





On the theory of Quantum Fluids of Light Part 3/3

Iacopo Carusotto

INO-CNR BEC Center and Università di Trento, Italy







Horizon 2020 European Union funding for Research & Innovation





Part 4:

<u>The future:</u>

Strongly interacting fluids of light

Mott insulators & Fractional Quantum Hall liquids

<u>Photon blockade</u>

Driven-dissipative Bose-Hubbard model:

$$H_0 = \sum_i \hbar \omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar rac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i F_i(t) \hat{b}_i + h.c.$$

- Array of single-mode cavities at ω_0 , tunnel coupling J, losses γ
- Polariton interactions: on-site interaction U due to optical nonlinearity
- If $U >> \gamma \& J$, coherent pump resonant with $0 \rightarrow 1$, but not with $1 \rightarrow 2$.

Photon blockade \rightarrow <u>Effectively impenetrable photons</u> Opposite regime than non-interacting photons of Maxwell's eqs.

Single-cavity blockade observed in many platforms since the 2000s, present challenge \rightarrow scale up to many-cavity geometry



IC-Ciuti, Quantum Fluids of Light, RMP 85, 299 (2013)



Fluid of spin excitations in lattice of Rydberg atoms. (Broways, Lukin,...)



<u>1D array of coupled cavities</u>

- generalized Bose-Hubbard model
- Polariton interactions: strong on-site repulsion U
- Tunneling: Josephson coupling J
- Incident laser: coherent external driving F_p
 - > monochromatic at ω_{p} , plane wave $k_{p}=0$



- Weak losses $\gamma \ll J$, U crucial to determine non-equilibrium steady-state
- Light collected in reflection and/or transmission geometry
 - far-field emission: k-selection from orbital angular momentum
 - > near-field emission: spatial resolution
- Strongly interacting regime J << U

<u>Impenetrable bosons (U=∞):</u> <u>Girardeau's Bose-Fermi mapping</u>

Many-body, bosonic wavefunction written in terms of non-interacting Fermions $\psi(x_1, x_2, ...) = \epsilon[\sigma(x_1, x_2, ...)] \psi_F(x_1, x_2, ...)$

 \checkmark density and density fluctuations preserved in Bose-Fermi mapping

× coherence function $g^{(l)}(x,x')$ and momentum distribution not preserved

Many-body states labeled by momenta q of occupied Fermi orbitals $|q_{I'}q_{2'}q_{3'}...>$ Periodic boundary cond.: Fermi orbitals satisfy PBC/anti-PBC for odd/even N Energy (and momentum) preserved by mapping $E = \sum_{i} \frac{\hbar^2 q_i^2}{2m}$

Example: two-particle states $\psi(i_1, i_2) = \frac{1}{\sqrt{2}L} \sin\left|\frac{(2n+1)\pi}{L} |i_1 - i_2|\right|$

M. D. Girardeau, Phys. Rev. 1, 516 (1960)

<u>Impenetrable "fermionized" photons in 1D necklaces</u>

Many-body eigenstates of Tonks-Girardeau gas of impenetrable photons

Coherent pump selectively addresses specific many-body states



Transmission spectrum as a function pump frequency for fixed pump intensity:

- each peak corresponds to a Tonks-Girardeau many-body state $|q_1,q_2,q_3...>$
- q_i quantized according to PBC/anti-PBC depending on N=odd/even
- U/J >> 1: efficient photon blockade, impenetrable photons.

N-particle state excited by N photon transition:

- Plane wave pump with $k_p = 0$: selects states of total momentum P=0
- Monochromatic pump at ω_p : resonantly excites states of many-body energy E such that $\omega_p = E / N$

IC, D. Gerace, H. E. Türeci, S. De Liberato, C. Ciuti, A. Imamoglu, PRL **103**, 033601 (2009) See also related work D. E. Chang et al, Nature Physics (2008)

State tomography from emission statistics





- Finite U/J, pump laser tuned on two-photon resonance
- intensity correlation between the emission from cavities i_1, i_2
- at large U/ γ , larger probability of having N=0 or 2 photons than N=1
 - > low U<<J: bunched emission for all pairs of i_1, i_2
 - > large U>>J: antibunched emission from a single site positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

<u> Photon blockade + synthetic gauge field = FQHE for light</u>

Bose-Hubbard model:

$$H_0 = \sum_i \hbar \omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j \underbrace{e^{i\varphi_{ij}}}_{\bullet} + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

with usual coherent drive and dissipation \rightarrow look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$egin{aligned} \psi_l(z_1,...,z_N) &= \mathcal{N}_L F_{ ext{CM}}^{(l)}(Z) e^{-\pi lpha \sum_i y_i^2} \ & imes \ \prod_{i < j}^N \left(artheta \left[rac{1}{2} \ rac{1}{2}
ight] \left(rac{z_i - z_j}{L} \Big| i
ight)
ight)^2 \end{aligned}$$

• no need for adiabatic following, etc....





Continuous space FQH physics

Single cylindrical cavity. No need for cavity array



same form
Coriolis
$$F_c = -2m\Omega \times v$$

Lorentz $F_L = e \vee x B$

Photon gas injected by Laguerre-Gauss pump with finite orbital angular momentum Strong repuls. interact., e.g. layer of Rydberg atoms Resonant peak in transmission due to Laughlin state: $\psi(z_1,...,z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i \le i} (z_i - z_j)^2$



Experiment @ Chicago

A far smarter design

Non-planar ring cavity:

- Parallel transport \rightarrow synthetic B
- Landau levels for photons observed

Crucial advantages:

- Narrow frequency range relevant
- Integrated with Rydberg-EIT reinforced nonlinearities

Polariton blockade on lowest (0,0) mode

• Equivalent to $\Delta_{\text{Laughlin}} > \gamma$

Easiest strategy for Laughlin

- Coherent pumping \rightarrow multi-photon peaks to few-body states
- Laughlin state \rightarrow quantum correlations between orbital modes (Umucalilar-Wouters-IC, PRA 2014)

Breaking news: 2-photon Laughlin state realized (Clark et al., Nature 2020)

> Figures from J. Simon's group @ U. Chicago Schine et al., Nature 2016; Jia et al. 1705.07475







Experiment @ Chicago (II)

PHYSICAL REVIEW A 89, 023803 (2014)

Probing few-particle Laughlin states of photons via correlation measurements

R. O. Umucalılar^{*} and M. Wouters TQC, Universiteit Antwerpen, Universiteitsplein 1, B-2610 Antwerpen, Belgium

I. Carusotto INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy (Received 29 November 2013; published 5 February 2014)

We propose methods to create and observe Laughlin-like states of photons in a strongly nonlinear optical cavity. Such states of strongly interacting photons can be prepared by pumping the cavity with a Laguerre-Gauss beam, which has a well-defined orbital angular momentum per photon. The Laughlin-like states appear as sharp resonances in the particle-number-resolved transmission spectrum. Power spectrum and second-order correlation function measurements yield unambiguous signatures of these few-particle strongly correlated states.





Quantum optical tricks to highlight generation of two-photon Laughlin state

<u>Challenge:</u> scale up to larger number of particles

Coherent pump scheme scales very bad with N for topological states

L. W. Clark, N. Schine, C. Baum, N. Jia, J. Simon, Observation of Laughlin states made of light, Nature 2020

<u>Conservative dynamics in circuit-QED experiment:</u> interplay of strong interactions & synthetic magnetic field

Ring-shaped array of qubits in a superconductor-based circuit-QED platform

- Transmon qubit: two-level system
 → Impenetrable microwave photons
- Time-modulation of couplings
 → synthetic gauge field
- > Independently initialize sites
- Follow unitary evolution until bosons lost (microwave photons → long lifetime)
- Monitor site occupation in time



"Many"-body effect:

two-photon state → opposite rotation compared to one-photon state (similar to cold-atom experiment in Greiner's lab: Tai et al., Nature 2017)

How to access larger particle numbers

Coherent pump only able to selectively excite few-photon states

 \rightarrow Frequency-dependent incoherent pumping, e.g. collection of inverted emitters

- Lorentzian emission line around ω_{at} sophisticated schemes \rightarrow other spectral shapes
- Emission only active if many-body transition is near resonance
- Injects photons until band is full (MI) or many-body gap develops (FQH)
- Many-body gap blocks excitation of higher states







Umucalilar-IC, PRA 2017 Lebreuilly, Biella et al., PRA 2017

<u>Mott insulators of light</u>

- non-Markovian master equation: frequency-dependent emission → rescaled jump operators
- driven-dissipative steady state stabilizes strongly correlated many-body states e.g. Mott-insulator, FQH...
- resembles low-T equilibrium (but interesting deviations in some cases)
- (in principle) no restriction to small N_{ph} only requirement → many-body energy gap



First expt: Ma et al. Nature 2019

$$\bar{\mathcal{L}}_{em}(\rho_{ph}) = \frac{\Gamma_{em}}{2} \sum_{i=1}^{k} \left[2\bar{a}_{i}^{\dagger}\rho_{ph}\bar{a}_{i} - \bar{a}_{i}\bar{a}_{i}^{\dagger}\rho_{ph} - \rho_{ph}\bar{a}_{i}\bar{a}_{i}^{\dagger} \right]$$
$$\langle f' | \bar{a}_{i}^{\dagger} | f \rangle = \frac{\Gamma_{pump}/2}{\sqrt{(\omega_{at} - \omega_{f',f})^{2} + (\Gamma_{pump}/2)^{2}}} \langle f' | a_{i}^{\dagger} | f \rangle$$



Lebreuilly, Biella et al., 1704.01106 & 1704.08978 (published on PRA, 2017)

Related work in Kapit, Hafezi, Simon, PRX 2014

What about large FQH fluids?

Coherent pump:

- Able to selectively generate few-body states
- Limited by (exponentially) decreasing matrix element for larger systems

Frequency-dependent incoherent pump:

- Interactions \rightarrow many-body gap Δ
- Edge excitations not gapped. Hard-wall confinement gives small δ
- Non-Markovianity blocks excitation to higher states

Calculations only possible for small systems:

- Large overlap with Laughlin states
- Excitations localized mostly on edge

Open question: what are ultimate limitations of this pumping method?

R. O. Umucalilar, IC, Generation and spectroscopic signatures of a fractional quantum Hall liquid of photons in an incoherently pumped optical cavity, PRA 2017. R. O Umucalilar, J. Simon, IC, Autonomous stabilization of photonic Laughlin states through angular momentum potentials, PRA 2022 Kurilovich et al., Stabilizing the Laughlin state of light: dynamics of hole fractionalization, arXiv:2111.01157





<u>A long-term objective:</u> <u>Probing anyonic statistics</u> <u>of quasi-holes</u>

What is an anyon?

<u>Quantum mechanics of anyons (I) – single particle</u>

Laughlin wavefunction of Fractional Quantum Hall:

- quasi-holes \rightarrow no E_{kin} , no independent life
- dressed by heavy impurity \rightarrow anyonic molecule
- full-fledged mechanical degree of freedom

Born-Oppenheimer approx:

- Heavy impurity \rightarrow slow Degree of Freedom
- Light FQH particles \rightarrow fast DoF

$$H_{\text{eff}} = \frac{\left[-i\nabla_{\mathbf{R}} - (Q - \nu q)\,\mathbf{A}(\mathbf{R})\right]^2}{2\mathcal{M}}$$

- Mass $M \rightarrow M$ (impurity) + QH dragging effect
- Impurity & FQH particles feel (Synth-)B, so synth-Charge $\rightarrow Q$ (impurity) – v q (QH)

Cyclotron orbit \rightarrow fractional charge and BO mass correction

A. Muñoz de las Heras, E. Macaluso, IC, PRX 2020



Quantum mechanics of anyons (II) – two particles

Each particle \rightarrow attached flux

$$\begin{aligned} \mathcal{A}_{j}(\mathbf{R}) &= \mathcal{A}_{q}(\mathbf{R}_{j}) + \mathcal{A}_{\text{stat},j}(\mathbf{R}) \\ &= \frac{\mathcal{B}_{q}}{2} \mathbf{u}_{z} \times \mathbf{R}_{j} + (-1)^{j} \frac{\nu}{R_{\text{rel}}^{2}} \mathbf{u}_{z} \times \mathbf{R}_{\text{rel}} \end{aligned}$$

 $H_{\rm rel} = \frac{\left[\mathbf{P}_{\rm rel} + \mathbf{A}_{\rm rel}(\mathbf{R}_{\rm rel})\right]^2}{2\mathcal{M}_{\rm rel}}$

 $+ V_{\rm ii}(R_{\rm rel})$

Relative motion:

- inter-particle potential
- statistical A_{rel} due to attached flux

- fringes in differential cross section
- fringe position depends on attached flux, i.e. measure fractional statistics
- Scheme works with polar molecules (heavy + long-range interactions) in atoms (light FQH gas)
- > What about Rydberg polaritons?



A. Muñoz de las Heras, E. Macaluso, IC, PRX 2020



<u>Optical signatures of the anyonic braiding phase</u>



Anyonic statistics of quasi-holes: many-body Berry phase ϕ_{Br} when positions swapped during braiding

In a photonic system:

- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
 - \rightarrow create quasi-hole excitation in quantum Hall liquid
 - \rightarrow position of holes adiabatically braided in space
- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period T_{rot} along circle

 $\phi_{\rm Br} \equiv (\Delta \omega_{\rm oo} - \Delta \omega_{\rm o}) T_{\rm rot} [2 \pi]$

How to measure ϕ_{Br} without an actual braiding?





R. O. Umucalilar and IC, Anyonic braiding phases in a rotating strongly correlated photon gas, arXiv:1210.3070

<u>A simpler strategy: observing anyonic statistics in ToF measurements</u>



Braiding phase \rightarrow Berry phase when two quasi-holes are moved around each other $\varphi_{\rm B}(R) = i \oint_R \langle \Psi(\theta) | \partial_\theta | \Psi(\theta) \rangle d\theta$

Braiding operation can be generated by rotations, so braiding phase related to L_z

$$\varphi_{\rm B}(R) = \frac{1}{\hbar} \oint_R \langle \Psi(\theta) | L_z | \Psi(\theta) \rangle d\theta = \frac{2\pi}{\hbar} \langle L_z \rangle$$

Self-similar expansion of lowest-Landau-levels $\rightarrow L_z$ can be measured in time-of-flight via size of the expanding cloud

$$\langle r^2 \rangle_{\rm tof} = \frac{1}{N} \left(\frac{\hbar t}{\sqrt{2}M l_B} \right)^2 \left(\frac{\langle L_z \rangle}{\hbar} + N \right) = \left(\frac{\hbar t}{2M l_B^2} \right)^2 \langle r^2 \rangle$$

Can be applied to both cold atoms or to fluids of light looking at far-field emission pattern Difficulty \rightarrow small angular momentum difference of QH compared to total L_z

Umucalilar, Macaluso et al., Observing anyonic statistics via time-of-flight measurements, PRL (2018)

Quasi-Hole structure vs. anyon statistics (I)

• Compare (two) single quasi-holes and overlapping pair of quasi-holes:

$$\frac{\varphi_{\rm br}}{2\pi} = \frac{1}{\hbar} \left[\langle \hat{L}_z \rangle_{|\eta_1| = |\eta_2|} - \langle \hat{L}_z \rangle_{\eta_1 = \eta_2} \right].$$

• Relates to difference of density profiles:

$$\frac{\varphi_{\rm br}}{2\pi} = \frac{N}{2l_B^2} \left[\langle r^2 \rangle_{|\eta_1| = |\eta_2|} - \langle r^2 \rangle_{\eta_1 = \eta_2} \right],$$

- Incompressibility \rightarrow external region unaffected
- Statistics inferred from local density difference around QH core, i.e. variance of density depletion
- Insensitive to spurious excitation of (ungapped) edge states
- Proposal realizable in Chicago's twisted cavity set-up
- Numerical calculation using Moore-Read wavefunction allows to distinguish fusion channels of even/odd total particle number





E. Macaluso, T. Comparin, L. Mazza, IC, Fusion channels of non-Abelian anyons from angular-momentum and density-profile measurements, PRL 2019

Quasi-Hole structure vs. anyon statistics (II)



Discrete lattice model \rightarrow Harper-Hofstadter-Bose-Hubbard

Ground state using Tree-Tensor-Network ansatz

- experimentally realistic "large" system
- open boundary conditions with harmonic trap
- repulsive potentials to pin quasi-holes

Apply discretized version of braiding phase formula

$$\frac{\varphi_{\rm br}}{2\pi} = \frac{N}{2l_B^2} \left[\langle r^2 \rangle_{|\eta_1| = |\eta_2|} - \langle r^2 \rangle_{\eta_1 = \eta_2} \right],$$

to physical ground state wavefunction

 \rightarrow Accurate reconstruction of anyonic statistics

→ Experiment accessible in state-of-the-art circuit-QED systems

E. Macaluso *et al.*, *Charge and statistics of lattice quasiholes from density measurements: a Tree Tensor Network study*, Phys. Rev. Research (2020)



On-going work: Linear and nonlinear edge dynamics of FQH clouds Chiral Luttinger liquids and beyond

- A. Nardin, IC, Non-linear edge dynamics of an integer quantum Hall fluid, Europhys. Lett. 132, 10002 (2020).
- A. Nardin, IC, *Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm*, arXiv:2203.02539
- Z. Bacciconi, *Fractional quantum Hall edge dynamics from a quantum optics perspective*, MSc thesis at UniTrento (2021); arXiv:2111.05858

Response of trapped FQH cloud to external potential (I)



Trapping potential $V_{conf}(r) = \lambda r^{\delta}$

Ab initio ED calculations by MC evaluation of matrix elements via Metropolis (upto ~50 particles)

Time-dependent perturbation $U(r,\theta;t)$:

• generates oscillatory perturbation on edge

<u>Weak perturbation \rightarrow chiral Luttinger liquid behaviour</u>

- linear response proportional to filling $\boldsymbol{\nu}$
- related to quantized transverse conductivity of FQH
- matches with numerics for long wavelength excitations...



A. Nardin, IC, Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm, arXiv:2203.02539

Response of trapped FQH cloud to external potential (II)



Linear response matches χ LL...but much more physics hidden in edge perturbation $\sigma(z,t)$:

- free oscillation frequency shift $\sim k^3 \rightarrow$ group velocity dispersion
- nonlinear effects \rightarrow frequency shift proportional to amplitude σ (due to radially increasing trapping force)



 $t \ (\times 10^{-3})$

A. Nardin, IC, Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm, arXiv:2203.02539

Response of trapped FQH cloud to external potential (III)



Broadening associated to damping captured by quantum- χ LL

$$\hat{H}_{\chi \text{LL}}^{NL} = \int d\zeta \left[\frac{\pi v_0}{\nu} \, \hat{\sigma}^2 \left(\frac{\pi \beta_m \tilde{c}_0}{\nu} \left(\frac{\partial \hat{\sigma}}{\partial \zeta} \right)^2 \left(\frac{2\pi^2 \tilde{c}_0}{3\nu^2} \hat{\sigma}^3 \right) + U(\zeta, t) \, \hat{\sigma} \right]$$

with
$$[\hat{\sigma}(\zeta), \hat{\sigma}(\zeta')] = -i \frac{\nu}{2\pi} \partial_{\zeta} \delta(\zeta - \zeta')$$



A. Nardin, IC, Linear and nonlinear edge dynamics of trapped fractional quantum Hall droplets beyond the chiral Luttinger liquid paradigm, arXiv:2203.02539



(Preliminary) Quantum dynamics at constriction

 χ LL dynamics of edge + intrinsic nonlinearity at junction quasi-particle tunneling H_{int}~exp[i ($\varphi(0)-\varphi(1)$)]+ h.c.

 \rightarrow Complex nonlinear dynamics

$$\partial_t \rho_s(x,t) + v \partial_x \rho_s(x,t) = -\Gamma \sin(2\pi q_s(t)) (\delta(x) - \delta(x-l))$$

Truncated-Wigner description of bosonic χ LL: classical noise describes quantum statistics

$$\boldsymbol{p}_{s}(\boldsymbol{x},t=0) = \sum_{k>0} r_{k} \boldsymbol{\alpha}_{k} \boldsymbol{e}^{ikx} + h.c. \qquad \langle \boldsymbol{\rho}_{s}(\boldsymbol{0}^{-},\boldsymbol{0}) \boldsymbol{\rho}_{s}(\boldsymbol{0}^{-},t) \rangle \propto -\frac{1}{mt^{2}}$$

Reproduces many features of FQH edge dynamics:

- Crystallization of high-charge wavepackets
- Gives hints of fractional shot-noise in current
- Better performance for large 1/v: fractional charge v of FQH makes semi-classical picture more accurate



Z. Bacciconi, *Fractional quantum Hall edge dynamics from a quantum optics perspective*, MSc thesis at UniTrento (2021); arXiv:2111.05858 Z. Bacciconi, A. Nardin, IC, in preparation (2022)

<u>If you wish to know more...</u>

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY-MARCH 2013

Quantum fluids of light

lacopo Carusotto*

INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy

Cristiano Ciuti[†]

Laboratoire Matériaux et Phénomènes Quantiques, Université Paris Diderot-Paris 7 et CNRS, Bâtiment Condorcet, 10 rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France

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I. Carusotto, C. Ciuti, Rev. Mod. Phys. 85, 299 (2013)

Iacopo Carusotto¹, Andrew A. Houck⁰², Alicia J. Kollár^{3,4}, Pedram Roushan⁵, David I. Schuster^{6,7} and



Come and visit us in Trento!

REVIEWS OF MODERN PHYSICS. VOLUME 91

Topological photonics

Review article arXiv:1802.04173 by Ozawa, Price, Amo, Goldman, Hafezi, Lu, Rechtsman, Schuster, Simon, Zilberberg, IC, RMP 91, 015006 (2019)

We acknowledge generous financial support from:

electrodynamics

Jonathan Simon^{™6,7} ⊠

nature

physics



Review article on Nature Physics (2020)

PROVINCIA AUTONOMA DI TRENTO

Photonic materials in circuit quantum





Horizon 2020 European Union funding for Research & Innovation

Non-equilibrium Bose–Einstein condensation in photonic systems

Jacqueline Bloch 1^{1} , lacopo Carusotto 1^{2} and Michiel Wouters 3^{3} Review article on Nat. Rev. Phys. (2022)



