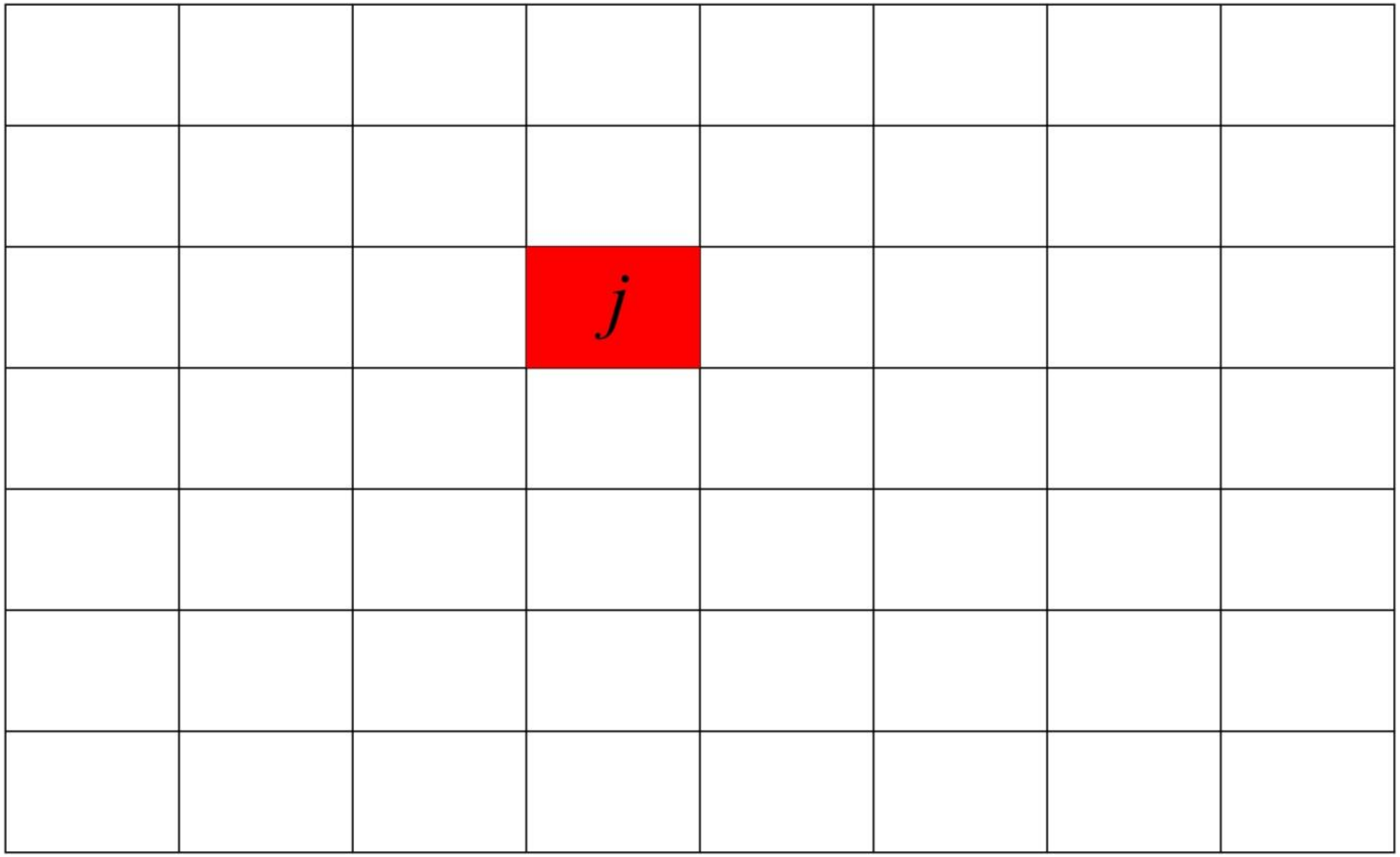
A	**area of the fault plane**
$M_0(t)$	**scalar moment as a function of time**
μ	**rigidity of the material**

For any of the sub-faults, we have

$$D_j(t) = \frac{M_j(t)}{\mu A_j}$$

j **represents any of the sub-faults**

A fault can be divided into numerous sub-faults



The observed STF at any of the stations has the following relation with the STFs of all the sub-faults

$$S_i(t) = \sum_{j=1}^{J} w_j s_j \left(t - \tau_{ij}\right)$$

$$w_j = \frac{M_j}{M_0}$$

S_i **observed STF at the station *i***

s_j **STF of the sub-fault *j***

τ_{ij} **difference of arrival time at the station *i* between the first and the *j*-th sub-fault**

Let

$$m_j(t) = w_j s_j(t)$$

$m_j(t)$ **weighted far-field STF of the *j*-th sub-fault.**

Apparently,

$$w_j = \int_0^{\infty} m_j(t)\, dt$$

Since

$$\int_0^{\infty} s_j(t)\, dt = 1$$

Therefore,

$$S_i(t) = \sum_{j=1}^{J} m_j\left(t - \tau_{ij}\right)$$

And the slip rate of the *j*-th sub-fault can be written as

$$\dot{D}_j(t) = \frac{M_0}{\mu A_j} m_j(t)$$

The slip or slip-rate of the *j*-th sub-fault will be determined if the function of the moment rate of the sub-fault is given.

To get the moment-rate function, we write the right hand side of the above equation as

$$S_i(t) = \sum_{j=1}^{J} \delta\left(t - \tau_{ij}\right) * m_j(t)$$

$$\tau_{ij} = \frac{r_j}{v_i}$$

r_j is the distance between the *j*-th subfault and the reference point on the fault plane and v_i is the apparent velocity, which depends on the wave velocity and propagation direction and he location of the *j*-th sub-fault.

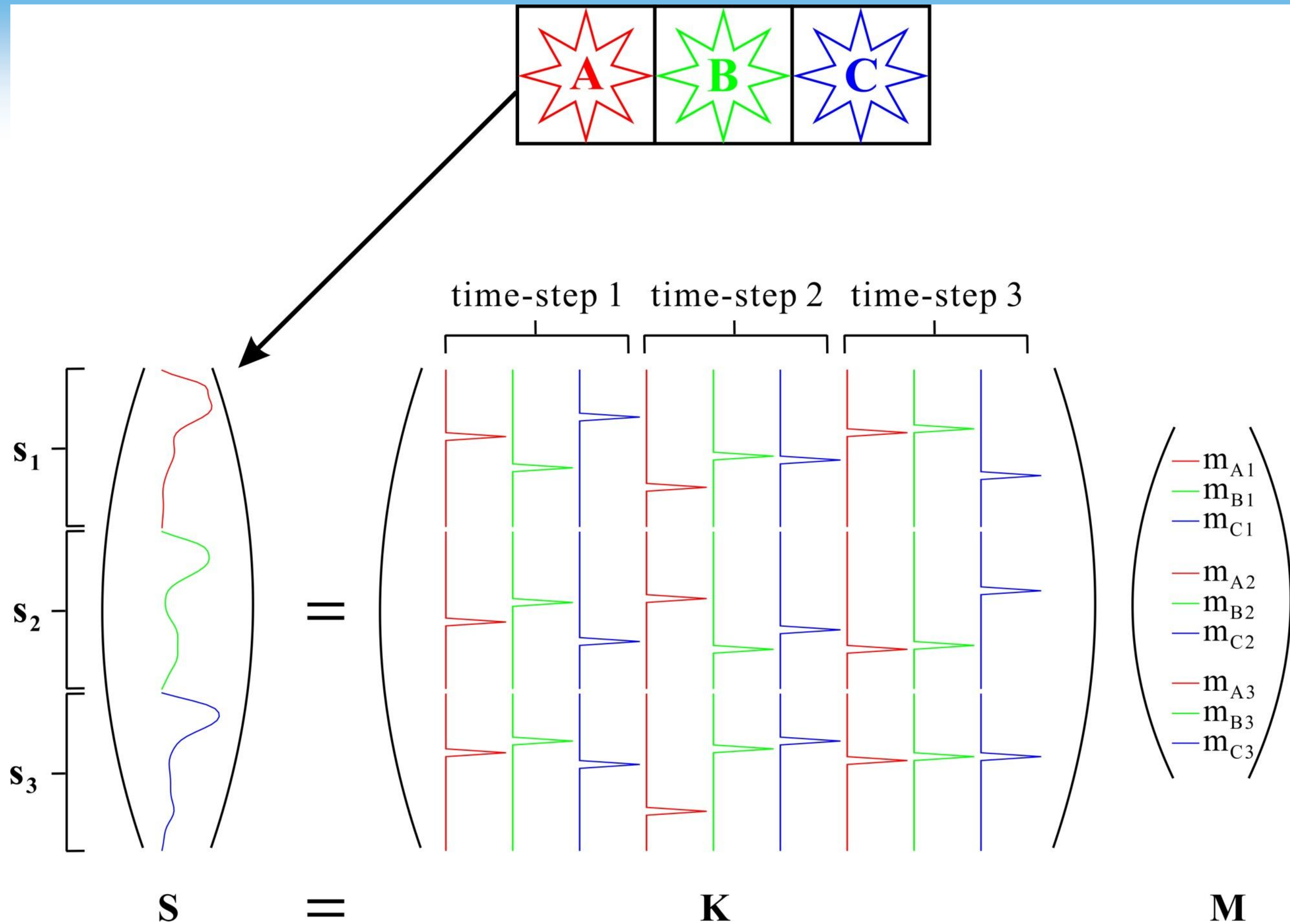
In matrix form

$$\mathbf{S} = \mathbf{K}\mathbf{M}$$

S data vector, consisting of the observed STFs

M the unkown vector made up of the weighted STFs or moment-rate functions of all the sub-faults

K coefficient matrix determined by the time delay associated with the wave propagation.


A
B
C
time-step 1
time-step 2
time-step 3
s_1
s_2
s_3
=
m_{A1}
m_{B1}
m_{C1}
m_{A2}
m_{B2}
m_{C2}
m_{A3}
m_{B3}
m_{C3}
S
=
K
M

Note: K is a sparse matrix consisting of zeros or ones as the following

$$\delta(t-\tau_{ij}) = \begin{cases} 1 & t = \tau_{ij} \\ 0 & t \neq \tau_{ij} \end{cases}$$

In inersion，a non-negative constraint is usually imposed to get a physically meaningful result.

$$m_j(t) \geq 0$$

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For a finite fault, the *n*-component of the seismic displacement at the station x can be expressed as

$$u_n(\mathbf{x},t)=\sum_{k=1}^{K} M_{pq}(\xi_k,t)*G_{np,q}(\mathbf{x},t;\xi_k,0)$$

$u_n(x,t)$	**The *n*-component of displacement at the station *x***
$M_{pq}(\xi_k,t)$	**The moment tensor of the sub-fault *k* at location** ξ
$G_{np,q}(x,t;\xi_k,0)$	**Green's function**
$*$	**convolution**

If the components of the moment tensor have the same time history, then

$$u_n(\mathbf{x},t)=\sum_{k=1}^{K}\left[M_{pq}(\xi_k)\cdot s_k(t)\right]*G_{np,q}(\mathbf{x},t;\xi_k,0)$$

$s_k(t)$ **The source time function of the *k*- subfault**

For a double-couple source, we have

$$M_{pq}(\xi_k) = M_0(\xi_k)\left(e_p(\xi_k)\nu_q(\xi_k) + e_q(\xi_k)\nu_p(\xi_k)\right)$$

$M_0(\xi_k)$ **scalar seismic moment of the *k*-th sub-fault**

$e_{p(q)}$ **component of the slip vector**

$\nu_{p(q)}$ **components of the normal vector**

In this case,

$$u_n(\mathbf{x},t)=\sum_{k=1}^{K}\hat{s}_k(t)*\left[\left(e_p(\xi_k)\nu_q(\xi_k)+e_q(\xi_k)\nu_p(\xi_k)\right)\cdot G_{np,q}(\mathbf{x},t;\xi_k,0)\right]$$

where

$$\hat{s}_k(t)=M_0(\xi_k)s_k(t)$$

1.4.1 case of the unchangeable focal mechanisms

If all the sub-faults have the same focal mechanism, the slip vectors and normal vectors will be unchangeable for all the sub-faults, then

$$u_n\left(\mathbf{x},t\right)=\sum_{k=1}^{K}\hat{s}_k\left(t\right)*G_{nk}\left(\mathbf{x},t\right)$$

where

$$G_{nk}\left(\mathbf{x},t\right)=\left(e_p\nu_q+e_q\nu_p\right)\cdot G_{np,q}\left(\mathbf{x},t;\boldsymbol{\xi}_k,0\right)$$

In matrix form, the above equation is

$$\mathbf{u} = \mathbf{G}\hat{\mathbf{s}}$$

On the left is a vector of observation data, the first item on the right is the matrix of Green's function, and the second is a vector of unknowns, consisting of the moment rate functions of sub-faults.

1.4.2 Case of the changeable rakes

If the rake of each sub-fault is allowed to be changeable, one may always decompose the slip vector into two components perpendicular to each other in an orthogonal source Cartesian coordinate system (*x*, *y*, *z*), in which *x* and *y* indicate two orthogonal slip directions on fault plane, and *z* indicates normal direction of the fault plane.

In this case,

$$u_n\left(\mathbf{x},t\right)=\sum_{k=1}^{K}G_{knx,z}\left(\mathbf{x},t\right)*s_{kx}\left(t\right)+G_{kny,z}\left(\mathbf{x},t\right)*s_{ky}\left(t\right)$$

Since

$$\nu_x=\nu_y=e_z=0$$

$$\nu_z=1$$

$$\hat{s}_{kx}\left(t\right)=e_xM_{k0}s_k\left(t\right)$$

$$\hat{s}_{ky}\left(t\right)=e_yM_{k0}s_k\left(t\right)$$

Accordingly, the matrix equation can be changed as

$$\mathbf{u} = \begin{pmatrix} \mathbf{G}_x & \mathbf{G}_y \end{pmatrix} \begin{pmatrix} \hat{\mathbf{s}}_x \\ \hat{\mathbf{s}}_y \end{pmatrix}$$

In order to mitigate the impact caused by the errors in observation and Green's functions and to obtain physically acceptable solution, we introduce constraints such as the minimization of scalar seismic moment and the smoothness in time and space.

Taking into account the above constraints, we expand the above matrix equations into the following forms.

For the first case

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \mathbf{G} \\ \lambda_1 \mathbf{T} \\ \lambda_2 \mathbf{D} \\ \lambda_3 \mathbf{Z} \end{pmatrix} \hat{\mathbf{s}}$$

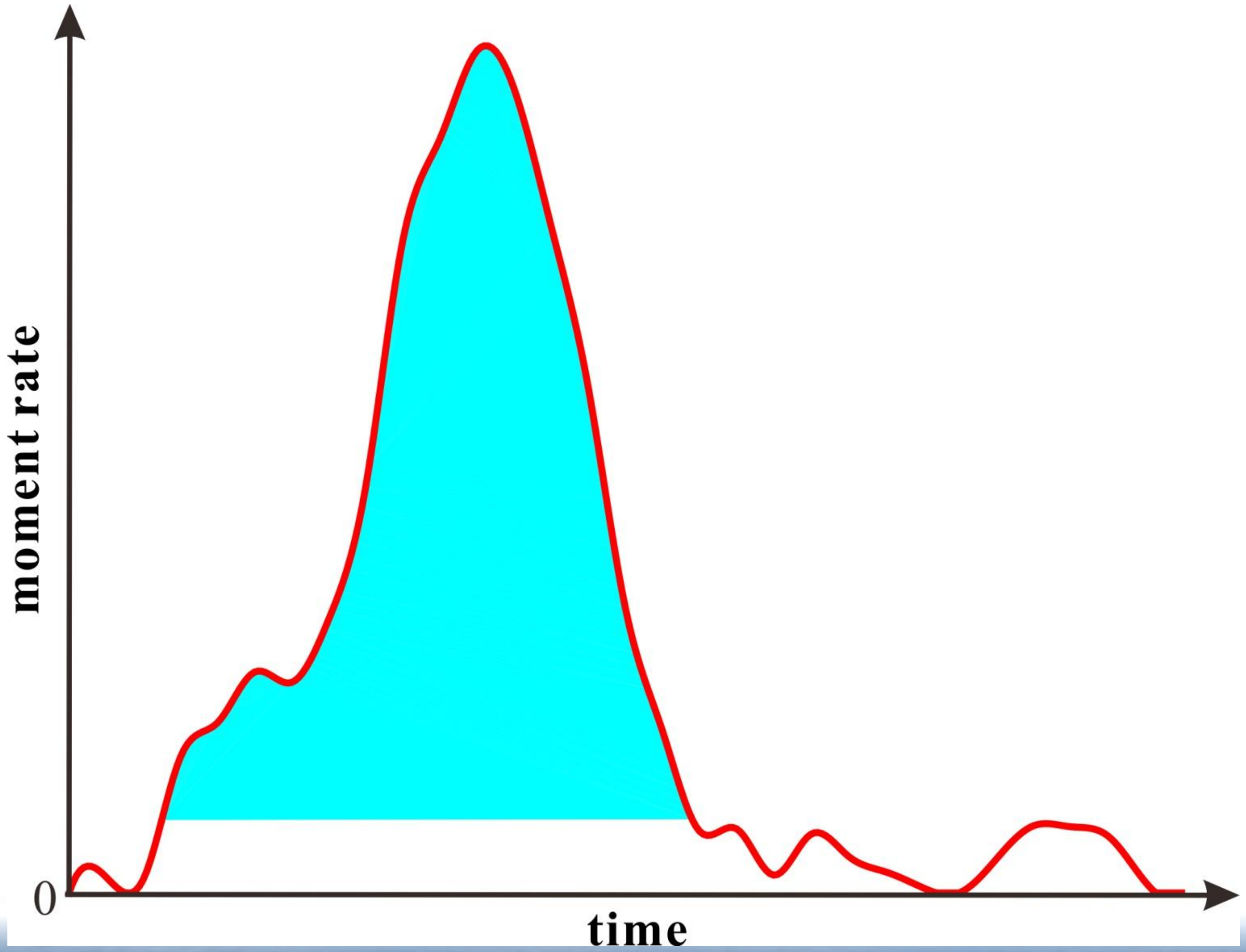
For the second case

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \mathbf{G}_x & \mathbf{G}_y \end{pmatrix} \\ \lambda_1 \begin{pmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{0} & \mathbf{T} \end{pmatrix} \\ \lambda_2 \begin{pmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix} \\ \lambda_3 \begin{pmatrix} \mathbf{Z} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{s}}_x \\ \hat{\mathbf{s}}_y \end{pmatrix}$$

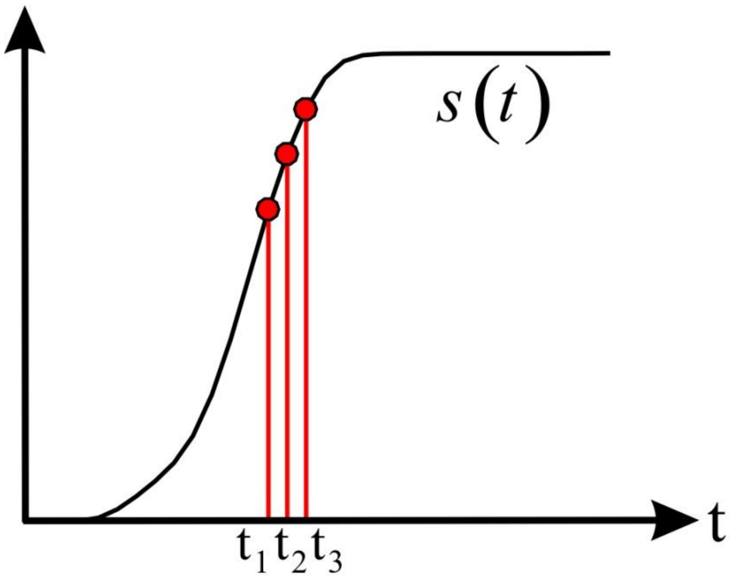
where

$\mathbf{T}$	**A matrix of temporal smoo**thness
$\mathbf{D}$	**A matrix of spatial smoothness**
$\mathbf{Z}$	**A matrix of the minimization of scalar seismic moment**
λ_1	**The weight**
λ_2	**The weight**
λ_3	**The weight**

Minimization of the seismic scalar moment

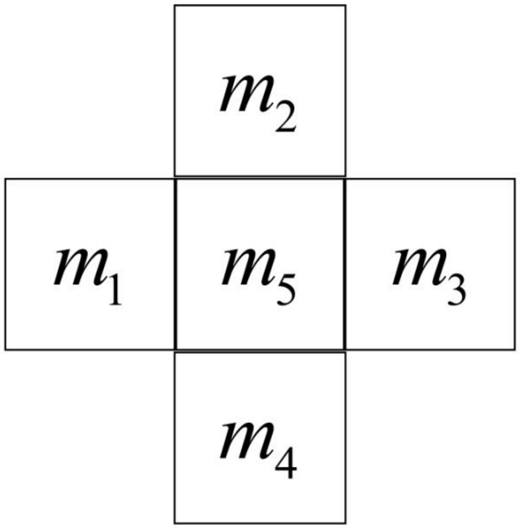


Temporal smoothing



$$s(t_2) = \frac{s(t_1) + s(t_3)}{2}$$

Spatial smoothing



$$m_5 = \frac{m_1 + m_2 + m_3 + m_4}{4}$$

Summary

We briefly overviewed the theory of inversion of earthquake rupture process, and emphasized the needs for the joint inversion of the needs of using both of the seismic and geodetic data


谢谢！
Thank you!
Спасибо!