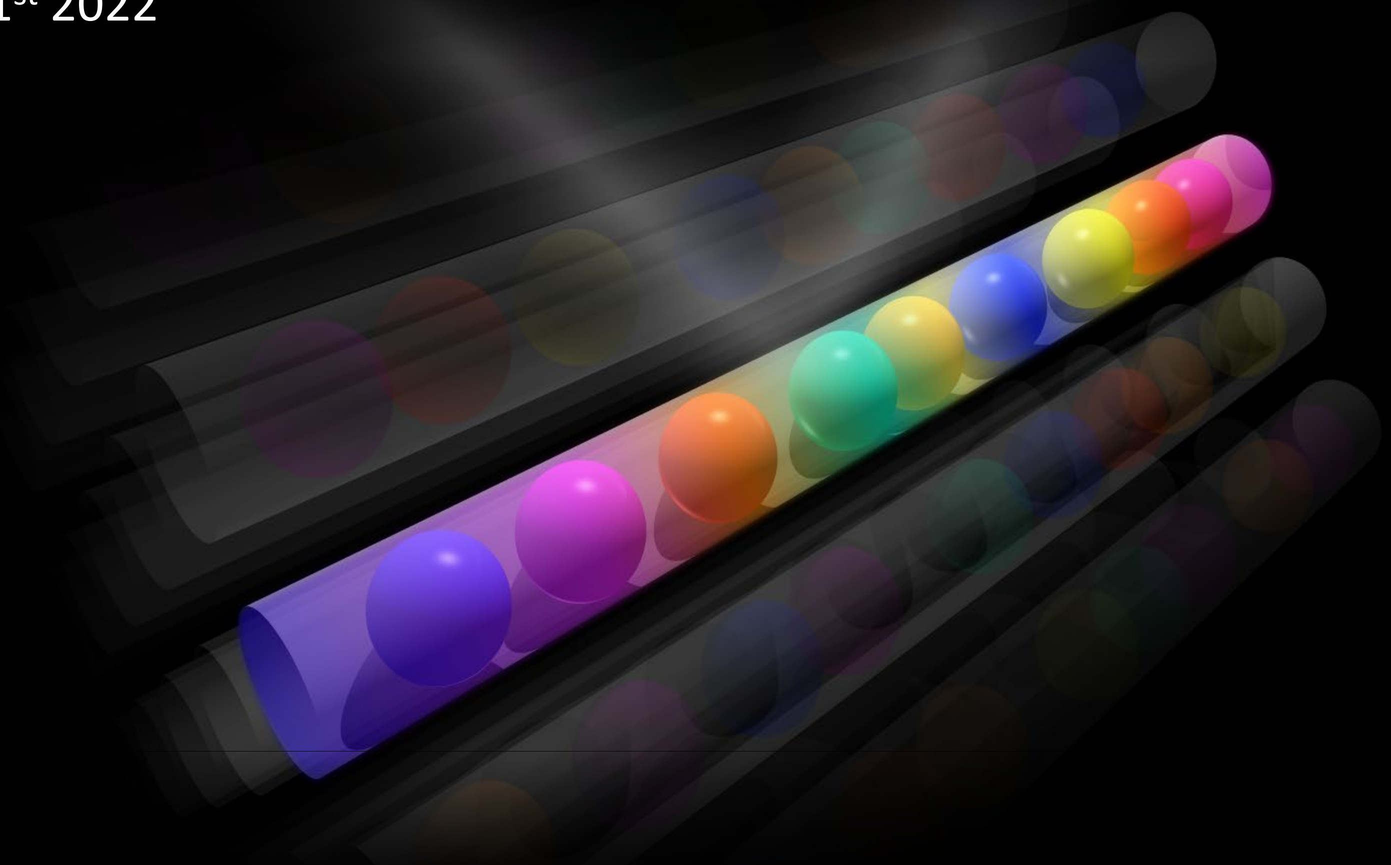
Multicomponent spin mixtures of two-electron fermions

Enrico Fermi School on Quantum Mixtures
Varenna, July 21st 2022



Leonardo Fallani

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LENS European Laboratory for Nonlinear Spectroscopy

Lecture 1

- Introduction to multicomponent quantum gases
- Interactions in two-electron fermions and SU(N) physics
- Experimental techniques
- **EXP**: SU(N) physics in low dimensions
- **EXP**: SU(N) Fermi-Hubbard and breaking SU(N) physics

Lecture 2

- Multicomponent systems with coherent coupling
- Synthetic dimensions and artificial magnetic fields
- **EXP**: Chiral edge currents in synthetic ladders
- **EXP**: Synthetic Hall effect

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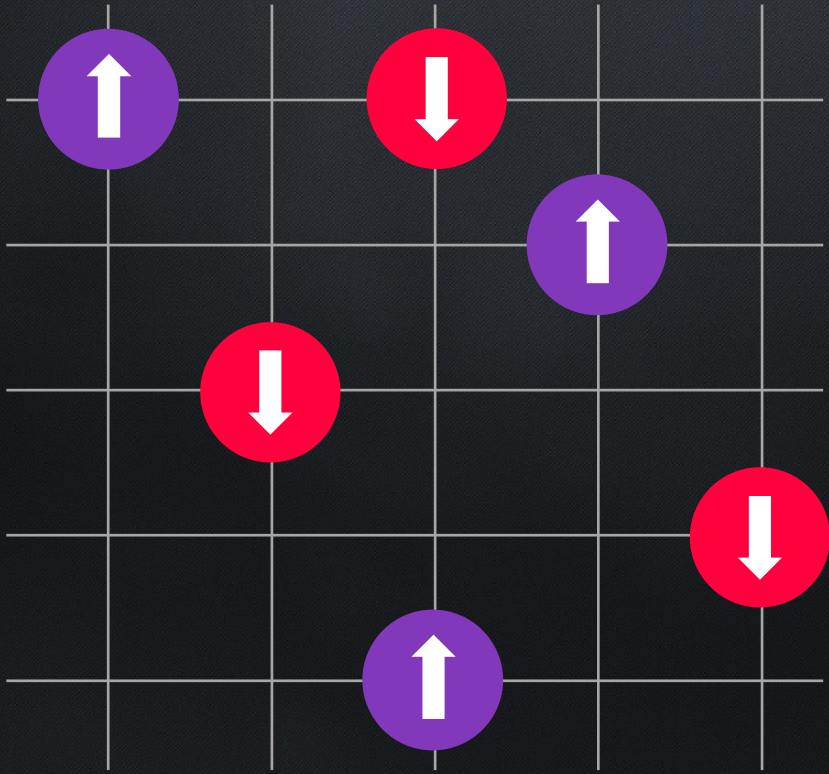
Two quantum states

Two-level systems are everywhere in Nature
and in the way we model and take advantage of quantum processes

Two quantum states

Two-level systems are everywhere in Nature
and in the way we model and take advantage of quantum processes

Spin of elementary particles (e.g. electrons)



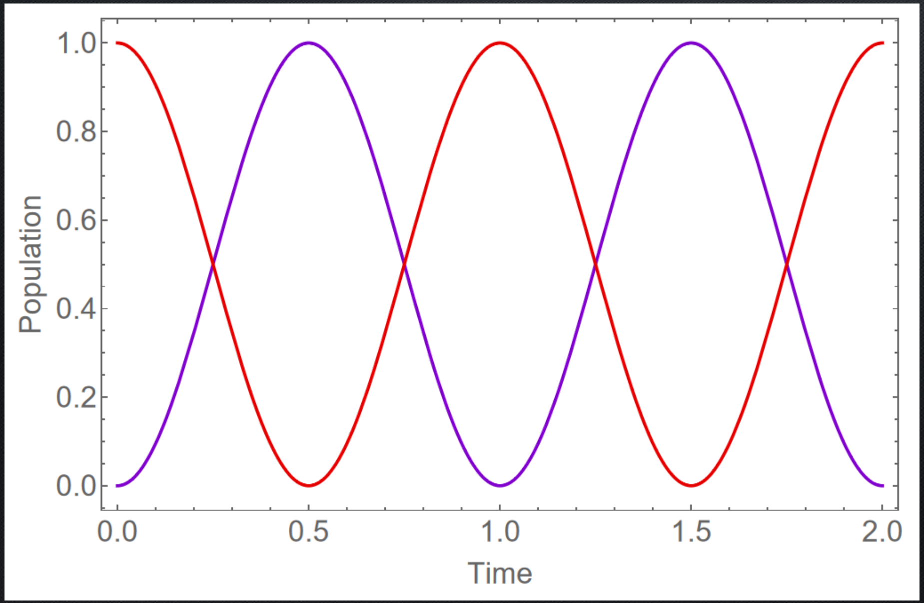
Superconductivity
Magnetism
...

and their realization in binary ultracold mixtures

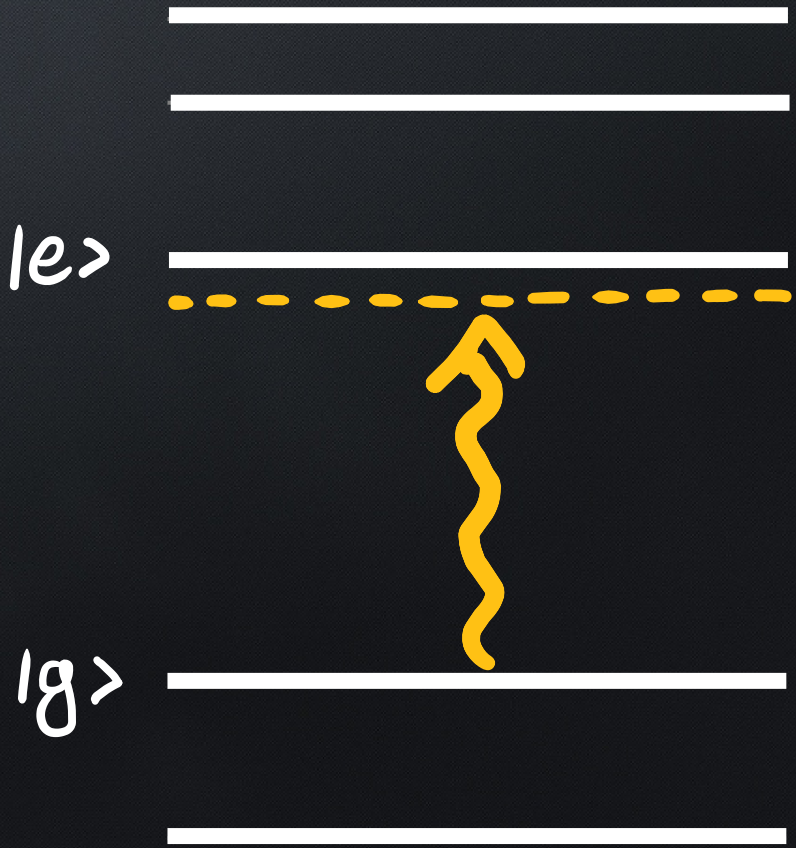
Two quantum states

Two-level systems are everywhere in Nature
and in the way we model and take advantage of quantum processes

Light-matter interaction



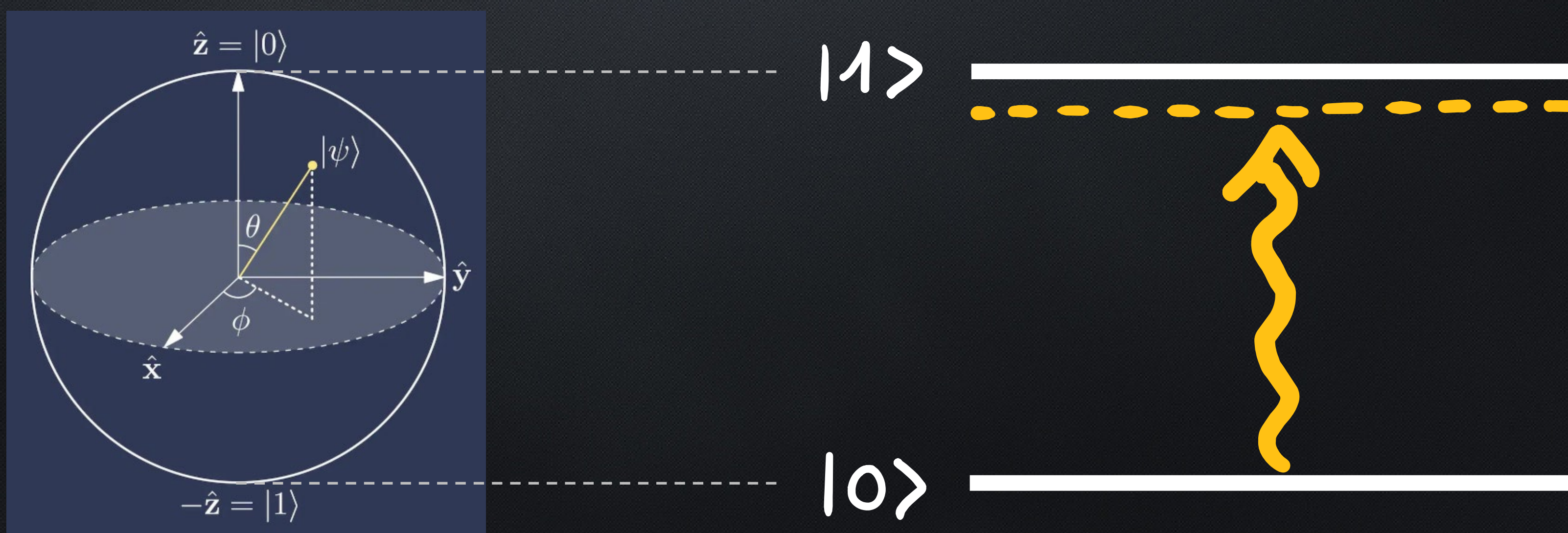
two-level coherent dynamics



Two quantum states

Two-level systems are everywhere in Nature
and in the way we model and take advantage of quantum processes

Quantum information



qubit
Bloch sphere

More states!

Extend the Hilbert space!

Larger space: more directions to explore, more fun

$|\searrow\rangle$ $|c\rangle$ $|4\rangle$

$|\rightarrow\rangle$ $|b\rangle$ $|3\rangle$

$|\nearrow\rangle$ $|a\rangle$ $|2\rangle$

$|\uparrow\rangle$ $|e\rangle$ $|1\rangle$

$|\downarrow\rangle$ $|g\rangle$ $|0\rangle$

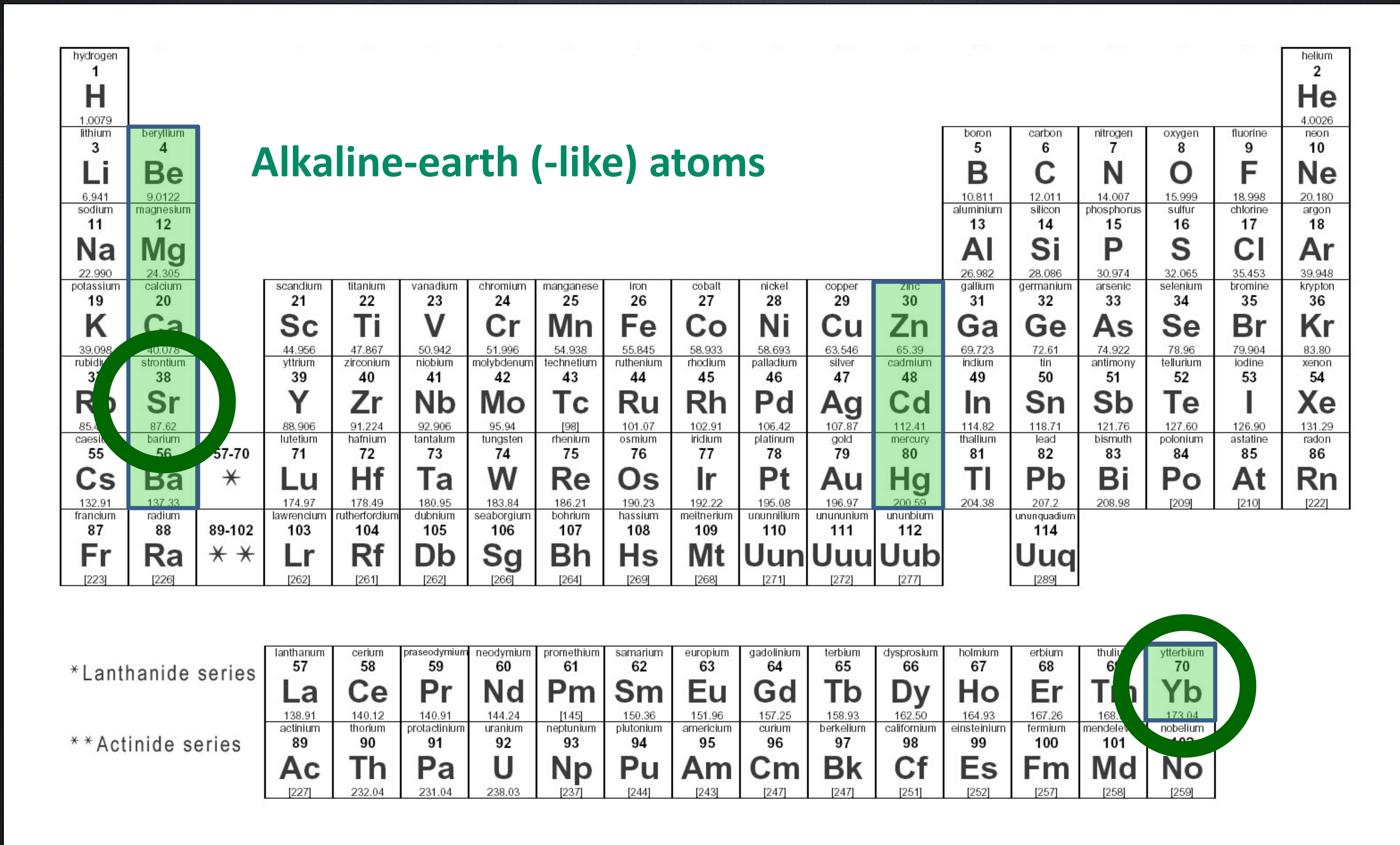
Spin mixtures

Atoms are not elementary particles:
2F+1-dimensional spin manifold available

^{173}Yb
F = 5/2

^{87}Sr
F = 9/2

Spin mixture of two-electron fermions



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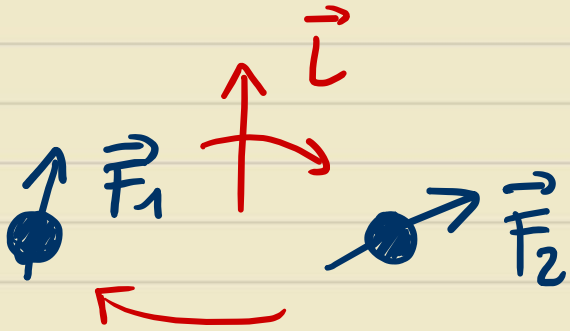
Interactions in multicomponent mixtures



$\vec{L} = 0$ S-WAVE COLLISIONS

$\vec{F} = \vec{F}_1 + \vec{F}_2$ CONSERVED

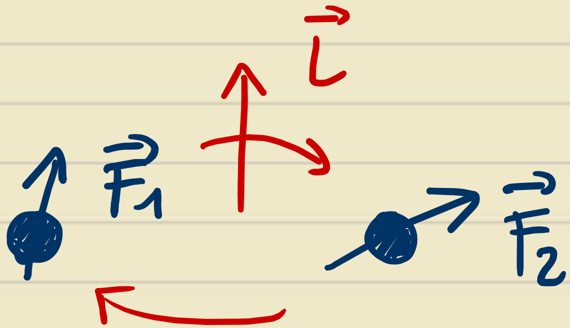
$$V(\vec{r}) = \sum_F g_F P_F \delta(\vec{r})$$ SHORT-RANGE INTERACTIONS

$$g_F = \frac{4\pi\hbar^2 a_F}{\mu}$$

$$P_F = \sum_{M=-F}^{F} |FM\rangle\langle FM|$$

(see Ueda's lectures)

Interactions in multicomponent mixtures



$\vec{L} = 0$ S-WAVE COLLISIONS

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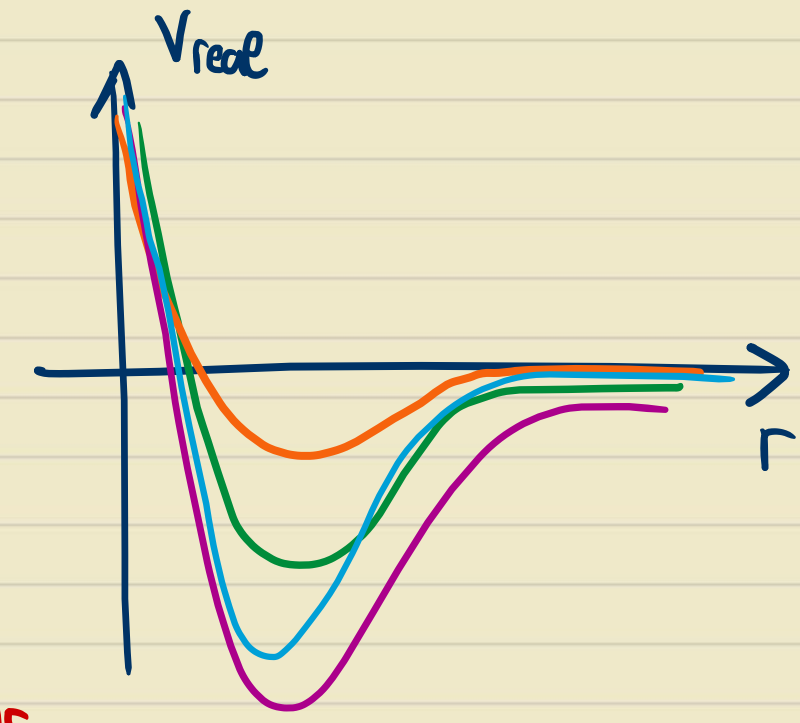
$$V(\vec{r}) = \sum_F g_F P_F \delta(\vec{r})$$

$$g_F = \frac{4\pi\hbar^2 a_F}{m}$$

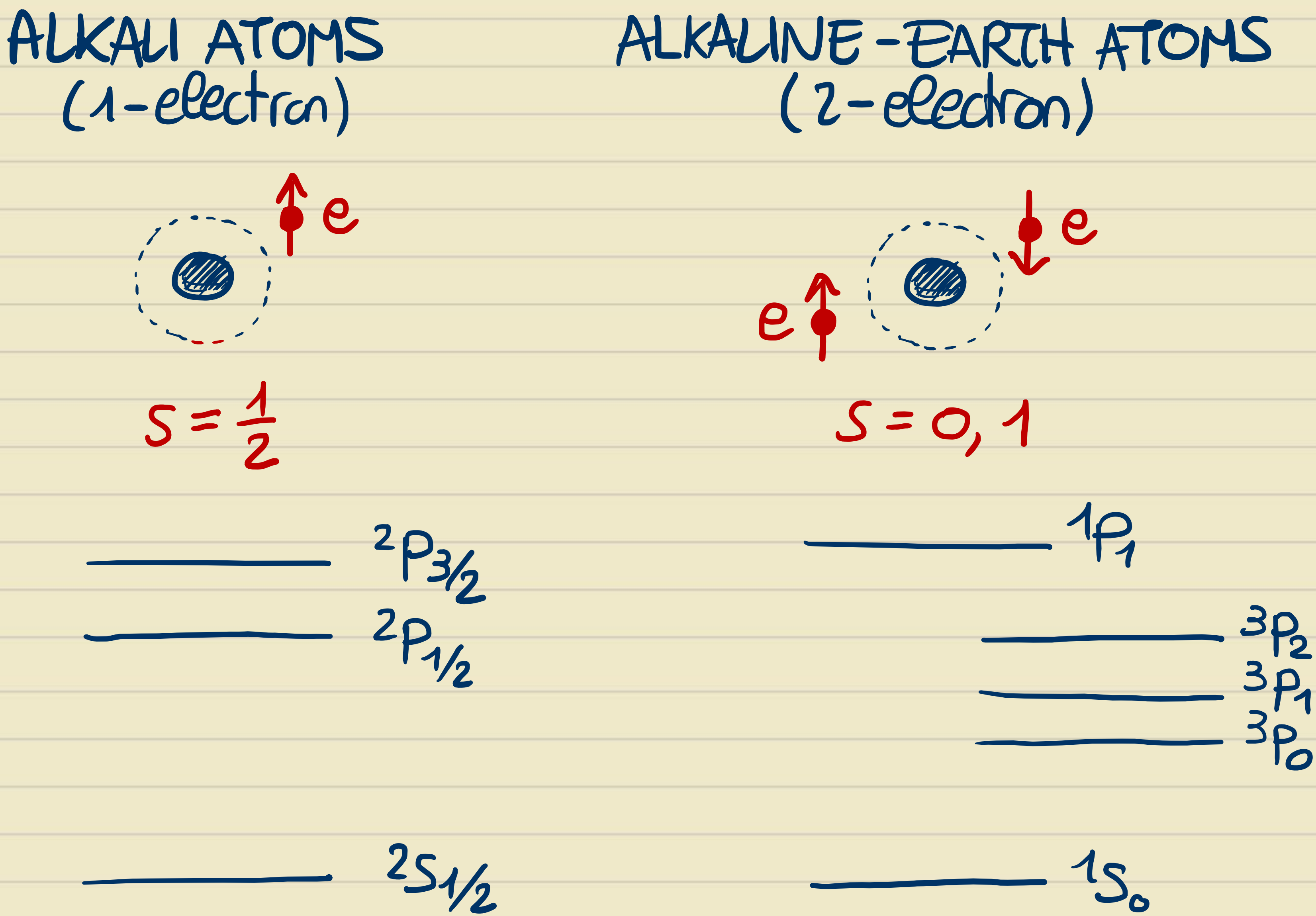
generally depends on F

↓

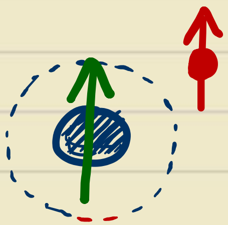
SPIN-DEPENDENT INTERACTIONS



Alkali and alkaline-earth atoms



ALKALI ATOMS
(1-electron)



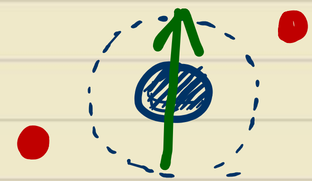
$J = S = \frac{1}{2}$

$I \neq 0$

$H_{hf} \sim \vec{I} \cdot \vec{J} \neq 0$

$^{2}S_{1/2}$

ALKALINE-EARTH ATOMS
(2-electron)



$J = S = 0$

$I \neq 0$ FERMIONS

$I = 0$ BOSONS

$H_{hf} \sim \vec{I} \cdot \vec{J} = 0$

$^{1}S_{0}$

Alkaline-earth fermions

ALKALINE-EARTH ATOMS
(2-electron)

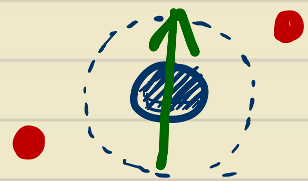
NO ELECTRON SPIN
IN GROUND STATE
(ONLY NUCLEAR SPIN)

↓

NO HYPERFINE INTERACTION

↓

NUCLEAR SPIN DECOUPLED
FROM ELECTRON D.O.F.



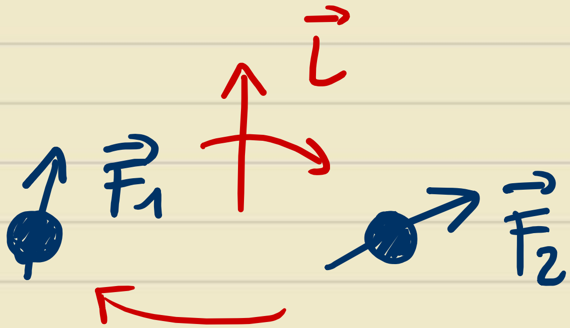
$J = S = 0$

$I \neq 0$ FERMIONS

$H_{hf} \sim \vec{I} \cdot \vec{J} = 0$

$^1S_0$

Interactions in alkaline-earth fermions



$\vec{L} = 0$ S-WAVE COLLISIONS

$\vec{F} = \vec{F}_1 + \vec{F}_2$ CONSERVED

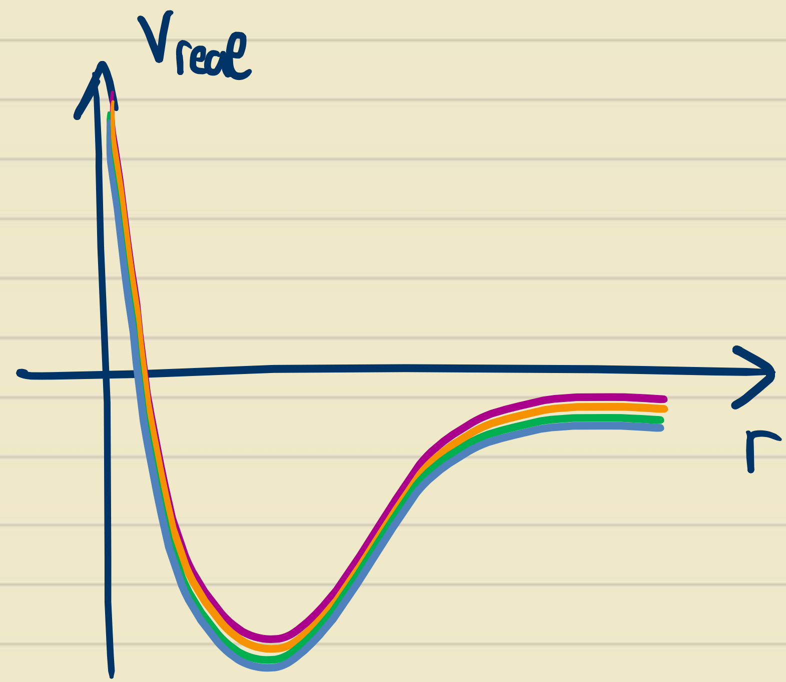
$$V(\vec{r}) = \sum_F g_F P_F \delta(\vec{r})$$

$$g_F = \frac{4\pi\hbar^2 a_F}{\mu}$$

does not depend on F

↓

SPIN-INDEPENDENT INTERACTIONS



SU(N) fermions

INTERACTING MIXTURE OF TWO-ELECTRON FERMIONS $(m = -F \dots F)$

$$\hat{H} = \int d\vec{r} \left(\frac{\hbar^2}{2M} \sum_m \nabla\psi_m^{\dagger} \nabla\psi_m + \frac{g}{2} \sum_{m,n} \psi_m^{\dagger} \psi_n^{\dagger} \psi_n \psi_m \right)$$

$\hat{H}$ COMMUTES WITH SPIN-PERMUTATION OPERATORS

$$\hat{S}_{mn} = \int d\vec{r}\, \psi_m^{\dagger} \psi_n \qquad [\hat{S}_{mn}, \hat{S}_{pq}] = \delta_{mq} S_{pn} - \delta_{pn} S_{mq}$$

GENERATORS OF SU(N)

$[\hat{H}, \hat{S}_{mn}] = 0 \rightarrow$ SU(N) SYMMETRY $\qquad (N = 2F + 1)$

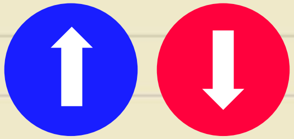
$[\hat{H}, \hat{S}_{mm}] = 0 \rightarrow N_m = \langle \hat{S}_{mm} \rangle$ NUMBER OF ATOMS IN EACH m IS CONSTANT (NO SPIN EXCHANGE)

M. Cazalilla and A. M. Rey, Rep. Prog. Phys. **77**, 124401 (2014).

SU(N) fermions

N = 2 COMPONENTS → SU(2)

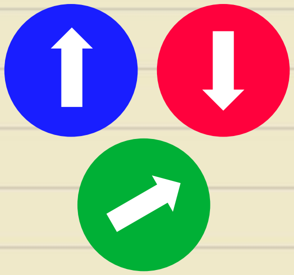
$N^2 - 1 = 3$ generators → PAULI MATRICES



$$\sigma_1 \equiv \sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 \equiv \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 \equiv \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

N = 3 COMPONENTS → SU(3)

$N^2 - 1 = 8$ generators → GELL-MANN MATRICES



$$\lambda_1 := \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 := \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_4 := \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 := \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda_7 := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 := \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

SU(N) fermions

Fermions with (large) nuclear spin: **tunable SU(N) interaction symmetry**

Stable multicomponent mixtures

Larger Hilbert space

Spectral degeneracies

Exotic quantum phases

Role of quantum statistics

Connections with high-energy physics

…

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Two-electron fermions

Nuclear spin: all-optical manipulation

No magnetic trapping!
(weak magnetic dipoles $\mu_N = \mu_B$ / 1836: $\Delta E = m_F \times 200$ Hz/G)

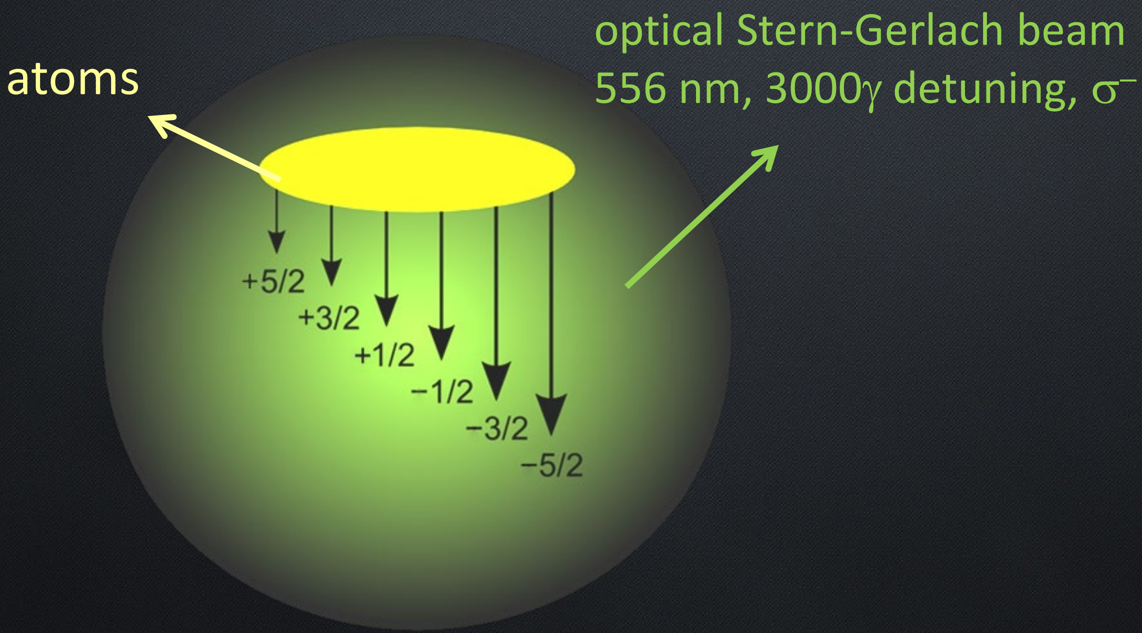
No Feshbach resonances!
(no role of nuclear spin in collisions)

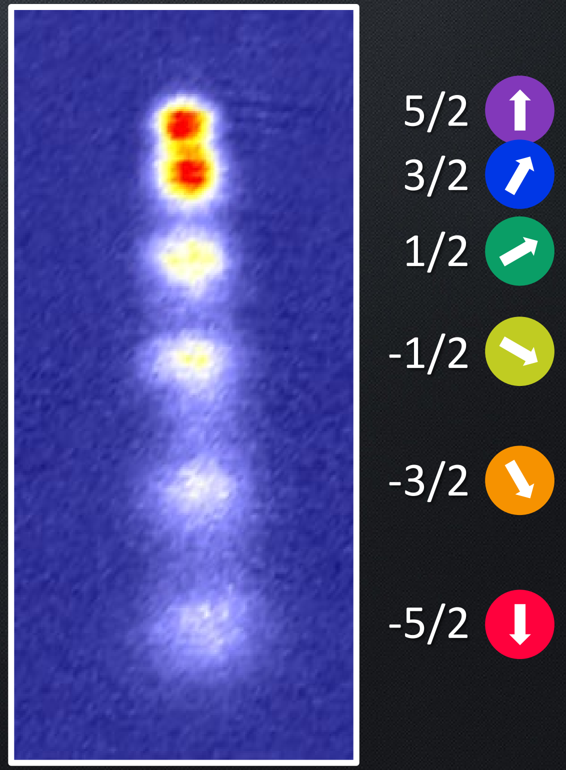
Long nuclear spin coherence times
(weak effect of magnetic field noise)

Spin detection and manipulation

Optical Stern-Gerlach detection Taie et al., PRL (2010)

State-dependent optical dipole force



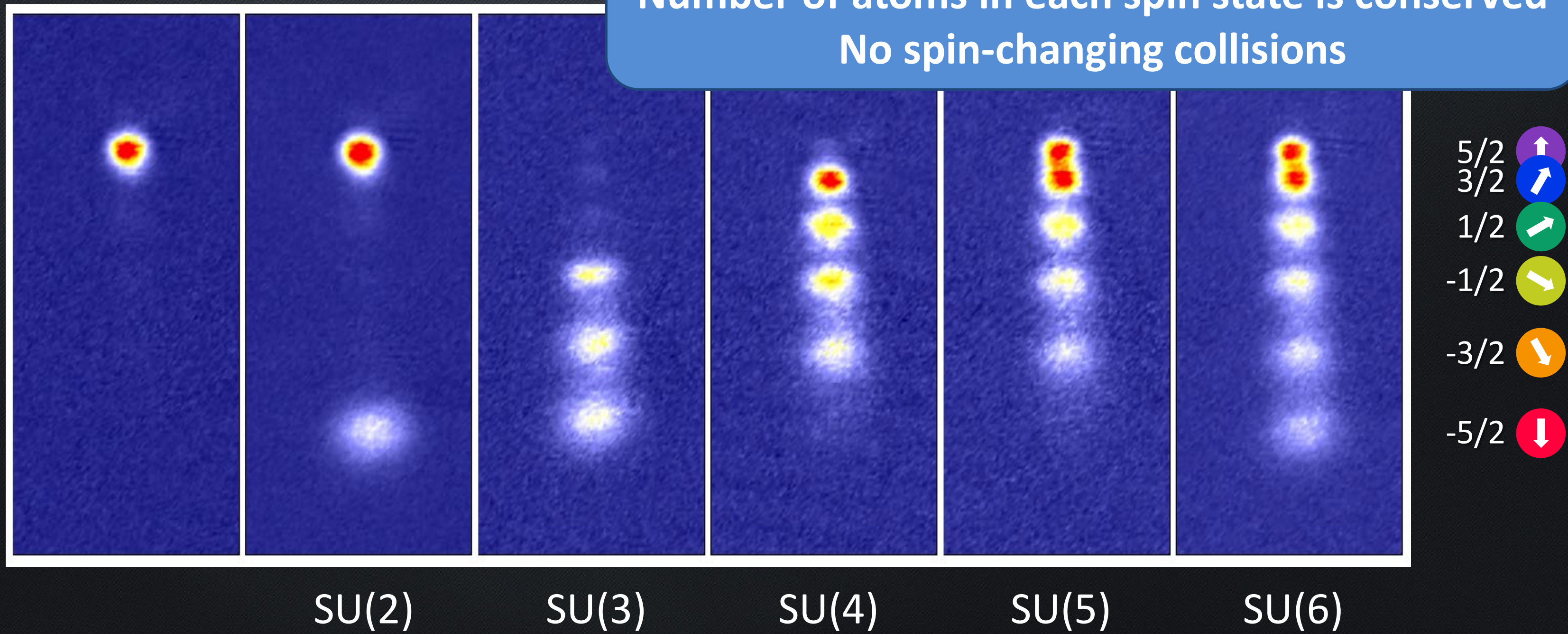


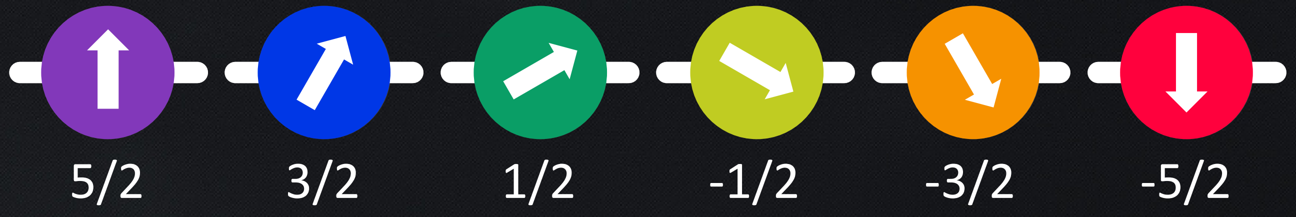
Spin detection and manipulation

Spin-selective preparation with optical pumping

Arbitrary-number spin mixtures:

Number of atoms in each spin state is conserved
No spin-changing collisions





^{173}Yb

Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:

Coherent spin-flip transitions

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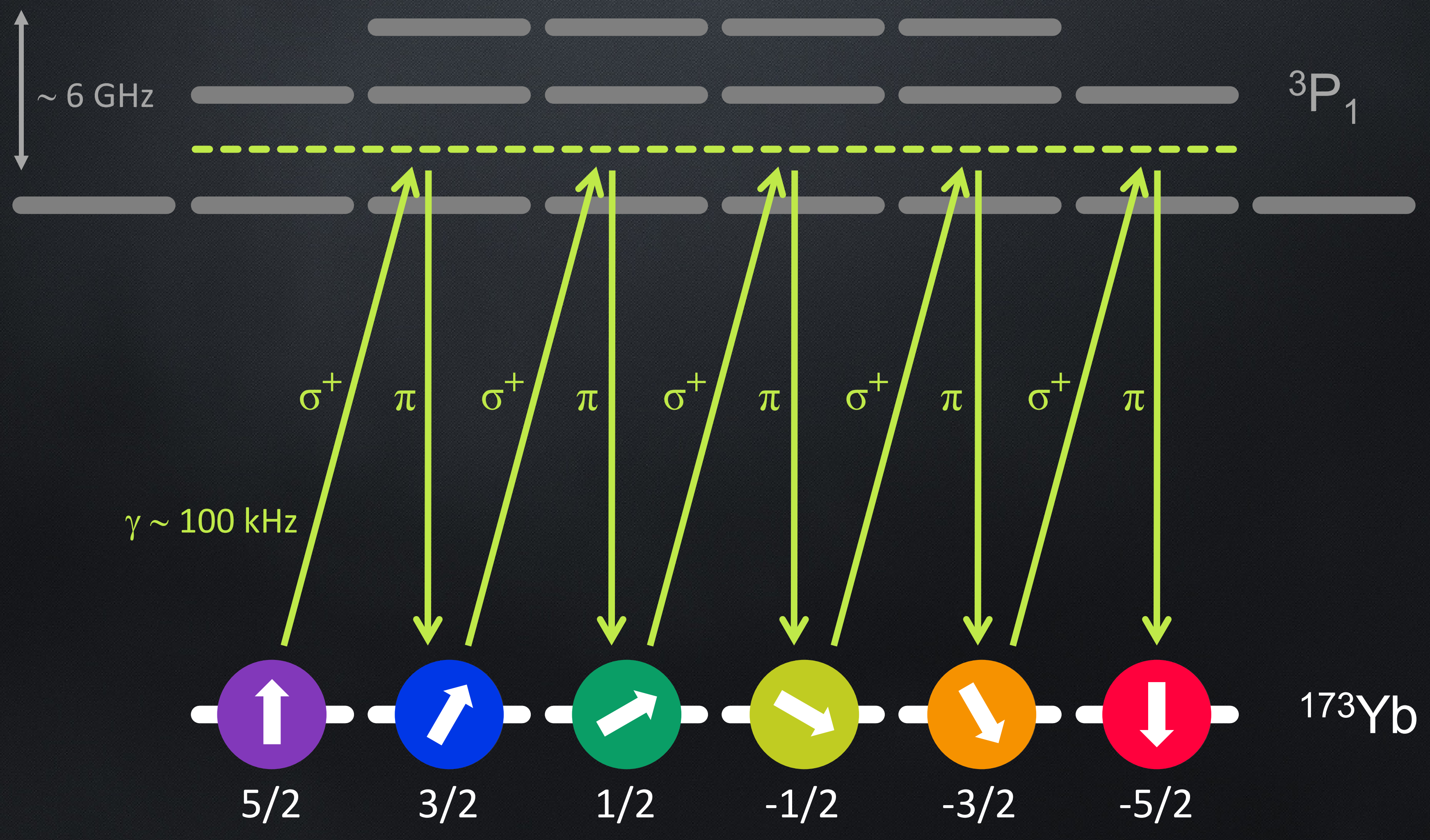

Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:

Coherent spin-flip transitions

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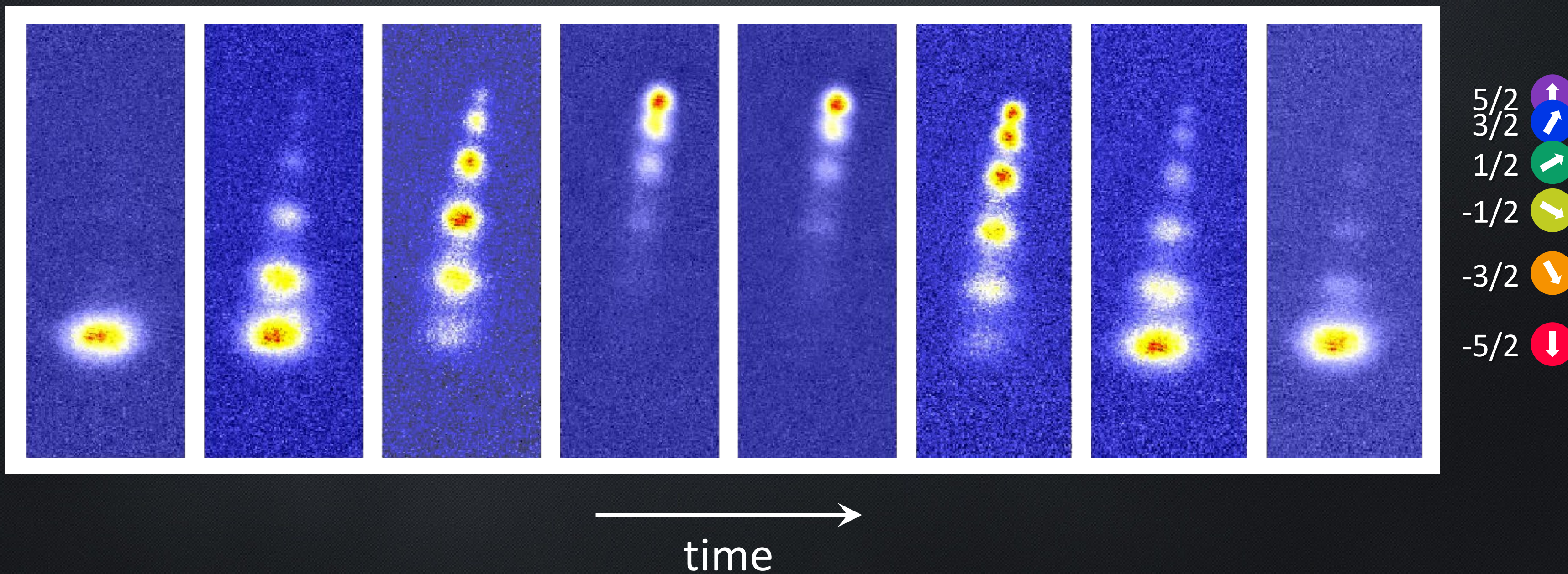
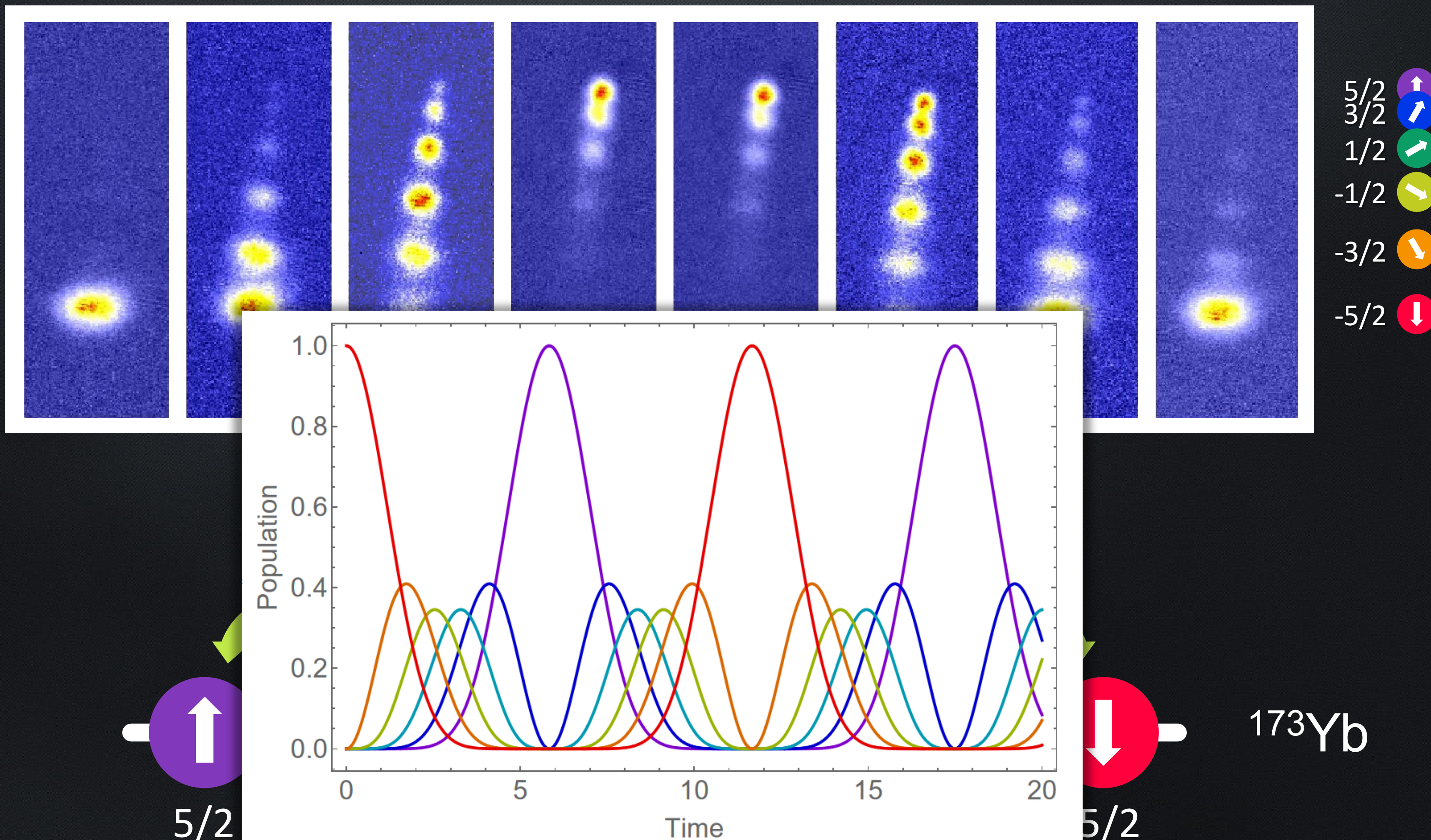
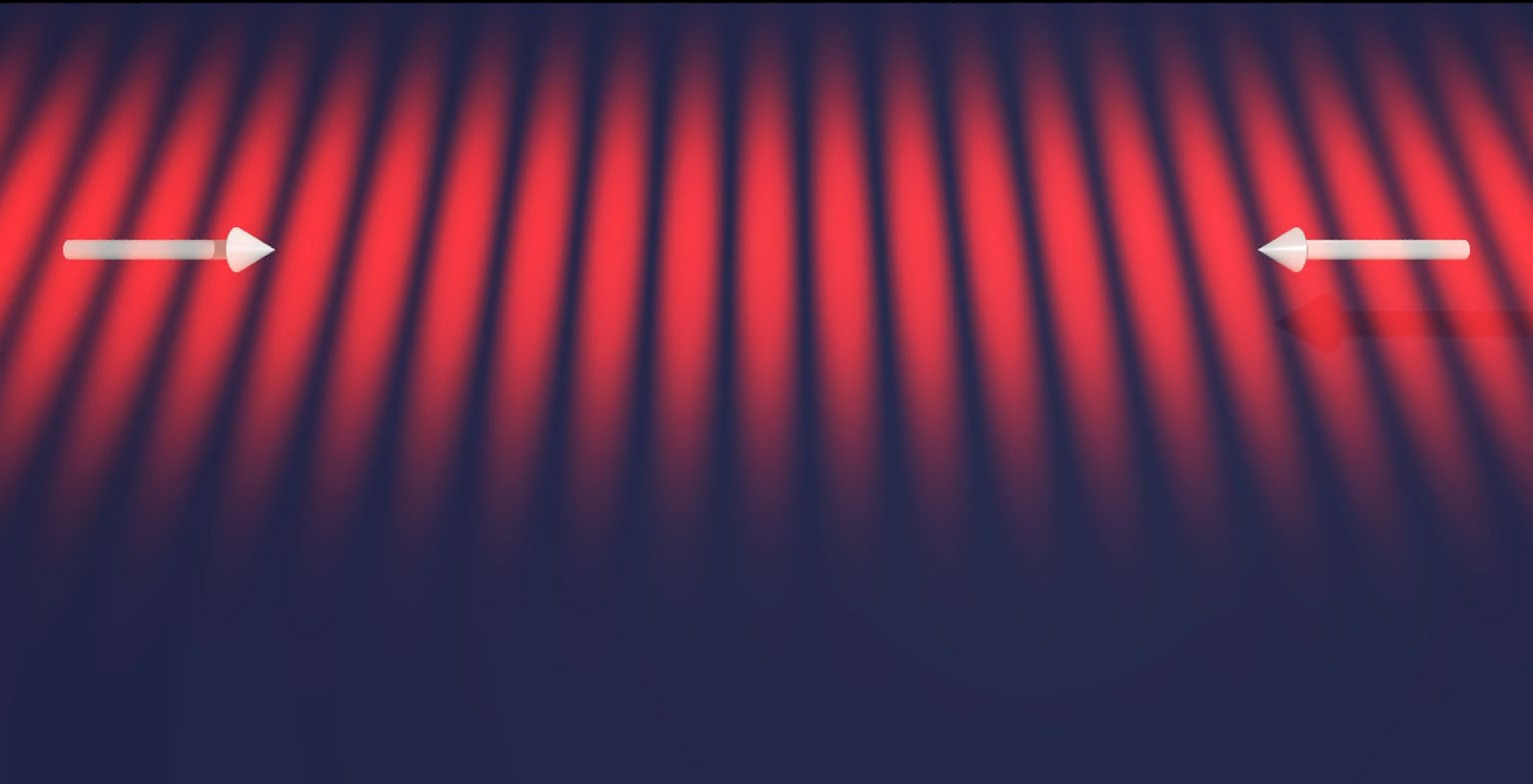


Coherent spin-flip transitions

Raman transitions coupling coherently different nuclear spin states:

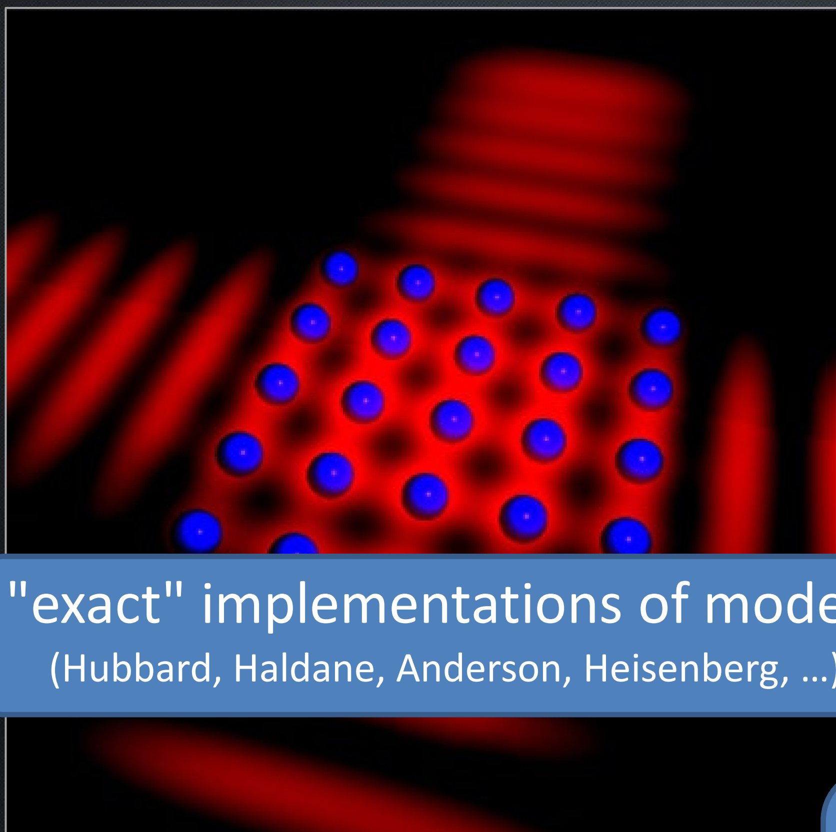


Optical lattices

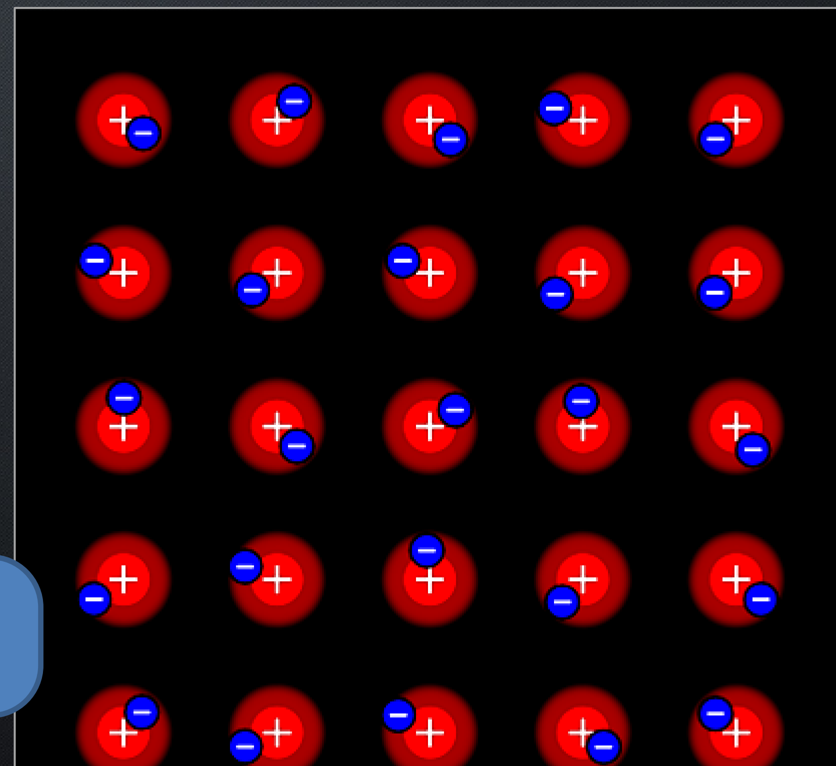


Quantum simulation of condmat systems

atoms in optical lattices



electrons in solids

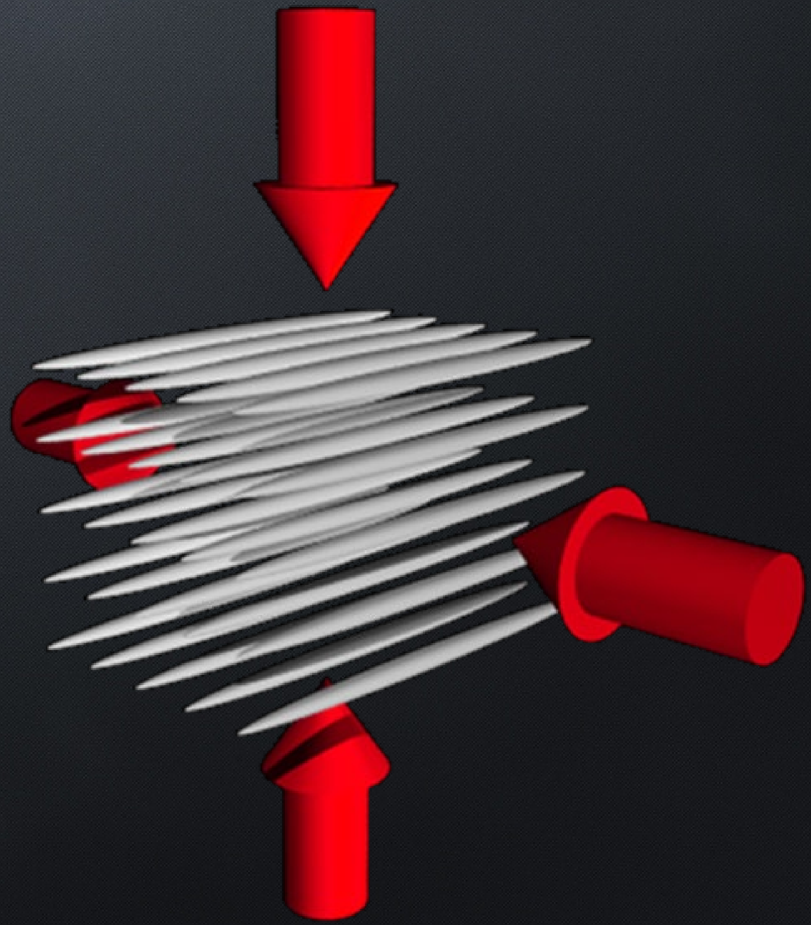


"exact" implementations of models
(Hubbard, Haldane, Anderson, Heisenberg, ...)

new "extreme" states of matter

Low-dimensional quantum systems

2D systems



1D systems

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- **EXP**: SU(N)

One-dimensional fermions with tunable SU(N) symmetry
G. Pagano et al., Nature Phys. **10**, 198 (2014)

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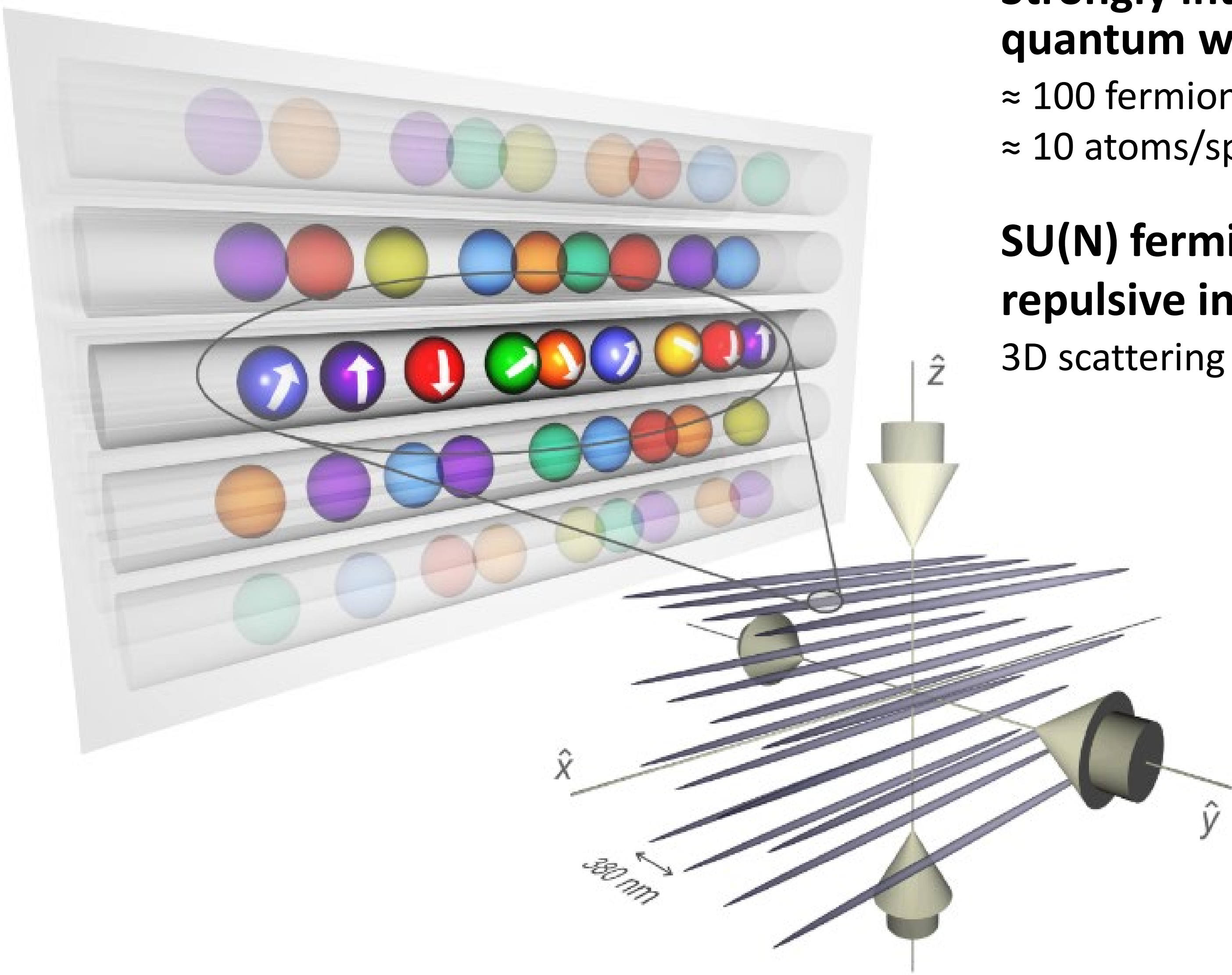
SU(N) quantum wires



Strongly interacting quantum wires

≈ 100 fermionic wires

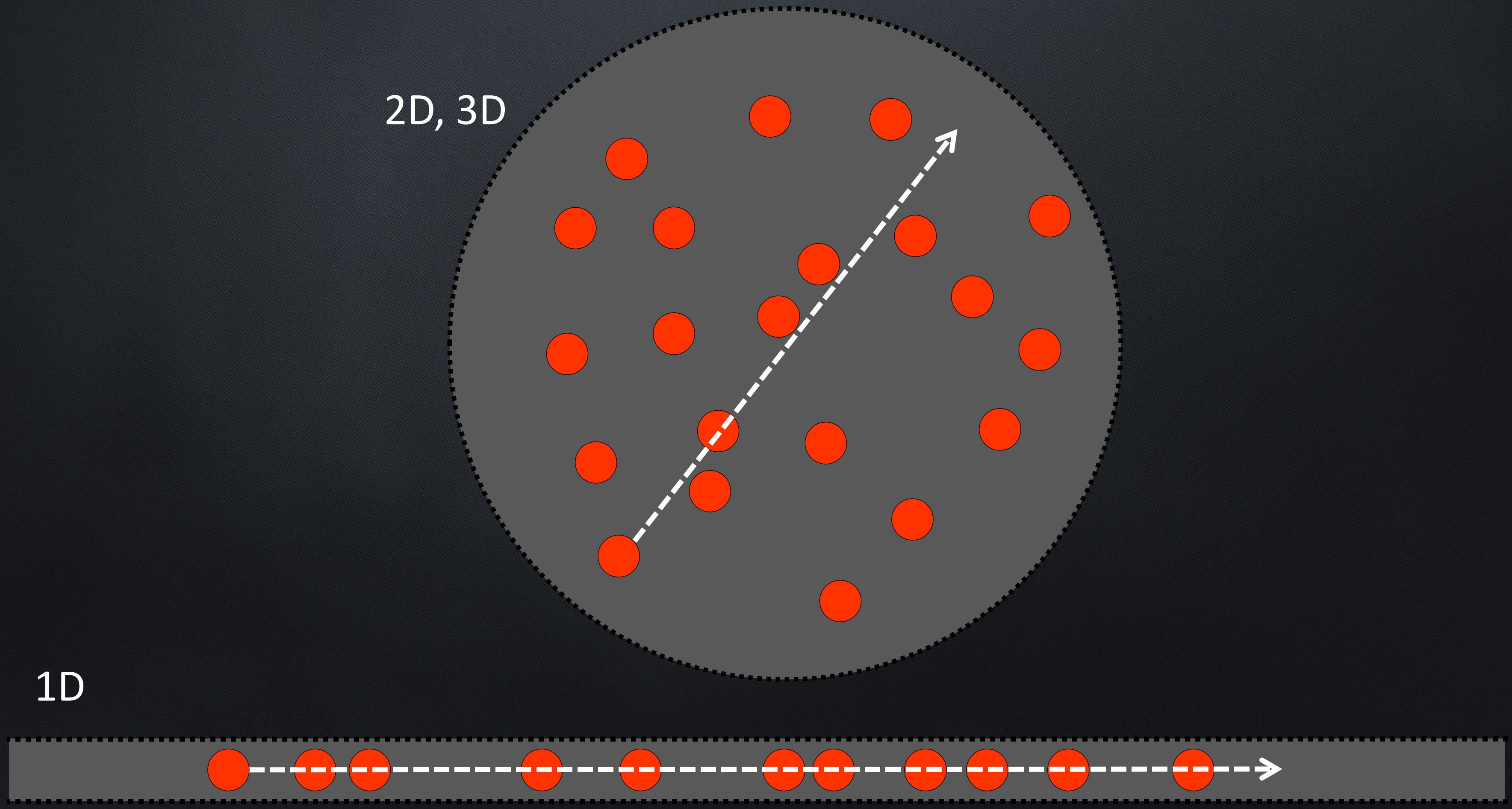
≈ 10 atoms/spin/wire

SU(N) fermions with repulsive interactions

3D scattering length a = +200 a_0

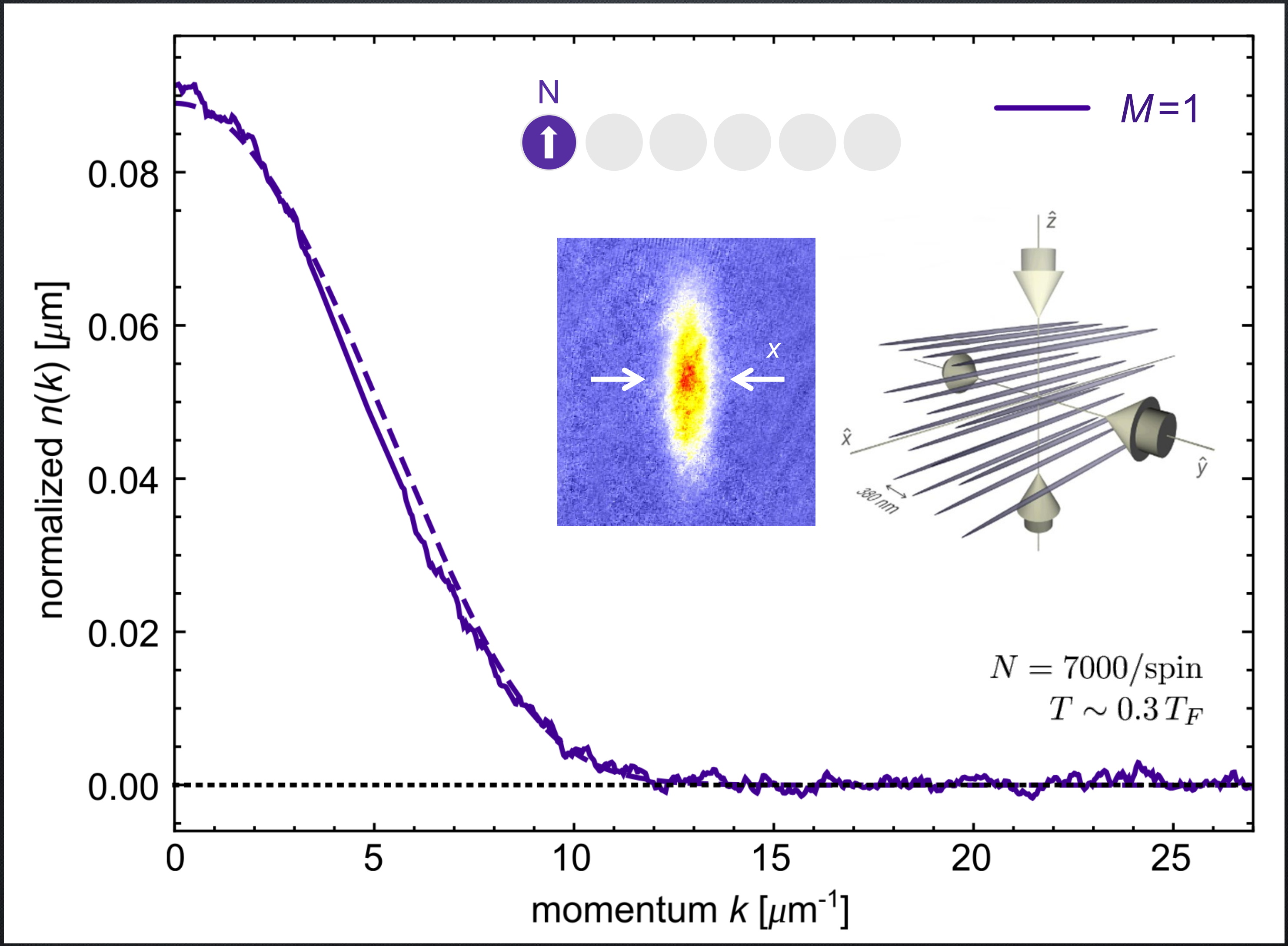
1D physics

Low dimensions strongly amplify the effects of interactions between particles



Momentum distribution

Momentum distribution measured after time-of-flight expansion:



G. Pagano et al., Nature Phys. ***10****, 198 (2014)*

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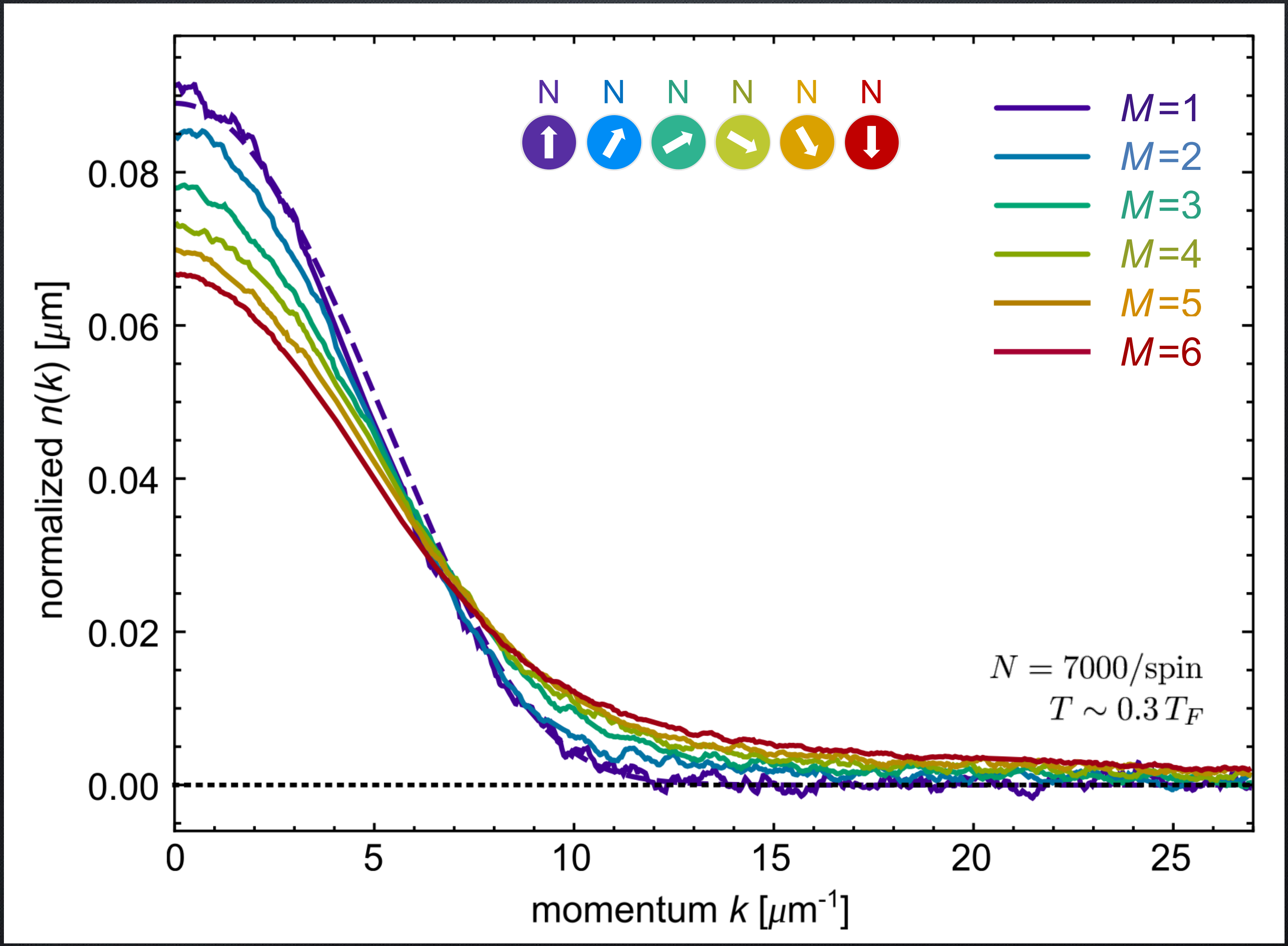
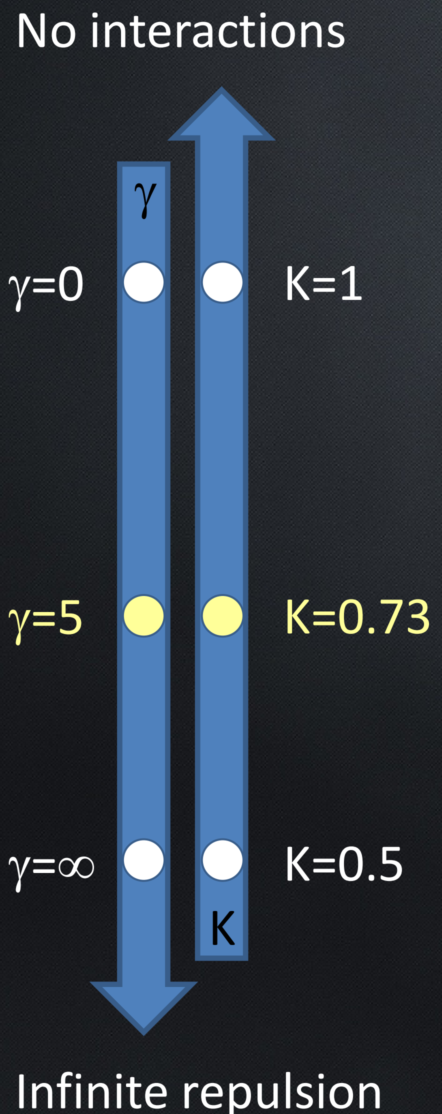
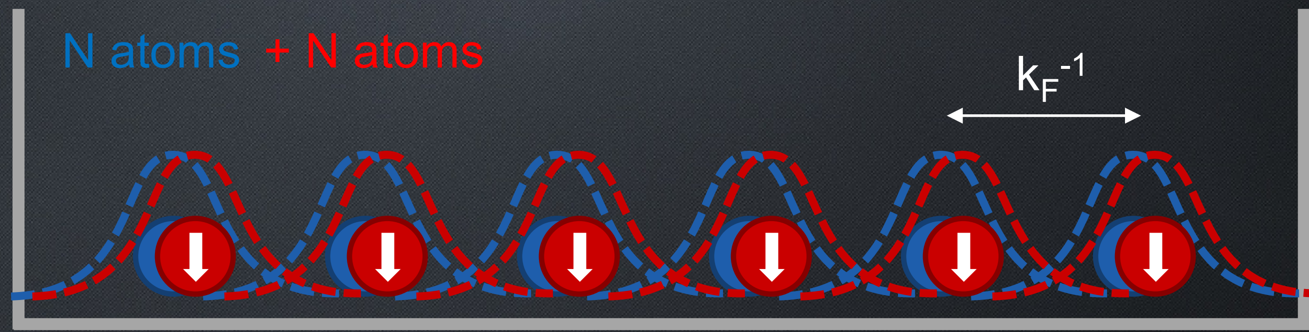
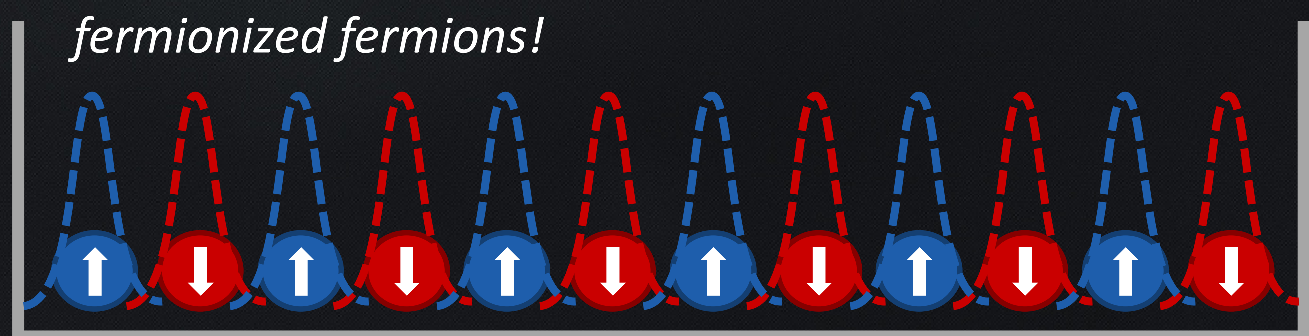


Quantum correlations in 1D fermions

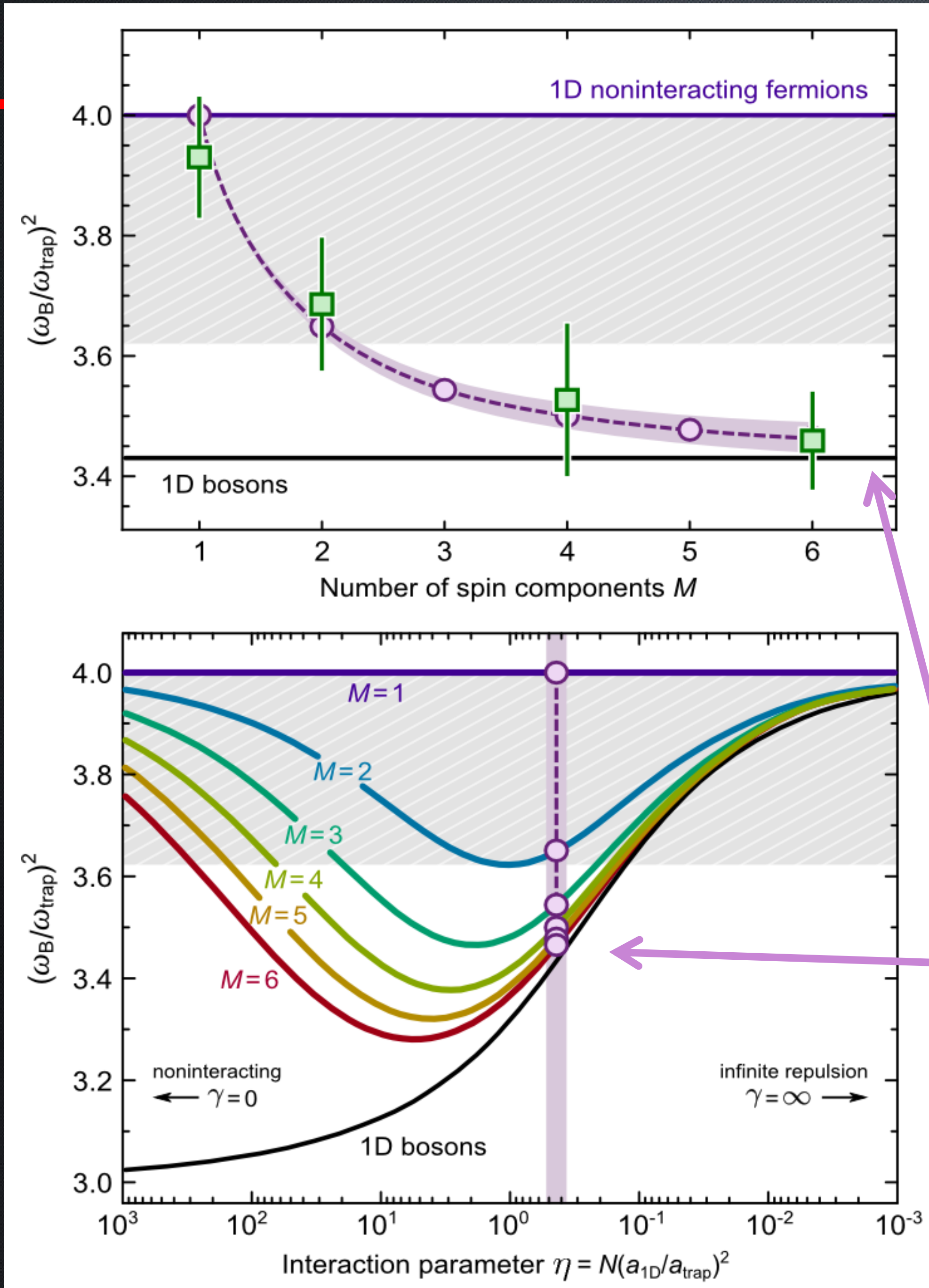


Experiment – strongly correlated regime

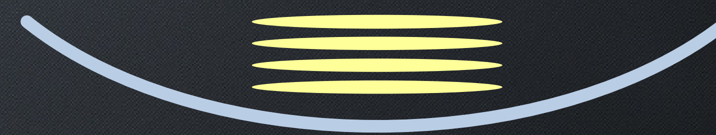




Collective dynamics



Interaction-induced redshift of breathing frequency in harmonic trap



Theory by H. Hu & X.-J. Liu (Swinburne)
Bethe Ansatz + LDA + Hydrodynamic

For $M \to \infty$ the breathing frequency approaches that of spinless bosons

«bosonization» of large-spin fermions

Interplay between interactions and spin multiplicity (distinguishability)

"Bosonization" of large-spin fermions

A very general result first demonstrated by C. N. Yang

C. N. Yang & Y. Yi-Zhuang, Chin. Phys. Lett. 28, 020503 (2011)

One-Dimensional w-Component Fermions and Bosons with Repulsive Delta Function Interaction *

C. N. YANG(杨振宁)[1,2]**, YOU Yi-Zhuang(尤亦庄)[1]

[1] *Institute for Advanced Study, Tsinghua University, Beijing 100084*

[2] *Institute of Theoretical Physics, Chinese University of Hong Kong, Hong Kong*

The ground state energy for such a system was studied in the 1960s, first in 1963 by Lieb and Liniger[1] for spinless Bosons, then in 1967 by Yang[2] for spin 1/2 Fermoins, finally in 1968 by Sutherland[3] for 3 component Fermions. This last result is readily generalizable to any value of w = number of components. In the present paper we *complete this series of studies* by solving the problem for w component Bosons via a *detour through* ∞ *component Fermions.*

Theorem 2.

$$Y_{F\infty}(Z) = Y_{B1}(Z).$$

See also measurement of Tan's contact (Hong Kong, Gyu-Boong Jo)

B. Song et al., PRX 10, 041053 (2020)