

Multicomponent spin mixtures of two-electron fermions

Enrico Fermi School on Quantum Mixtures

Varenna, July 21st 2022



European Laboratory for
Non-Linear Spectroscopy



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Lecture 1



Introduction to multicomponent quantum gases



Interactions in two-electron fermions and $SU(N)$ physics



Experimental techniques



EXP: $SU(N)$ physics in low dimensions



EXP: $SU(N)$ Fermi-Hubbard and breaking $SU(N)$ physics

Lecture 2



Multicomponent systems with coherent coupling



Synthetic dimensions and artificial magnetic fields



EXP: Chiral edge currents in synthetic ladders



EXP: Synthetic Hall effect

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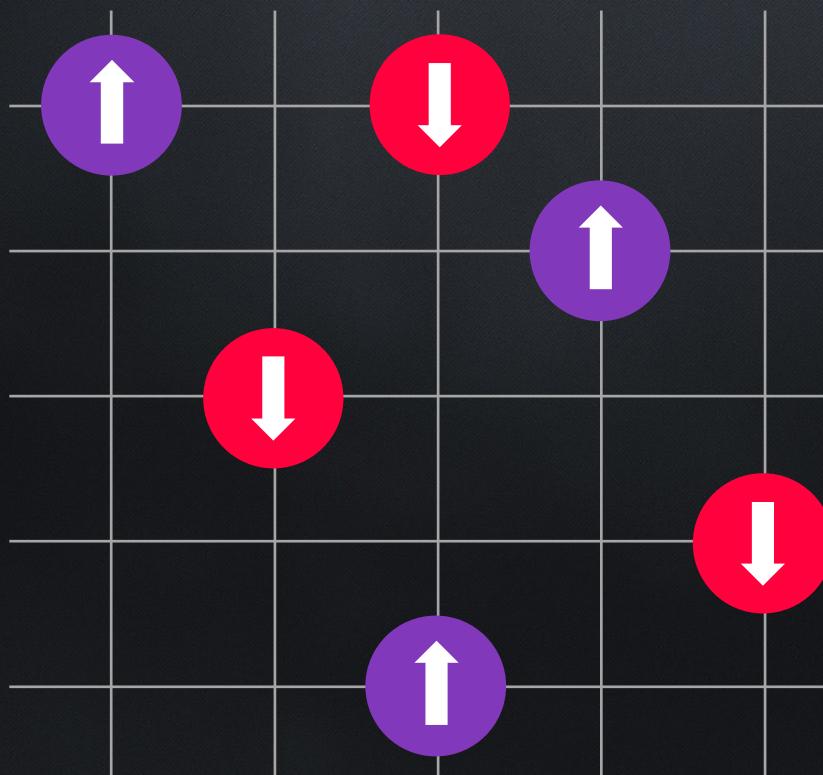
Two quantum states

Two-level systems are everywhere in Nature
and in the way we model and take advantage of quantum processes

Two quantum states

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and in the way we model and take advantage of quantum processes

Spin of elementary particles (e.g. electrons)



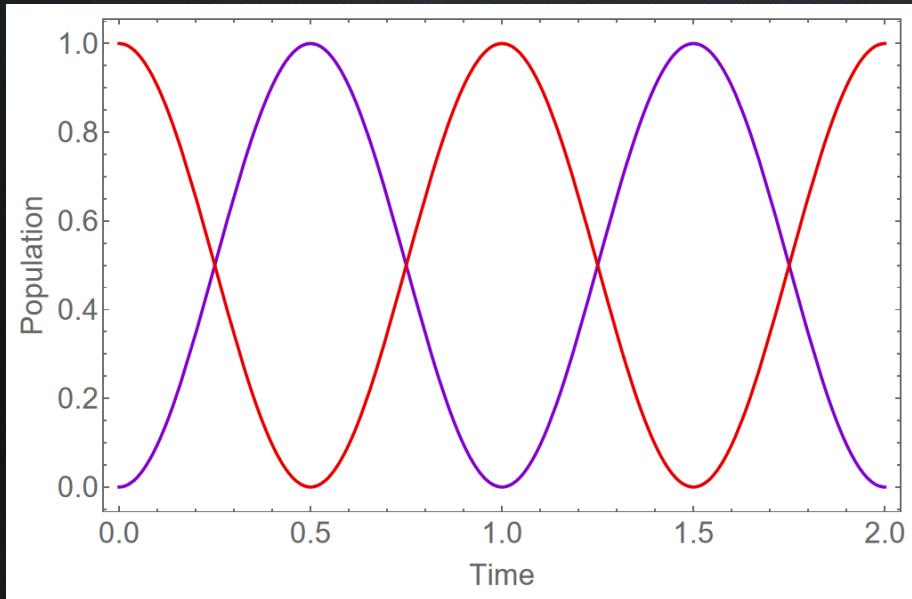
Superconductivity
Magnetism
...

and their realization in
binary ultracold mixtures

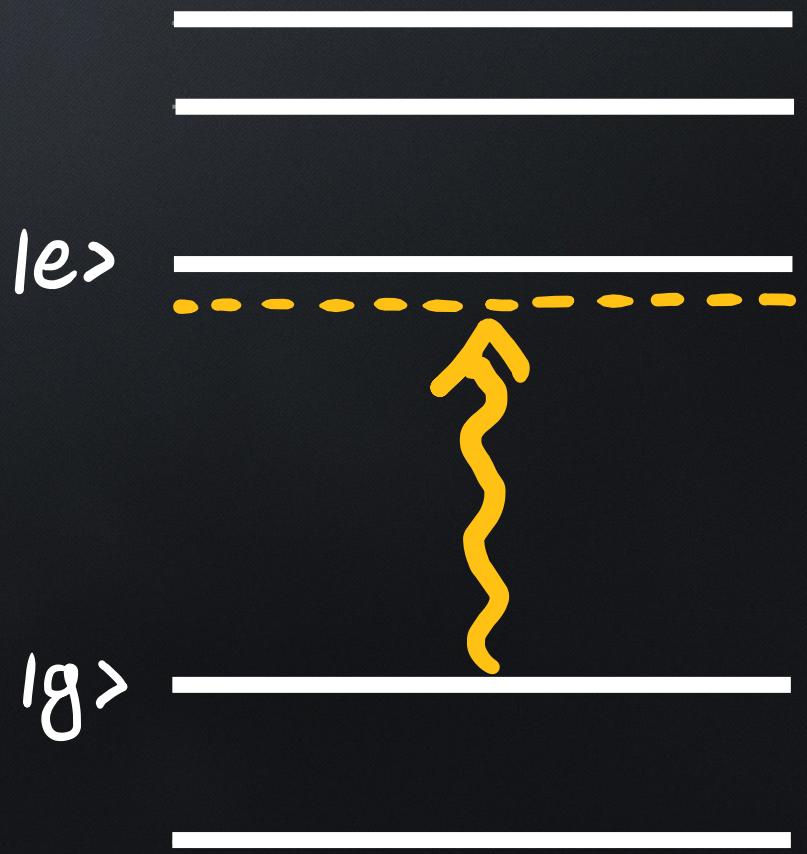
Two quantum states

Two-level systems are everywhere in Nature
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Light-matter interaction



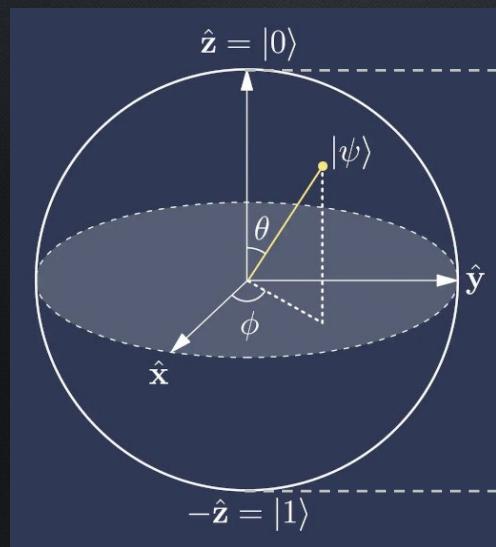
two-level coherent dynamics



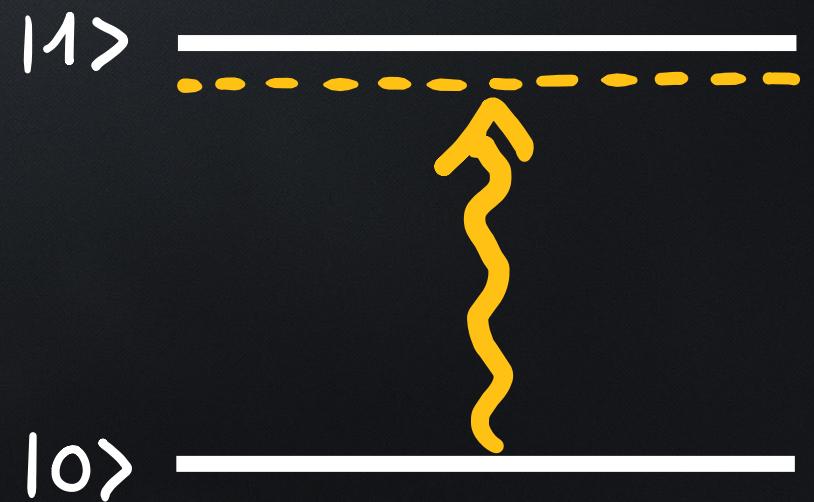
Two quantum states

Two-level systems are everywhere in Nature
and in the way we model and take advantage of quantum processes

Quantum information



qubit
Bloch sphere



More states!

Extend the Hilbert space!

Larger space: more directions to explore, more fun

$|{\downarrow}\rangle |c\rangle |4\rangle$



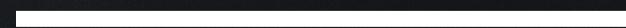
$|{\rightarrow}\rangle |b\rangle |3\rangle$



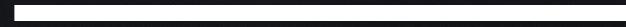
$|{\nearrow}\rangle |a\rangle |2\rangle$



$|{\uparrow}\rangle |e\rangle |1\rangle$



$|{\downarrow}\rangle |g\rangle |0\rangle$

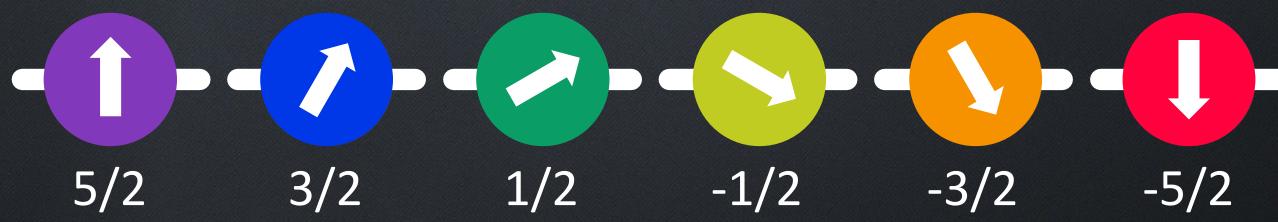






Spin mixtures

Atoms are not elementary particles:
 $2F+1$ -dimensional spin manifold available



^{173}Yb

$F = 5/2$



^{87}Sr

$F = 9/2$

Spin mixture of two-electron fermions

hydrogen 1 H 1.0079	lithium 3 Li 6.941	beryllium 4 Be 9.0122
sodium 11 Na 22.990	magnesium 12 Mg 24.305	
potassium 19 K 39.098	calcium 20 Ca 40.078	
rubidium 37 Rb 85.46	strontium 38 Sr 87.62	
caesium 55 Cs 132.91	barium 56 Ba 137.33	57-70
francium 87 Fr [223]	radium 88 Ra [226]	89-102 **

Alkaline-earth (-like) atoms

scandium 21 Sc 44.956	titanium 22 Ti 47.867	vanadium 23 V 50.942	chromium 24 Cr 51.996	manganese 25 Mn 54.938	iron 26 Fe 55.845	cobalt 27 Co 58.933	nickel 28 Ni 58.693	copper 29 Cu 63.546	Zinc 30 Zn 65.39
yttrium 39 Y 88.906	zirconium 40 Zr 91.224	niobium 41 Nb 92.906	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.91	palladium 46 Pd 106.42	silver 47 Ag 107.87	cadmium 48 Cd 112.41
lutetium 71 Lu 174.97	hafnium 72 Hf 178.49	tantalum 73 Ta 180.95	tungsten 74 W 183.84	rhenum 75 Re 186.21	osmium 76 Os 190.23	iridium 77 Ir 192.22	platinum 78 Pt 195.08	gold 79 Au 196.97	mercury 80 Hg 200.59
lawrencium 103 Lr [261]	rutherfordium 104 Rf [262]	dubnium 105 Db [266]	seaborgium 106 Sg [264]	bohrium 107 Bh [269]	hassium 108 Hs [268]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]	ununnilium 111 Uuu [272]	ununnilium 112 Uub [277]
francium 87 Fr [223]	radium 88 Ra [226]	lawrencium 103 Lr [261]	rutherfordium 104 Rf [262]	dubnium 105 Db [266]	seaborgium 106 Sg [264]	bohrium 107 Bh [269]	hassium 108 Hs [268]	meitnerium 109 Mt [268]	ununnilium 110 Uun [271]

boron 5 B 10.811	carbon 6 C 12.011	nitrogen 7 N 14.007	oxygen 8 O 15.999	fluorine 9 F 18.998	neon 10 Ne 20.180
aluminium 13 Al 26.982	silicon 14 Si 28.086	phosphorus 15 P 30.974	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.948
gallium 31 Ga 69.723	germanium 32 Ge 72.61	arsenic 33 As 74.922	selenium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.80
indium 49 In 114.82	tin 50 Sn 118.71	antimony 51 Sb 121.76	tellurium 52 Te 127.60	iodine 53 I 126.90	xenon 54 Xe 131.29
thallium 81 Tl 204.38	lead 82 Pb 207.2	bismuth 83 Bi 208.98	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]

* Lanthanide series

** Actinide series

lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europlium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	yterbium 70 Yb 173.04
actinium 89 Ac [223]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

Lecture 1



Introduction to multicomponent quantum gases



Interactions in two-electron fermions and SU(N) physics



Experimental techniques



EXP: SU(N) physics in low dimensions



EXP: SU(N) Fermi-Hubbard and breaking SU(N) physics

Lecture 2



Multicomponent systems with coherent coupling



Synthetic dimensions and artificial magnetic fields

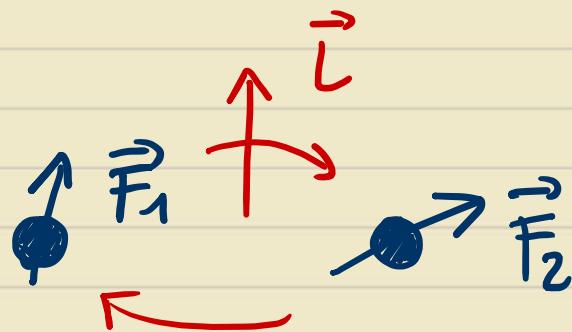


EXP: Chiral edge currents in synthetic ladders



EXP: Synthetic Hall effect

Interactions in multicomponent mixtures



$\vec{L} = 0$ S-WAVE COLLISIONS

$\vec{F} = \vec{F}_1 + \vec{F}_2$ CONSERVED

$$V(\vec{r}) = \sum_F g_F P_F \delta(\vec{r})$$

SHORT-RANGE INTERACTIONS

$$g_F = \frac{4\pi\hbar^2 a_F}{N}$$

$$P_F = \sum_{M=-F}^F |FM\rangle \langle FM|$$

(see Vedula's lectures)

Interactions in multicomponent mixtures



$$\vec{L} = 0 \quad \text{S-WAVE COLLISIONS}$$
$$\vec{F} = \vec{F}_1 + \vec{F}_2 \quad \text{CONSERVED}$$

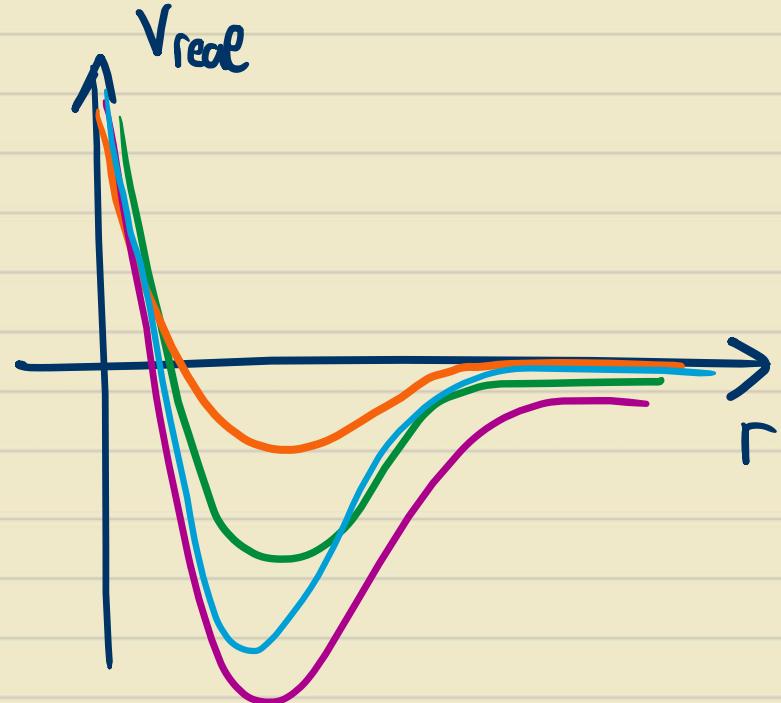
$$V(\vec{r}) = \sum_F g_F P_F \delta(\vec{r})$$

$$g_F = \frac{4\pi\hbar^2 a_F}{N}$$

generally depends on F

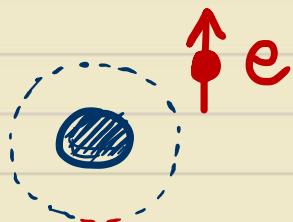


SPIN-DEPENDENT INTERACTIONS

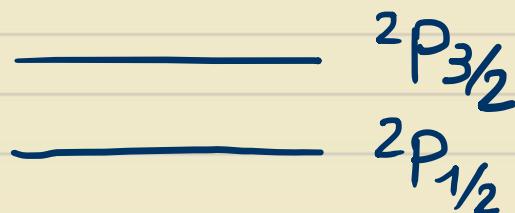


Alkali and alkaline-earth atoms

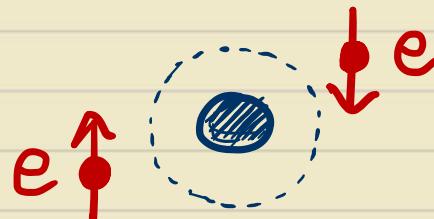
ALKALI ATOMS
(1-electron)



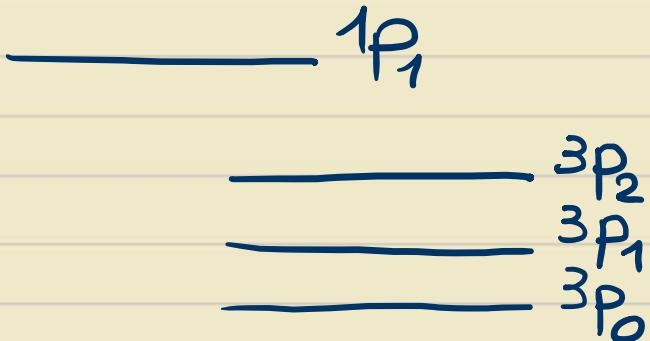
$$S = \frac{1}{2}$$



ALKALINE-EARTH ATOMS
(2-electron)

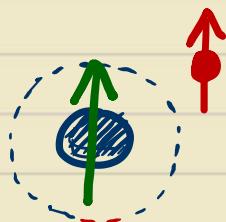


$$S = 0, 1$$



Alkali and alkaline-earth atoms

ALKALI ATOMS
(1-electron)



$$J = S = \frac{1}{2}$$

$$l \neq 0$$

$$H_{hf} \sim \vec{l} \cdot \vec{j} \neq 0$$

— $^2S_{1/2}$

ALKALINE-EARTH ATOMS
(2-electron)



$$J = S = 0$$

$l \neq 0$ FERMIONS
 $l = 0$ BOSONS

$$H_{hf} \sim \vec{l} \cdot \vec{j} = 0$$

— 1S_0

Alkaline-earth fermions

ALKALINE-EARTH ATOMS (2-electron)

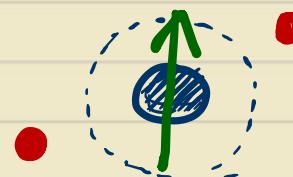
NO ELECTRON SPIN
IN GROUND STATE
(ONLY NUCLEAR SPIN)



NO HYPERFINE INTERACTION



NUCLEAR SPIN DECOUPLED
FROM ELECTRON D.O.F.



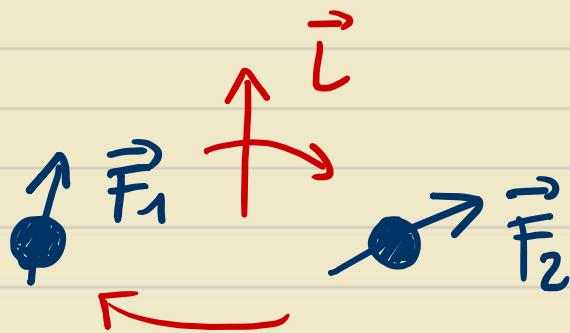
$$J = S = 0$$

$I \neq 0$ FERMIONS

$$H_{hf} \sim \vec{I} \cdot \vec{J} = 0$$

— 1S_0

Interactions in alkaline-earth fermions



$$\vec{L} = 0 \quad \text{S-WAVE COLLISIONS}$$
$$\vec{F} = \vec{F}_1 + \vec{F}_2 \quad \text{CONSERVED}$$

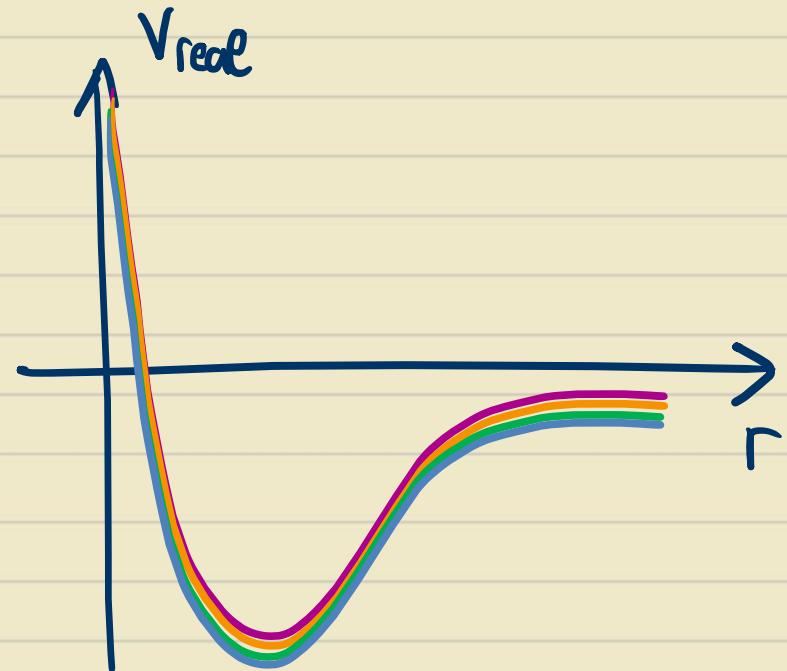
$$V(\vec{r}) = \sum_F g_F P_F \delta(\vec{r})$$

$$g_F = \frac{4\pi\hbar^2 a_F}{N}$$

does not depend on F



SPIN-INDEPENDENT INTERACTIONS



SU(N) fermions

INTERACTING MIXTURE OF TWO-ELECTRON FERMIONS ($m = -F \dots F$)

$$\hat{H} = \int d\vec{r} \left(\frac{\hbar^2}{2M} \sum_m \nabla \psi_m^+ \nabla \psi_m + \frac{g}{2} \sum_{m,n} \psi_m^+ \psi_n^+ \psi_n \psi_m \right)$$



\hat{H} COMMUTES WITH SPIN-PERMUTATION OPERATORS

$$\hat{S}_{mn} = \int d\vec{r} \psi_m^+ \psi_n$$

$$[\hat{S}_{mn}, \hat{S}_{pq}] = \delta_{mq} S_{pn} - \delta_{pn} S_{mq}$$

GENERATORS OF SU(N)

$$[\hat{H}, \hat{S}_{mn}] = 0 \rightarrow \text{SU}(N) \text{ SYMMETRY}$$

$$(N = 2F + 1)$$

$$[\hat{H}, \hat{S}_{mm}] = 0 \rightarrow N_m = \langle \hat{S}_{mm} \rangle$$

NUMBER OF ATOMS IN EACH m
IS CONSTANT (NO SPIN EXCHANGE)

SU(N) fermions

$N = 2$ components $\rightarrow \text{SU}(2)$

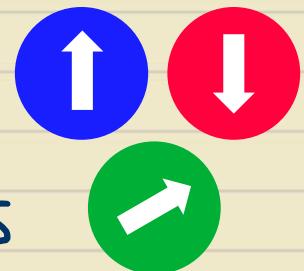
$N^2 - 1 = 3$ generators $\rightarrow \text{PAULI MATRICES}$



$$\sigma_1 \equiv \sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 \equiv \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 \equiv \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$N = 3$ components $\rightarrow \text{SU}(3)$

$N^2 - 1 = 8$ generators $\rightarrow \text{GELL-MANN MATRICES}$



$$\lambda_1 := \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 := \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 := \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 := \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

SU(N) fermions

Fermions with (large) nuclear spin: **tunable SU(N) interaction symmetry**

Stable multicomponent mixtures

Larger Hilbert space

Spectral degeneracies

Exotic quantum phases

Role of quantum statistics

Connections with high-energy physics

...



Lecture 1



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Lecture 2



Multicomponent systems with coherent coupling



Synthetic dimensions and artificial magnetic fields



EXP: Chiral edge currents in synthetic ladders



EXP: Synthetic Hall effect

Two-electron fermions

Nuclear spin: all-optical manipulation

No magnetic trapping!

(weak magnetic dipoles $\mu_N = \mu_B / 1836$: $\Delta E = m_F \times 200 \text{ Hz/G}$)

No Feshbach resonances!

(no role of nuclear spin in collisions)

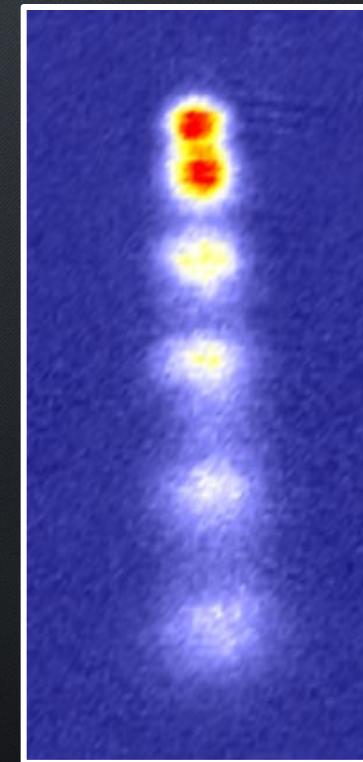
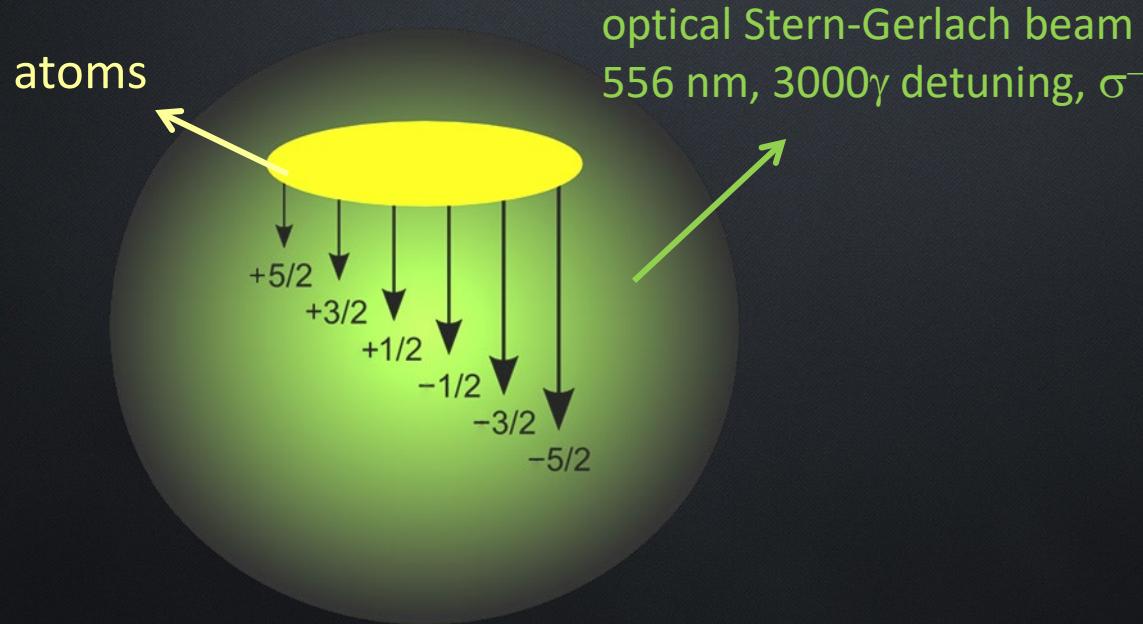
Long nuclear spin coherence times

(weak effect of magnetic field noise)



Spin detection and manipulation

Optical Stern-Gerlach detection Taie et al., PRL (2010)
State-dependent optical dipole force



$5/2$	↑
$3/2$	↗
$1/2$	→
$-1/2$	↘
$-3/2$	↙
$-5/2$	↓

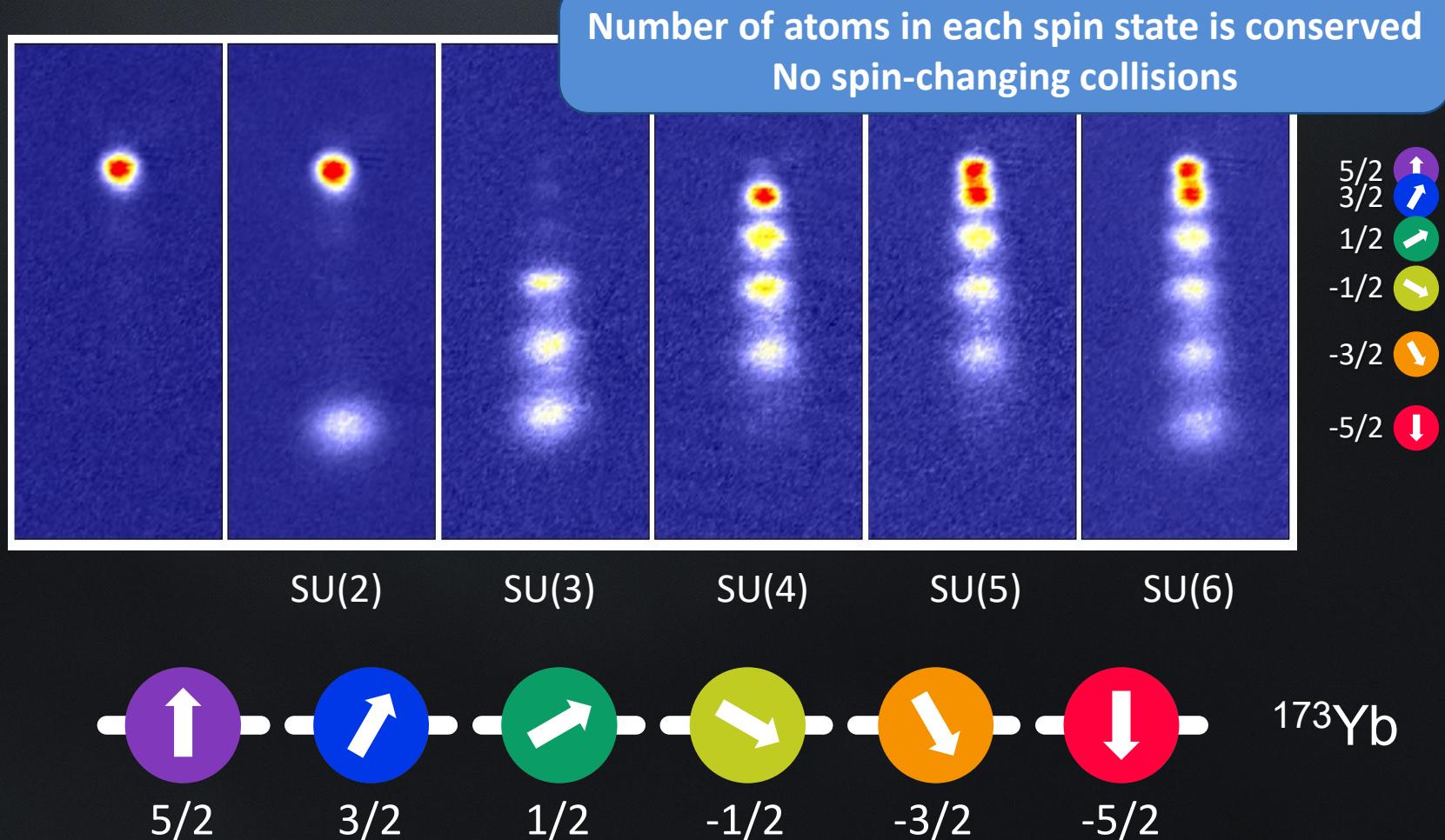


^{173}Yb

Spin detection and manipulation

Spin-selective preparation with optical pumping

Arbitrary-number spin mixtures:



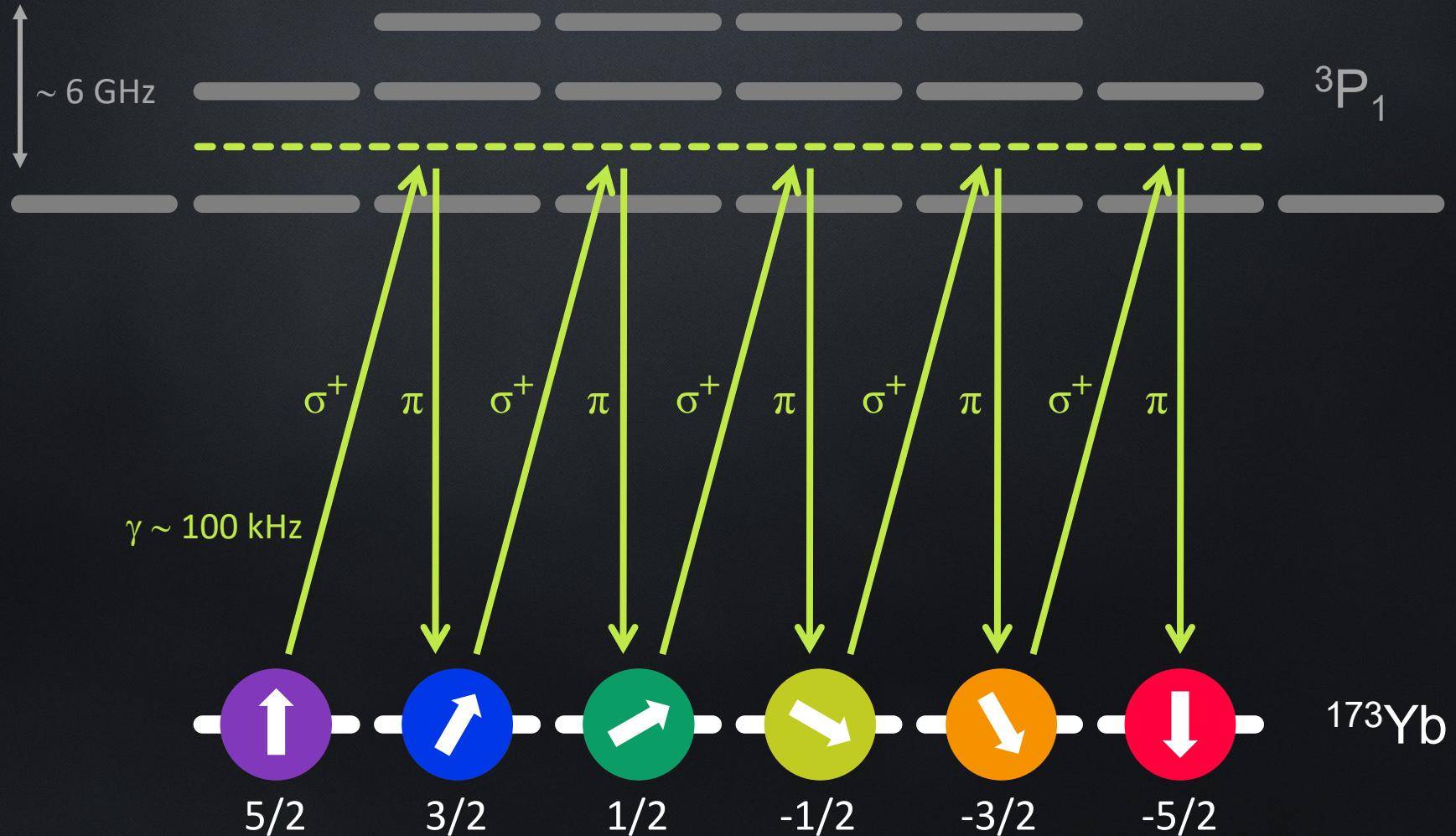
Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:



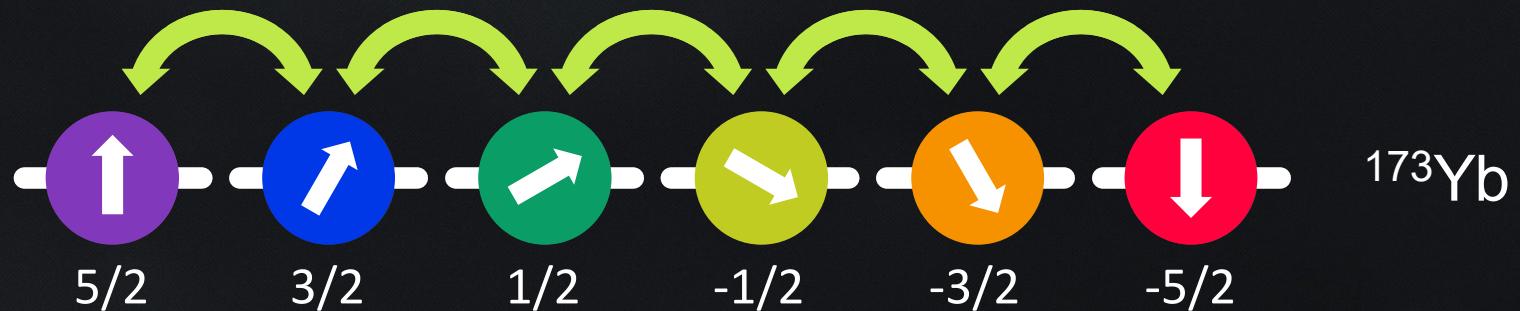
Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:



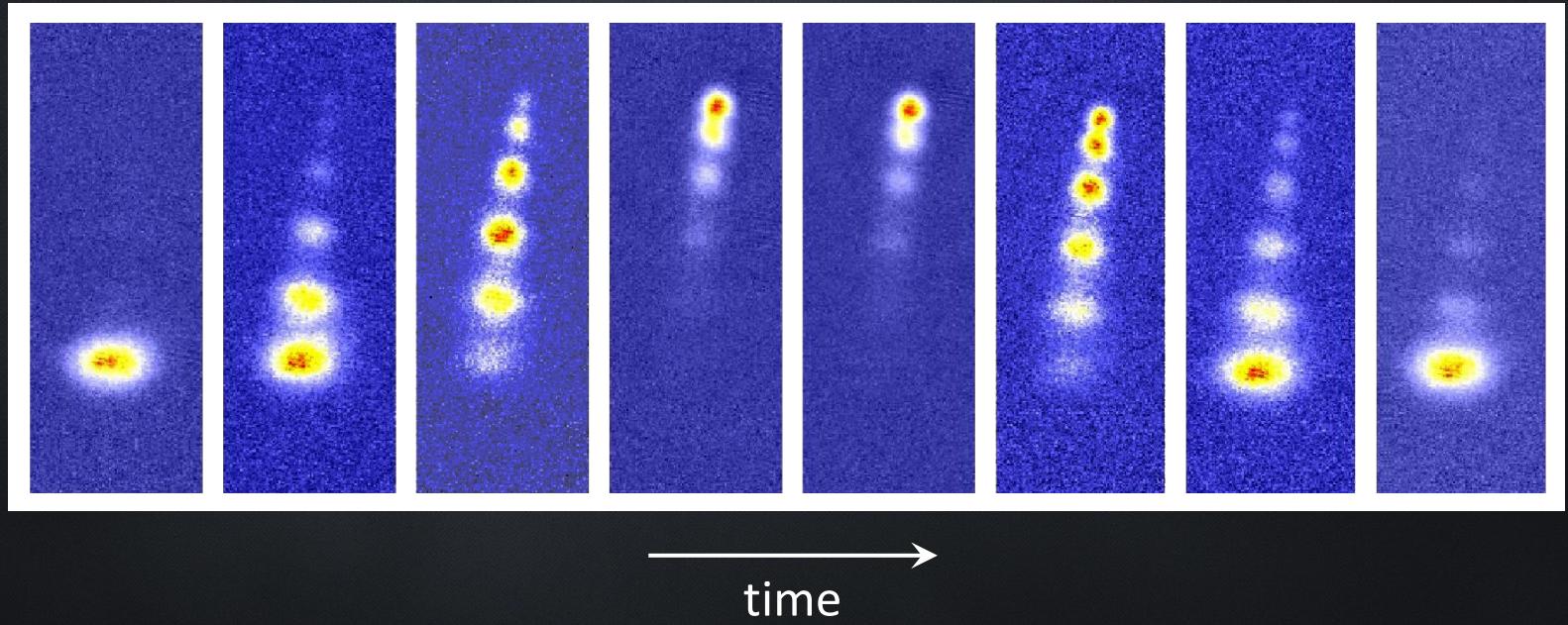
Coherent spin-flip transitions

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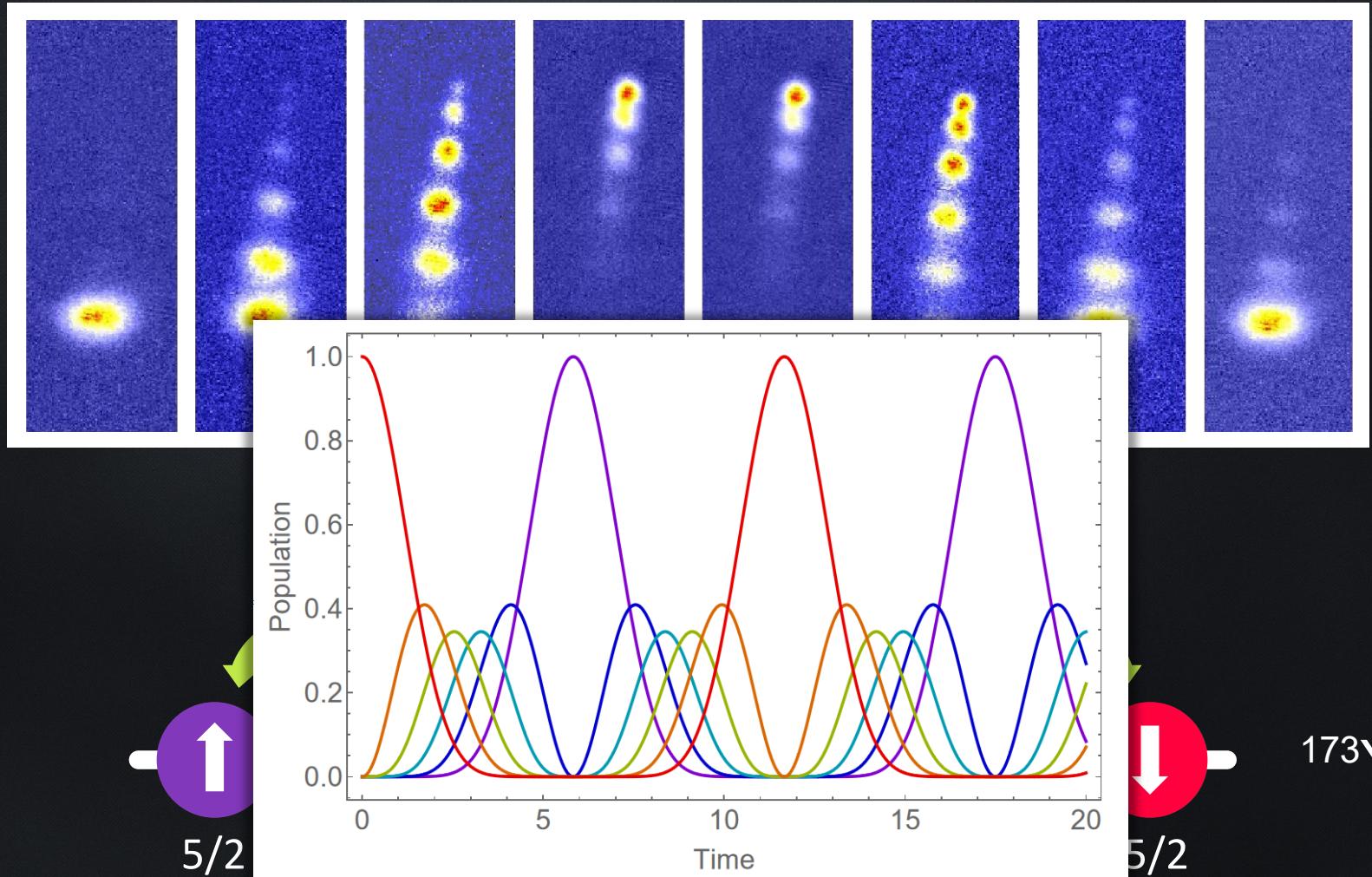
Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:

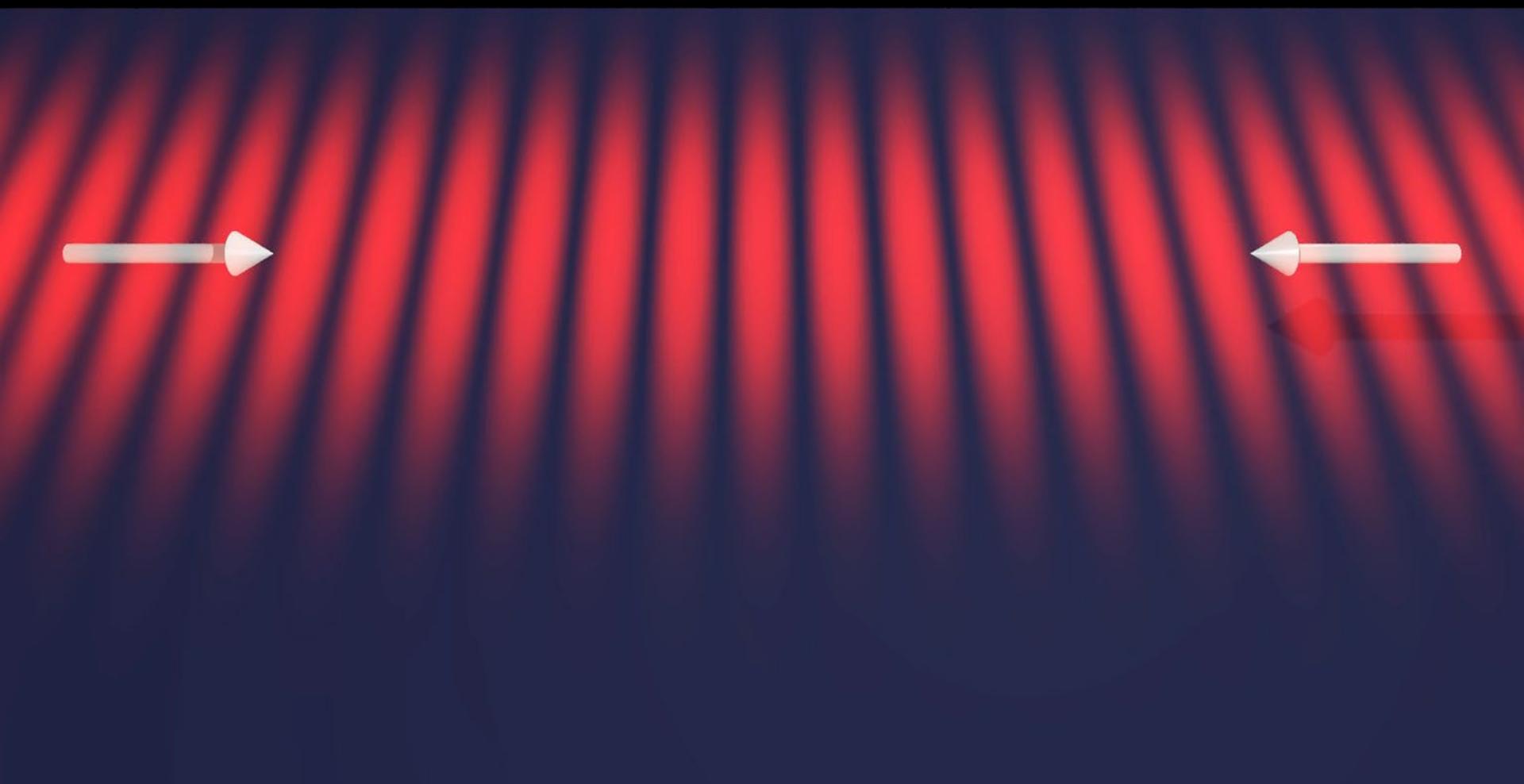


Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:

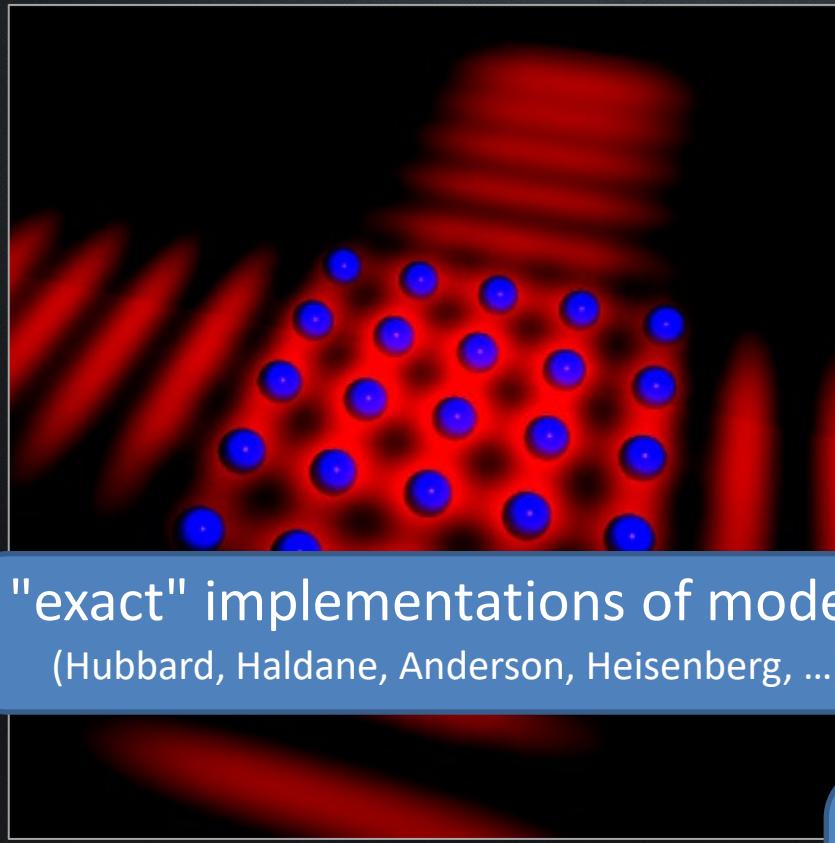


Optical lattices



Quantum simulation of condmat systems

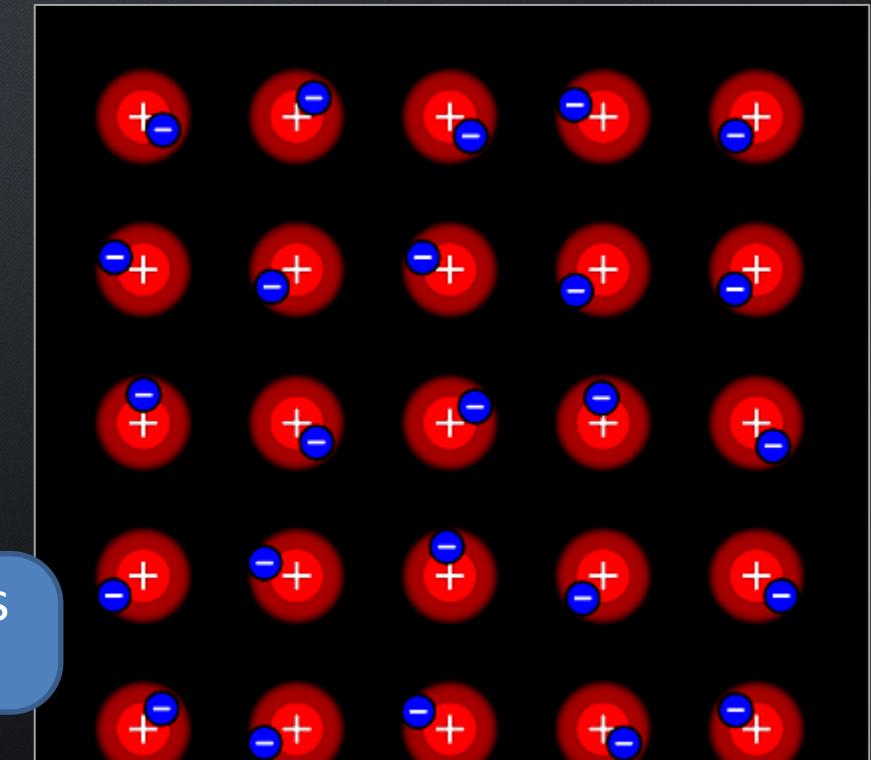
atoms in optical lattices



"exact" implementations of models

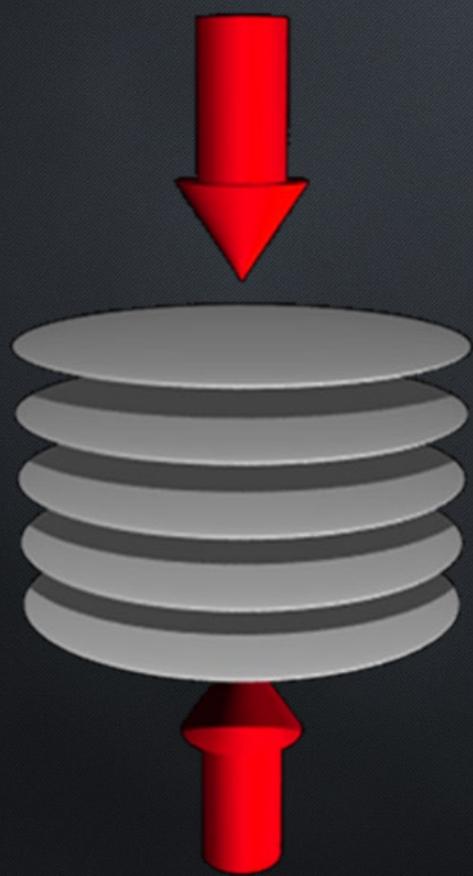
(Hubbard, Haldane, Anderson, Heisenberg, ...)

electrons in solids

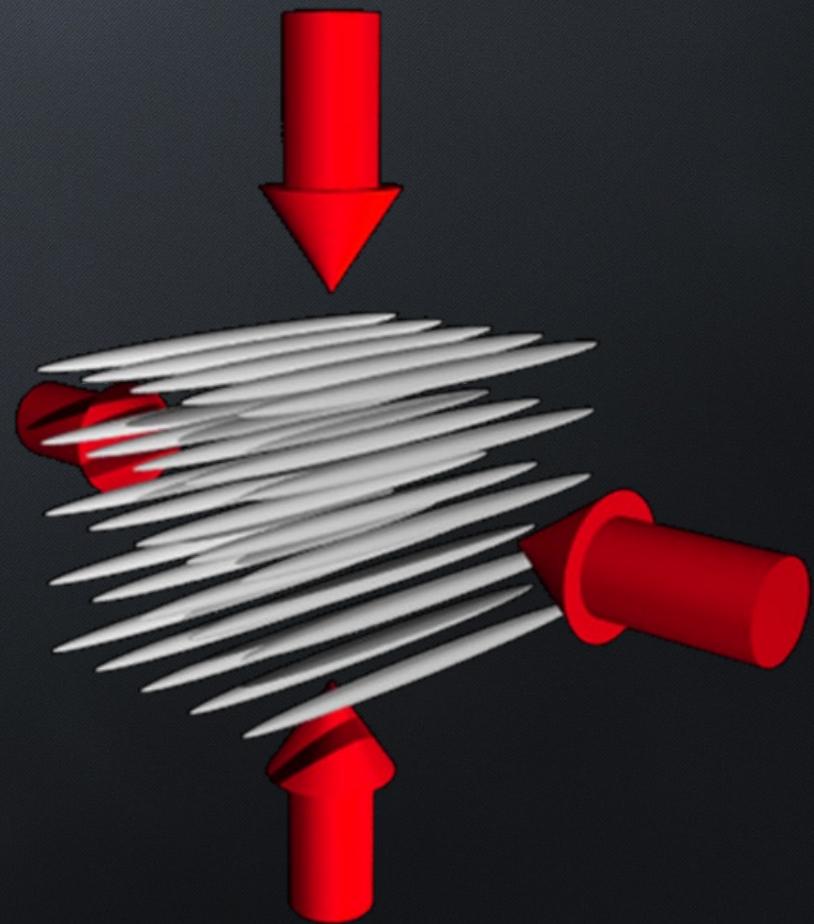


new "extreme" states of matter

Low-dimensional quantum systems



2D systems



1D systems

Lecture 1



Introduction to multicomponent quantum gases



Interactions in two-electron fermions and SU(N) physics



Experimental techniques



EXP: SU(N) physics in low dimensions



EXP: SU(N)

One-dimensional fermions with tunable SU(N) symmetry
G. Pagano et al., Nature Phys. **10**, 198 (2014)

Lecture 2



Multicomponent systems with coherent coupling



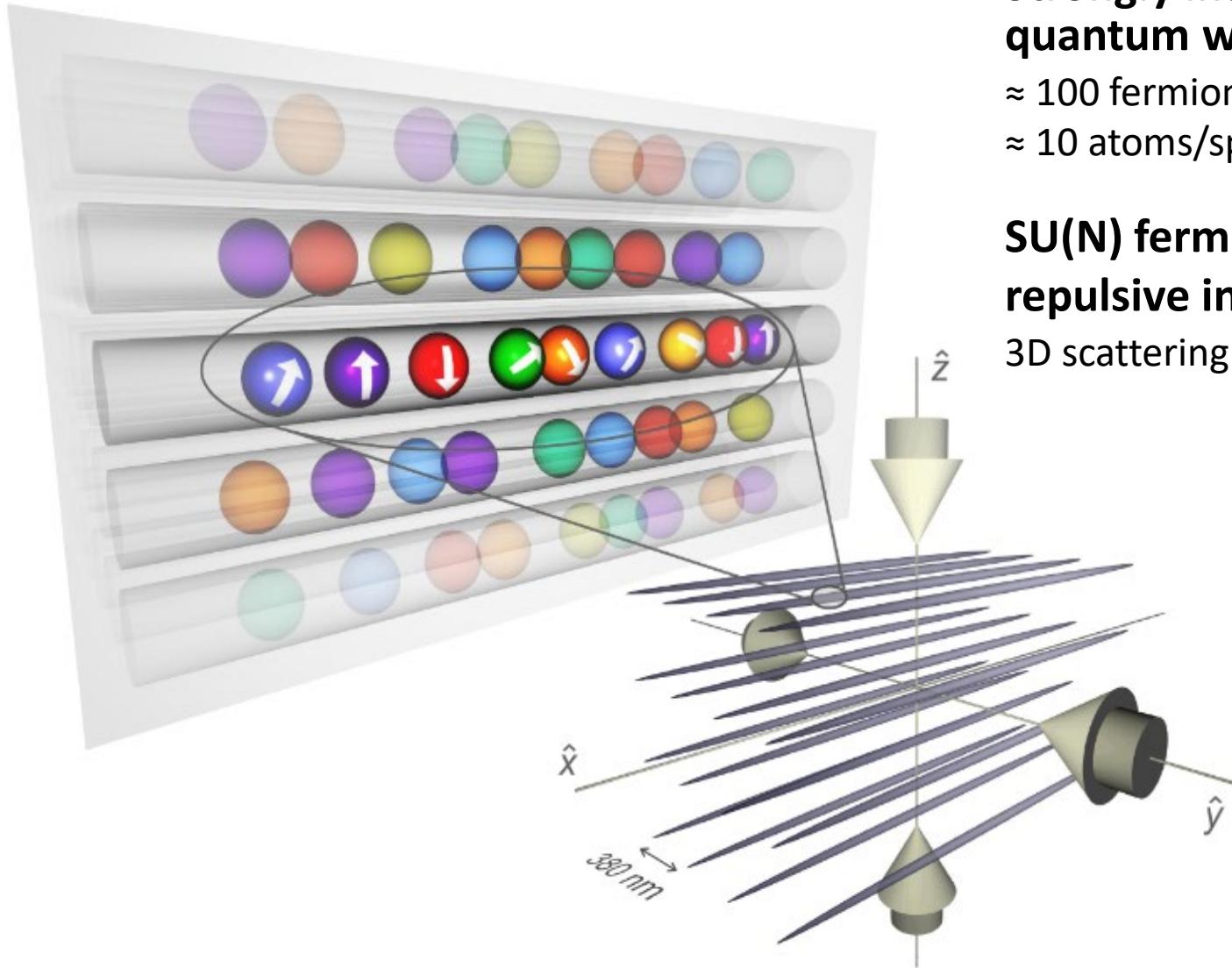
Synthetic dimensions and artificial magnetic fields



EXP: Chiral edge currents in synthetic ladders



EXP: Synthetic Hall effect



Strongly interacting quantum wires

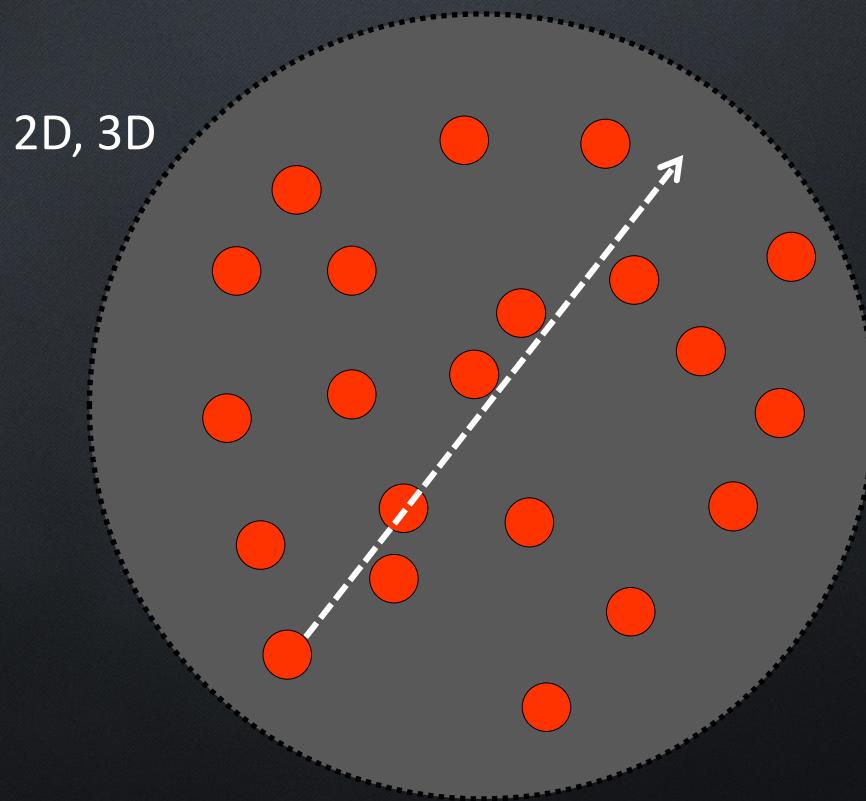
≈ 100 fermionic wires

≈ 10 atoms/spin/wire

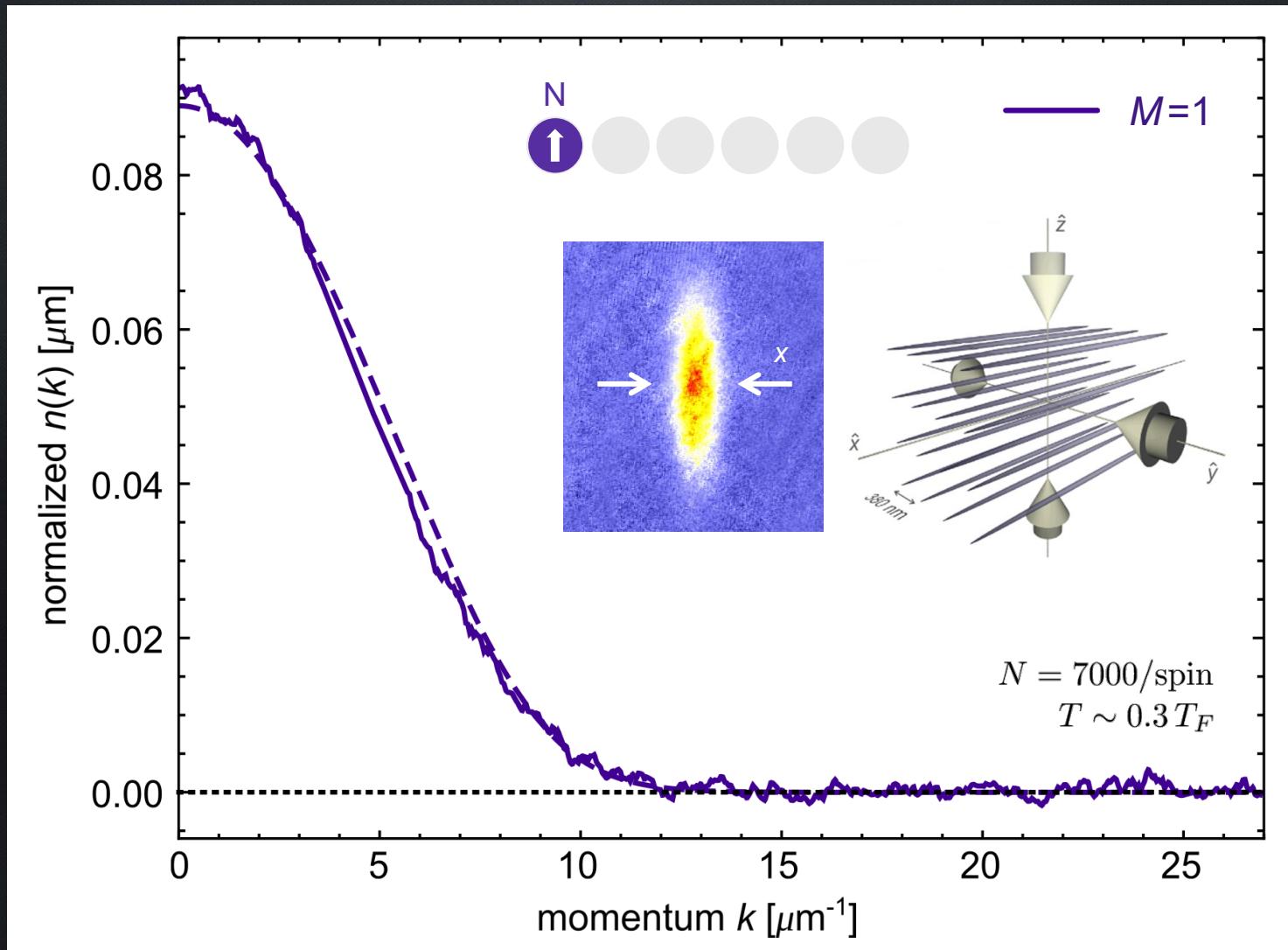
SU(N) fermions with repulsive interactions

3D scattering length $a = +200 a_0$

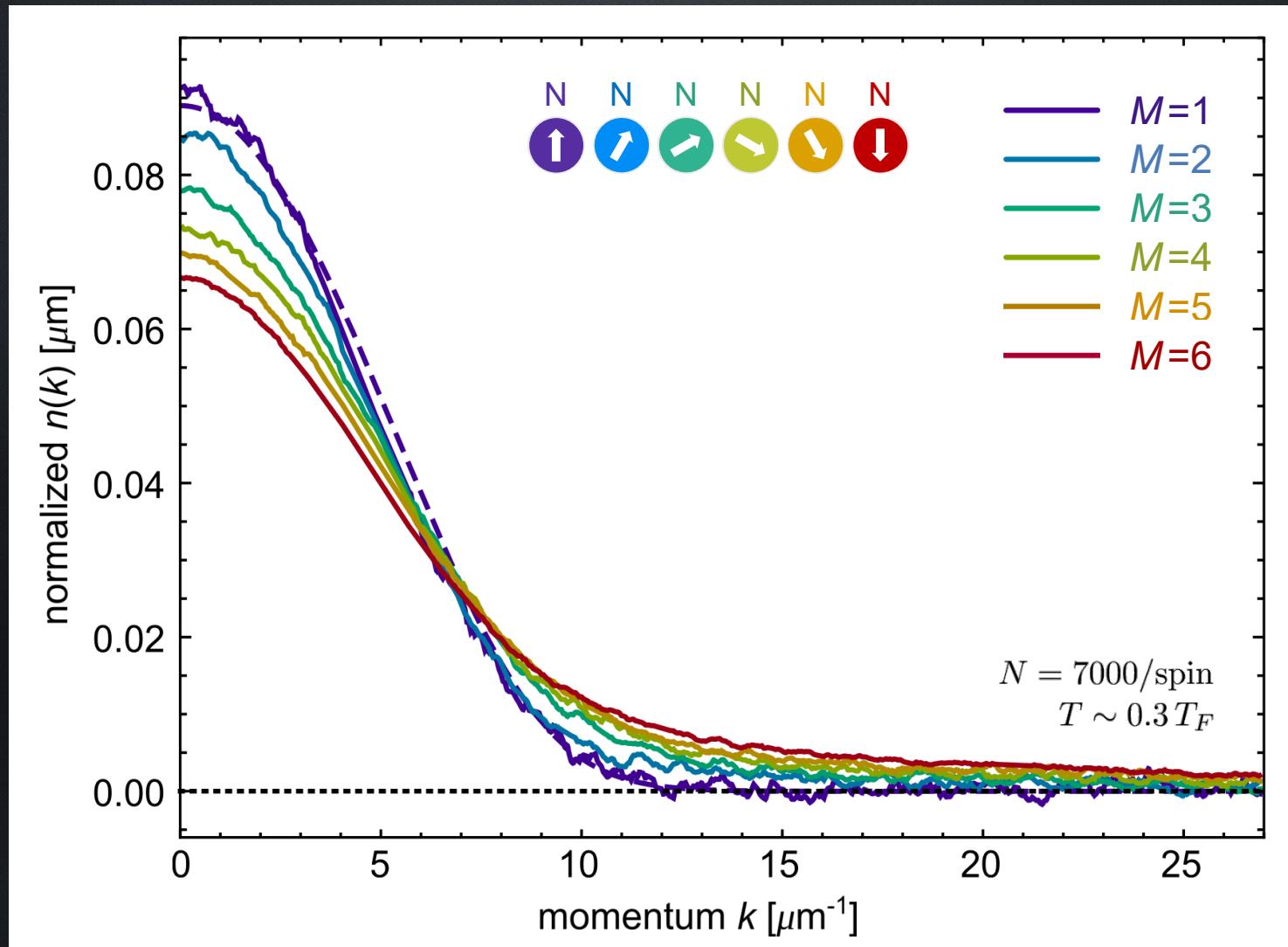
Low dimensions strongly amplify the effects of interactions between particles



Momentum distribution measured after time-of-flight expansion:

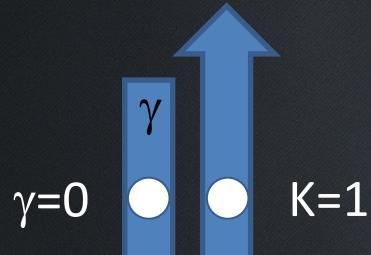


Momentum distribution measured after time-of-flight expansion:



Quantum correlations in 1D fermions

No interactions

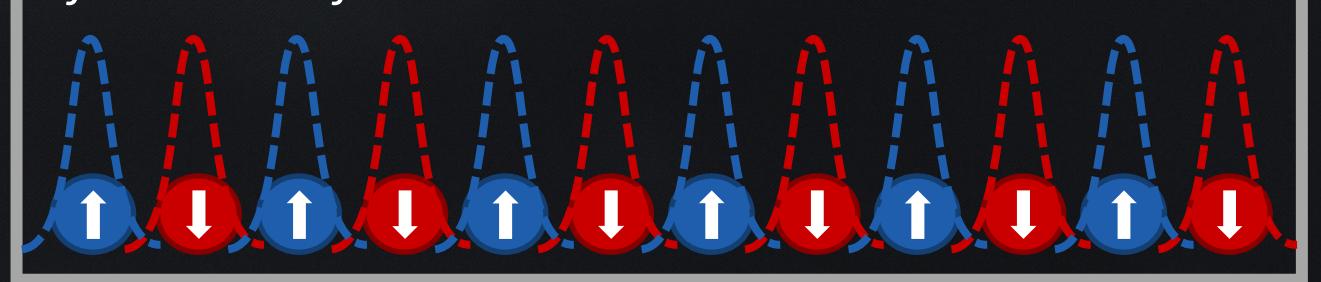


Infinite repulsion

N atoms + N atoms

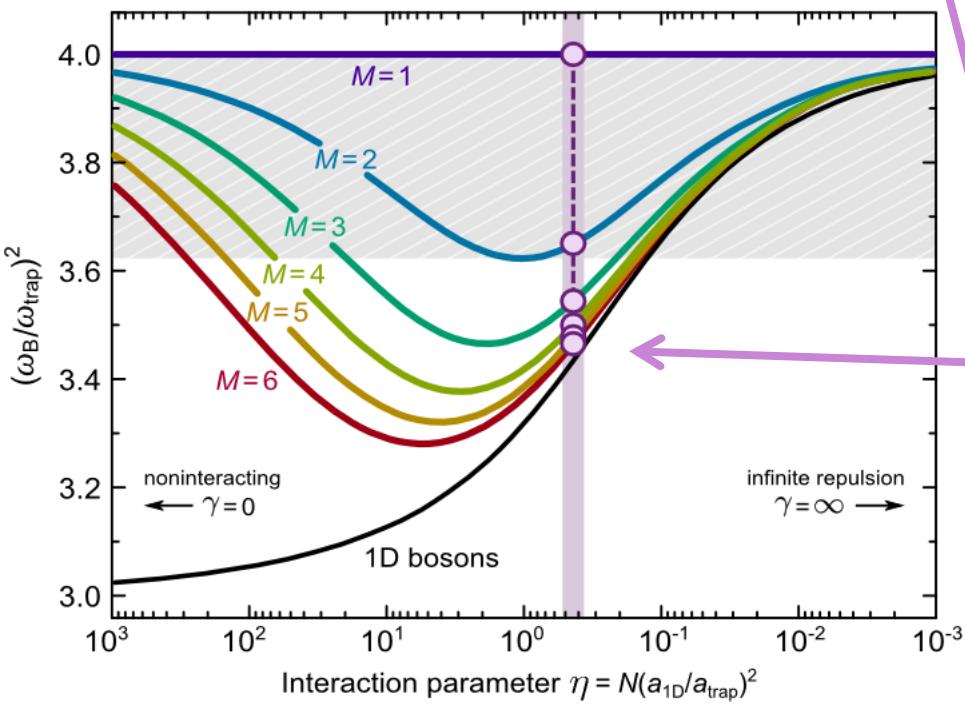
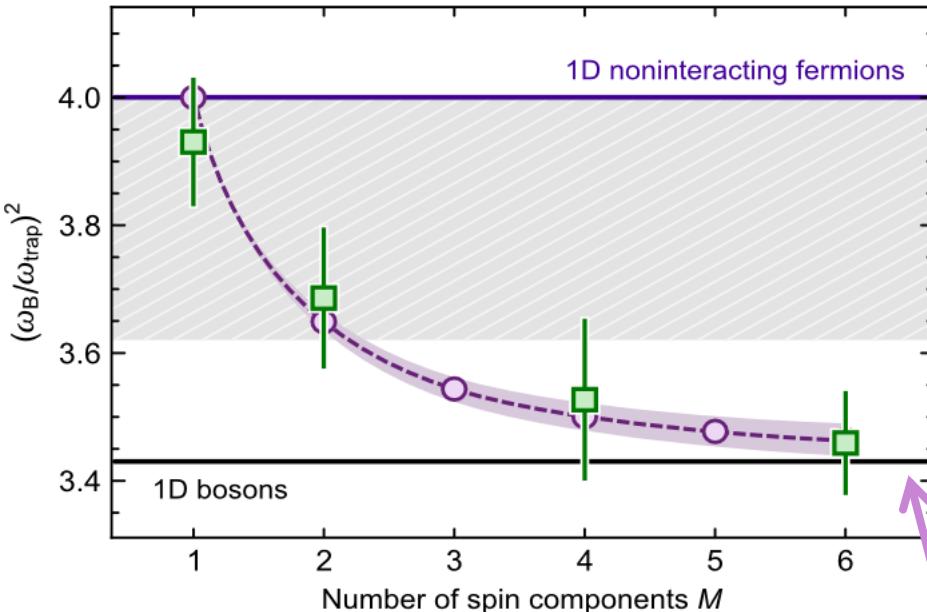


fermionized fermions!



Collective dynamics

Interaction-induced redshift of breathing frequency in harmonic trap



Theory by H. Hu & X.-J. Liu (Swinburne)
Bethe Ansatz + LDA + Hydrodynamic

For $M \rightarrow \infty$ the breathing frequency approaches that of spinless bosons

«bosonization» of large-spin fermions

Interplay between interactions and spin multiplicity (distinguishability)



“Bosonization” of large-spin fermions

A very general result first demonstrated by C. N. Yang

C. N. Yang & Y. Yi-Zhuang, Chin. Phys. Lett. 28, 020503 (2011)

One-Dimensional w -Component Fermions and Bosons with Repulsive Delta Function Interaction *

C. N. YANG(杨振宁)^{1,2**}, YOU Yi-Zhuang(尤亦庄)¹

¹Institute for Advanced Study, Tsinghua University, Beijing 100084

²Institute of Theoretical Physics, Chinese University of Hong Kong, Hong Kong

The ground state energy for such a system was studied in the 1960s, first in 1963 by Lieb and Liniger^[1] for spinless Bosons, then in 1967 by Yang^[2] for spin 1/2 Fermions, finally in 1968 by Sutherland^[3] for 3 component Fermions. This last result is readily generalizable to any value of w = number of components. In the present paper we complete this series of studies by solving the problem for w component Bosons via a detour through ∞ component Fermions.

Theorem 2.

$$Y_{F\infty}(Z) = Y_{B1}(Z).$$



See also measurement of Tan's contact
(Hong Kong, Gyu-Boong Jo)

B. Song et al., PRX 10, 041053 (2020)

