On the theory of

Quantum Fluids of Light

Part 1/3

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Why not hydrodynamics of light ?

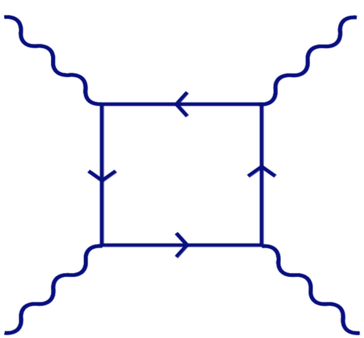
Light field/beam composed by a huge number of photons

- *in vacuo* photons travel along straight line at *c*
- (practically) do not interact with each other
- in standard cavity, thermalization via walls and absorption/emission

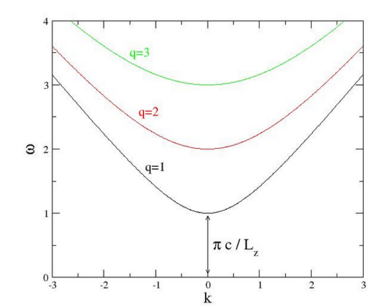
optics in vacuo typically dominated by single-particle physics

In suitable photonic structures:

- spatial confinement → effective photon mass
- $\chi^{(3)}$ nonlinearity → photon-photon interactions



Collective behaviour of *quantum fluid of light*



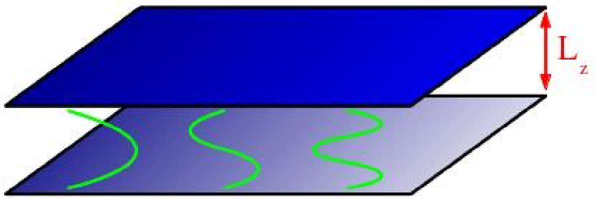
IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

What about mass?

In vacuo: photons massless, dispersion $\omega = c\,|k|$

In planar cavity → confinement along *z*, free propagation along *x,y*

Quantization along *z*: $k_z^{(q)} = q\,\pi\,/\,L_z$

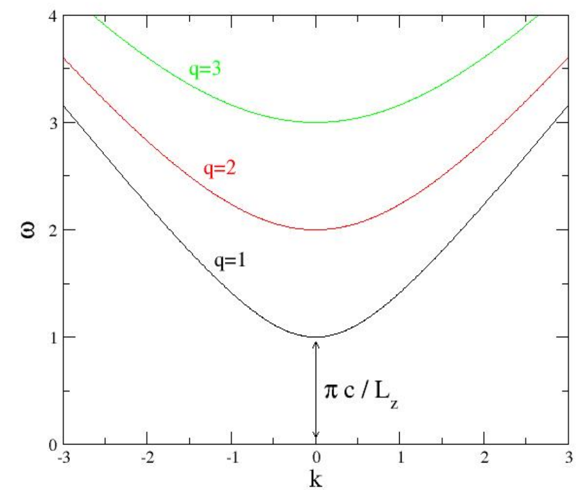


Massive dispersion along *x,y*:

$$\omega^{(q)}(\mathbf{k}_\parallel) = c\sqrt{[k_z^{(q)}]^2 + \mathbf{k}_\parallel^2} = c\sqrt{\left(\frac{q\pi}{L_z}\right)^2 + \mathbf{k}_\parallel^2} \simeq ck_z^{(q)} + \frac{c}{2k_z^{(q)}}\mathbf{k}_\parallel^2$$

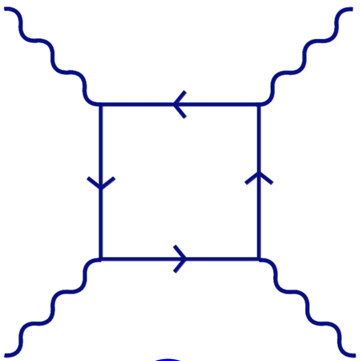
Confinement gives effective photon mass $m_{ph}c^2 = \hbar c k_z^o$

- Rest mass → cut-off in the dispersion
- Inertial mass → curvature of dispersion



IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

What about interactions?



Photon-photon interactions exist in QED:

Heisenberg-Euler processes via electron-positron exchange

… but cross section ridiculously small for visible light

(recent experiment in accelerator → Nat. Phys. 2017)

$$\sigma \sim \alpha^4 \left(\frac{\hbar^2}{m^2 c^2}\right)\left(\frac{\hbar\omega}{mc^2}\right)^6$$

Compton λ → pm range

<u>How to enhance it ?</u>

Replace electron-positron pair (E~1MeV) with
electron-hole pair (E~1eV) → gain factor $(10^6)^6 = 10^{36}$!!

In optical language:

- $\chi^{(3)}$ nonlinearity ↔ local photon-photon interactions
- typical material → spatially local (or quasi-local) $\chi^{(3)}$

Modern exceptional media:

- <u>Rydberg atoms</u> (Hannes' lectures)
  - Ultra-large, long-range nonlinearity in Rydberg-EIT config.
- <u>Superconducting circuits</u> (Steven's lectures)
  - Strong coupling to macroscopic oscillation mode of superconductor device

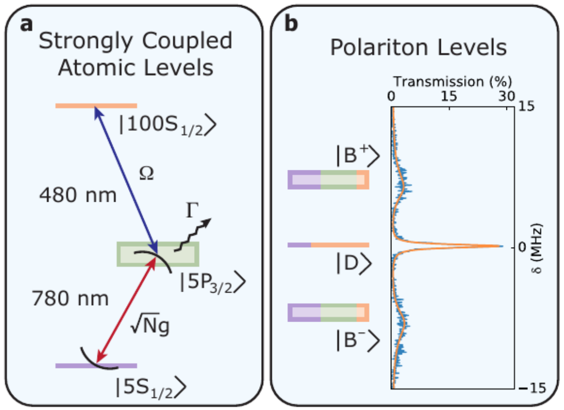
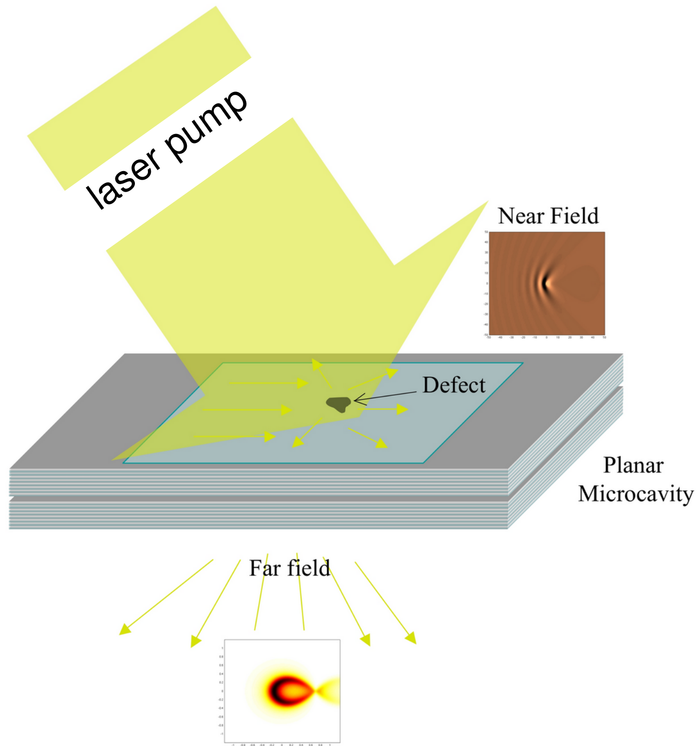


Figure: J. Simon's group @ U. Chicago

How to create and detect the photon gas?



Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

- Coherent laser pump: directly injects photon BEC in cavity, may lock BEC phase
- Incoherent (optical or electric) pump: BEC transition similar to laser threshold
  spontaneous breaking of U(1) symmetry

Classical and quantum correlations of in-plane field directly transfer to emitted radiation

IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

Part 2:

Weakly interacting fluids of light

Superfluid light &

Non-Equilibrium Statistical Mechanics

Mean-field theory: generalized GPE

$$i\frac{d\psi}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|\psi|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|\psi|^2}-\gamma\right)\right\}\psi+F_{ext}$$

Time-evolution of macroscopic wavefunction ψ of photon/polariton condensate

- standard terms: kinetic energy, external potential V_{ext}, interactions g, losses γ
- under coherent pump: forcing term (also known as Lugiato-Lefever eq.)
- under incoherent pump: polariton-polariton scattering from thermal component give saturable amplification term as in semiclassical theory of laser

→ a sort of Complex Landau-Ginzburg equation

To go beyond mean-field theory:

- Exact diagonalization, Wigner representation, Keldysh diagrams, ...

IC-Ciuti, *Quantum Fluids of Light*, RMP **85**, 299 (2013)

2006 - Photon Bose-Einstein condensation

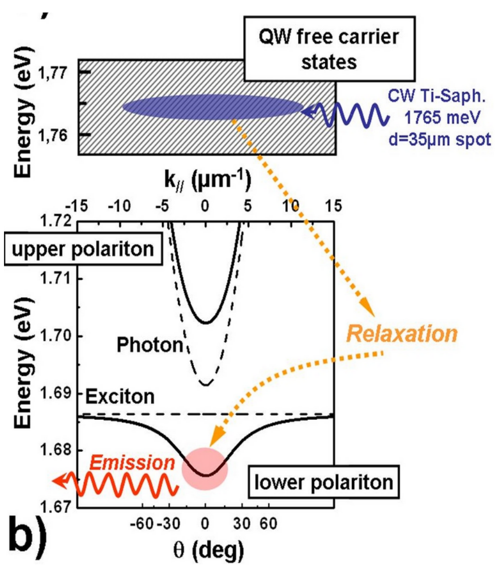
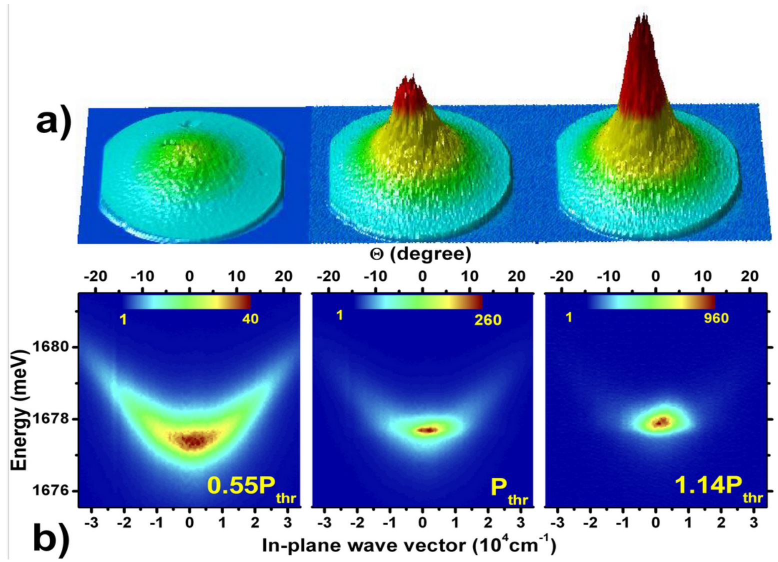
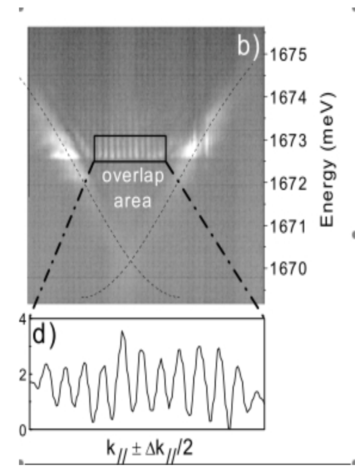


Figure from Kasprzak et al., Nature 2006



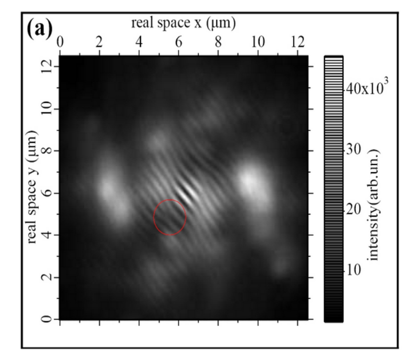
Momentum distribution

Kasprzak et al., Nature **443**, 409 (2006)



Interference

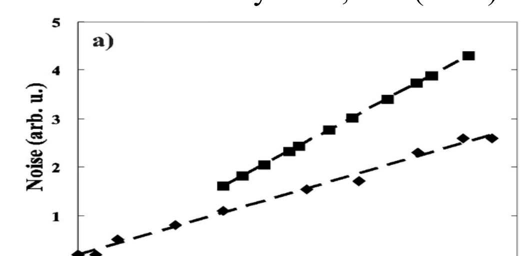
Richard et al., PRL **94**, 187401 (2005)



Quantized vortices

K. Lagoudakis *et al.*

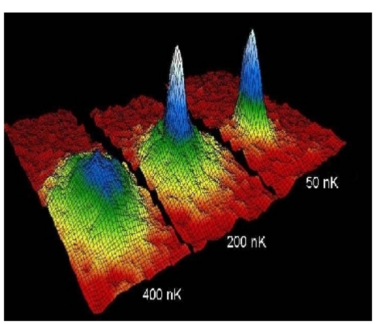
Nature Physics 4, 706 (2008).



Suppressed fluctuations

A. Baas et al., PRL **96**, 176401 (2006)

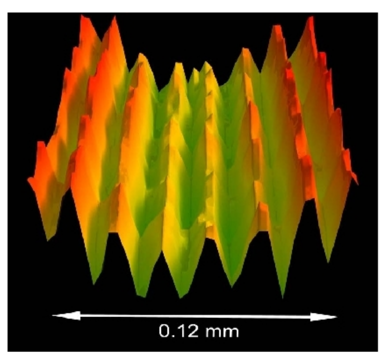
Many features very similar to atomic BEC



The first atomic BEC

M. H. Anderson et al.

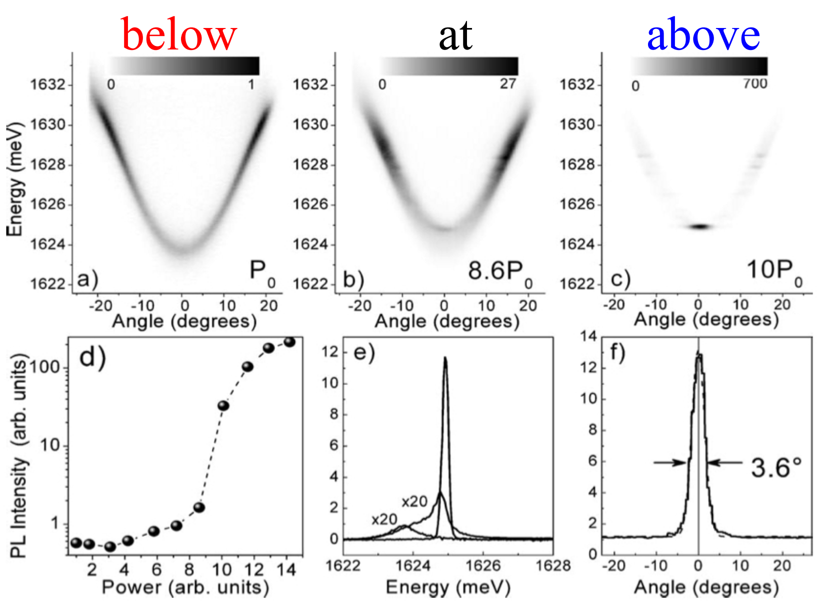
Science **269**, 198 (1995)

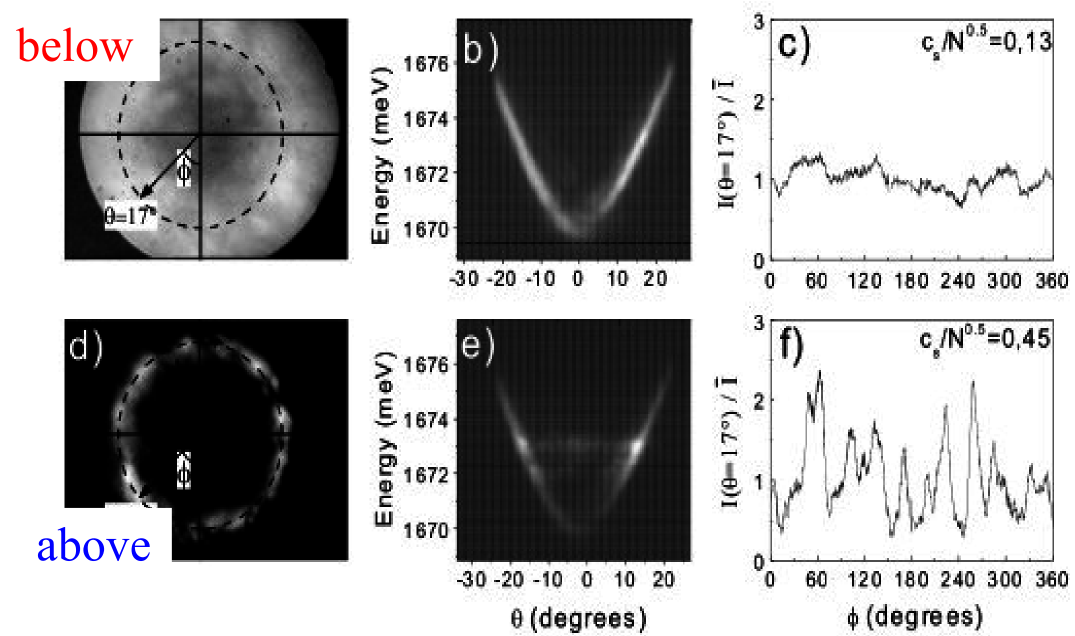


Interference pattern of two expanding atomic BECs

M. R. Andrews, Science **275**, 637 (1995)

A crucial difference: the shape of a non-equilibrium BEC





wide pump spot: 20 μm

M. Richard et al., PRB **72**, 201301 (2005)

narrow pump spot: 3 μm

M. Richard et al., PRL **94**, 187401 (2005)

Experimental observations under non-resonant pump:

- condensate shape depends on pump spot size
  - wide pump spot: condensation at k=0
  - narrow pump spot: condensation on a ring of modes at finite |k|

Analogous experiments in J. Bloch's group @ LPN in 1D geometry: Wertz et al. Nat. Phys 2008

Physical interpretation of condensation at $k \neq 0$

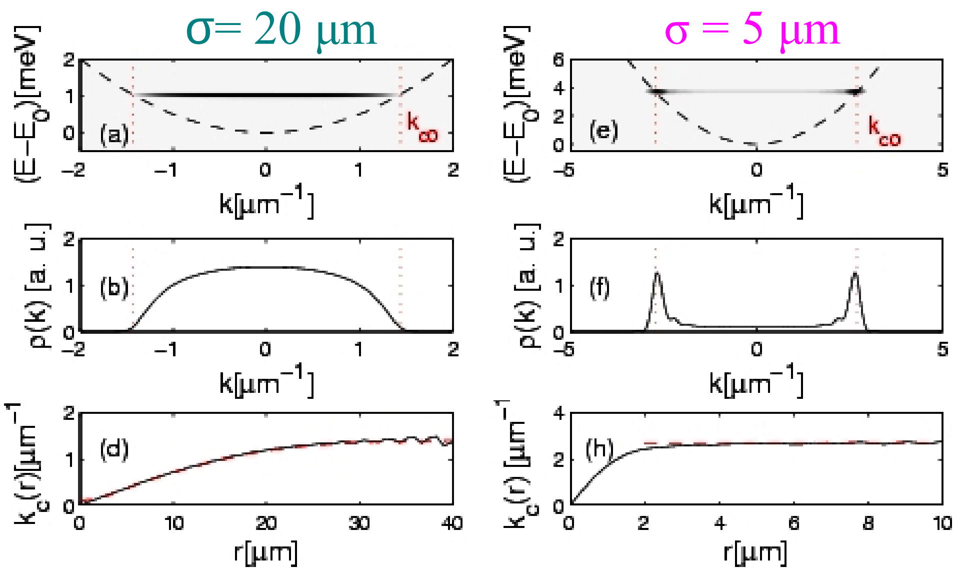
Repulsive interactions

- outward radial acceleration
- energy conservation

$$E = k^2/2m + U_{int}(r)$$

→ radially increasing flow velocity

→ coherent ballistic flow

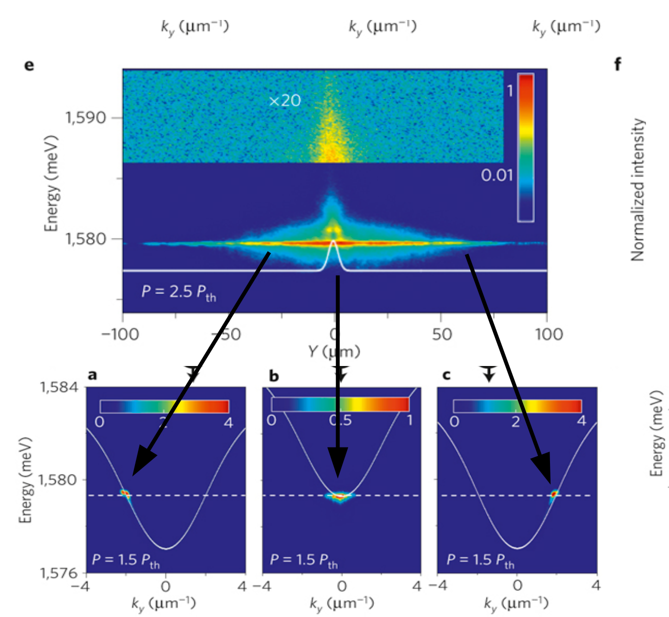


M. Wouters, IC, and C. Ciuti, PRB **77**, 115340 (2008)

Narrow spot:

- ballistic free flight outside pump spot $U_{int}(r)=0$
- emission mostly on free particle dispersion

Later expts confirm mechanism →



Wertz et al., Nat. Phys. 6, 860 (2010)

T-reversal breaking:

- allowed by non-equilibrium
- allows for non-zero current
- also visible as $n(k) \neq n(-k)$

Quasi-condensation features

At equilibrium → Hohenberg-Mermin-Wagner theorem

- BEC at low T in d≥3
- BKT transition in d=2
- no phase transition in d<2

Z. Physik 237, 31 – 46 (1970)

Laserlight — First Example of a Second-Order Phase Transition Far Away from Thermal Equilibrium*

R. GRAHAM and H. HAKEN
I. Institut für theoretische Physik der Universität Stuttgart

What about non-equilibrium?

- no BEC in 1D: exponential decay of $g^{(1)}(x)$
  (Graham-Haken, 1970; Wouters-IC, PRB 2006)

  Calculations in linearized Bogoliubov theory,
  but robust against Kardar-Parisi-Zhang nonlinearities of phase dynamics

$$\partial_{\tilde{t}}\tilde{\phi} = \nu\partial_{\tilde{x}}^2\tilde{\phi} + \frac{\lambda}{2}(\partial_{\tilde{x}}\tilde{\phi})^2 + \sqrt{\mathcal{D}}\xi_1$$

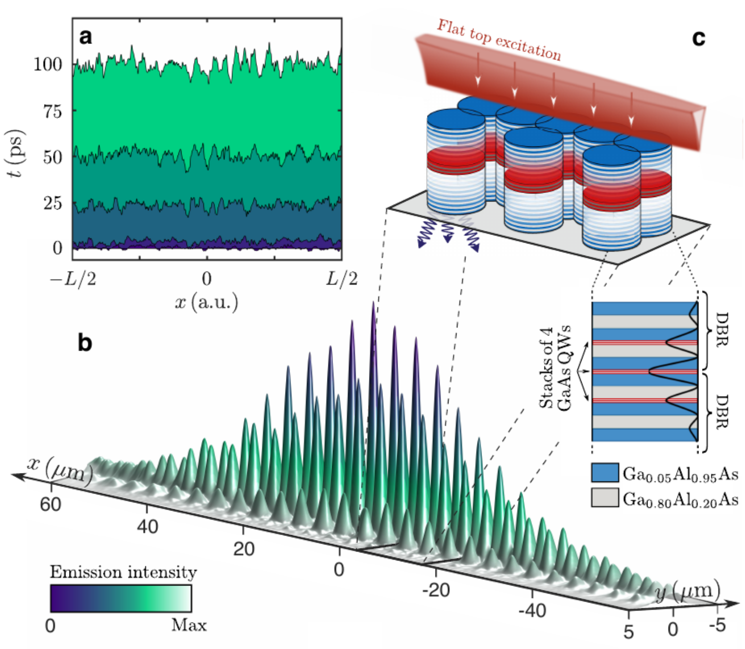
  Still, peculiar statistical features expected (ask Ivan) !

- debate in 2D: KPZ nonlinearities destroy BKT transition?
  (Dagvadorj et al., PRX 2015; Altman et al., PRX 2015; Zamora et al., PRX 2017)

- standard BEC in 3D but … how to realize it? Tomoki will tell you...

KPZ features in non-equilibrium 1D polariton BEC

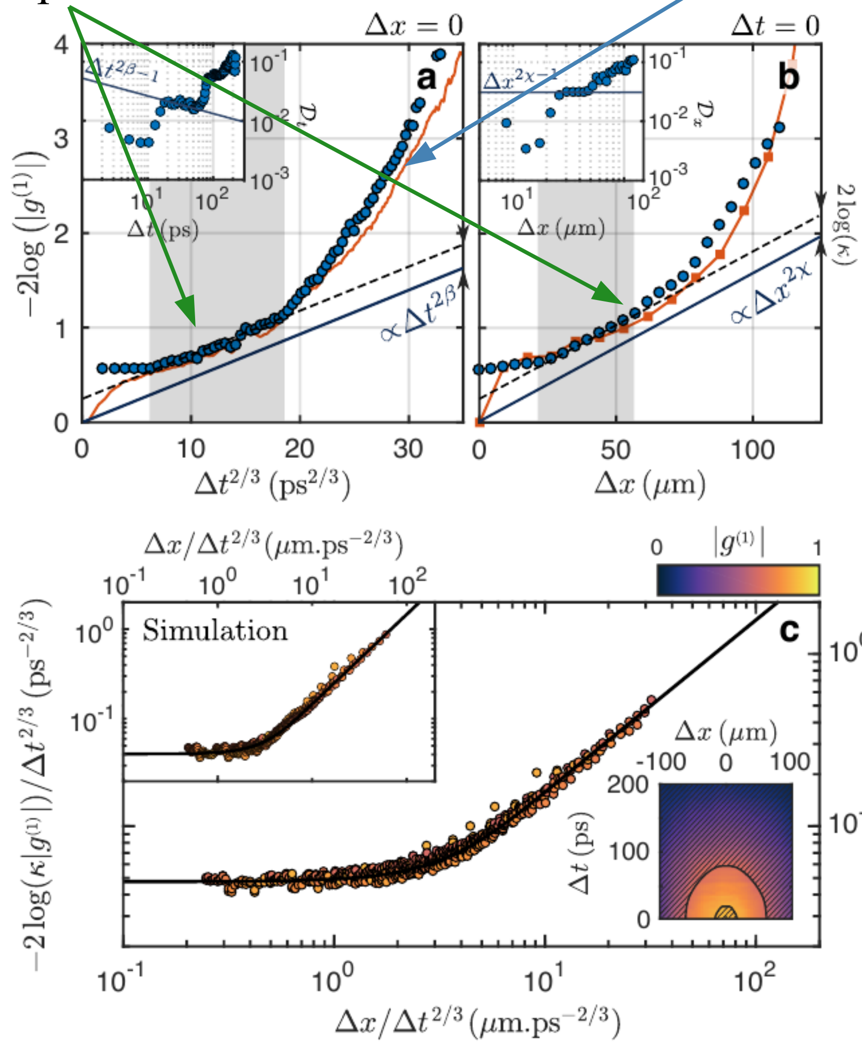
Polariton lasing in array of micropillars



Observable → decay of $g^{(1)}(\Delta x,\Delta t)$
at large space and times

KPZ exponent

Modified Schawlow-Townes decay



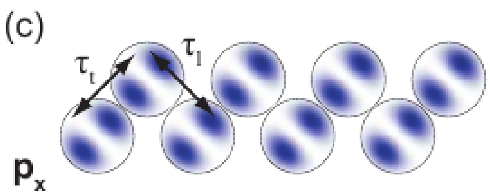
g$^{(1)}(\Delta x,\Delta t)$ cuts collapse on Kardar-Parisi-Zhang universality

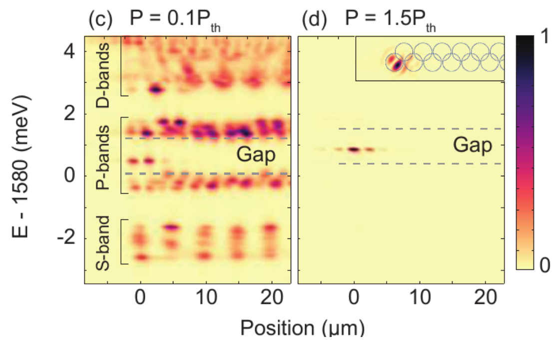
BEC / lasing = limit cycle. Correlations show robust/weak-ness of limit cycle in 1D (quasi-BEC)
Full stability recovered in thermodynamic limit in 3D systems (not so straightforward in expt...)

Fontaine et al., *Observation of KPZ universal scaling in a one-dimensional polariton condensate*, arXiv:2112.09550. Accepted on Nature

The ultimate 1D BEC: Topological lasing
a.k.a. non-equilibrium BEC in a topological edge state

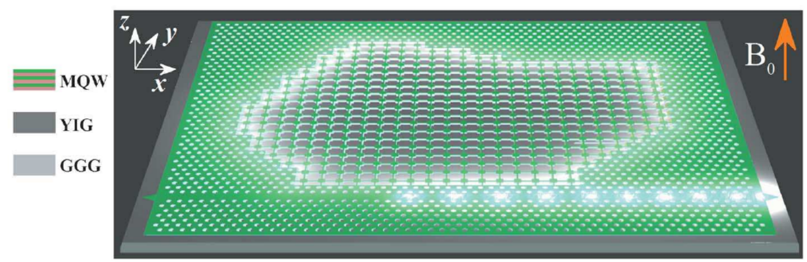
In other words: what happens if gain added to topological photonics model ?

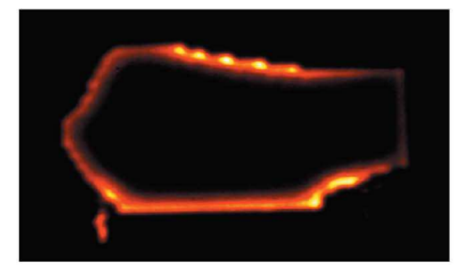


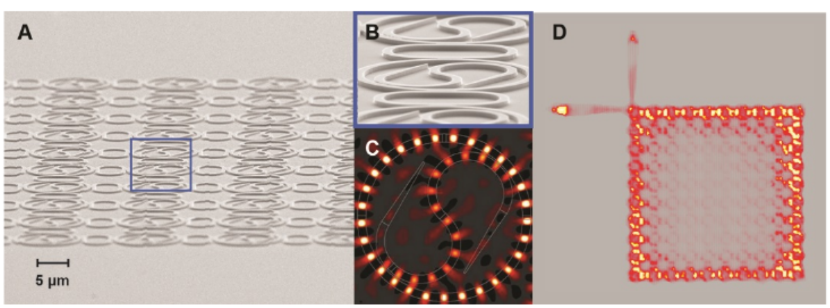


St. Jean, et al., Nat. Phot. ‘17
System: 1D SSH array of micropillar cavities for exciton-polaritons under incoherent pump

Bahari et al., Science 2017
System: 2D photonic crystal slab, amplification by QWs, magnetic field to break T







Bandres et al., Science 2018
System: array of Si-based ring resonators with optically pumped III-V amplifier layer.
Tai-Ji shape to break inversion symmetry

Exciting for opto-electronic applications:

- robust platform to ensure large-area coherence in high-power laser source
- Ultimate limitations to coherence KPZ stat-mech effects – Amelio-IC, PRX 2021

More details: Price et al., *Roadmap in topological photonics*, J. Phys.: Photonics 2022

Superfluids of light: an ongoing construction...

Superfluidity checklist

Laboratoire Kastler Brossel
Physique quantique et applications

CONDENSED-MATTER PHYSICS

Going with the flow

Jonathan Keeling and Natalia G. Berloff   Nature **457,** 273 (2009)

Table 1 | Superfluidity checklist

	Quantized vortices	Landau critical velocity	Metastable persistent flow	Two-fluid hydro-dynamics	Local thermal equilibrium	Solitary waves
Superfluid ^{4}He/cold atom Bose-Einstein condensate	✓	✓	✓	✓	✓	✓
Non-interacting Bose-Einstein condensate	✓	X	X	X	✓	X
Classical irrotational fluid	X	✓	X	✓	✓	✓
Incoherently pumped polariton condensates	✓	X	?	?	X	?
Parametrically pumped polariton condensates	✓	✓	✓	?	X	✓
Resonantly pumped polariton condensates	?	✓	?	?	X	✓

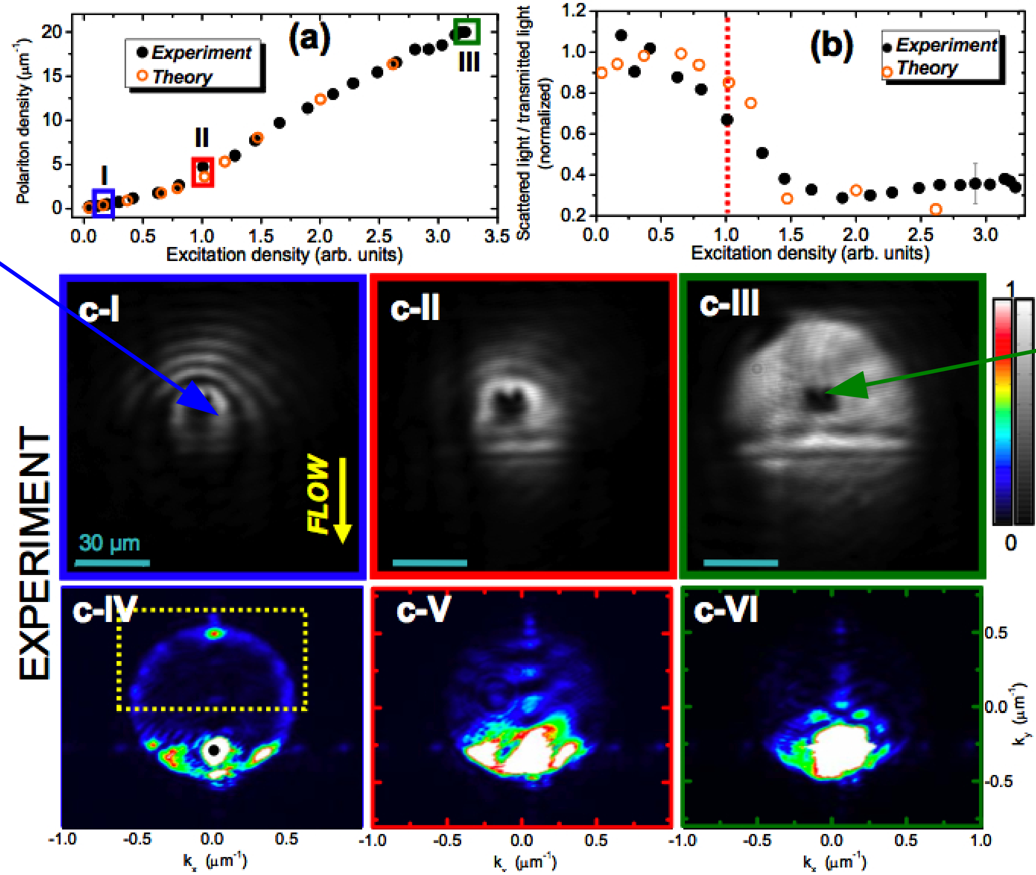
Amo, Lefrère, *et al.*, *Nature Physics*, (in press).

I Carusotto *talk at ICSCE 4 conference (Cambridge, UK, 2008), available at*

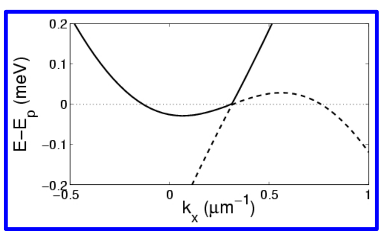
<u>Another missing item:</u> disappearance of mechanical friction on physical object...

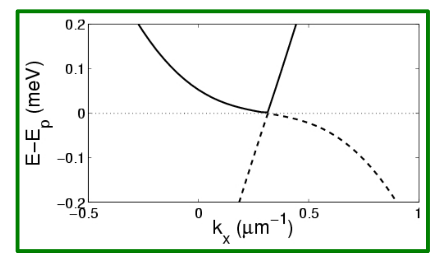
2008 - Superfluid light (under coherent pump)

scattering
on weak defect



superfluid flow





increase polariton density

Expt: A.Amo, J. Lefrère, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)
Theory: IC and C. Ciuti, PRL **93**, 166401 (2004).

Analogies and differences with wake behind swimming duck?

Wake behind swimming duck

Photograph courtesy of Fabrice Neyret (ARTIS-CNRS, France)

Sound in photon BECs: non-equilibrium effects

Polariton BEC regime under incoherent pump (i.e. polariton lasing)

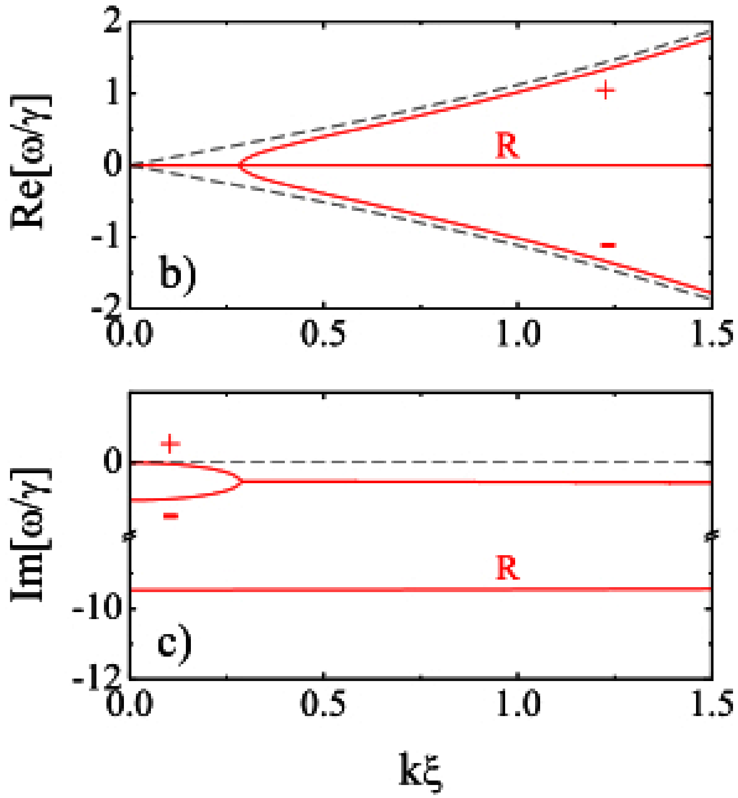
Linearize GPE around steady state

- Reservoir R mode at $-i\,\gamma_R$
- Condensate density and phase modes at:

$$\omega_\pm(k) = -\frac{i\Gamma}{2} \pm \sqrt{[\omega_{Bog}(k)]^2 - \frac{\Gamma^2}{4}}$$

with:

$$\omega_{Bog}(k) = \sqrt{\frac{\hbar k^2}{2m_{LP}}\left(\frac{\hbar k^2}{2m_{LP}} + 2\,\mu\right)}$$



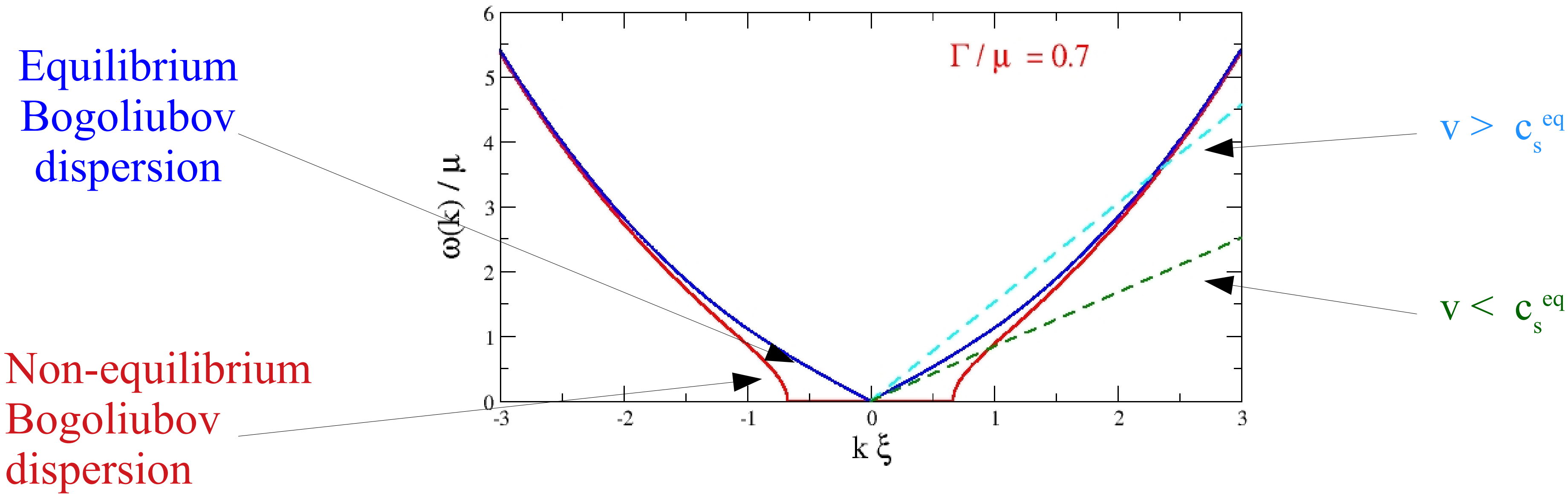
- density (-) and phase (+) oscillations decoupled around k=0
- Goldstone phase (+) mode is diffusive

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, Phys. Rev. Lett. **99**, 140402 (2007)

Similar results in: M. H. Szymanska, J. Keeling, P. B. Littlewood, PRL **96**, 230602 (2006)

Not yet experimentally observed

Consequences on superfluidity



Long-range coherence → metastability of supercurrents (mode stability of ring lasers)

Interaction with defect: naïf Landau argument

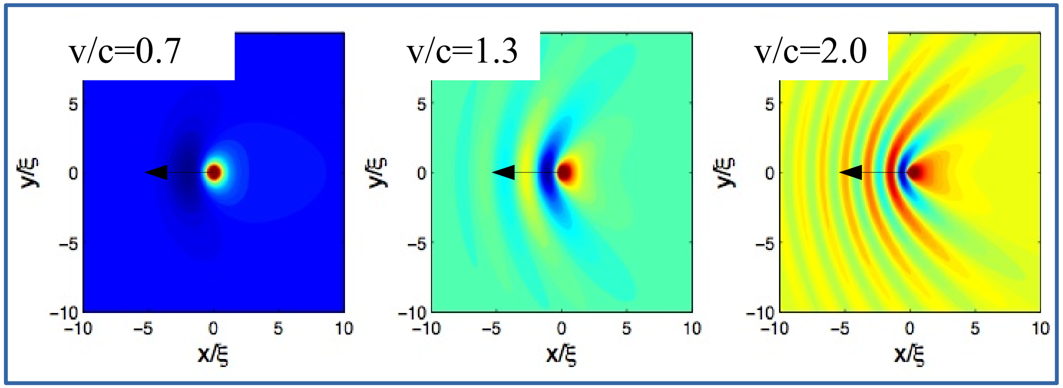
- Landau critical velocity $v_L = \min_k[\omega(k) / k] = 0$ at non-equilibrium BEC
- Any moving defect expected to emit phonons

M. Wouters and IC, *Excitations in a non-equilibrium polariton BEC*, Phys. Rev. Lett. **99**, 140402 (2007)
Similar results in: M. H. Szymanska, J. Keeling, P. B. Littlewood, PRL **96**, 230602 (2006)

But nature is always richer than expected...

Steady-state → well defined ω
Defect → k not a good quantum number

(Complex) k vs.(real) ω dispersion

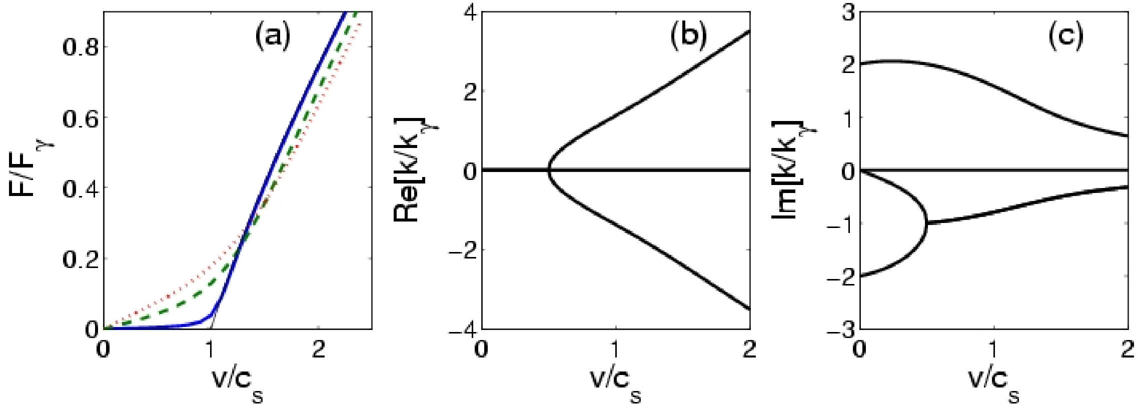


Low v :

- emitted $k_{\parallel}$ purely imaginary
- no real propagating phonons
- perturbation localized around defect

Critical velocity $v_c < c$:

- corresponds to bifurcation point
- decreases with Γ / μ



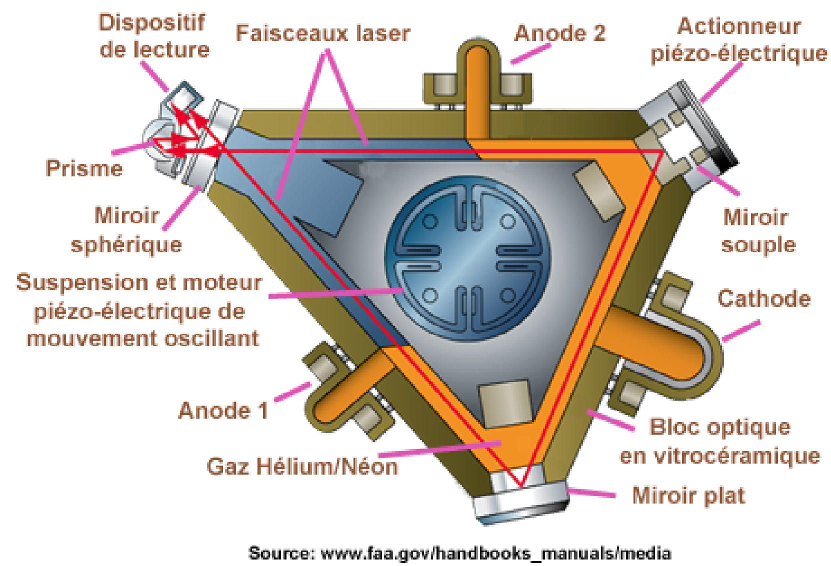
High v:

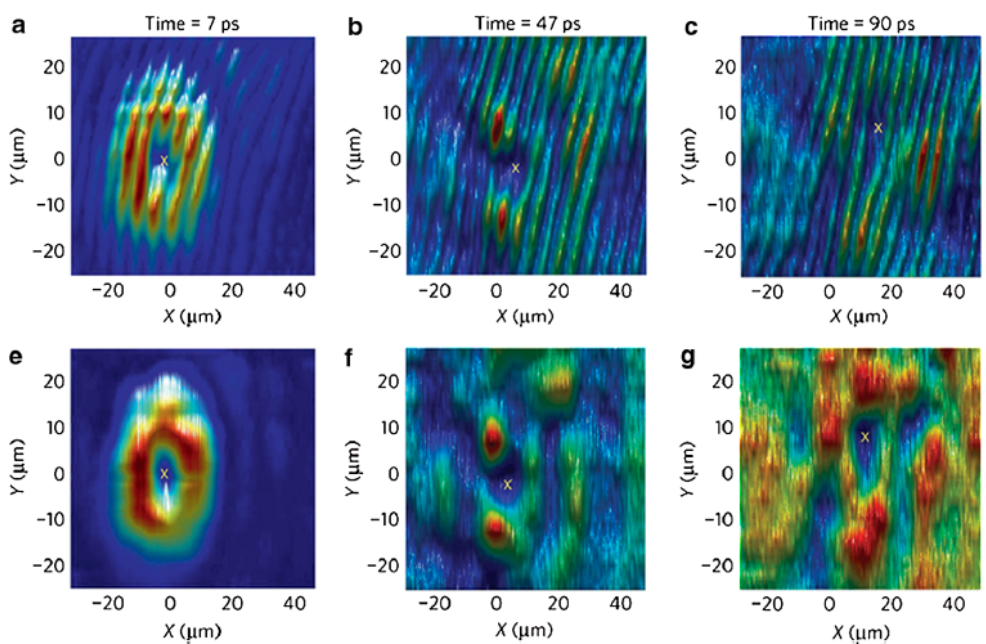
- emitted propagating phonons:
  → Cerenkov cone
  → parabolic precursors
- spatial damping of Cerenkov cone

M. Wouters, IC, *Superfluidity and Critical Velocities in Nonequilibrium BECs*, PRL **105**, 020602 (2010).

Metastability of supercurrents

Gyrolaser





Polariton condensate

Sanvitto et al., Nat. Phys. 2010

If you wish to know more...

BEC

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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I. Carusotto, C. Ciuti, Rev. Mod. Phys. **85**, 299 (2013)

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https://doi.org/10.1038/s41567-020-0815-y

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Photonic materials in circuit quantum electrodynamics

Iacopo Carusotto[1], Andrew A. Houck[2], Alicia J. Kollár[3,4], Pedram Roushan[5], David I. Schuster[6,7] and Jonathan Simon[6,7]

Review article on Nature Physics (2020)

REVIEWS OF MODERN PHYSICS, VOLUME 91

Topological photonics

Review article arXiv:1802.04173 by Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC, RMP **91**, 015006 (2019)

Non-equilibrium Bose–Einstein condensation in photonic systems

Jacqueline Bloch[1], Iacopo Carusotto[2] and Michiel Wouters[3]

Review article on Nat. Rev. Phys. (2022)

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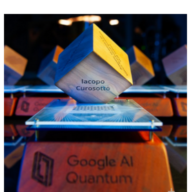

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