Inversion of Earthquake Rupture Process：Theory and Applications

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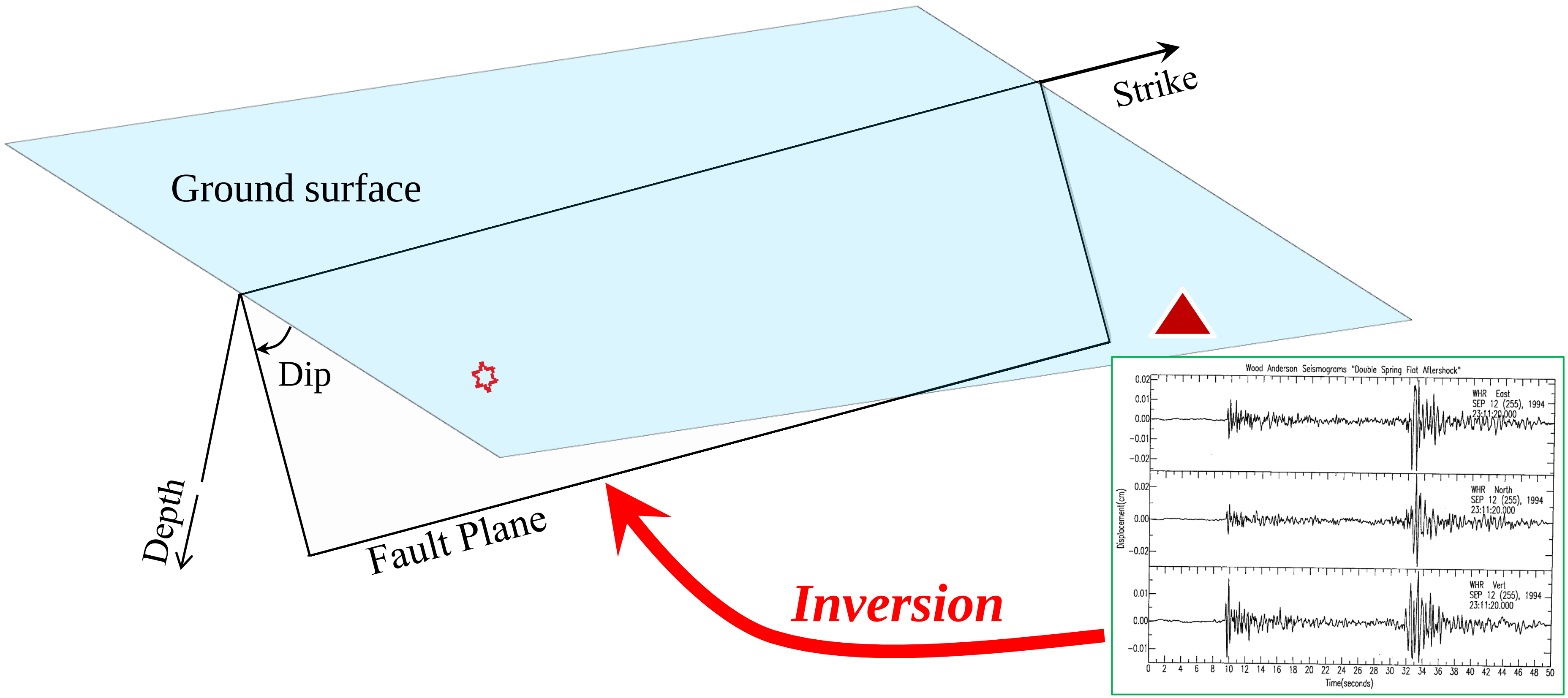
Inversion of Earthquake Rupture Process：Theory and Applications

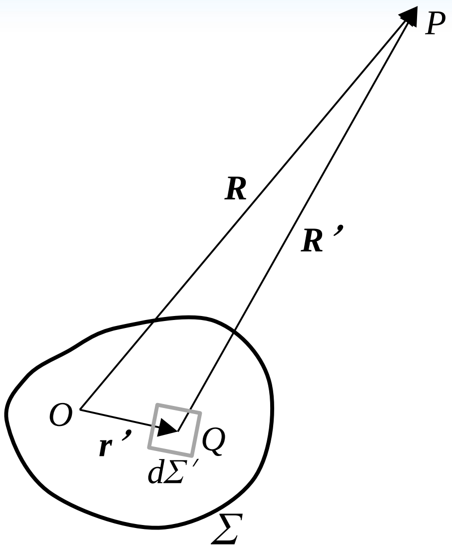
2. Theory and Techniques

2.1 Seismic Inversion


Strike
Ground surface
Dip
Depth
Fault Plane
Inversion
Wood Anderson Seismograms "Double Spring Flat Aftershock"
WHR East
SEP 12 (255), 1994
23:11:20.000
WHR North
SEP 12 (255), 1994
23:11:20.000
WHR Vert
SEP 12 (255), 1994
23:11:20.000
Displacement(cm)
Time(seconds)

2.1.1 Inversion with Rake Fixed

The seismic waves radiated from a finite-fault can be represented as



$$U_n(P,t) = \iint_{\Sigma} G_{np,q}(P,t;Q,0) * m_{pq}(Q,t) d\Sigma'$$

If a purely shear dislocation is assumed

$$m_{pq}(Q,t) = M_0(Q,t)(e_p v_q + e_q v_p)$$

Then we have

$$U_n(P,t) = \iint_{\Sigma} G_{np,q}(P,t;Q,0) * [M_0(Q,t)(e_p v_q + e_q v_p)] d\Sigma'$$

$$U_n(P,t) = \iint_{\Sigma} G_{np,q}(P,t;Q,0) * [M_0(Q,t)(e_p v_q + e_q v_p)] d\Sigma'$$

If assume

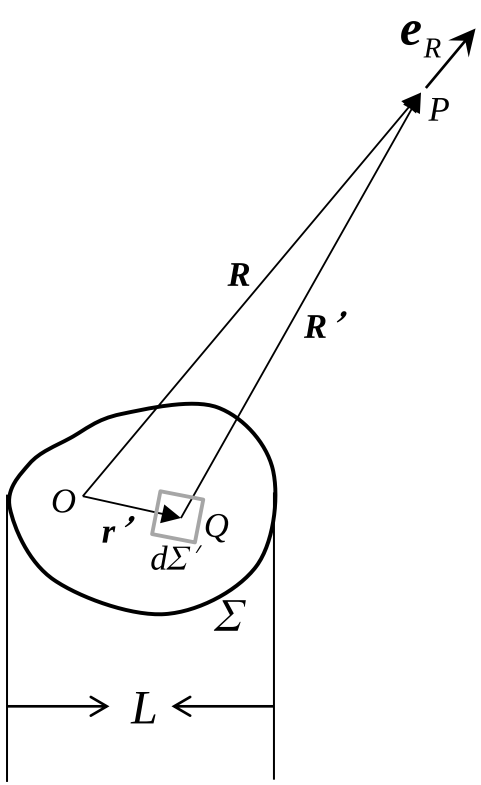
$$g_n(P,t;Q,0) = G_{np,q}(P,t;Q,0) \cdot (e_p v_q + e_q v_p)$$

It becomes

$$U_n(P,t) = \iint_{\Sigma} g_n(P,t;Q,0) * M_0(Q,t) d\Sigma'$$

The seismograms can be recorded during an earthquake, the Green's functions can be calculated with a preferred earth model, the unknowns are the seismic moment functions, which are dependent on place and time.

$$U_n(P,t) = \iint_{\Sigma} g_n(P,t;Q,0) * M_0(Q,t) d\Sigma'$$



For a case of spatial point source (R>>L), L is the source dimension), a path approximation can be assumed

$$g_n(P,t;Q,0) \cong g_n(P,t-\tau;O,0)$$

Where $$\tau = \frac{R' - R}{c}$$

When $$\frac{\lambda R}{2} >> L^2, \qquad \tau = \frac{R' - R}{c} = -\frac{\boldsymbol{r}' \cdot \boldsymbol{e}_R}{c}$$

With the path approximation

$$U_n(P,t) = \iint_{\Sigma} [g_n(P,t-\tau;O,0) * M_0(Q,t)] d\Sigma'$$

It can be rewritten as

$$U_n(P,t) = g_n(P,t;O,0) * \iint_{\Sigma} [\delta(t-\tau) * M_0(Q,t)] d\Sigma'$$

The integration on the right is the apparent source time function (ASTF)

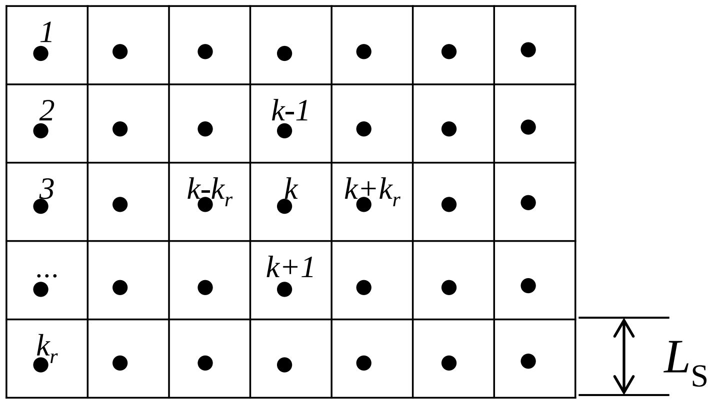
$$S_A(P,t) = \iint_{\Sigma} [\delta(t-\tau) * M_0(Q,t)] d\Sigma'$$

So two conditions are demanded for the ASTF

$R >> L$ At far-field distances

$\tau = \frac{R' - R}{c}$ For single phase

To perform the inversion, it is necessary to discretize the fault plane into sub-faults



Since the calculations of Green's functions is done based on point sources, a point source approximation is needed for the sub-faults. It means the sub-fault size can be neglectable. It demands

$R >> L_S$ Spatial point source

$\lambda >> L_S$ Temporal point source

With the point source approximation, the integrations on the fault can be replaced as summations

$$U_n(P,t)=\iint_{\Sigma}[g_n(P,t-\tau;O,0)*M_0(Q,t)]d\Sigma'$$

$$U_n(P,t)=g_n(P,t;O,0)*\iint_{\Sigma}[\delta(t-\tau)*M_0(Q,t)]d\Sigma'$$

$$S_A(P,t)=\iint_{\Sigma}[\delta(t-\tau)*M_0(Q,t)]d\Sigma'$$

They are

$$U_n^m(t)=\sum_k[g_n^m(t-\tau^{mk})*M_0^k(t)]$$

$$U_n^m(t)=g_n^m(t)*S_A^m(t)$$

$$S_A^m(t)=\sum_k[\delta(t-\tau^{mk})*M_0^k(t)]$$

Two inversion ways can be classified

I. ASTF inversion: get the ASTFs by deconvolving the Green’s function from the seismograms, and then invert the ASTFs for rupture model

$$U_n^m(t) = g_n^m(t) * S_A^m(t)$$

$$S_A^m(t) = \sum_k [\delta(t - \tau^{mk}) * M_0^k(t)]$$

II. Seismogram inversion

$$U_n^m(t) = \sum_k [g_n^m(t - \tau^{mk}) * M_0^k(t)]$$

① Apparent Source Time Function Inversion

$$S_A^m(t) = \sum_k [\delta(t-\tau^{mk}) * M_0^k(t)]$$

The matrix equation

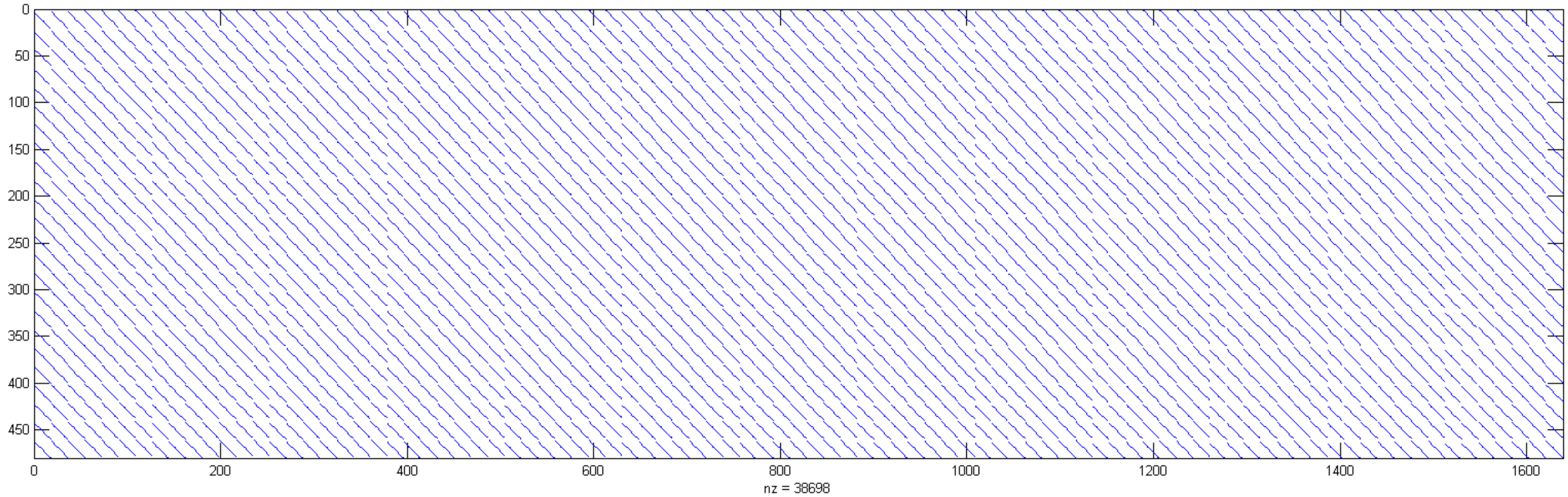
$$[\mathbf{S}_A] = [\boldsymbol{\delta}][\mathbf{M_0}]$$

Where [$\boldsymbol{\delta}$] consists of block matrixes

$$[\boldsymbol{\delta}] = \begin{bmatrix} \boldsymbol{\delta}_B^{11} & \boldsymbol{\delta}_B^{12} & & \boldsymbol{\delta}_B^{1K} \\ \boldsymbol{\delta}_B^{21} & & & \\ \dots & & \boldsymbol{\delta}_B^{MK} & \\ \boldsymbol{\delta}_B^{M1} & & & \boldsymbol{\delta}_B^{MK} \end{bmatrix}$$

And the block matrixes [$\boldsymbol{\delta}_B$] is

$$[\boldsymbol{\delta}_B(t)] = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \dots & \\ & & & 1 \end{bmatrix} \quad [\boldsymbol{\delta}_B(t-1)] = \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ & \dots & \dots & \\ & & 1 & 0 \end{bmatrix} \quad [\boldsymbol{\delta}_B(t+1)] = \begin{bmatrix} 0 & 1 & & \\ & 0 & \dots & \\ & & \dots & 1 \\ & & & 0 \end{bmatrix}$$



An example of the matrix. Blue dots denote the non-zero elements

② Waveform Inversion

$$U_n(P,t)=\iint_{\Sigma}[g_n(P,t-\tau;O,0)*M_0(Q,t)]d\Sigma'$$

The matrix equation

$$\mathbf{[U]=[g][M_0]}$$

Where [**g**] consists of block matrixes

$$\mathbf{[g]}=\begin{bmatrix}\mathbf{g}^{11} & \mathbf{g}^{12} & \mathbf{...} & \mathbf{g}^{1K}\\ \mathbf{g}^{21} & \mathbf{g}^{22} & \mathbf{...} & \mathbf{g}^{2K}\\ \mathbf{...} & \mathbf{...} & \mathbf{g}^{mk} & \mathbf{...}\\ \mathbf{g}^{M1} & \mathbf{g}^{M2} & \mathbf{...} & \mathbf{g}^{MK}\end{bmatrix}$$

When the length of sub-fault STF is shorter than that of the Green's function

And the block matrixes [$\mathbf{g}_B$] is

$$[\mathbf{g}^{mk}(t)]=\begin{bmatrix}g^{mk}(1) & & & \\ g^{mk}(2) & g^{mk}(1) & & \\ \mathbf{...} & & \mathbf{...} & \\ g^{mk}(L_g) & \mathbf{...} & g^{mk}(2) & g^{mk}(1)\end{bmatrix}$$

1.2 Inversion with Rake Variation

$$U_n(P,t) = \iint_{\Sigma} G_{np,q}(P,t;Q,0) * m_{pq}(Q,t) d\Sigma'$$

The slip vector on the fault ***e*** can change with time

$$m_{pq}(Q,t) = M_0(Q,t)[e_p(t)v_q + e_q(t)v_p]$$

Then

$$U_n(P,t) = \iint_{\Sigma} G_{np,q}(P,t;Q,0) * \{M_0(Q,t) \cdot [e_p(t)v_q + e_q(t)v_p]\} d\Sigma'$$

$$U_n(P,t) = \iint_{\Sigma} G_{np,q}(P,t;Q,0) * \{M_0(Q,t) \cdot [e_p(t)v_q + e_q(t)v_p]\} d\Sigma'$$

It can be written as

$$U_n(P,t) = \iint_{\Sigma} [g_{n1}(P,t;Q,0) * M_{01}(Q,t) + g_{n2}(P,t;Q,0) * M_{02}(Q,t)] d\Sigma'$$

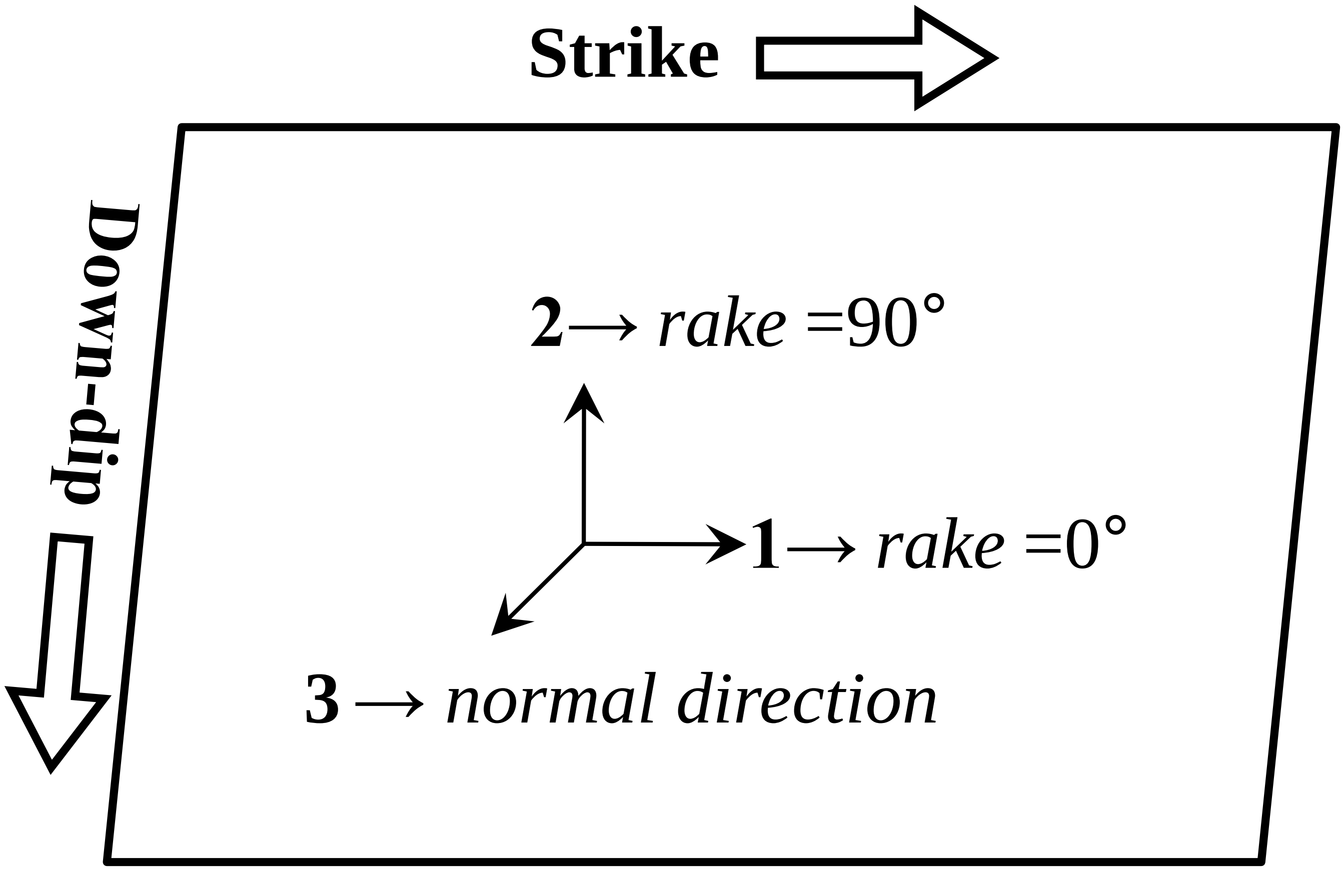
Where

$$g_{n1}(P,t;Q,0) = G_{n1,3}(P,t;Q,0) \cdot v_3, \;\; M_{01}(Q,t) = M_0(Q,t) \cdot e_1(t)$$

$$g_{n2}(P,t;Q,0) = G_{n2,3}(P,t;Q,0) \cdot v_3, \;\; M_{02}(Q,t) = M_0(Q,t) \cdot e_2(t)$$

The inversion equation is

$$[\mathbf{U}] = [\mathbf{g}_1 \; \mathbf{g}_2] \begin{bmatrix} \mathbf{M}_{01} \\ \mathbf{M}_{02} \end{bmatrix}$$



Strike
Down-dip
2→ rake =90°
1→ rake =0°
3 → normal direction

The inversion equations

ASTF inversion	Seismogram inversion with rake fixed	Seismogram inversion with rake variation
$[\mathbf{S}_A] = [\delta][\mathbf{M_0}]$	$[\mathbf{U}] = [\mathbf{g}][\mathbf{M_0}]$	$[\mathbf{U}] = [\mathbf{g}_1\ \mathbf{g}_2]\begin{bmatrix}\mathbf{M}_{01} \\ \mathbf{M}_{02}\end{bmatrix}$

- The unknown parameters in the 3 equations are the history of scalar seismic moment.
- When the Green's functions are calculated by considering the step response, the unknowns become the moment rate function (source time function).
- The unknowns of moment rate can be transferred into fault slip-rate by multiplying the Green's functions by the shear modulus and the sub-fault area.

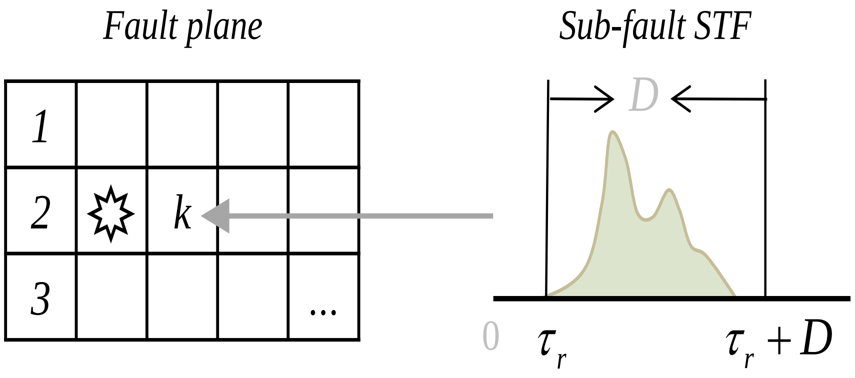
The inversion equations

ASTF inversion	Seismogram inversion with rake fixed	Seismogram inversion with rake variation
$[\mathbf{S}_A] = [\boldsymbol{\delta}][\mathbf{s}]$	$[\mathbf{U}] = [\mathbf{g}][\mathbf{s}]$	$[\mathbf{U}] = [\mathbf{g}_1\ \mathbf{g}_2]\begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix}$

2.1.3

Some limitations or constraints are needed to stabilize the solution

- Limitation of the maximum rupture velocity
- Limitation of the maximum rupture duration of the sub-fault
- Limitation of non-negative solution



A sketch of the rupture initiation time (τ_r) and rupture duration (D) of a sub-fault STF

1						
2			*k-1*			
3		$k\text{-}k_r$	*k*	$k+k_r$		
...			*k+1*			
k_r						

- Spatial smoothing

$$4s^k(t)-[s^{k-1}(t)+s^{k+1}(t)+s^{k-k_r}(t)+s^{k+k_r}(t)]=0$$

$$\mathbf{[D][s]}=\mathbf{[}0\mathbf{]}$$

- Temporal smoothing

$$2s^k(t)-[s^k(t-1)+s^k(t+1)]=0$$

$$\mathbf{[T][s]}=\mathbf{[}0\mathbf{]}$$

- Scalar moment minimization

$$s^k\mathbf{(}t\mathbf{)}=0$$

$$\mathbf{[Z][s]}=\mathbf{[}0\mathbf{]}$$

2.1.4 Equations for the three kinds of inversions

The inversion equations

ASTF inversion

$$\begin{bmatrix} \mathbf{S_A} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta} \\ \kappa_1 \mathbf{D} \\ \kappa_2 \mathbf{T} \end{bmatrix} [\mathbf{s}]$$

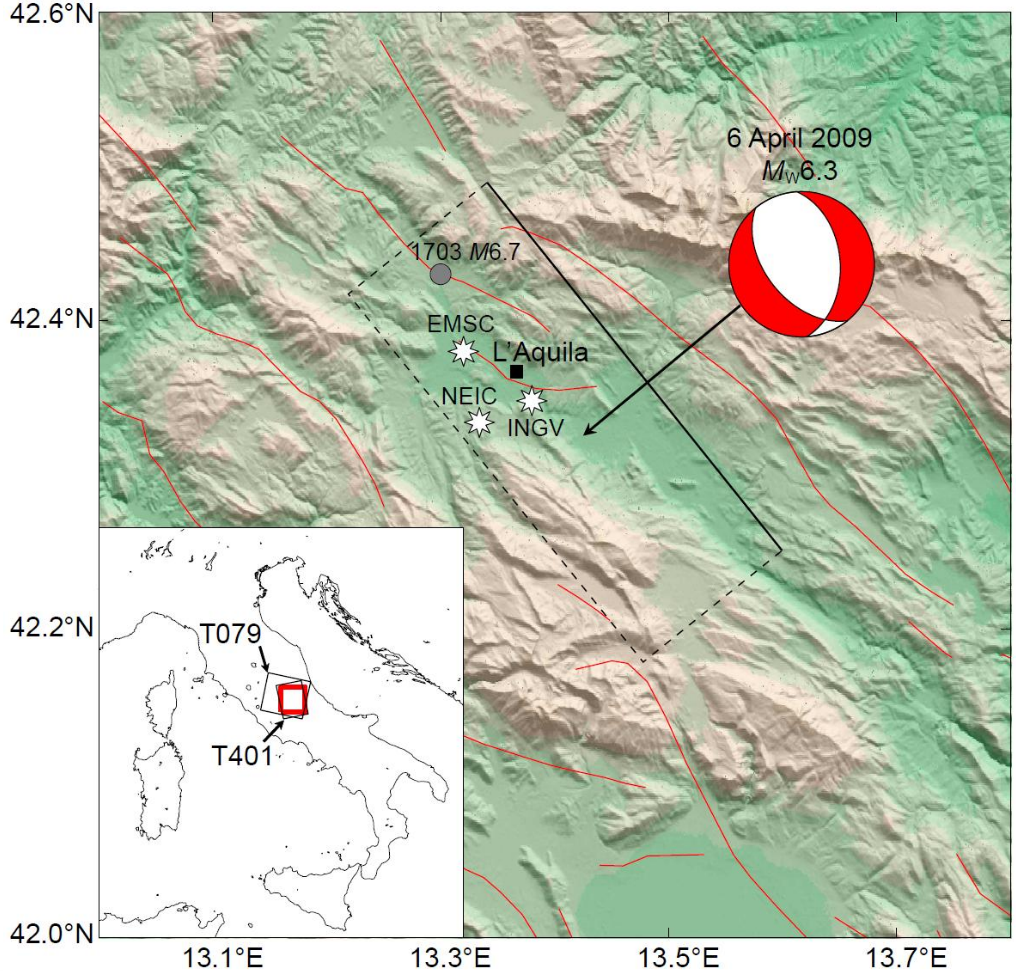
Seismogram inversion with rake fixed

$$\begin{bmatrix} \mathbf{U} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{g} \\ \kappa_1 \mathbf{D} \\ \kappa_2 \mathbf{T} \\ \kappa_3 \mathbf{Z} \end{bmatrix} [\mathbf{s}]$$

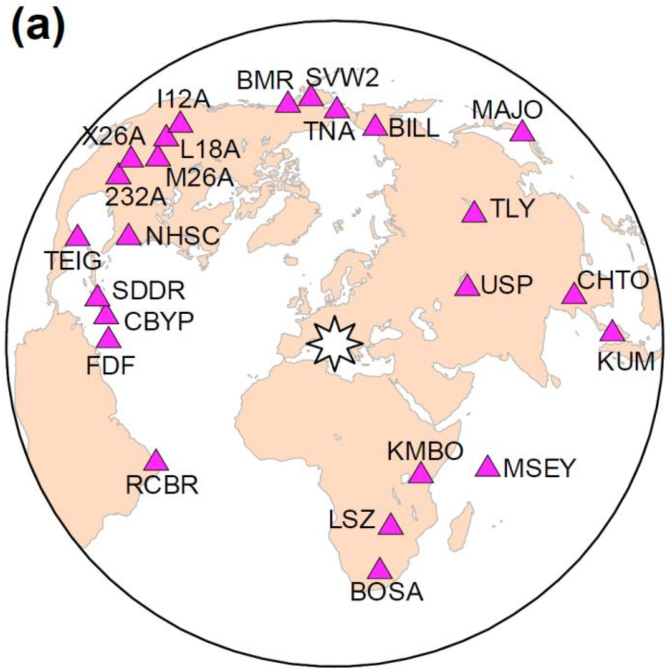
Seismogram inversion with rake variation

$$\begin{bmatrix} \mathbf{U} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1 \quad \mathbf{g}_2 \\ \kappa_1 \begin{bmatrix} \mathbf{D} & \\ & \mathbf{D} \end{bmatrix} \\ \kappa_2 \begin{bmatrix} \mathbf{T} & \\ & \mathbf{T} \end{bmatrix} \\ \kappa_3 \begin{bmatrix} \mathbf{Z} & \\ & \mathbf{Z} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix}$$

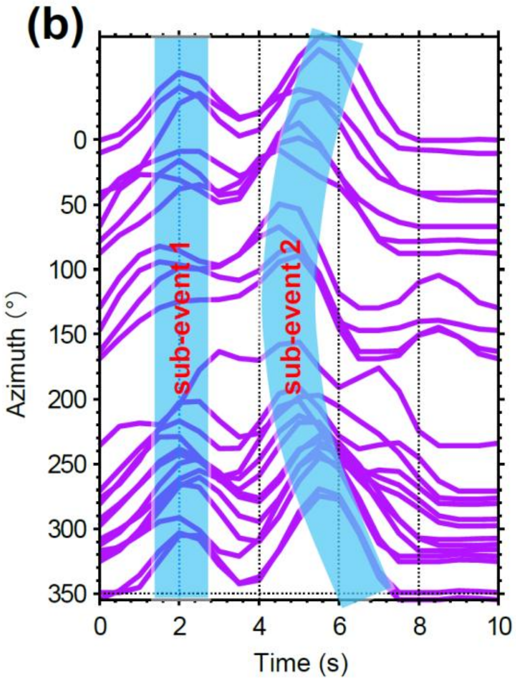
2.1.5 An Example: The 2009 M_W6.3 L'Aquila earthquake



An example: The 2009 M_W6.3 L'Aquila earthquake



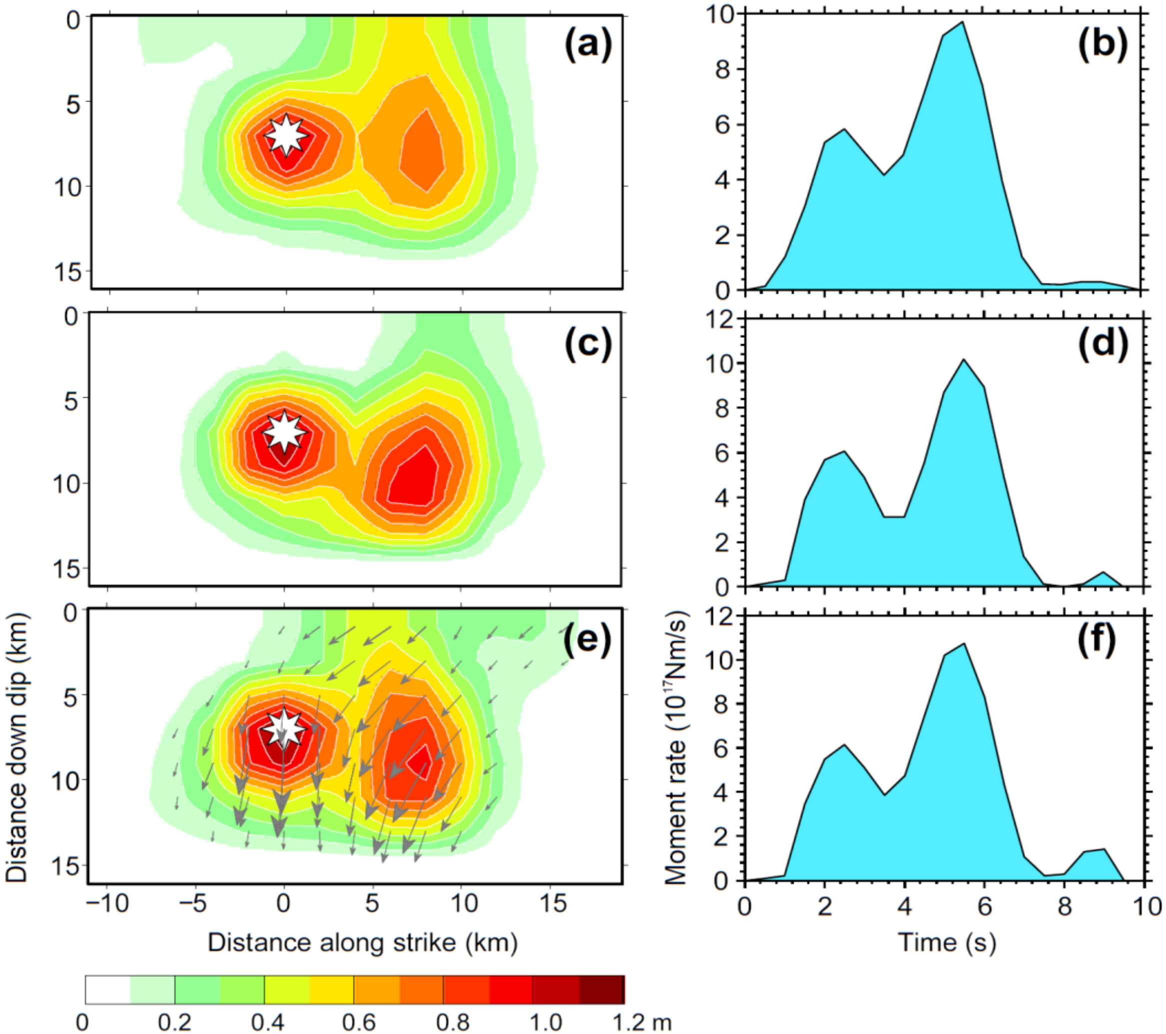
Teleseismic stations



Azimuth-dependent ASTFs

Fault slip distributions

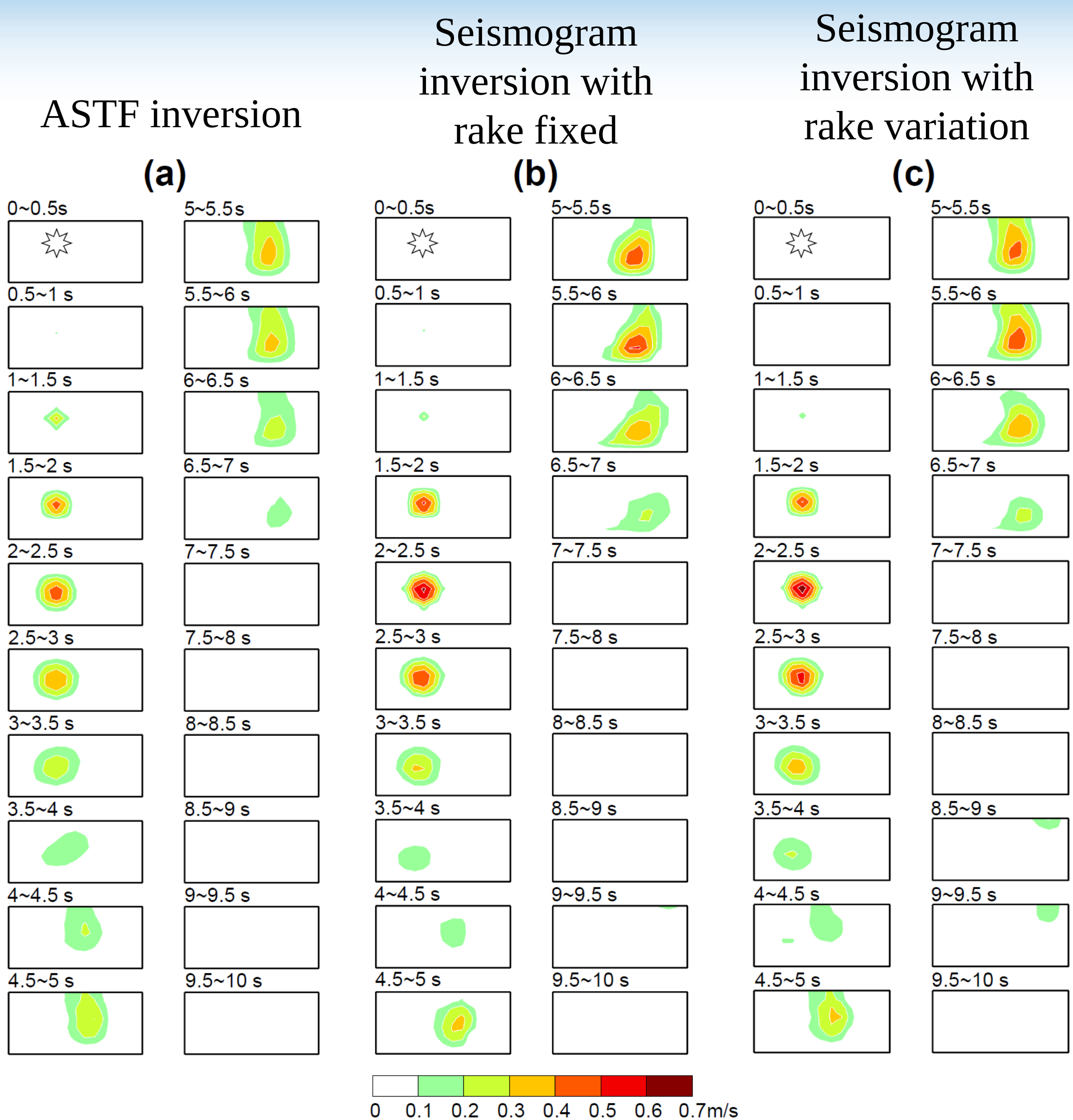
STFs

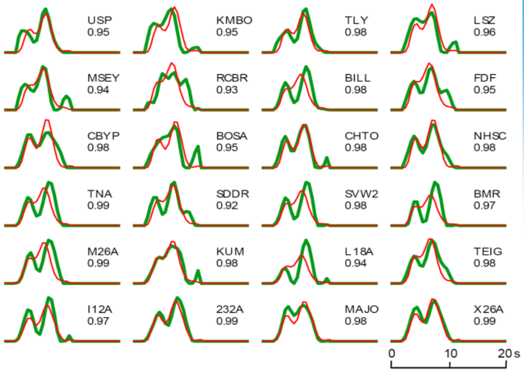


ASTF inversion

Seismogram inversion with rake fixed

Seismogram inversion with rake variation


ASTF inversion
Seismogram inversion with rake fixed
Seismogram inversion with rake variation
(a)
(b)
(c)
0~0.5s
0.5~1 s
1~1.5 s
1.5~2 s
2~2.5 s
2.5~3 s
3~3.5 s
3.5~4 s
4~4.5 s
4.5~5 s
5~5.5s
5.5~6 s
6~6.5 s
6.5~7 s
7~7.5 s
7.5~8 s
8~8.5 s
8.5~9 s
9~9.5 s
9.5~10 s
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7m/s

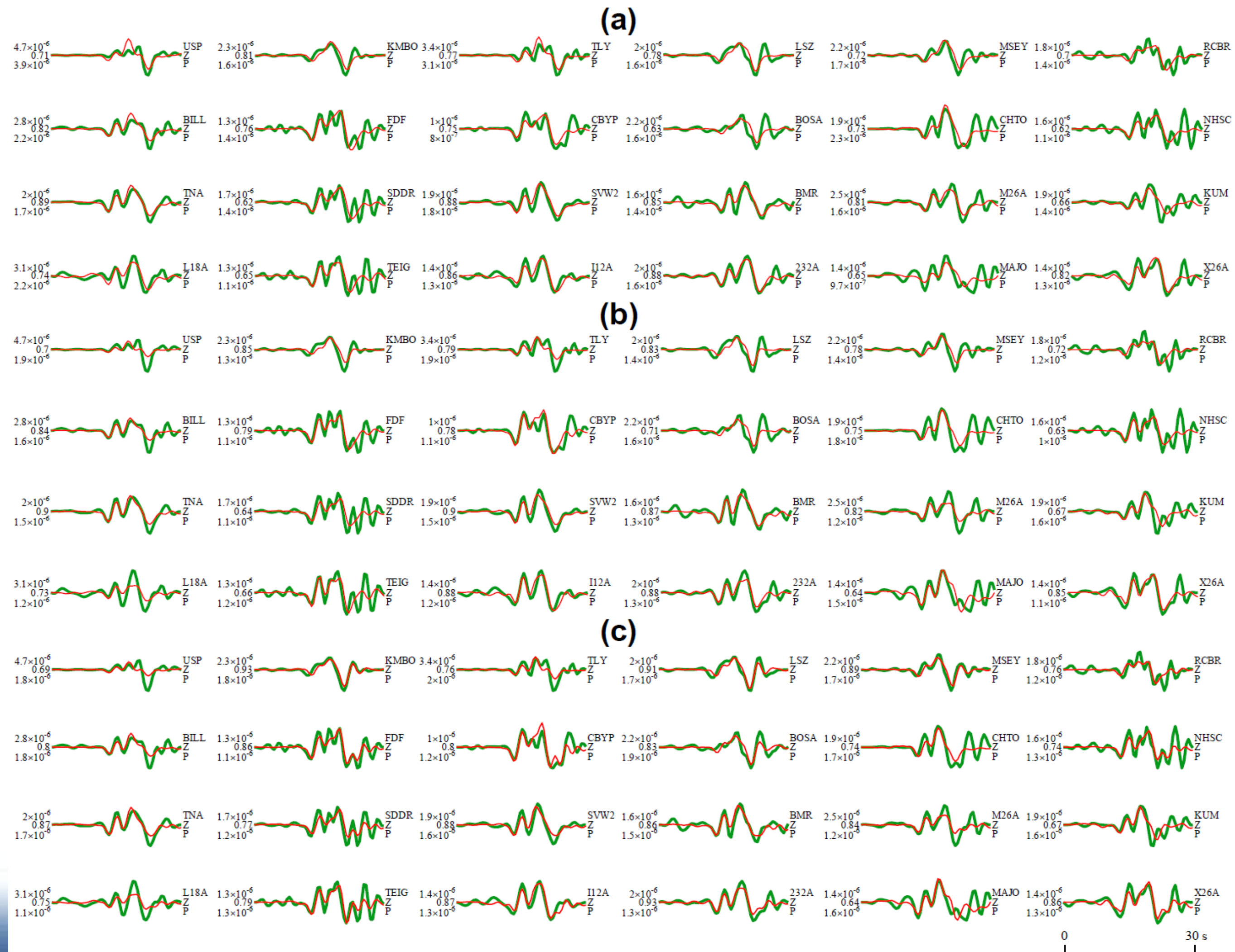


Retrieved and synthetic ASTFs

ASTF inversion

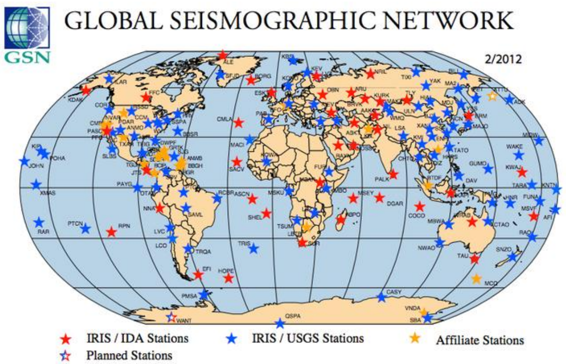
Seismogram inversion with rake fixed

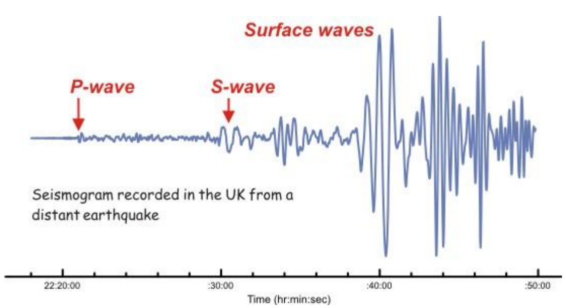
Seismogram inversion with rake variation



2.2 Joint Inversion of seismic and geodetic data

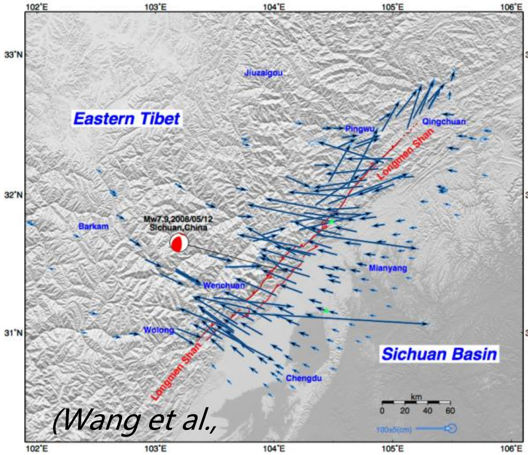
Geodetic data





Seismic data

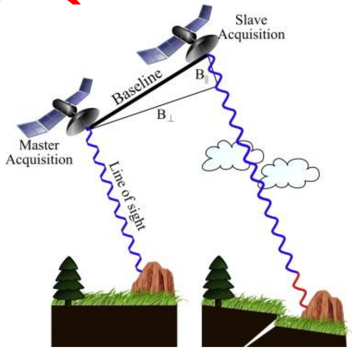
+

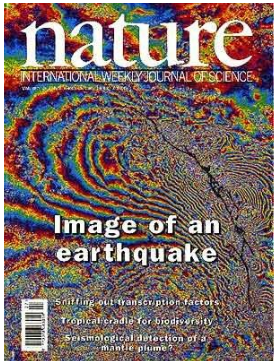


(Wang et al., 2011)

GNSS data

+





InSAR Data

Fault slip inversion with geodetic data when considering the rake variation

$$[\mathbf{E}] = [\mathbf{B}_1\ \mathbf{B}_2]\begin{bmatrix}\mathbf{f}_1 \\ \mathbf{f}_2\end{bmatrix}$$

The fault slip of each sub-fault is the summation of slip-rate

$$\begin{bmatrix}\mathbf{f}_1 \\ \mathbf{f}_2\end{bmatrix} = \begin{bmatrix}\mathbf{J} & 0 \\ 0 & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{s}_1 \\ \mathbf{s}_2\end{bmatrix}$$

The matrix [**J**] is a sparse matrix, in which the non-zero elements equal 1

$$[\mathbf{J}] = \begin{bmatrix} 1 & 1 & \cdots & 1 & & & & & & & & & & & \\ & & & & 1 & 1 & \cdots & 1 & & & & & & & \\ & & & & & & & & \cdots & \cdots & \cdots & \cdots & & & \\ & & & & & & & & & & & & 1 & 1 & \cdots & 1 \end{bmatrix} \begin{matrix} \leftarrow \text{Sub-fault 1} \\ \leftarrow \text{Sub-fault 2} \\ \ldots \\ \leftarrow \text{Sub-fault K} \end{matrix}$$

With the equations

$$[\mathbf{E}] = [\mathbf{B}_1\ \mathbf{B}_2]\begin{bmatrix}\mathbf{f}_1\\ \mathbf{f}_2\end{bmatrix} \qquad \begin{bmatrix}\mathbf{f}_1\\ \mathbf{f}_2\end{bmatrix} = \begin{bmatrix}\mathbf{J} & 0\\ 0 & \mathbf{J}\end{bmatrix}\begin{bmatrix}\mathbf{s}_1\\ \mathbf{s}_2\end{bmatrix}$$

The relation between the deformation data and the fault slip-rate is

$$[\mathbf{E}] = [\mathbf{H}_1\ \mathbf{H}_2]\begin{bmatrix}\mathbf{s}_1\\ \mathbf{s}_2\end{bmatrix}$$

Where

$$[\mathbf{H}_1] = [\mathbf{B}_1] * [\mathbf{J}], \quad [\mathbf{H}_2] = [\mathbf{B}_2] * [\mathbf{J}]$$

The equations of seismic and geodetic data inversion for slip-rate are

$$\begin{bmatrix} \mathbf{U} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{g}_2 \\ \kappa_1 \begin{bmatrix} \mathbf{D} & \\ & \mathbf{D} \end{bmatrix} \\ \kappa_2 \begin{bmatrix} \mathbf{T} & \\ & \mathbf{T} \end{bmatrix} \\ \kappa_3 \begin{bmatrix} \mathbf{Z} & \\ & \mathbf{Z} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix} \qquad [\mathbf{E}] = [\mathbf{H}_1 \ \mathbf{H}_2] \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix}$$

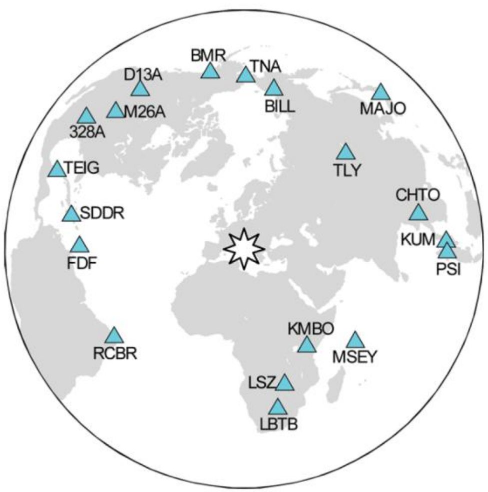
The joint inversion can be performed by solving

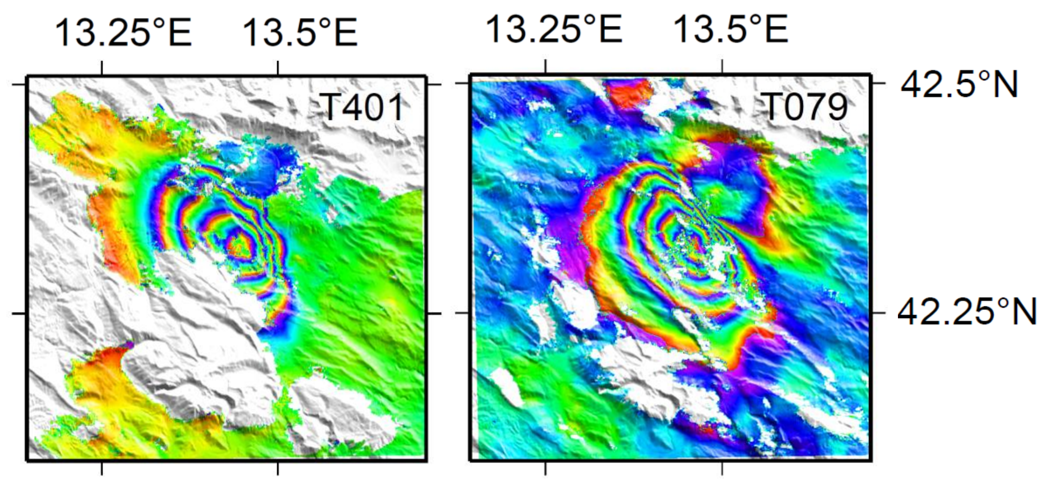
$$\begin{bmatrix} \mathbf{U} \\ 0 \\ 0 \\ 0 \\ \kappa \mathbf{E} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{g}_2 \\ \kappa_1 \begin{bmatrix} \mathbf{D} & \\ & \mathbf{D} \end{bmatrix} \\ \kappa_2 \begin{bmatrix} \mathbf{T} & \\ & \mathbf{T} \end{bmatrix} \\ \kappa_3 \begin{bmatrix} \mathbf{Z} & \\ & \mathbf{Z} \end{bmatrix} \\ \kappa [\mathbf{H}_1 \ \mathbf{H}_2] \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix}$$

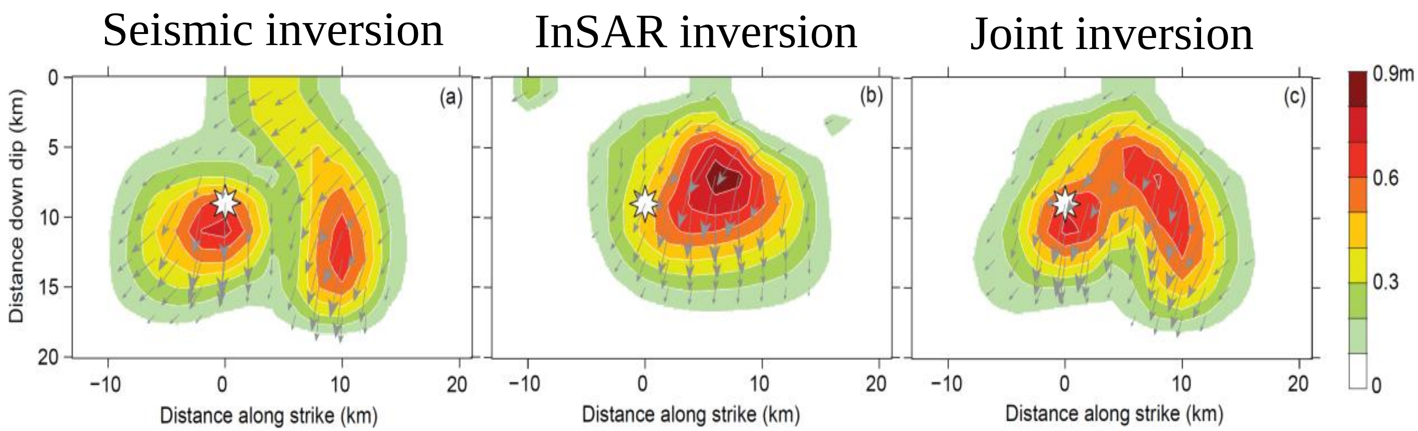
The difference in values of the seismic data *U* and the deformation data *E* may reach several orders of magnitude, a normalization is necessary to ensure that they are comparable in the joint inversion. Usually we normalize them by their square root of energy, It makes the two datasets be equally weighted in the least-square optimizations if

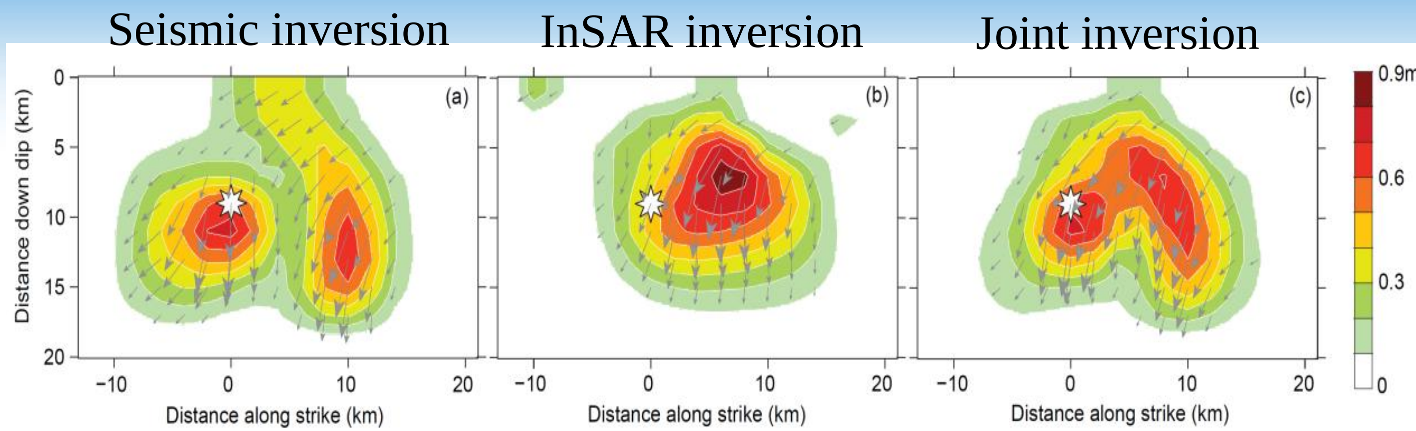
$$\kappa = 1 \qquad U = \frac{U}{\sqrt{\int U^2 dt}} \qquad E = \frac{E}{\sqrt{\int E^2 dt}}$$

An example: The 2009 M_W6.3 L'Aquila earthquake

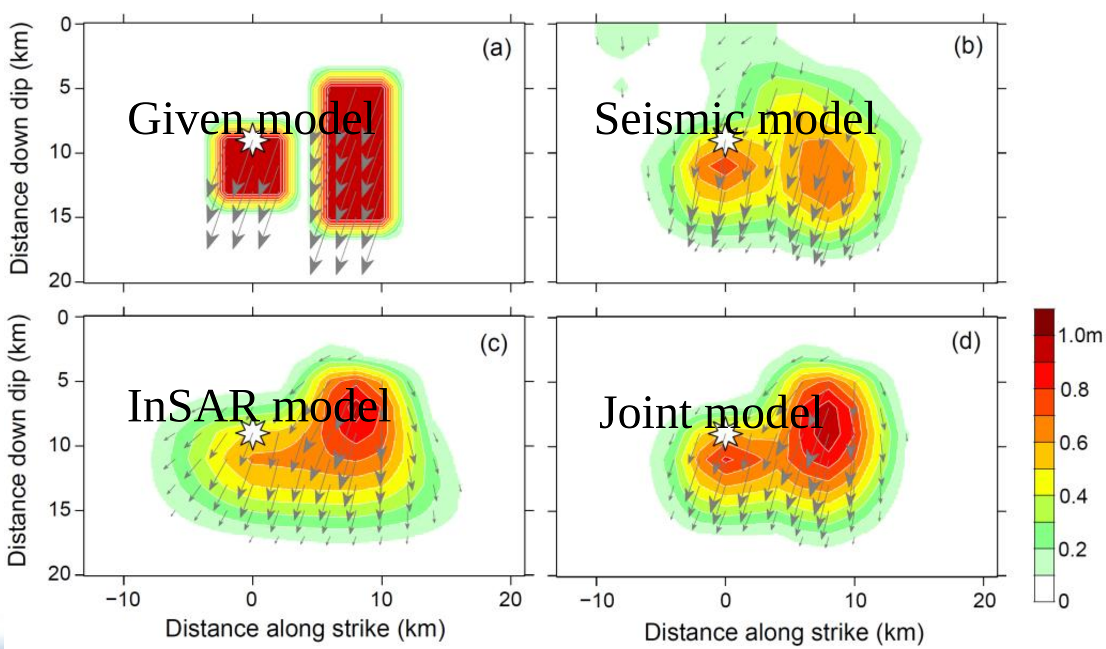




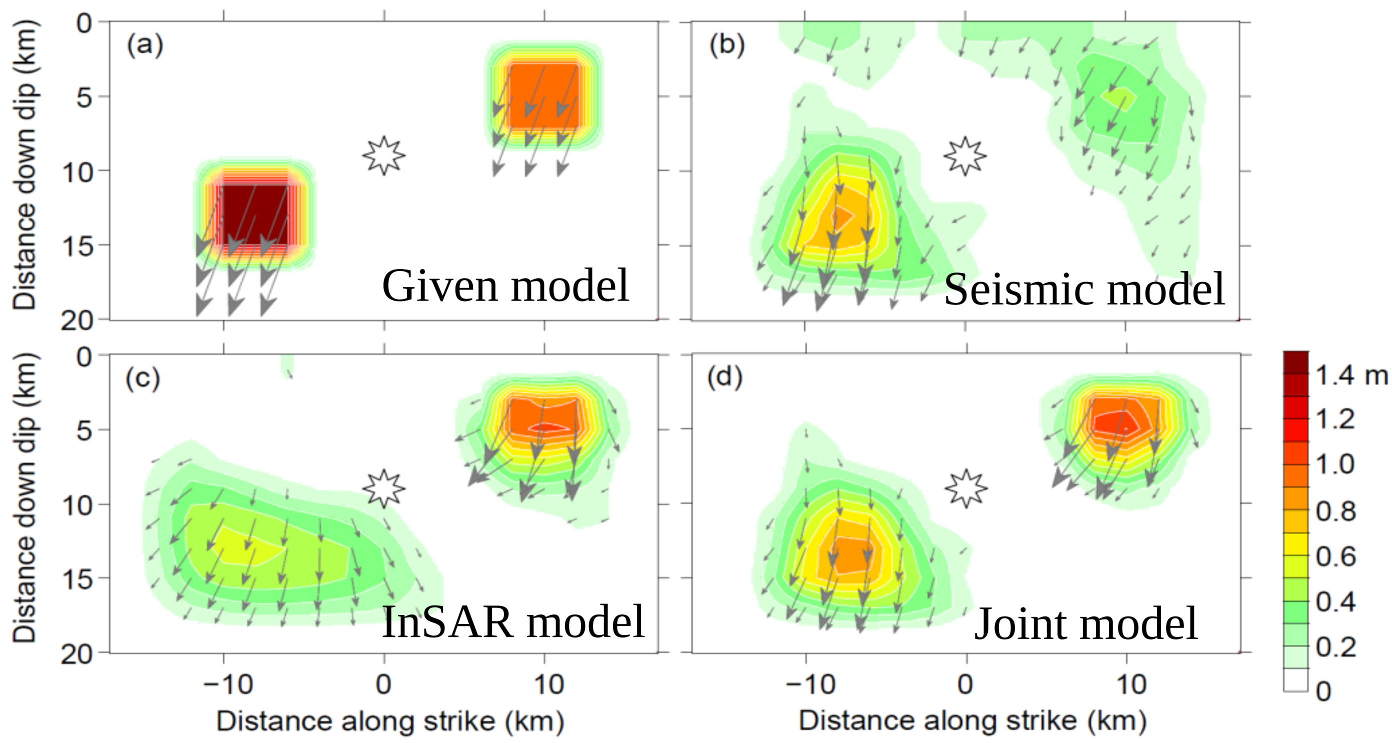




Fault slip distributions of the 2009 L'Aquila earthquake obtained by teleseismic data inversion (a), InSAR data inversion (b), and joint inversion (c).



A general tests of individual data inversion and joint inversion. Seismic data tends to recover the slip patch which released more moment, while geodetic InSAR data mainly constrain the shallow slips. The joint inversion synthesize the advantages of the two datasets.



Summary

We have described the theory and techniques of inversion of earthquake rupture process, and discussed the limitations and constraints existed in the seismic inversion.

Take the 2009 M_W6.3 L'Aquila earthquake as an example to ephamcize the needs in joint inversion using both of the seismic and geodetic.


谢谢！
Thank you!
Спасибо!