Particle non-conservation in multi-mode circuit QED: a probe of many-body localization

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A talk on this recent work:
N. Mehta, R. Kuzmin, CC, V. E. Manucharyan, arXiv:2203.17186
+ a second paper to be submitted

Seminar @QFLM2022
Varenna

The team

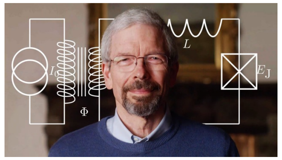
Dr. Nitish Mehta
(now @QC, Inc.)

Dr. Roman Kuzmin
(going to Univ. Wisconsin)

Prof. Vladimir E. Manucharyan
(just moved to EPFL)

N. Mehta, R. Kuzmin, CC, V. E. Manucharyan, arXiv:2203.17186

Connection to previous lectures and keywords



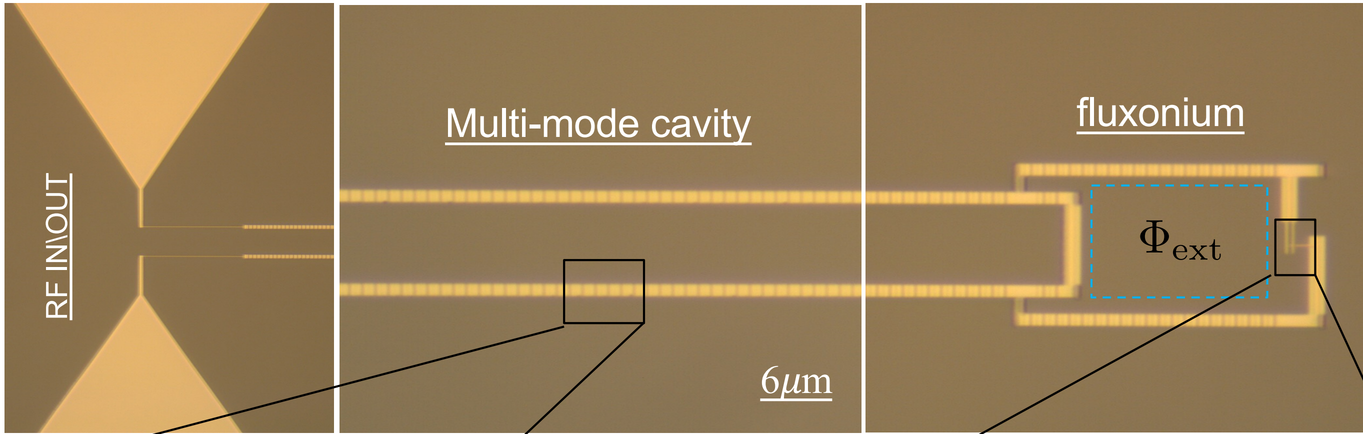
Steve Girvin: Circuit QED, photon manipulation by artificial atoms

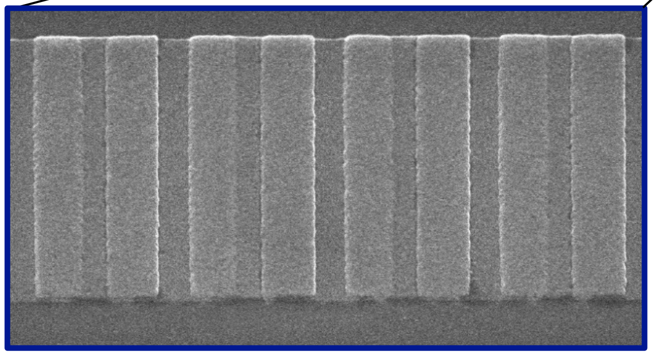
Luigi Lugiato: Down-conversion of photons (three-wave-mixing)

Iacopo Carusotto: many-body states of light

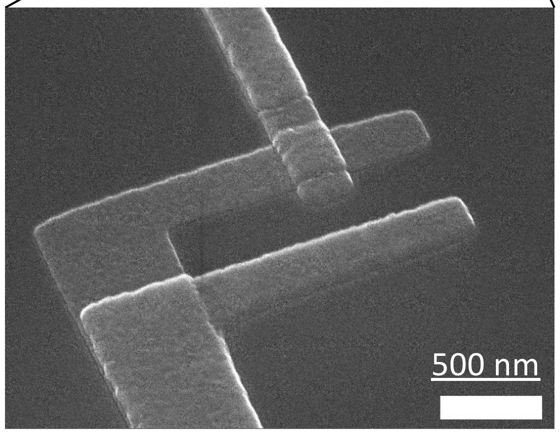
Atac Imamoglu: quantum impurity physics

The system: a multi-mode cavity coupled to a single qubit

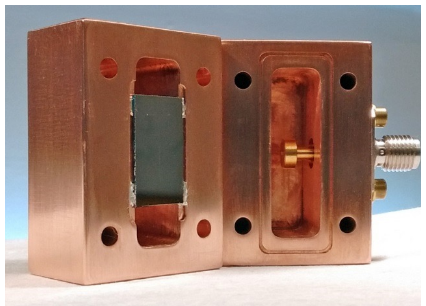


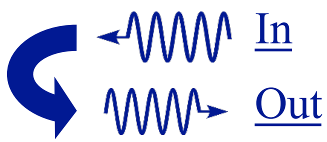


Transmission line (6 mm)
Made of 20 000
large Josephson junctions!
High impedance

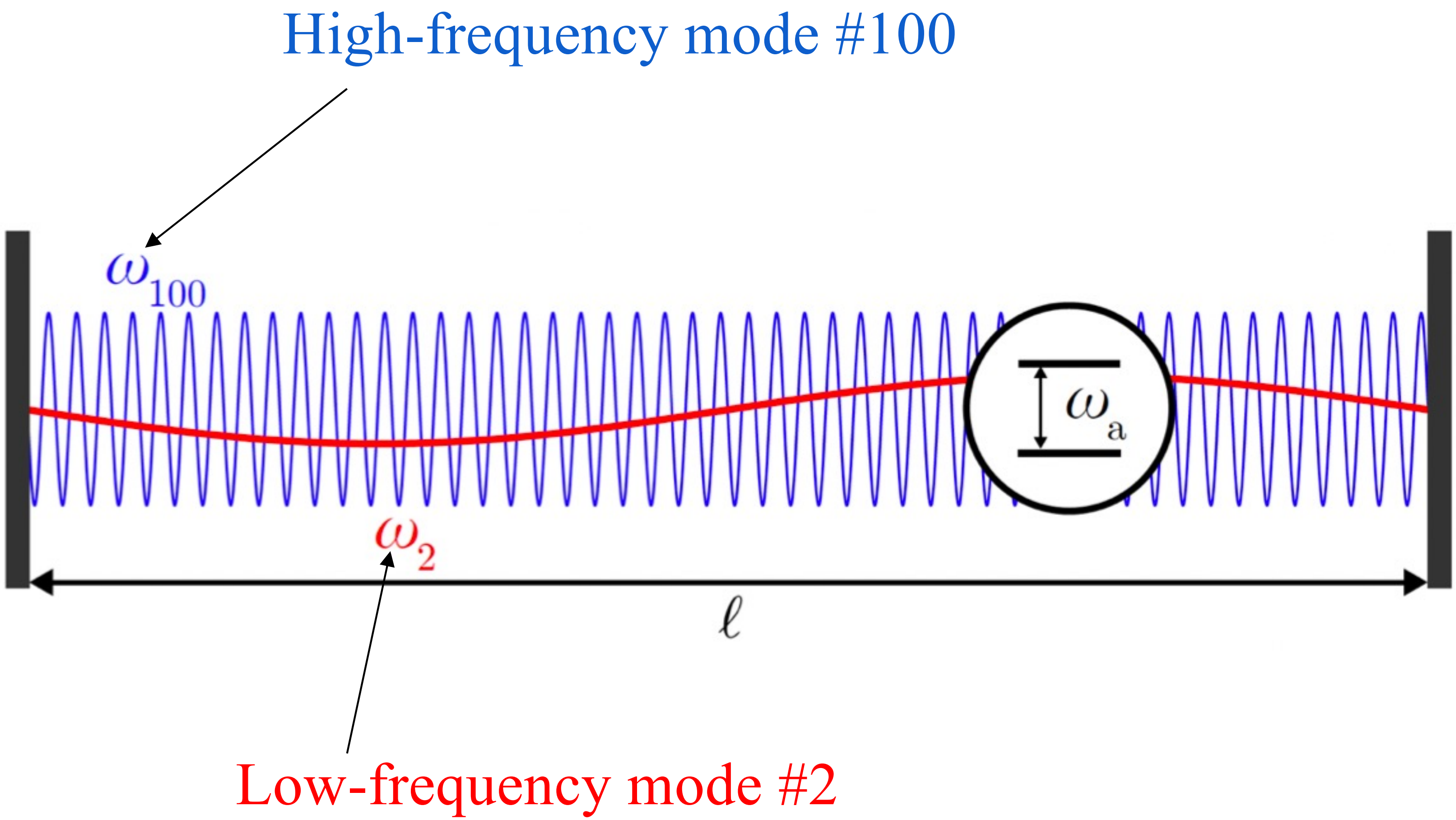


Small Josephson Junction

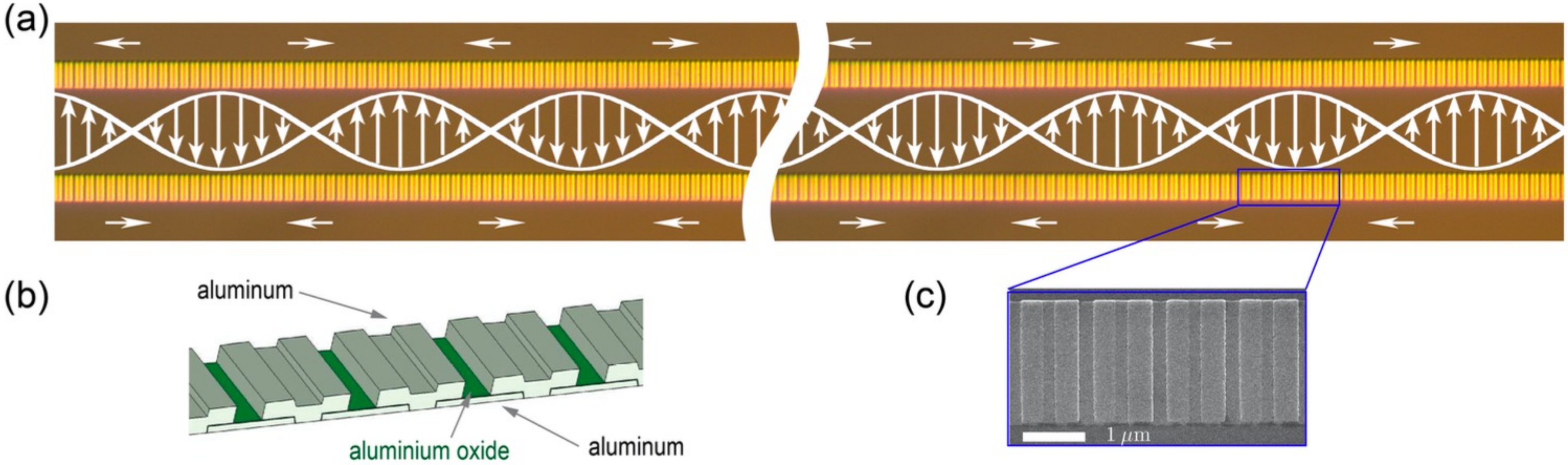




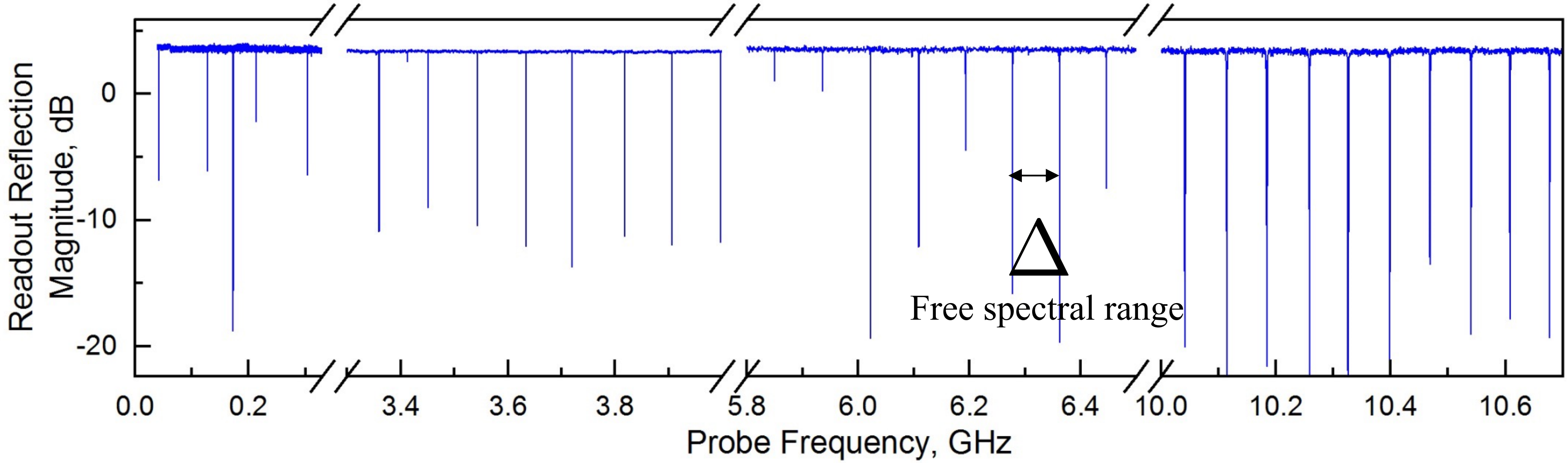
All in all: it's a fancy Fabry-Perot coupled to a single atom

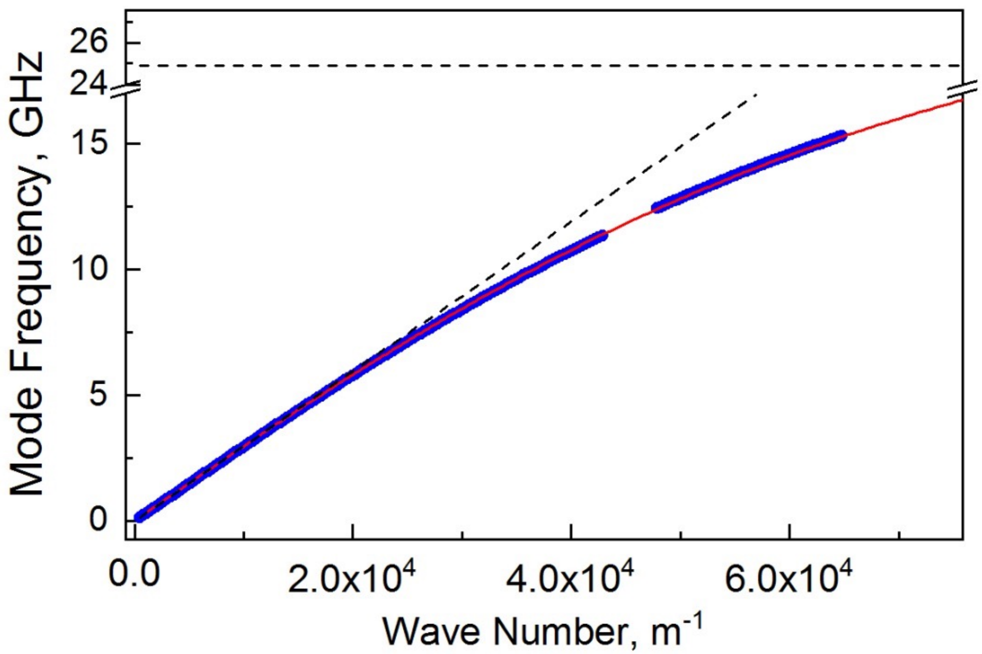


Closer look to transmission line with Josephson Junction arrays



Spectroscopy of the bare transmission line modes





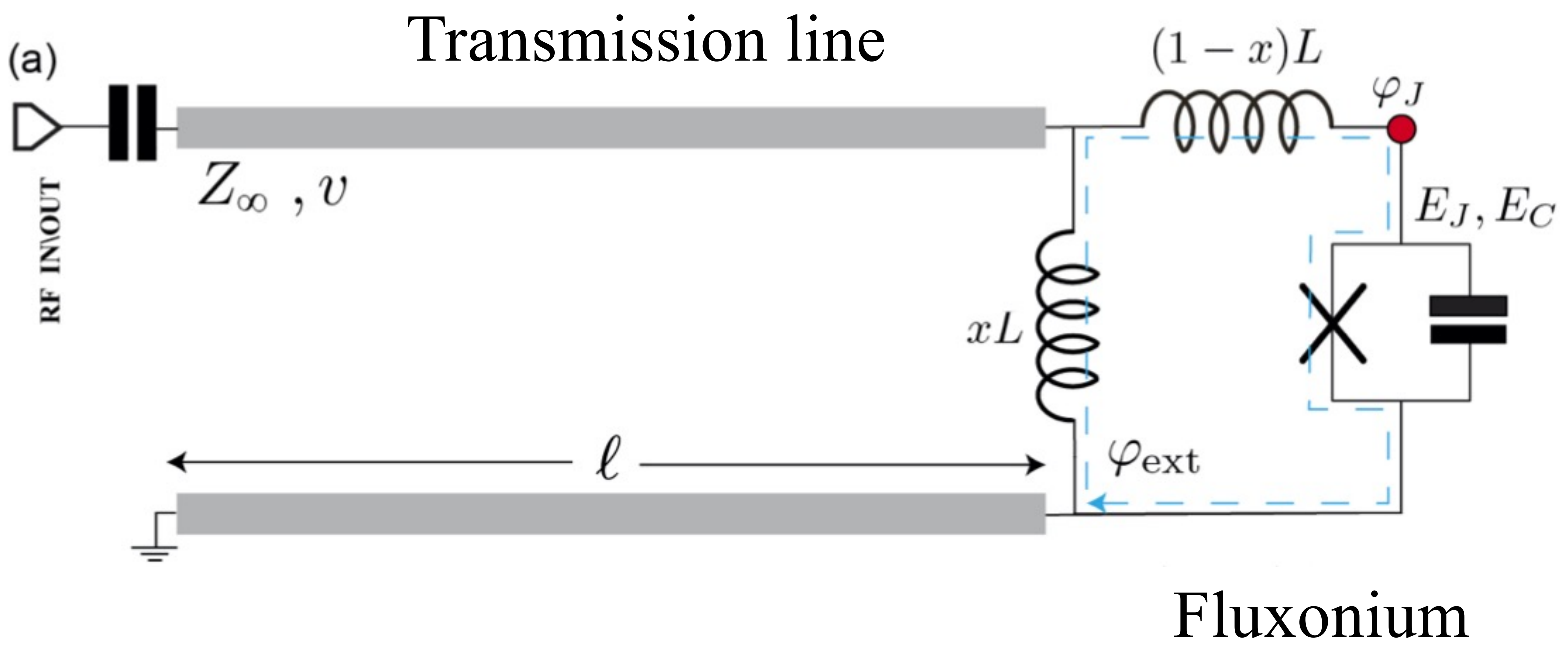
Slow light speed: v = 1/140 c

Linear Dispersion < 10 GHz Band

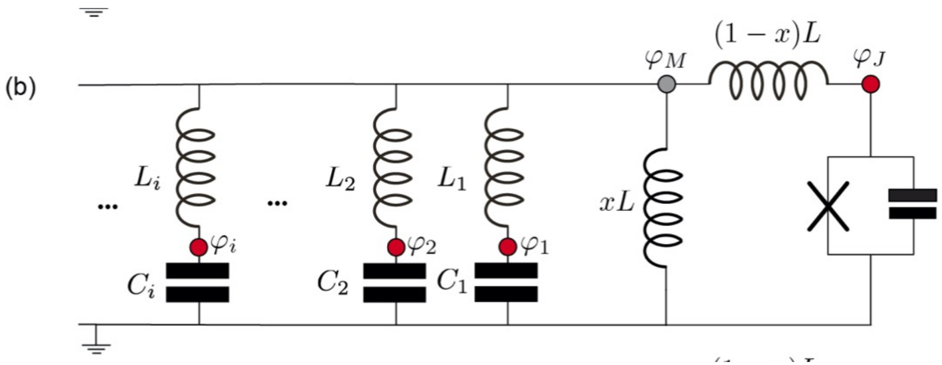
Band edge ~ 25 GHz

Z = 10 K

Equivalent circuit

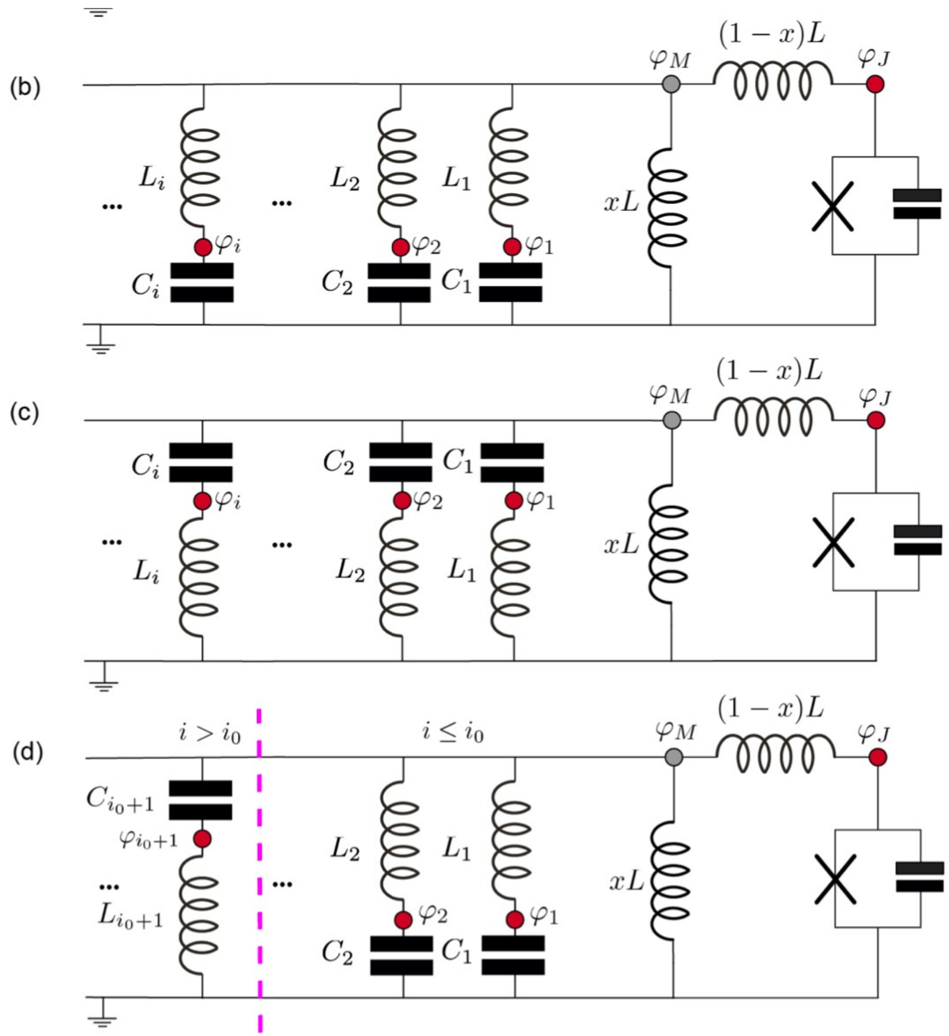


Equivalent lumped element circuits



‘Flux’ gauge

Equivalent lumped element circuits

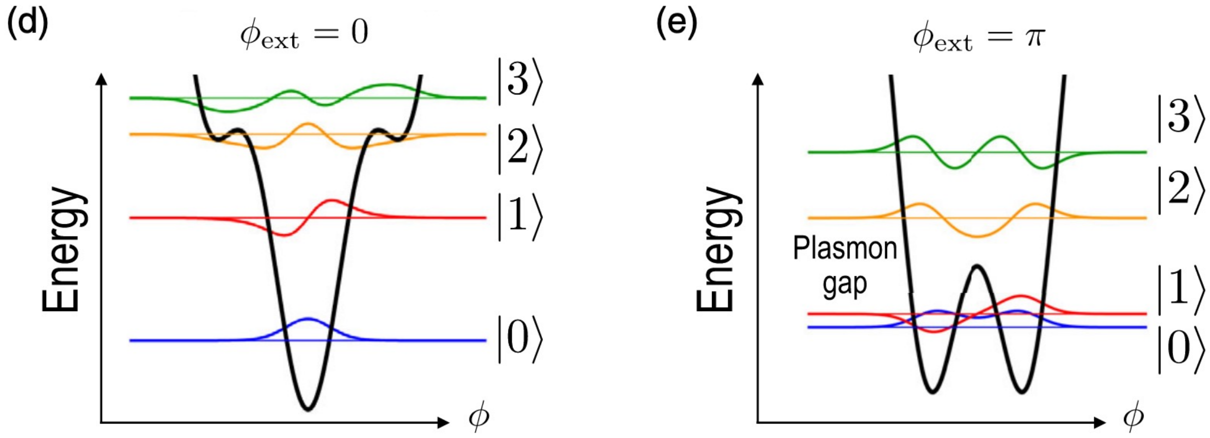


‘Flux’ gauge

‘Charge’ gauge

‘Hybrid’ gauge

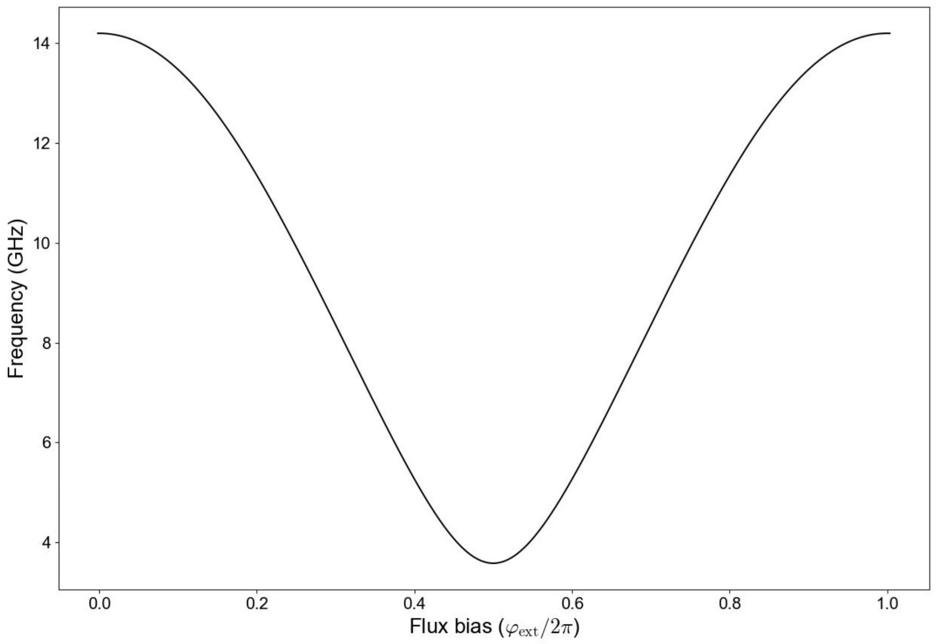
Fluxonium atom and its flux dependence

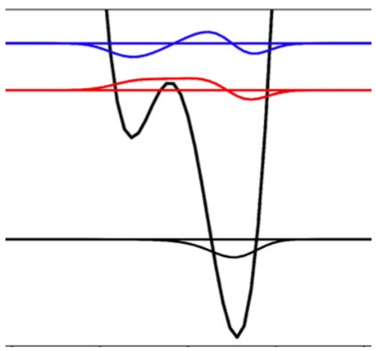


NOTE: potential ASYMMETRIC when

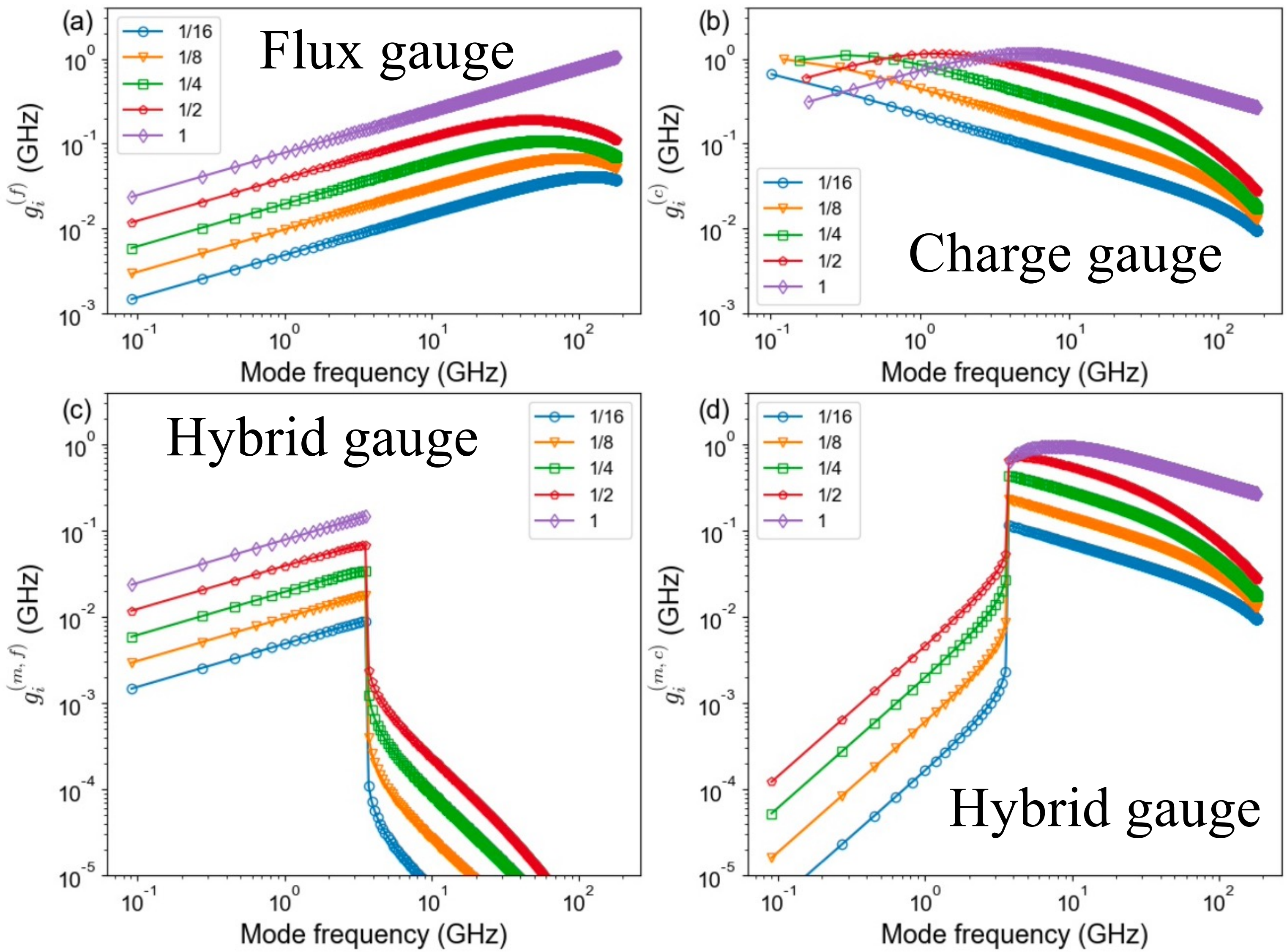
$$\phi_{ext} \neq 0, \pi, 2\pi$$

Frequency dispersion of qubit transition





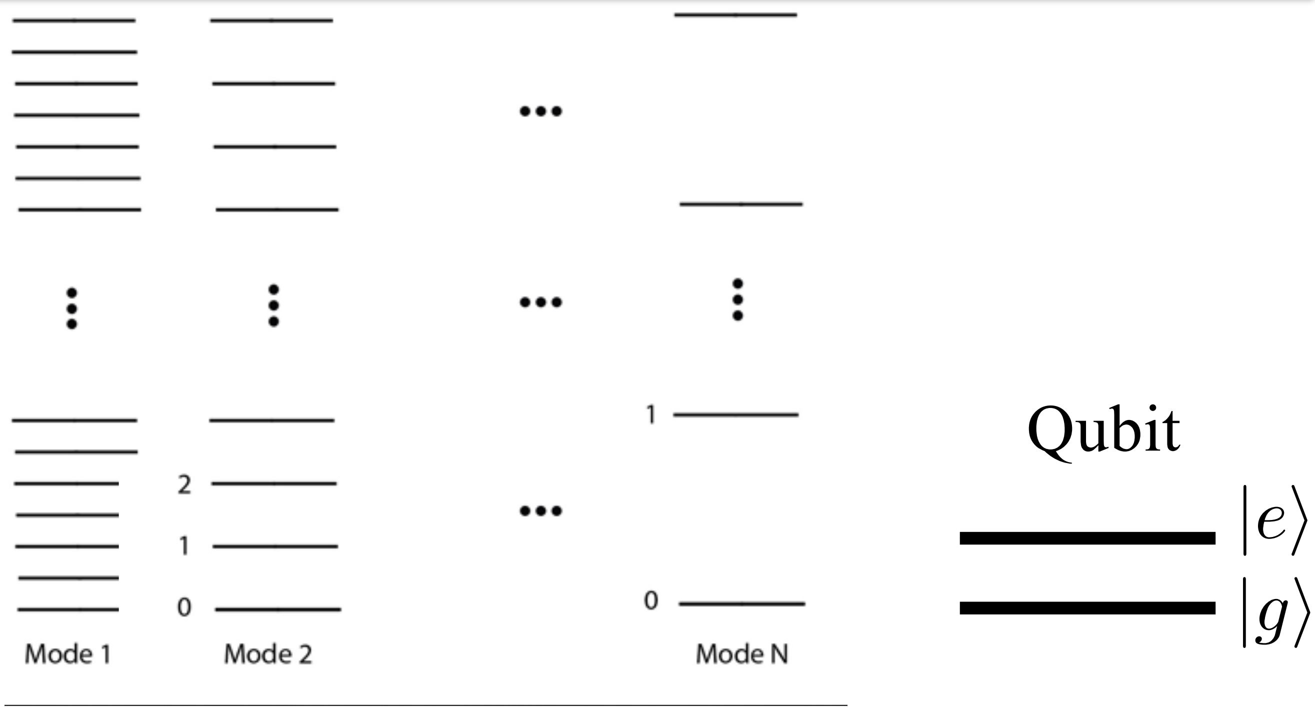
Ultrastrong couplings for different circuit gauges



The manybody problem

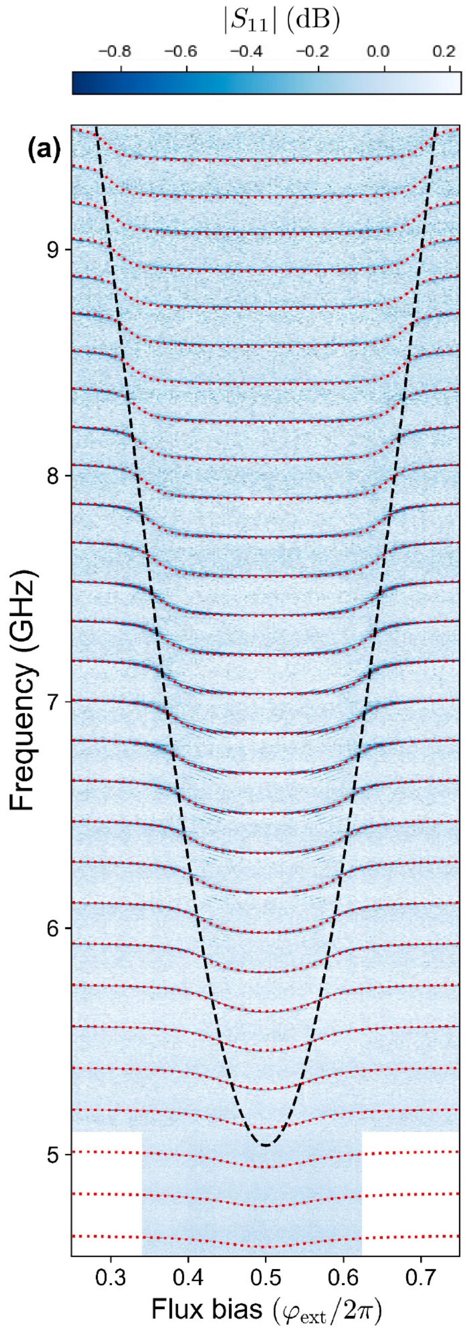
Manymode transmission line = **manybody problem (large Hilbert space)**
Related to the so-called quantum impurity problems

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N \otimes \mathcal{H}_{\text{atom}}$$



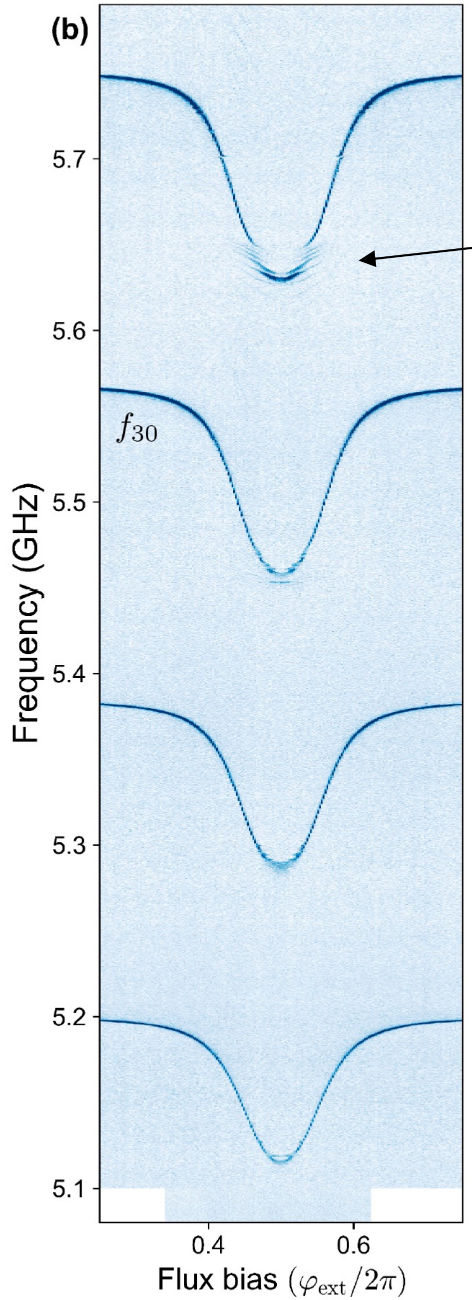
Connection to quantum impurity physics (spin-boson problem, Kondo physics,….)

Linear spectroscopy: a wide-frequency view of the "modes"



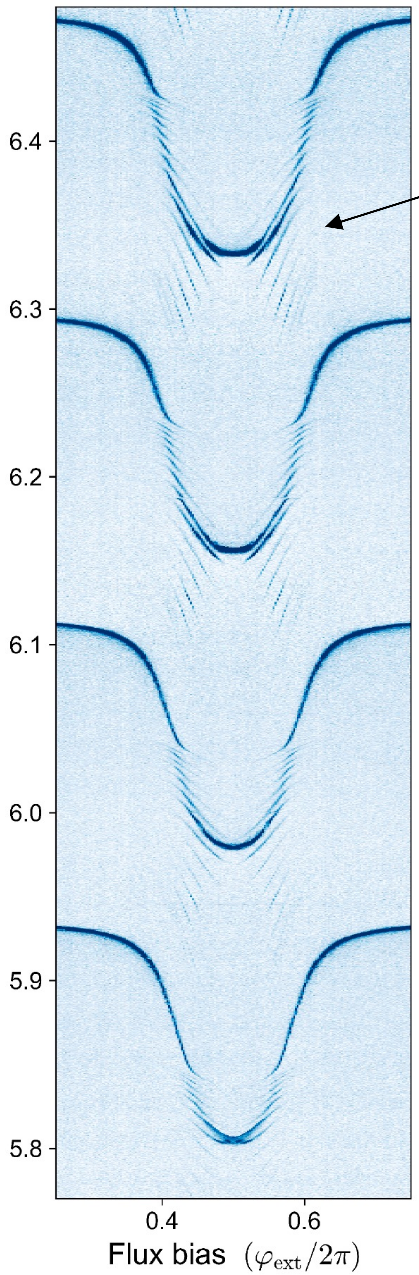
- Mode branches acquire a flux-dependent frequency
- Inflection points follow the qubit dispersion

Looking at low frequency branches



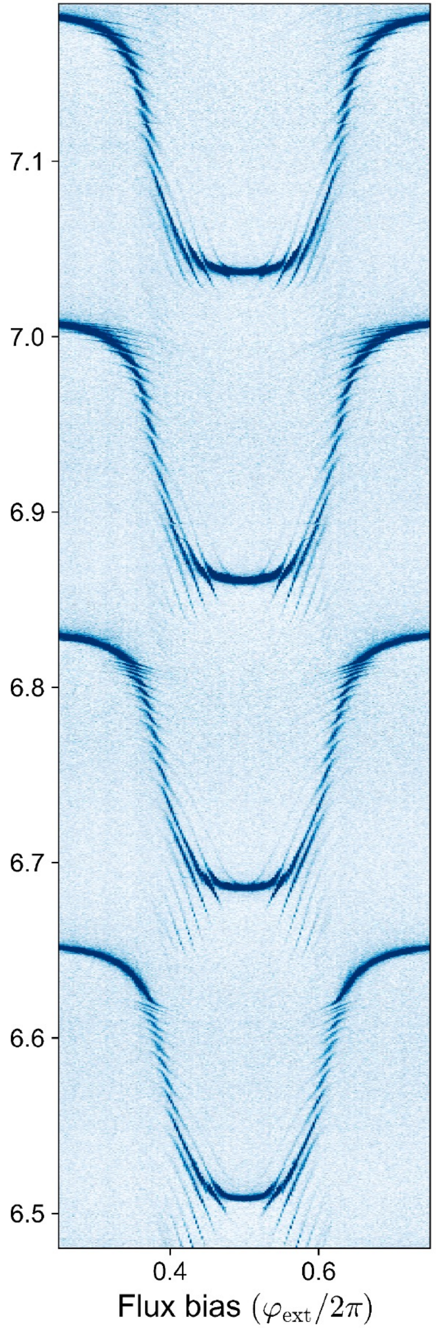
Some fine spectral structure appears

Going up: emerging spectral fine structure

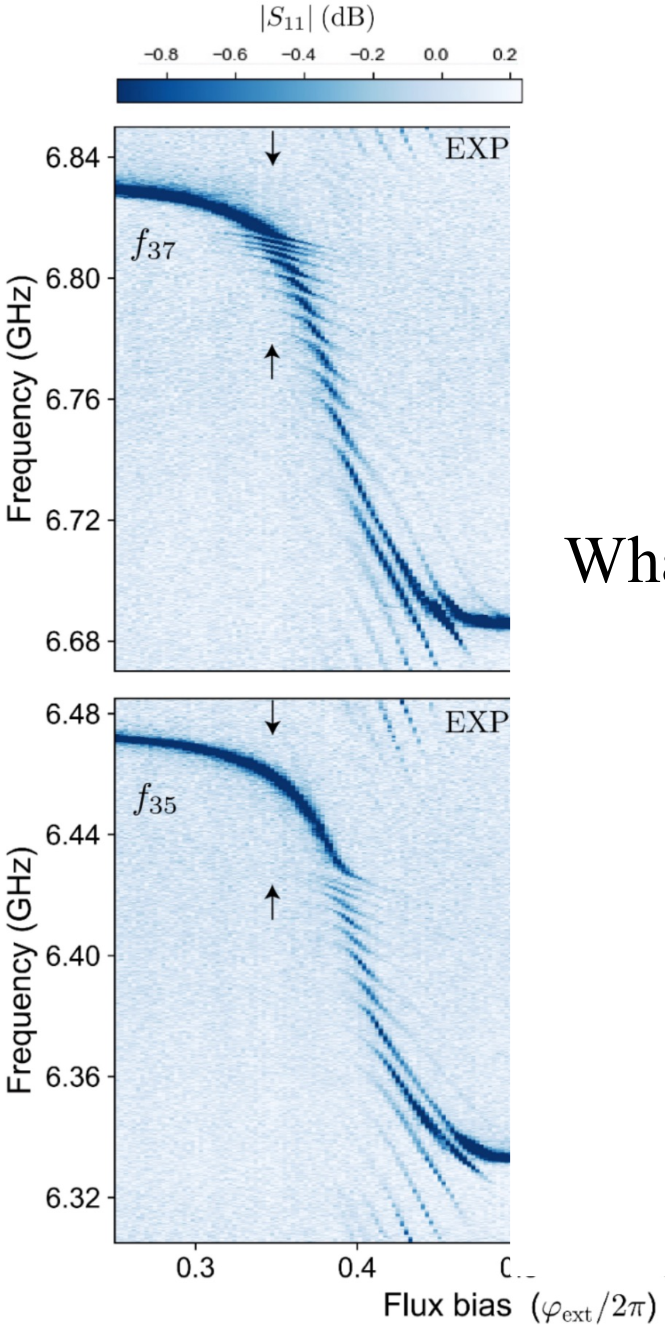


Richer fine structure

Going up: fine spectral structure getting richer



A jungle of resonances: to be revisited later…



What's going on?!?

What's going on? Let's start with the circuit QED Hamiltonian

$$\hat{H}_{\mathrm{QED}} = \hat{H}_{\mathrm{qubit}} + \hat{H}_{\mathrm{modes}} + \hat{H}_{\mathrm{int}}$$

Josephson's atom Hamiltonian

$$\hat{H}_{\mathrm{qubit}}/h = 4E_C\hat{n}_J^2 + \frac{1}{2}E_L\hat{\varphi}_J^2 - E_J\cos(\hat{\varphi}_J - \varphi_{\mathrm{ext}})$$

Josephson's atom Hamiltonian

$$\hat{H}_{\mathrm{modes}}/h = \sum_{i=1}^{+\infty} f_i^{(\mathrm{bare})}\hat{b}_i^\dagger\hat{b}_i$$

« Bare » transmission line modes (in reality they include diamagnetic-like renormalization)

$$\hat{H}_{\mathrm{int}}/h = -\hat{\varphi}_J\sum_{i=1}^{j_0} g_i^{(f)}(\hat{b}_i + \hat{b}_i^\dagger) + \mathrm{i}\hat{n}_J\sum_{i=j_0+1}^{+\infty} g_i^{(c)}(\hat{b}_i^\dagger - \hat{b}_i)$$

Atom-mode interaction (both rotating and counter-rotating wave terms) in a *hybrid gauge*

Counter-rotating wave-terms important in the ultra-strong coupling regime

Diagonalization in one-excitation subspace: the "polaritons"

$$\mathcal{B}_0 = \left\{ |\mathrm{e}\rangle|0\rangle, |\mathrm{g}\rangle\hat{b}_k^\dagger|0\rangle \right\}_{j_0 \leq k \leq N}$$

Basis of **single-excitation subspace**

$$\hat{a}_k^\dagger|\mathrm{g}\rangle|0\rangle = W_{k,0}|\mathrm{e}\rangle|0\rangle + \sum_{k'} W_{k,k'}|\mathrm{g}\rangle\hat{b}_{k'}^\dagger|0\rangle$$

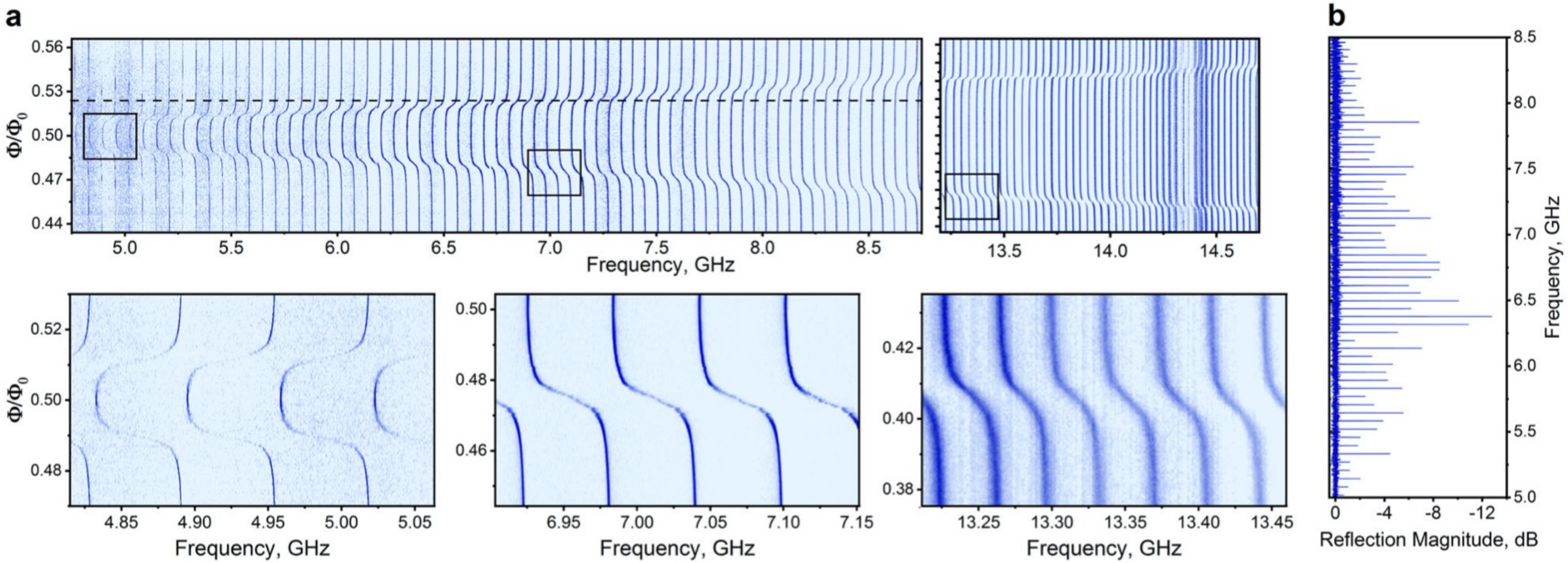
Polariton-like creation operator

At the single-photon level,
the 'atom' mediates an hybridization between several cavity modes

Superstrong coupling regime: already observed with transmons

Atom-induced multimode hybridization already observed with transmon atoms

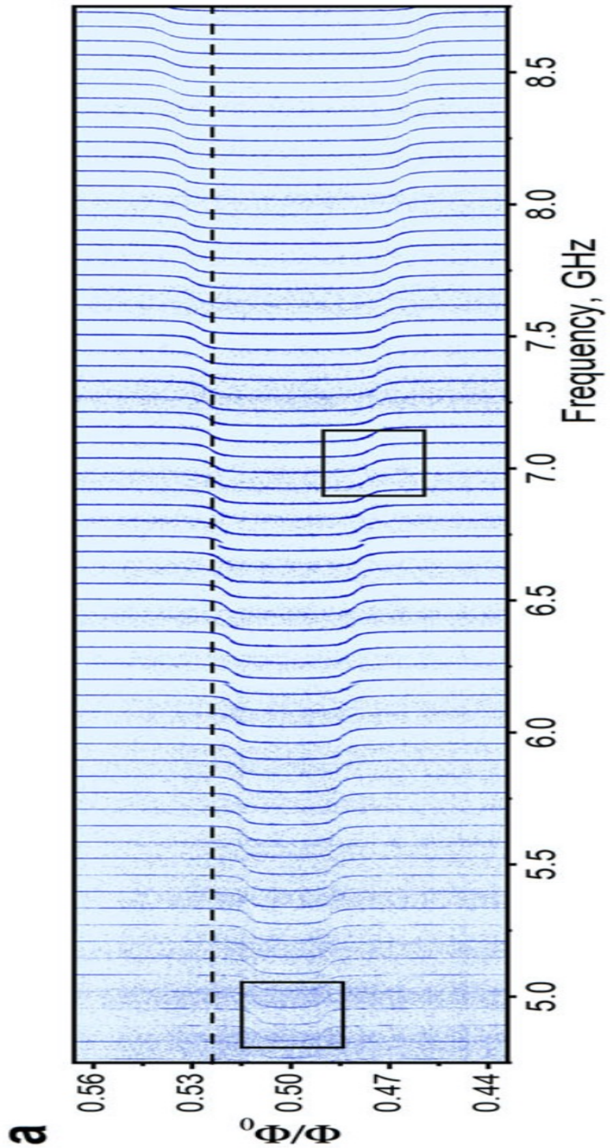
N.B. **Transmon is a weakly anharmonic oscillator**



R. Kuzmin, N. Mehta, N. Grabon, R. Mencia, and V. E. Manucharyan, npj Quantum Information (2019)5:20

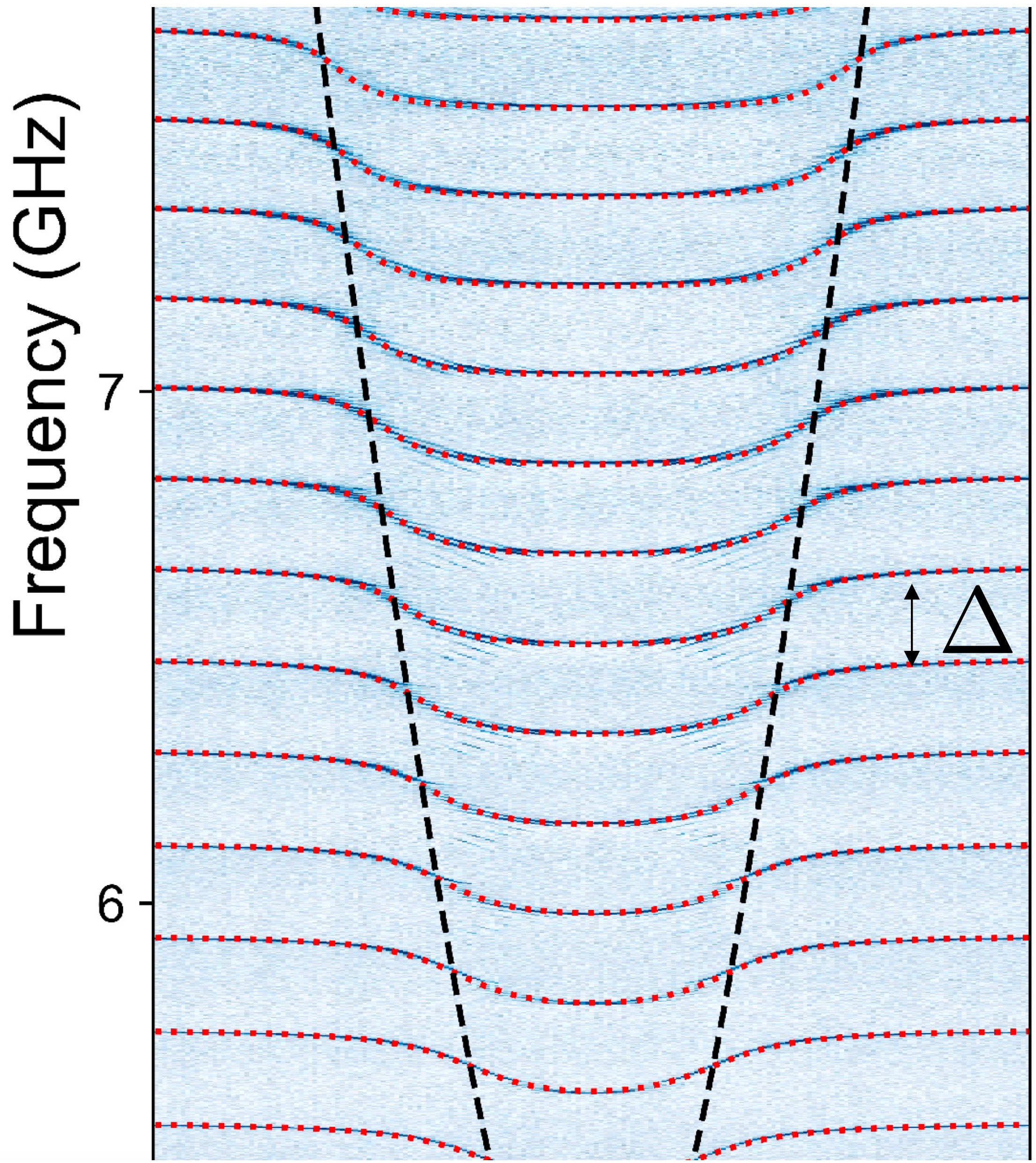
J. Puertas Martínez, Se. Léger, N. Gheeraert, R. Dassonneville, L. Planat, Farshad Foroughi, Yuriy Krupko, Olivier Buisson, Cécile Naud, Wiebke Hasch-Guichard, Serge Florens, Izak Snyman and Nicolas Roch, NPJ Quantum Information (2019)5:19

Superstrong coupling regime: already observed with transmons



R. Kuzmin, N. Mehta, N. Grabon, R. Mencia, and V. E. Manucharyan, npj Quantum Information (2019)5:20

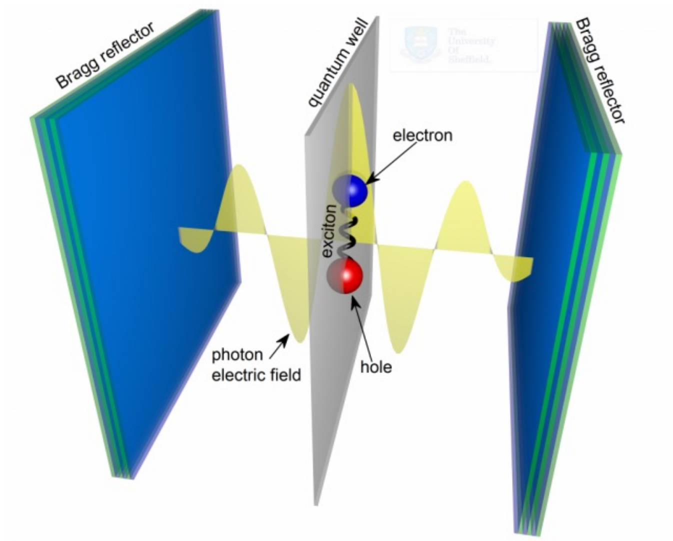
Also with fluxonium, single-excitation theory explains the "envelope" of the branches



Polariton Branches (dotted)

Fine structure cannot be explained at this level

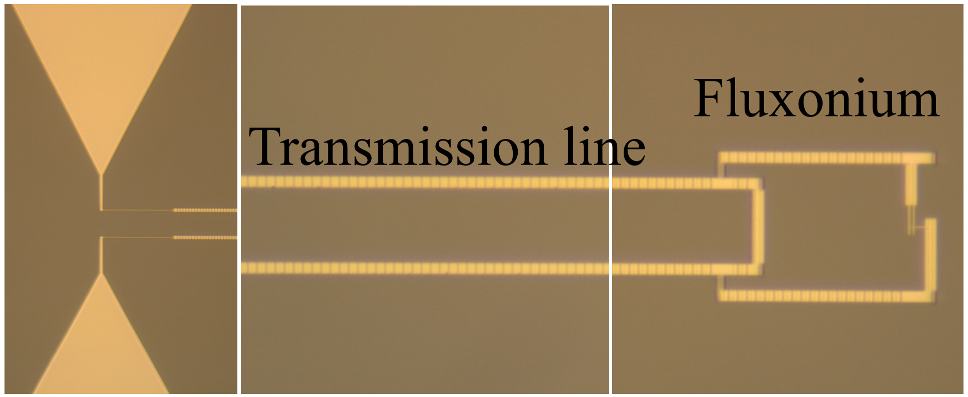
Disclaimer: different kinds of polaritons….



Hopfield polariton is a linear superposition of light and matter quanta (e.g., exciton-polariton)

Pasta + sauce

$$|\text{polariton}\rangle \propto |\text{photon}\rangle + |\text{exciton}\rangle$$



The 'polariton' here is the linear superposition of the quanta of 2 circuits

Pasta 1 + Pasta 2

$$\hat{a}_k^\dagger |\text{g}\rangle|0\rangle = W_{k,0}|\text{e}\rangle|0\rangle + \sum_{k'} W_{k,k'}|\text{g}\rangle\hat{b}_{k'}^\dagger|0\rangle$$

Multi-polariton subspaces

To understand the fine structure, multi-polariton states are needed:

$$\left\{\hat{a}_k^\dagger|0\rangle\right\}\bigcup\left\{\hat{a}_{k'}^\dagger\hat{a}_i^\dagger|0\rangle\right\}_{1\leq i\leq j_0}^{j_0<k'<k}$$

Basis of **two-excitation subspace**

$$\left\{\hat{a}_k^\dagger|0\rangle\right\}\bigcup\left\{\hat{a}_{k'}^\dagger\hat{a}_i^\dagger|0\rangle\right\}_{1\leq i\leq j_0}^{j_0<k'<k}\bigcup\left\{\hat{a}_{k'}^\dagger\hat{a}_{k''}^\dagger\hat{a}_i^\dagger|0\rangle\right\}_{1\leq i\leq j_0}^{j_0<k',k''<k}$$

Basis of **three-excitation subspace**

....

Subspaces with different number of particles are coupled
via the counter-rotating-wave terms of the atom-resonator interaction

$$\langle 0|\hat{a}_k\hat{H}_{int}\hat{a}_{k'}^\dagger\hat{a}_i^\dagger|0\rangle\neq 0$$

Effective Hamiltonian

$$\hat{V} = g \sum_{j\leq j_0}^{k,k'>j_0} \sqrt{j}\, A_{k,k'} \hat{a}_j^\dagger \hat{a}_{k'}^\dagger \hat{a}_k + h.c.$$

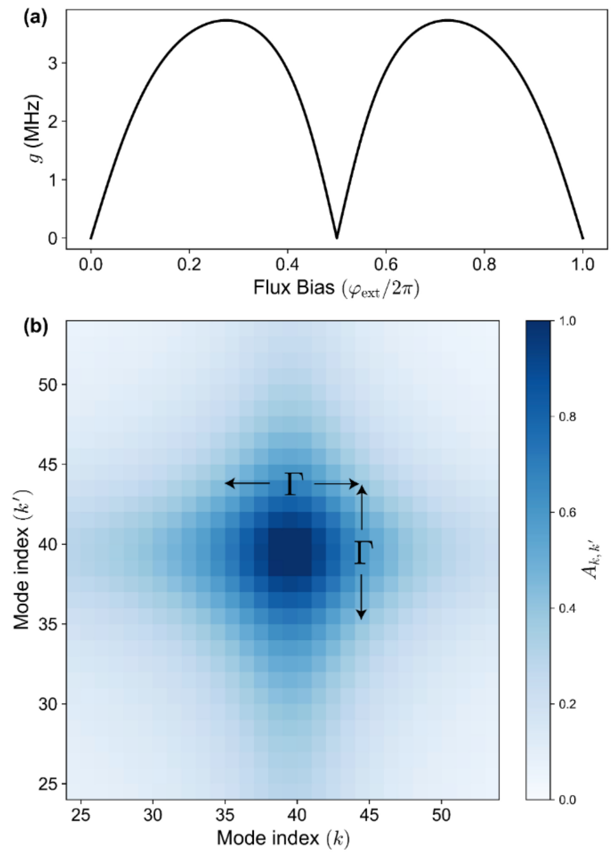
$$\frac{\hat{H}}{h} = \sum_{k>0} f_k \hat{a}_k^\dagger \hat{a}_k + \hat{V}$$

$$g \simeq \frac{\Delta^2}{\Gamma} \sqrt{\frac{h/(2e)^2}{32\pi Z_\infty}} (\langle e|\hat{\varphi}_J|e\rangle - \langle g|\hat{\varphi}_J|g\rangle)$$

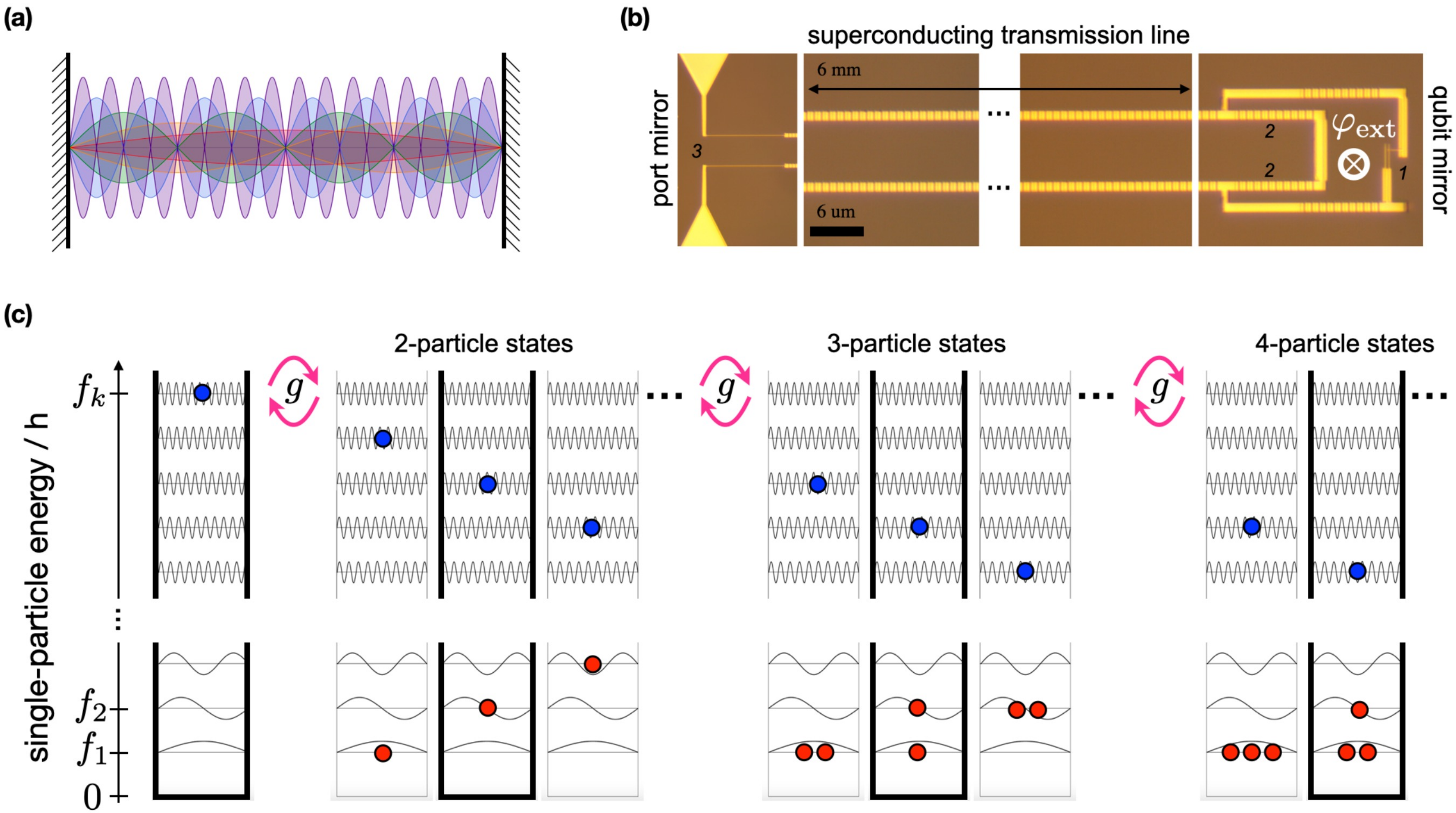
$$A_{k,k'} = \frac{1}{\max(W_{k,0}^* W_{k,0})} W_{k,0}^* W_{k',0}$$

Down-conversion Hamiltonian (three-wave-mixing)

Downconversion vanishes at symmetric points



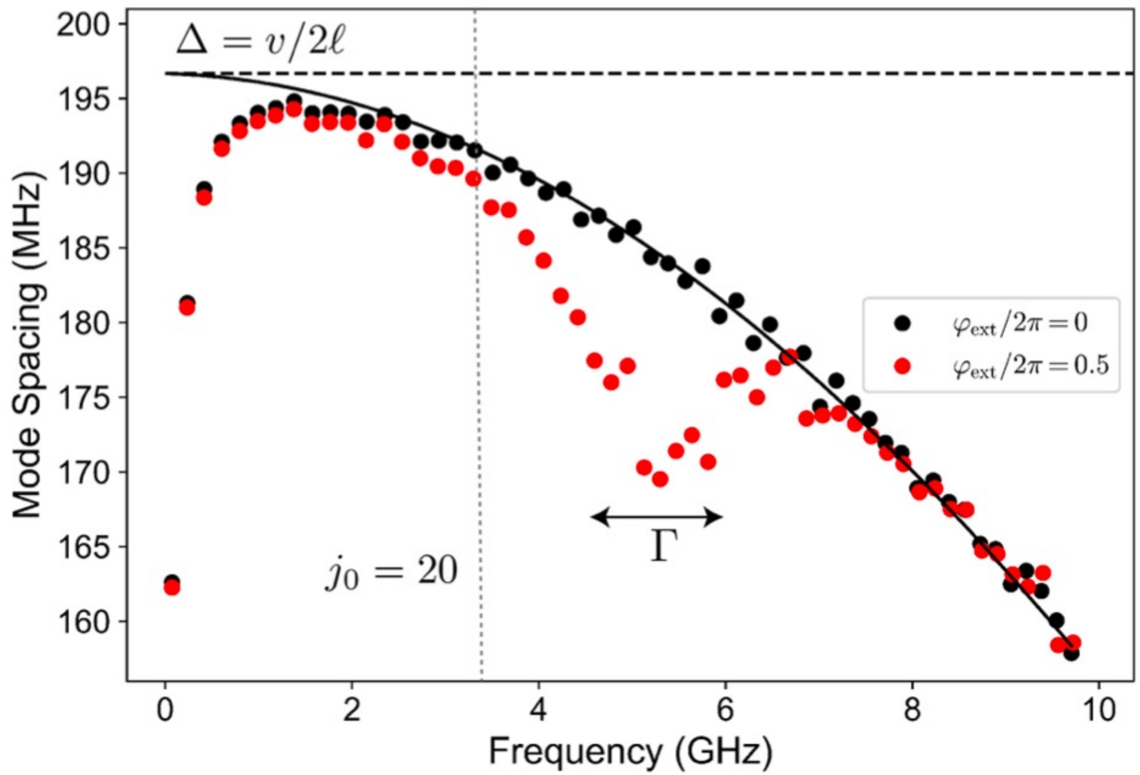
What is going on?



Free spectral range is not constant: pseudo-random disorder

Mode spacing is not constant -> degeneracy of multi-particle states is lifted

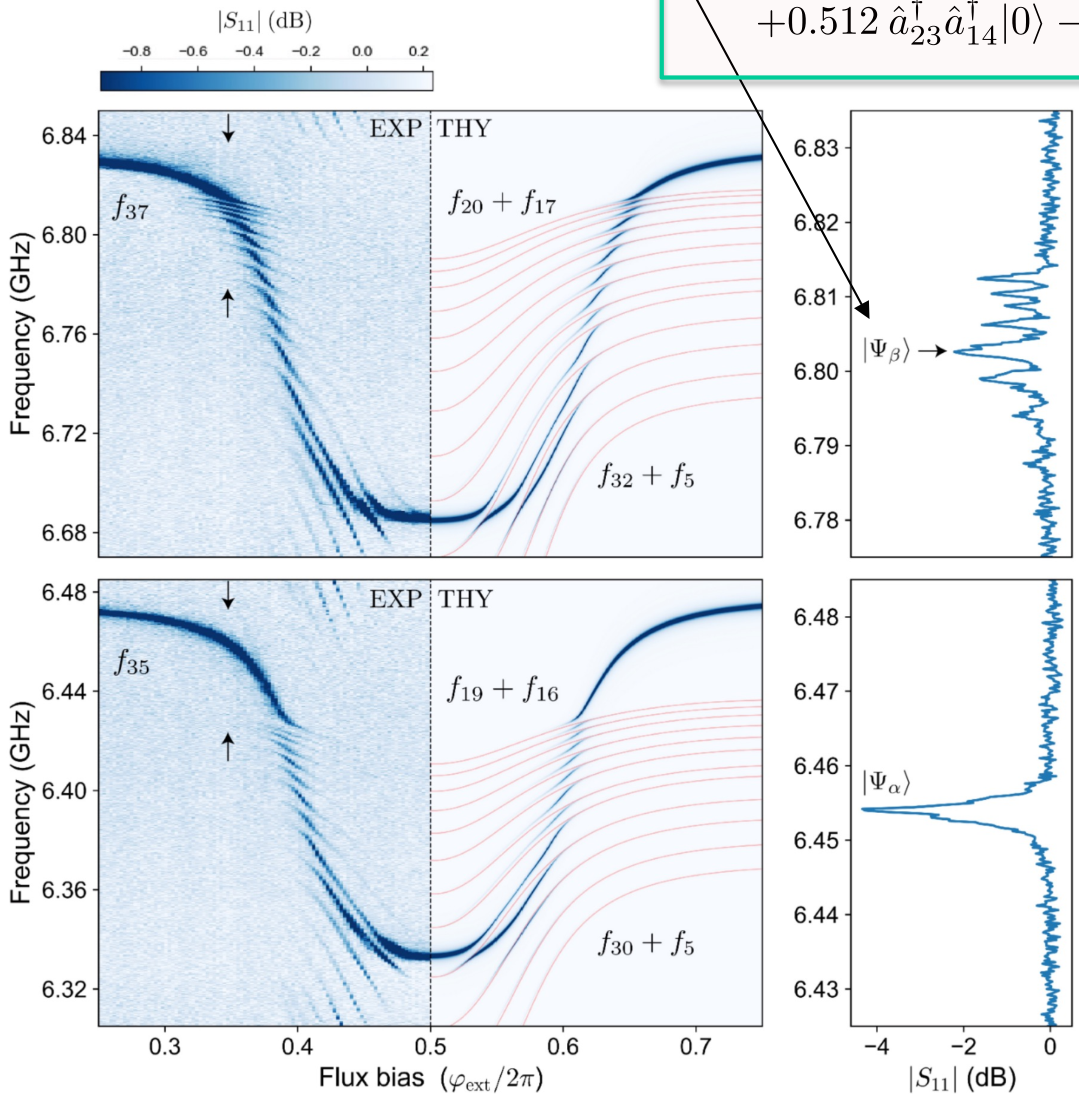
For example $f_k \neq f_{k-n} + f_n$



This is equivalent to a pseudo-random disorder for the multi-particle energies

Including two-photon states

$$|\Psi_\beta\rangle \approx 0.58\,\hat{a}^\dagger_{37}|0\rangle + 0.102\,\hat{a}^\dagger_{25}\hat{a}^\dagger_{12}|0\rangle + 0.184\,\hat{a}^\dagger_{24}\hat{a}^\dagger_{13}|0\rangle$$
$$+0.512\,\hat{a}^\dagger_{23}\hat{a}^\dagger_{14}|0\rangle - 0.53\,\hat{a}^\dagger_{22}\hat{a}^\dagger_{15}|0\rangle - 0.223\,\hat{a}^\dagger_{21}\hat{a}^\dagger_{16}|0\rangle - 0.125\,\hat{a}^\dagger_{20}\hat{a}^\dagger_{17}|0\rangle$$

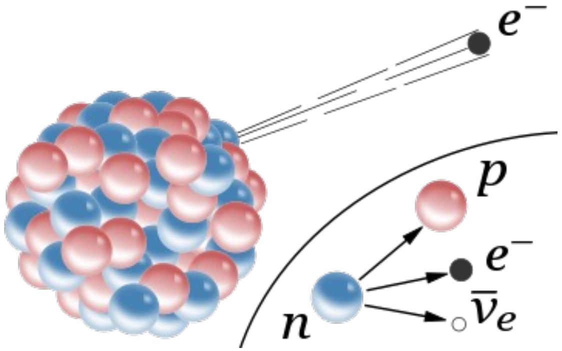


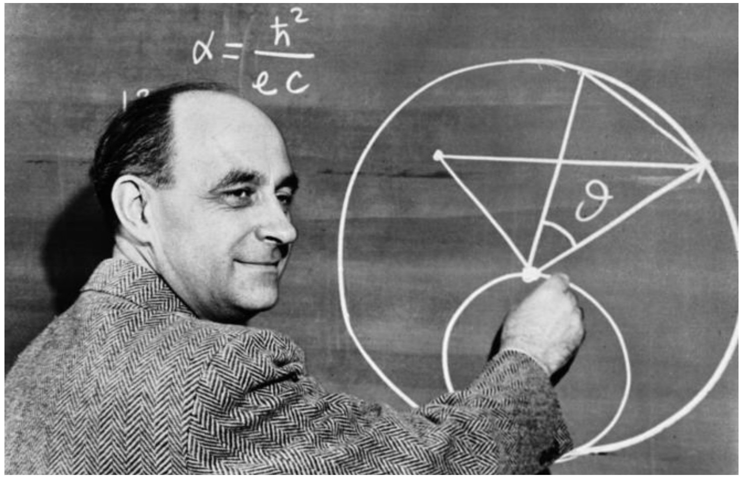
Photon in mode 37 attempts to decay, but fails to do so ….

Hybridization with two-photon states replaces the decay…

What about Fermi golden Rule?

$$\Gamma_{i,f} = \frac{2\pi}{\hbar} |\langle i|H|f\rangle|^2 \delta(E_f - E_i)$$





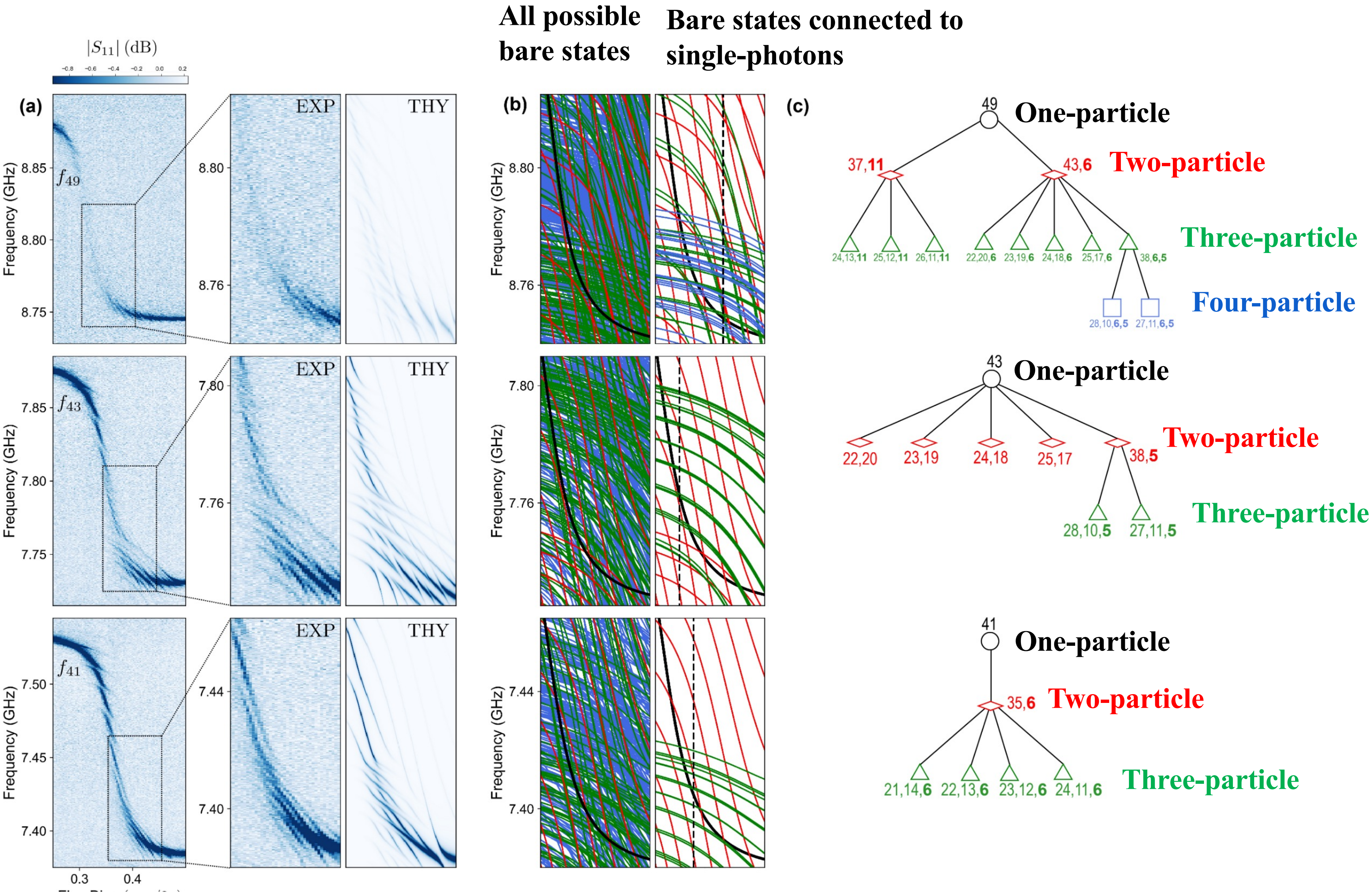
At Fermi's golden rule, particle decay is irreversible…..
Fermi's golden rule calculates damping rate

$$\Gamma_k \propto \sum_{k',j} |\langle 0|\hat{a}_k \hat{V} \hat{a}^\dagger_{k'} \hat{a}^\dagger_j |0\rangle|^2 \delta(f_k - f_{k'} - f_j)$$

The considered multi-mode circuit QED system has particle decay (down-conversion) that dramatically breaks Fermi's golden rule

States with three and four-particle components

Particle attempts to decay but fails… and hybridize with few-particle states



Connection to Altschuler et al. proposal with fermions

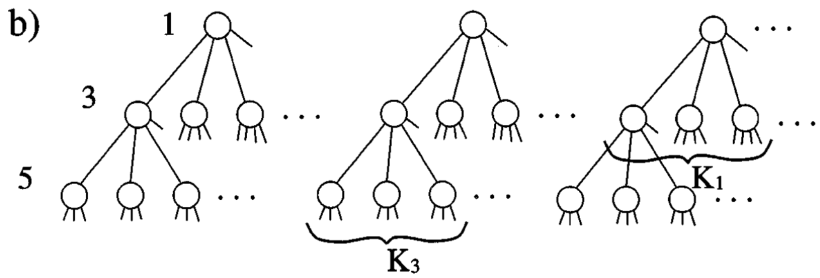
VOLUME 78, NUMBER 14 PHYSICAL REVIEW LETTERS 7 APRIL 1997

Quasiparticle Lifetime in a Finite System: A Nonperturbative Approach

Boris L. Altshuler,[1] Yuval Gefen,[2] Alex Kamenev,[2] and Leonid S. Levitov[3]
[1]NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540
[2]Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot, 76100, Israel
[3]Massachusetts Institute of Technology, 12-112, Cambridge, Massachusetts 02139
(Received 30 August 1996)

The problem of electron-electron lifetime in a quantum dot is studied beyond perturbation theory by mapping onto the problem of localization in the Fock space. Localized and delocalized regimes are identified, corresponding to quasiparticle spectral peaks of zero and finite width, respectively. In the localized regime, quasiparticle states are single-particle-like. In the delocalized regime, each eigenstate is a superposition of states with very different quasiparticle content. The transition energy is $\epsilon_c \simeq \Delta(g/\ln g)^{1/2}$, where Δ is mean level spacing, and g is the dimensionless conductance. Near ϵ_c there is a broad critical region not described by the golden rule. [S0031-9007(97)02895-0]

Quasiparticle in a Fermi liquid is not an eigenstate: it decays into two quasiparticles and a hole. In an infinite clean system, by using the golden rule (GR), quasiparticle decay rate is estimated as $\gamma(\epsilon) \sim \epsilon^2/\epsilon_F$, where ϵ is quasiparticle energy and ϵ_F is Fermi energy [1]. However, in a finite system the eigenstate spectrum is discrete. In this case, quasiparticles may be viewed as wave packets constructed of such states, the packet width being determined by the lifetime in an infinite system: $\delta\epsilon \simeq \gamma(\epsilon)$. In this paper we attempt to clarify the relation between quasiparticles and many-particle states, and find that at different energies it has different meanings.



$$\mathcal{H}_0 + \mathcal{H}_1 = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\gamma\delta}^{\alpha\beta} c_\gamma^\dagger c_\delta^\dagger c_\beta c_\alpha$$

Connection to Anderson localization

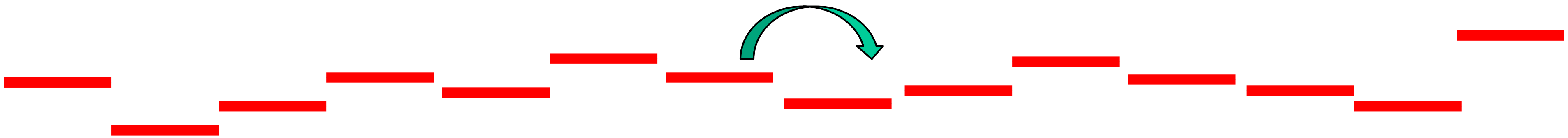
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

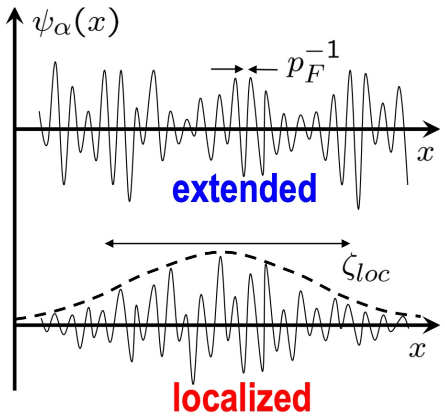
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

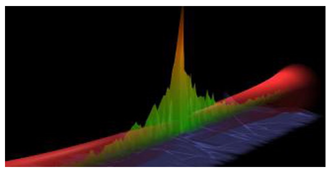
This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.



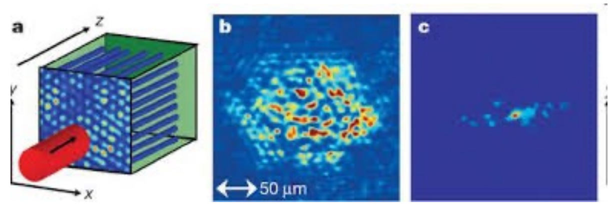
Single-particle** problem with a **random disordered potential



Atom localization

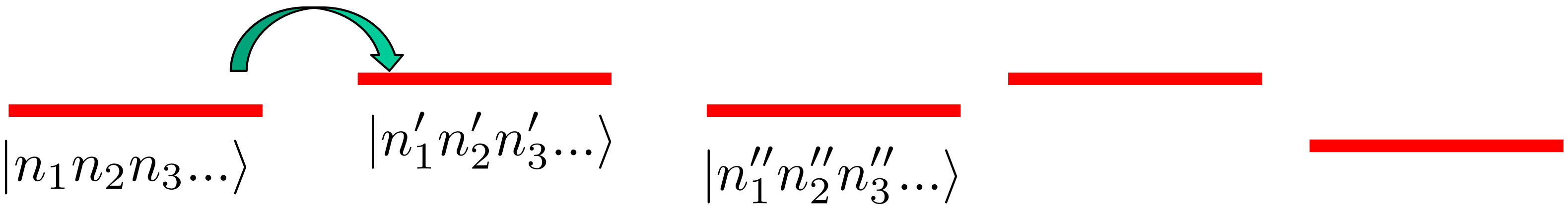


Photon localization



Connection to many-body localization

Manybody localization is a sort of Anderson localization in the Fock space



Anderson localization is a single-particle localization in a disordered lattice

For Anderson-like many-body localization, the following analogy holds:

- Lattice site ⟺ manybody Fock state
- Hopping ⟺ coupling due to particle-particle interaction
- Disorder ⟺ disorder for manybody energy states

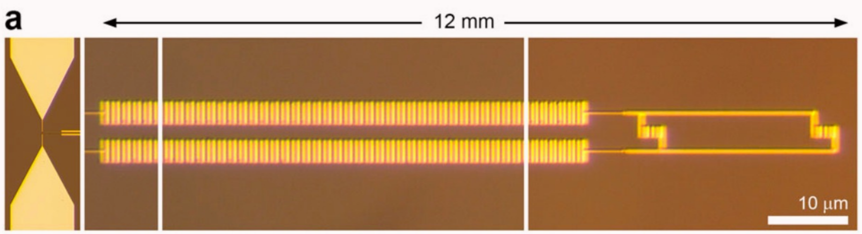
A manybody system initially prepared in an initial state remains 'localized' around it in Fock space

In the present circuit QED system, the initial single-photon state attempts to delocalize in Fock space by delocalizing into a shower of multiparticle states, but fails to do so

References:

- B. L. Altshuler, Y. Gefen, A. Kamenev, and L. S. Levitov, Quasiparticle lifetime in a finite system: A nonperturba- tive approach, Phys. Rev. Lett. 78, 2803 (1997)
- D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Colloquium: Many-body localization, thermalization, and entanglement, Rev. Mod. Phys. 91, 021001 (2019).
- S. Girvin, https://doi.org/10.36471/JCCM_April_2022_02

« Hopping connectivity » in Fock space depends on atom!

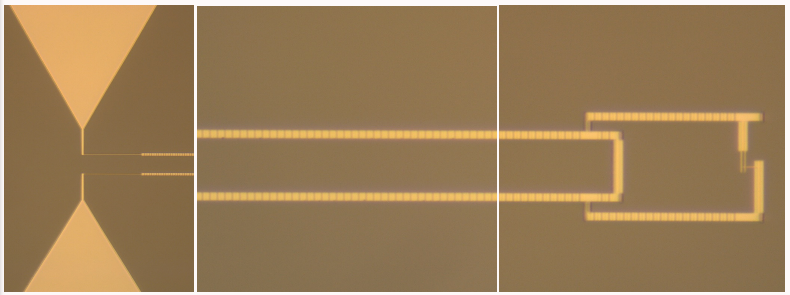


npj Quantum Information (2019)5:20

Transmon *(weakly anharmonic)*

↓

Negligible connectivity
(no fine spectral structure)

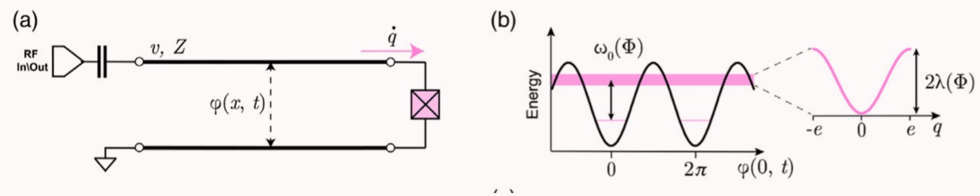


Present work

Fluxonium

↓

Finite connectivity
(fine spectral structure,
it remains many-body localized)



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Phase-slip junction

↓

All-to-all connectivity
(broadening,
Manybody delocalized)