Programmable Quantum Simulators for Condensed Matter and Lattice Gauge Models

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BROOKHAVEN NATIONAL LABORATORY https://www.bnl.gov/quantumcenter/

5 federal labs + 19 universities + IBM

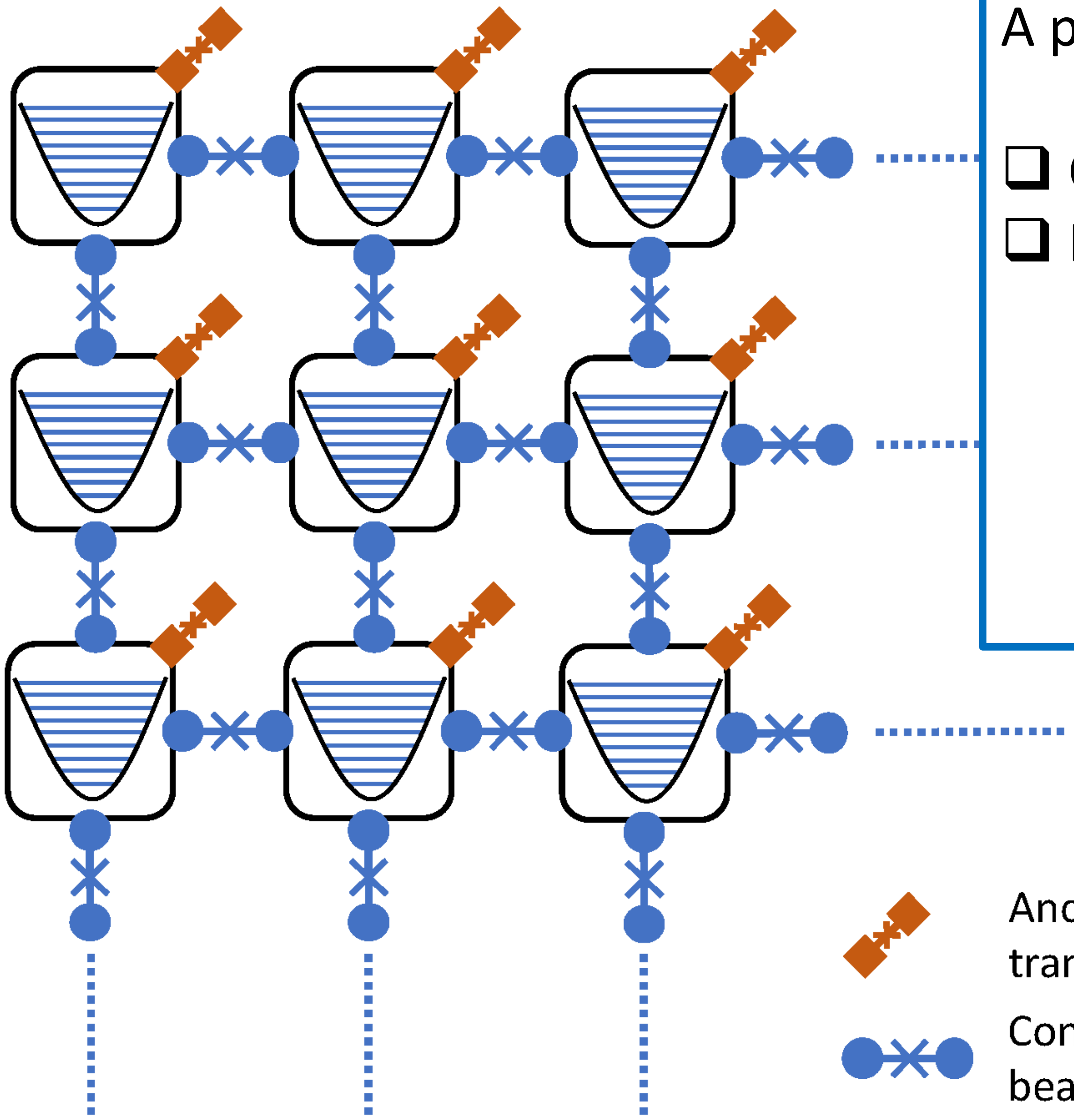
Two flavors of Quantum Simulators:

1. Simple, but non-programmable: use the 'natural' Hamiltonian of a synthetic system with similar degrees of freedom to the system being simulated.

2. Programmable: Requires universal control of all quantum degrees of freedom to synthesize arbitrary Hamiltonian time evolution.
   --Hamiltonian synthesis via 'digital' Trotter-Suzuki + Baker-Campbell-Hausdorff gate sequences and/or analog optimal control theory

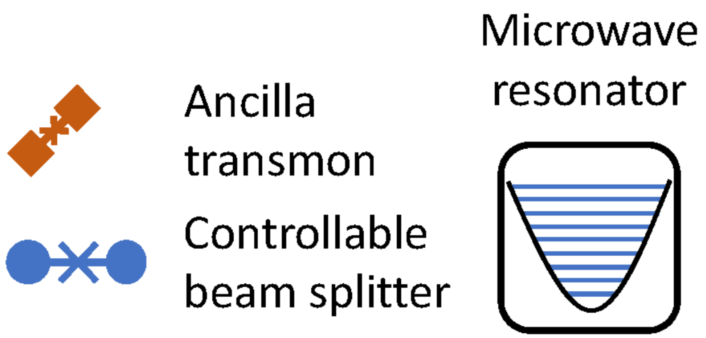
ALL simulators require the ability to make accurate and non-trivial measurements (hopefully beyond the capability of traditional experiment).

Error correction/mitigation will ultimately be needed in most cases.



A possible hybrid lattice architecture:

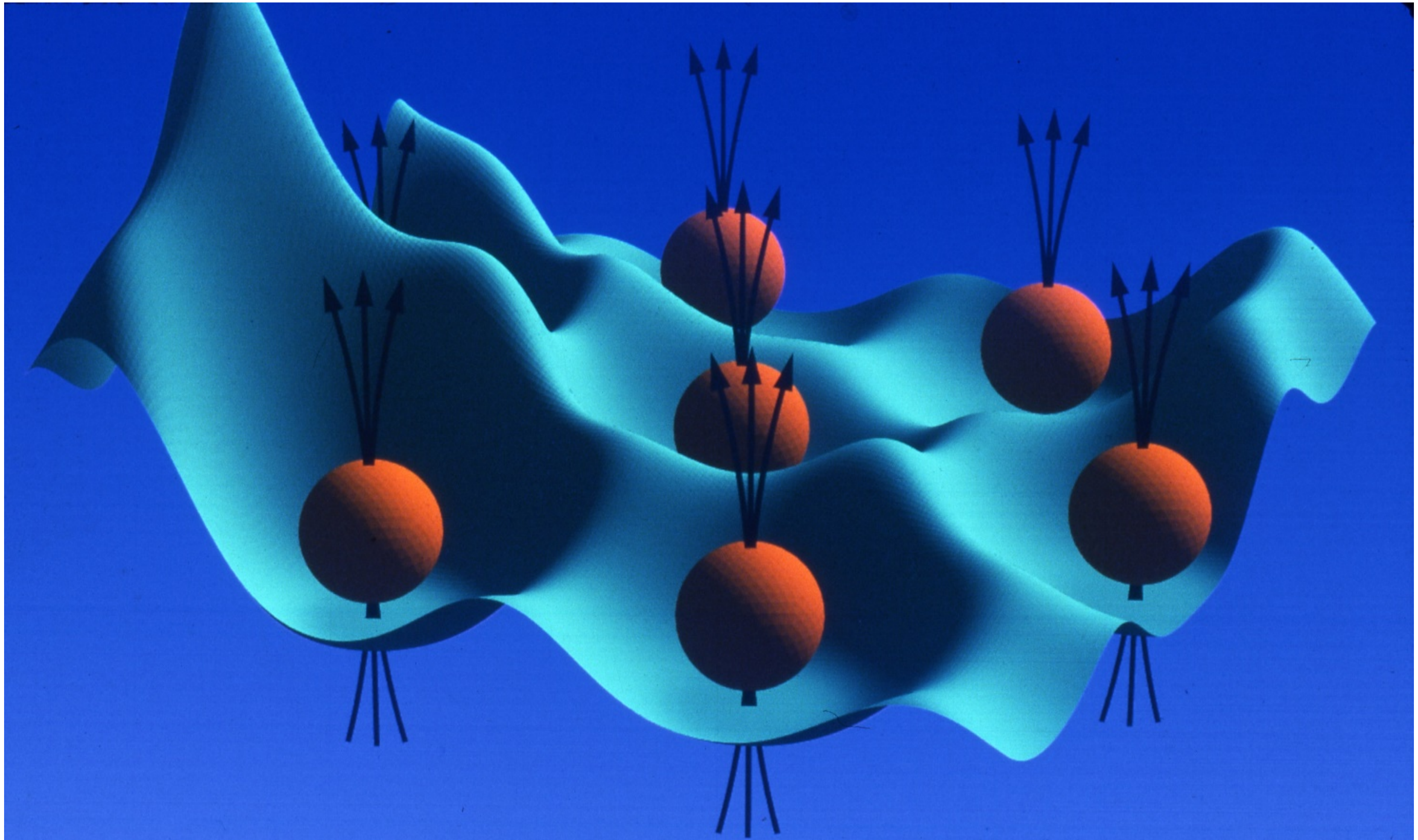
- ❑ Continuous variable [CV] oscillators (resonators)
- ❑ Discrete variable [DV] ancilla qubits (transmons)
  - Microwave photons stored in resonators [CV]
  - Controllable beam splitters for SWAP operations
  - Ancilla transmon qubits for control of resonators [DV]



Take-home message:

- **Hardware native bosonic modes offer advantages for:**
  - Efficient quantum error correction
  - Efficient quantum simulation of physical models containing bosons
- **Hybrid qubit/oscillator combinations can achieve universal control**
  - We need a simple instruction set architecture (ISA) in order to be able to develop algorithms and reason about circuit depth/complexity
  - Small ISA can be compiled to the control pulse level via OCT (optimal control theory) but entire algorithms cannot. We need an ISA to compile algorithms, estimate circuit costs and reason about error propagation.
- **Goals:**
  - Develop ISA: instruction set architecture(s); apply to quantum simulations, algorithms, and error correction
  - Represent the ISA in an extension of Qiskit that can treat bosonic modes; promulgate as a co-design tool for the community

Towards Many-Body Quantum Simulations of Interacting Bosons in Circuit QED



Modern Condensed Matter Physics (Cambridge Press, 2019)

Example target application:

FQHE for bosons (photons)

Can we convince microwave photons that they are charged particles in a magnetic field?

fractional statistics

$\nu = 1/2$ abelian semions

$\nu = 1$ non-abelian

REFERENCES:

K. Fang, Z. Yu and S. Fan, *Realizing effective magnetic field for photons by controlling the phase of dynamic modulation*, Nature Photonics **6**(11), 782 (2012).

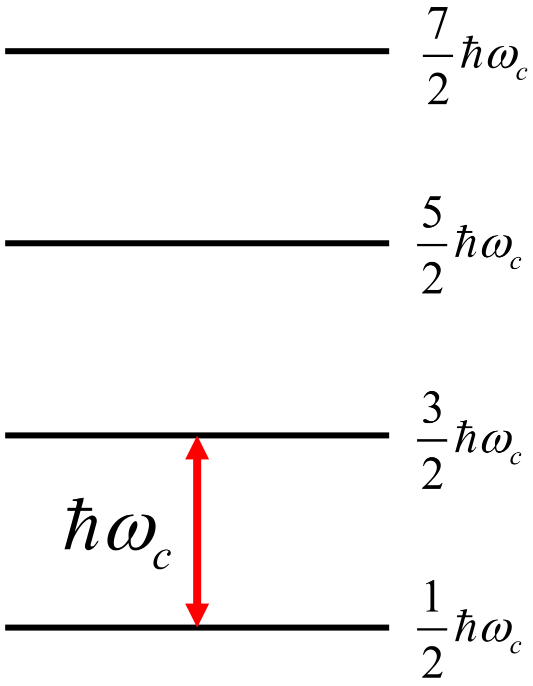
E. Kapit, *Quantum simulation architecture for lattice bosons in arbitrary, tunable, external gauge fields*, Phys. Rev. A **87**, 062336 (2013).

M. Hafezi, P. Adhikari and J. M. Taylor, *Engineering three-body interaction and Pfaffian states in circuit QED systems*, Phys. Rev. B **90**, 060503 (2014).

E. Kapit, *Universal two-qubit interactions, measurement, and cooling for quantum simulation and computing*, Phys. Rev. A **92**, 012302 (2015).

Kurilovich et al., 'Stabilizing the Laughlin state of light: dynamics of hole fractionalization,' arXiv:2111.01157

Single-particle wave functions in the lowest Landau Level (2DEG strong B field)



$$z = [x + iy] / \ell$$

$$\psi[z] = f[z] e^{-\frac{1}{4}|z|^2} \qquad f[z] = \text{poly[z]}$$

Laughlin correlated many-body ground state for Landau level filling factor $\nu = \frac{1}{m}$

$$\psi_m[z] = \prod_{i<j} (z_i - z_j)^m \; e^{-\frac{1}{4}\sum_k |z_k|^2}$$

$m =$ odd: fermions

$m =$ even: bosons

Laughlin correlated many-body ground state for Landau level filling factor $\nu = \dfrac{1}{m}$

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$m =$ odd: fermions

$m =$ even: bosons

Plasma analogy

2D Coulomb potential ('charge *m* rods')

2D Poisson equation

$$\nabla^2 \Phi = -2\pi\, \rho(\vec{r})$$

$$|\Psi_m[z]|^2 = e^{-\beta U_{\text{class}}},$$

where $\beta \equiv \frac{2}{m}$ and

$$U_{\text{class}} \equiv m^2 \sum_{i<j} (-\ln|z_i - z_j|) + \frac{m}{4} \sum_k |z_k|^2.$$

Potential from uniform compensating background charge density $\rho_0 = -\dfrac{1}{2\pi \ell^2}$. Charge neutrality sets the electron density ρ: $m\rho + \rho_0 = 0$ corresponding to Landau level filling $\nu = \dfrac{1}{m}$.

Fractionally charged quasi-hole excitations

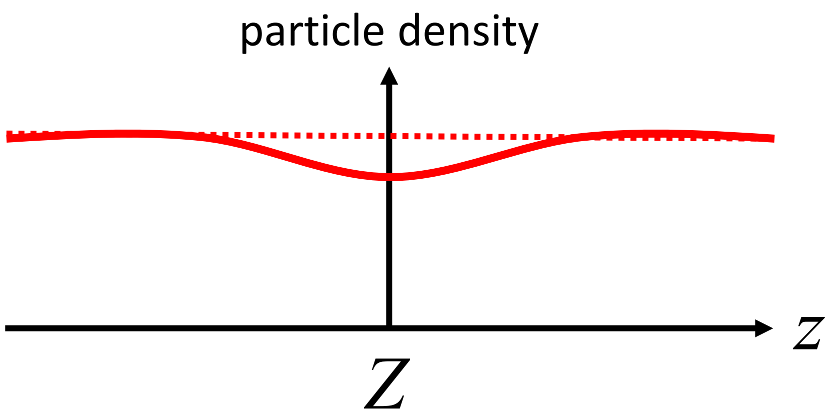
$$\psi_Z^+[z] = \left[\prod_{j=1}^{N}(z_j - Z)\right] \Psi_m[z]$$

Particles avoid the position of quasi-hole

$$|\psi_Z^+|^2 = e^{-\beta U_{\mathrm{class}}}\, e^{-\beta V}$$

$$V \equiv m \sum_{j=1}^{N} (-\ln|z_j - Z|)$$

Charge *m* particles repelled by a charge 1 impurity



Perfect screening in plasma implies local charge neutrality,

so the screening cloud has 'charge' $\delta q = m\,\delta n = -1$,

implying that the quasi-hole has net particle number $\delta n = -\dfrac{1}{m}$.

A similar calculation shows the quasi-holes obey fractional statistics.

Example target application: Bose-Hubbard/FQHE Hamiltonian Simulation

GOALS:

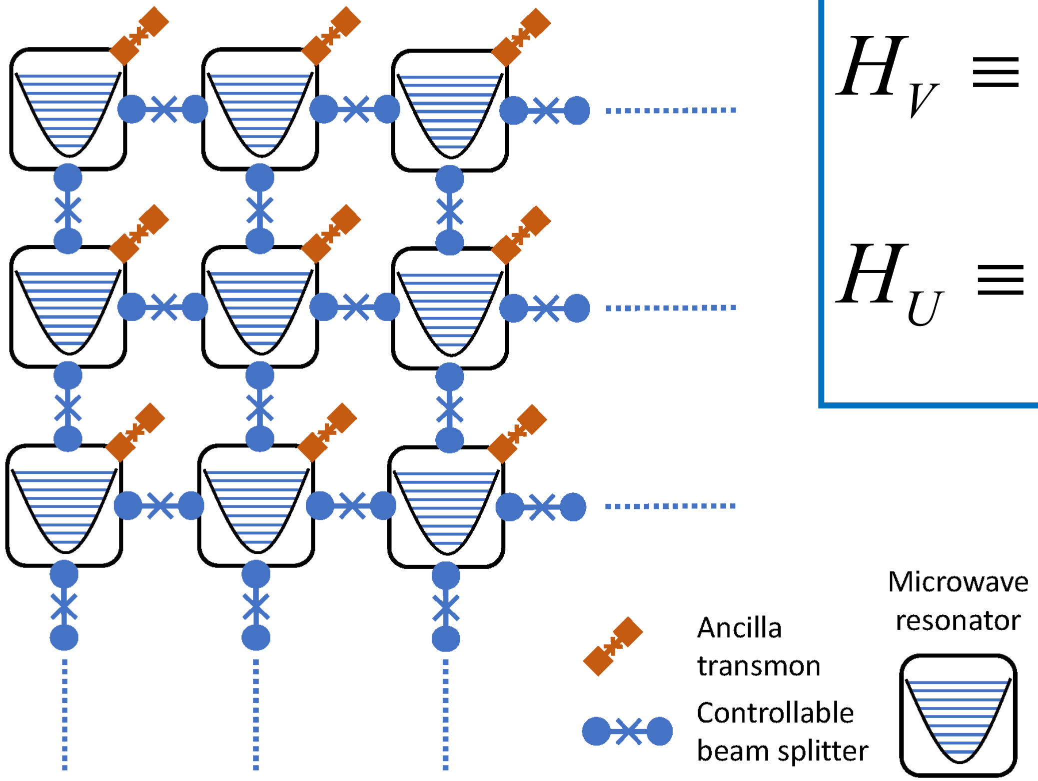
- ❑ Synthesize
  - Ground state (VQE)
  - Dynamics: $U(t) = e^{-iHt}$
- ❑ Measure observables

$$H = H_J + H_V + H_U$$

$$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$ boson hopping

$$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$ randomly disordered site energies

$$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$ Hubbard U boson repulsion



❑ Rich many-body phase diagram:

- Superfluid
- Mott insulator
- Anderson localization/Bose glass
- FQHE with fractional non-abelian excitations

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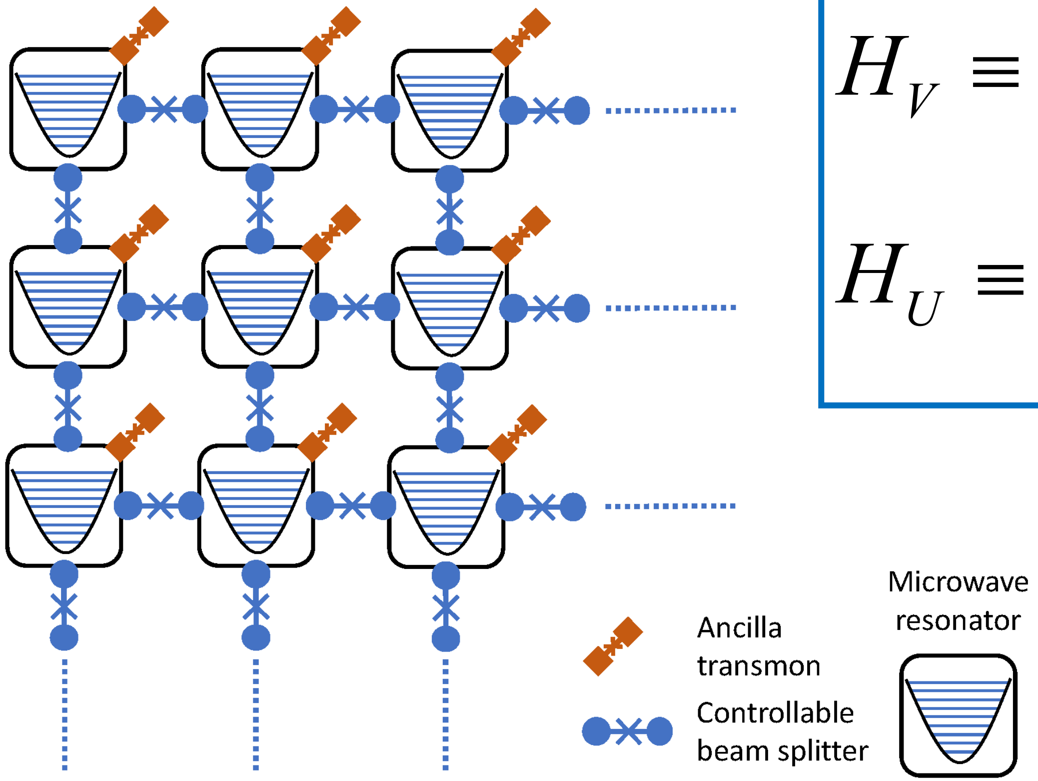


- ❑ Theory/Experiments with SC qubits rather than resonators: See Prof. Gorlach's talk.
- ❑ **Phys. Rev. Lett. 128, 213903 (2022)**
- ❑ **Phys. Rev. B 105, L081107 (2022)**

Controllable beam splitters to realize boson hopping

$$H = H_J + H_V + H_U$$

$$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$

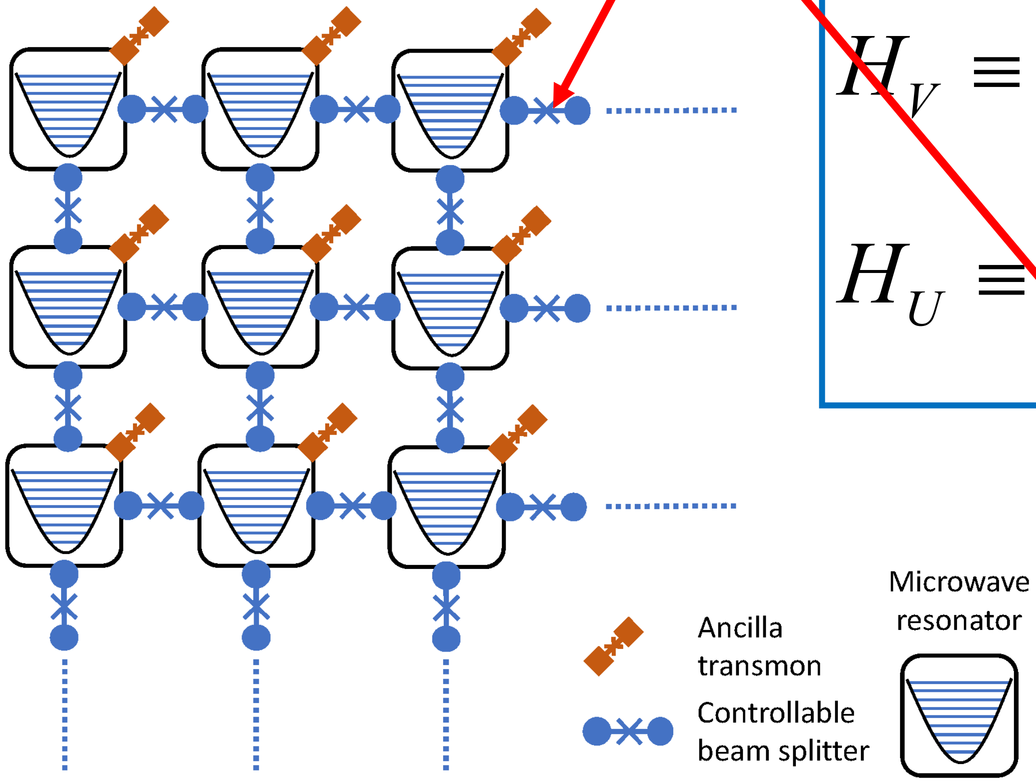
boson hopping

$$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$

randomly disordered site energies

$$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$

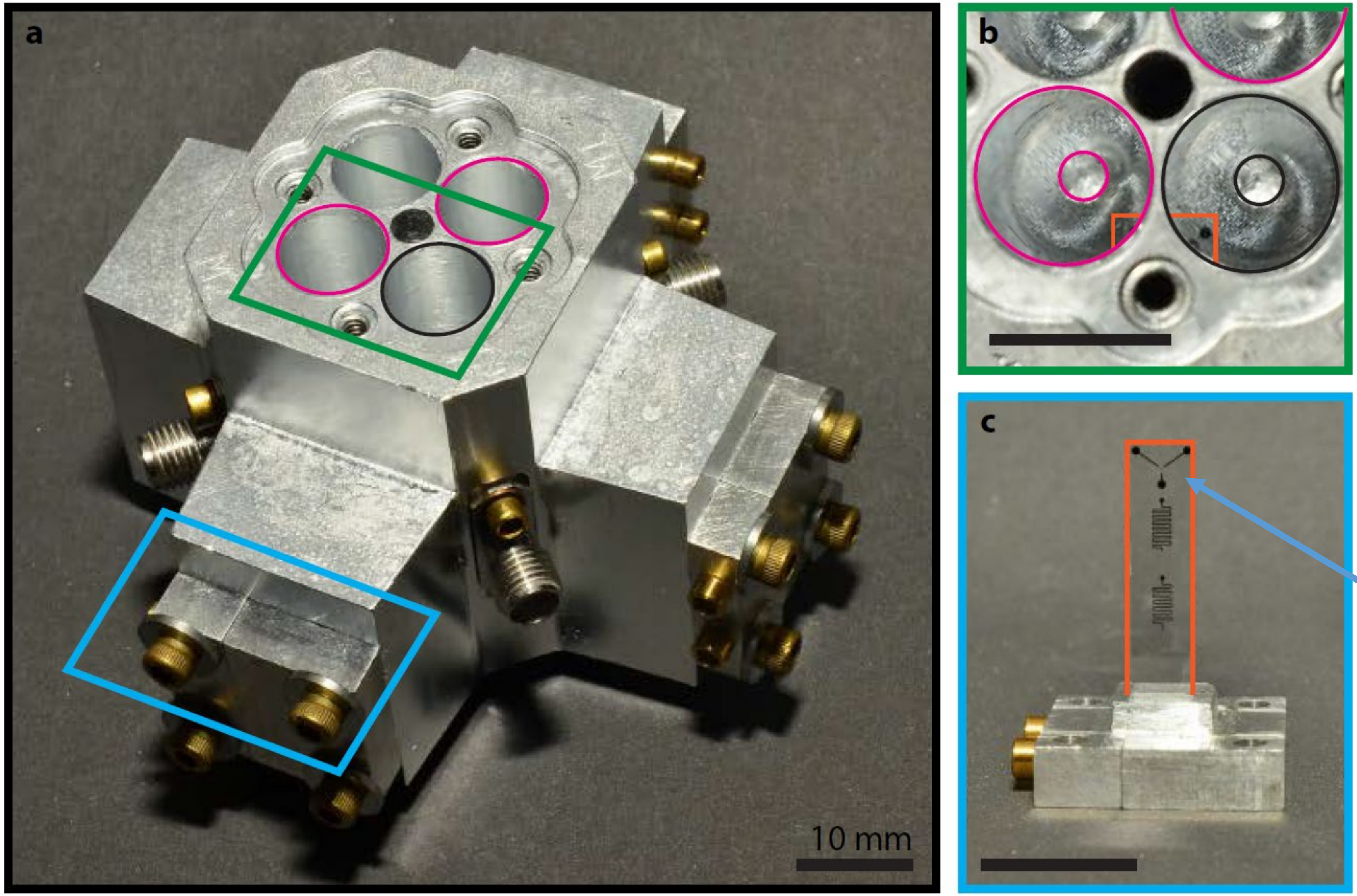
Hubbard *U* boson repulsion

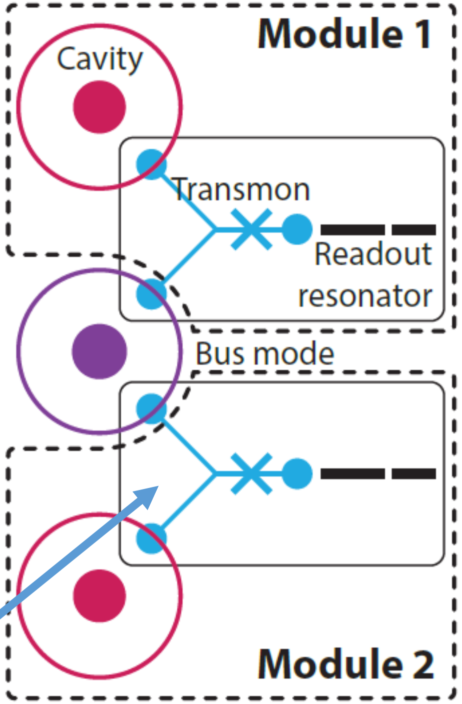


In quantum optics language, this is a beam-splitter Hamiltonian

Realizing a programmable beam splitter Hamiltonian

Deterministic teleportation of a quantum gate between two logical qubits, Kevin S. Chou et al., *Nature* **561**, 368 (2018)

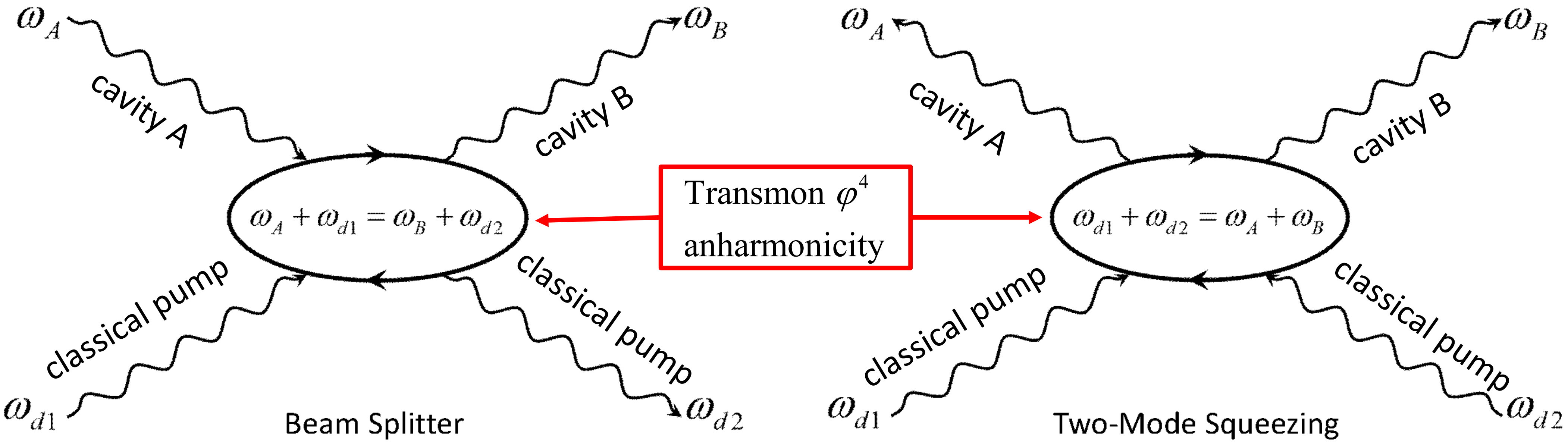




'Y-mon' transmon geometry

JJ is parametrically pumped to turn on the beam splitter between cavities.

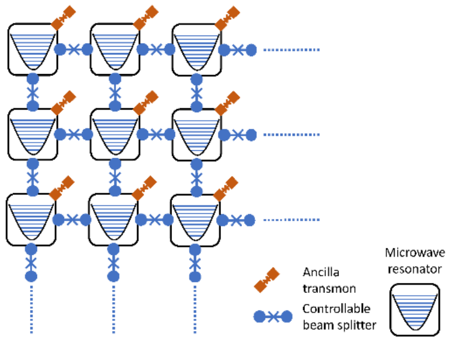
Two-mode Gaussian Operations via 4-wave Mixing with a Transmon Coupler
[All bilinear couplers that can be turned on and off by microwave pulses are necessarily pumped non-linear devices.]



$$\boxed{H_{\mathrm{BS}} = g_{\mathrm{BS}}(t) A B^\dagger + g^*_{\mathrm{BS}}(t) A^\dagger B.}$$

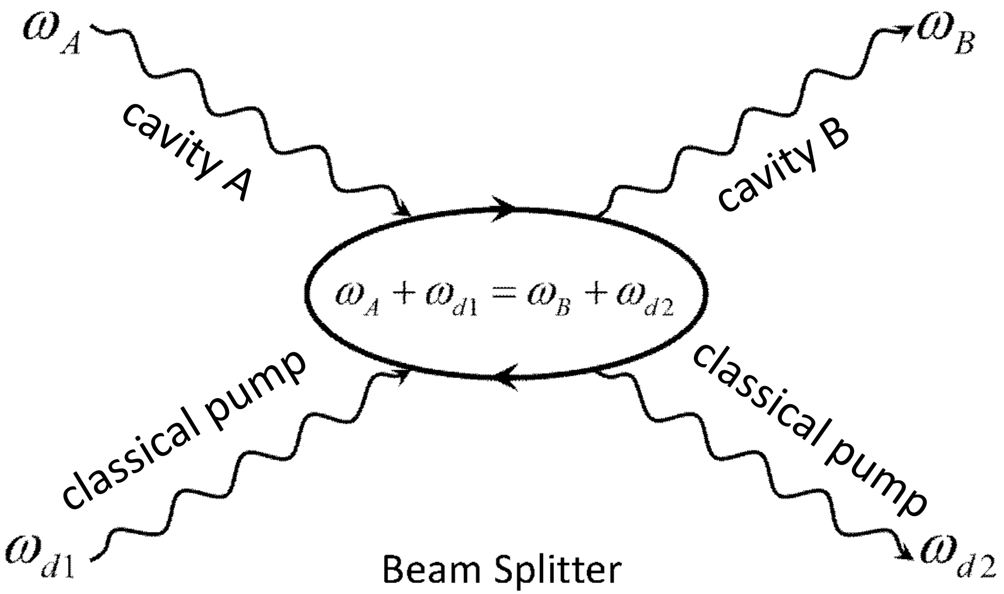
$$\boxed{H_{\mathrm{TMS}} = g_{\mathrm{TMS}}(t) A^\dagger B^\dagger + g^*_{\mathrm{TMS}}(t) A B.}$$

Each resonator has a distinct frequency to reduce cross-talk and enhance on/off ratio.



Phase and amplitude of the coupling is controlled by the choice of pump tone phases and amplitudes.
Pump supplies the energy change needed for the process to be resonant (e.g. frequency-converting beam splitter).
No need to fine tune the cavity manufacture.

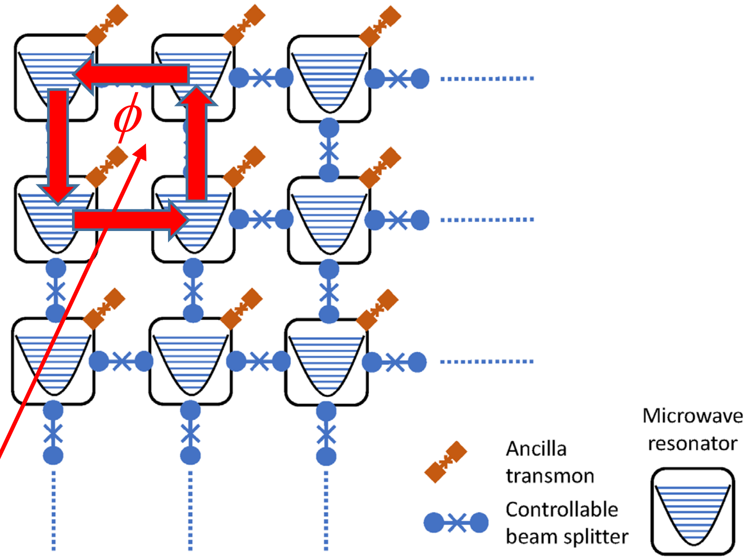
Two-mode Gaussian Operations via 4-wave Mixing



$$H_{\mathrm{BS}} = g_{\mathrm{BS}}(t) A B^\dagger + g_{\mathrm{BS}}^*(t) A^\dagger B.$$

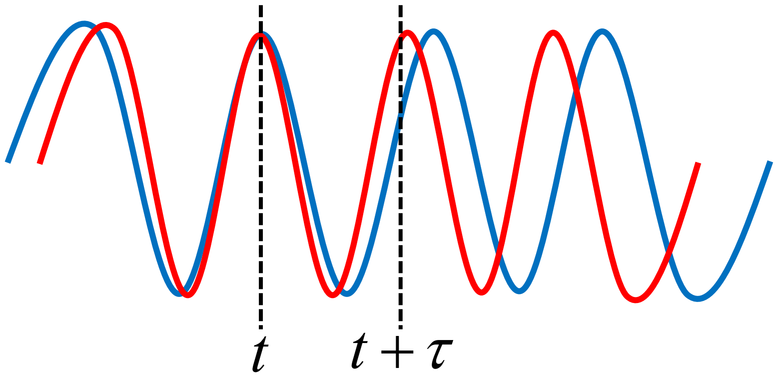
Beam splitter realizes the boson hopping term.

$$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$



Phase-locking the pump tones allows complex *J* and
Photon can acquire a non-zero phase around each plaquette.
Acts like a charged particle in a magnetic field $\phi = \frac{q}{\hbar} \oint d\vec{r} \cdot \vec{A}(\vec{r}) = \frac{q}{\hbar} \Phi$

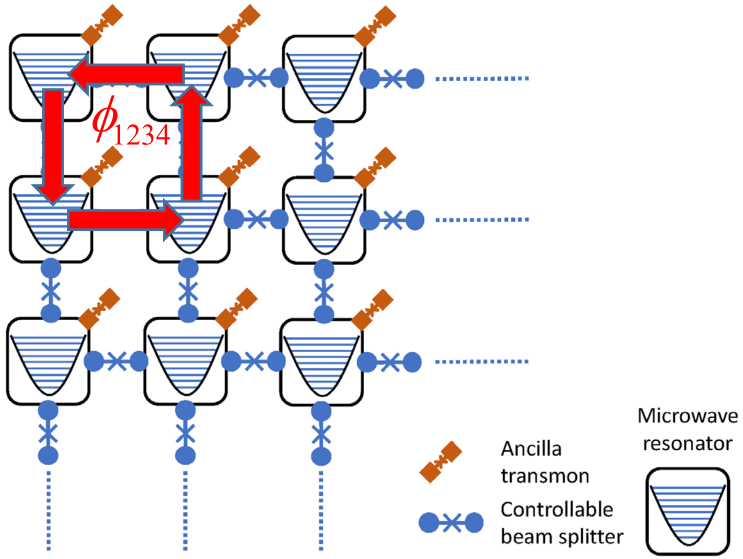
What does ‘phase locking’ mean?



Two-mode Gaussian Operations via 4-wave Mixing

Beam splitter realizes the boson hopping term.

$$H_J \equiv \sum_{\langle ij\rangle}\left\{J_{ij}b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i\right\}$$



Relative phase of two pump tones at different frequency is not unique—depends on choice of time origin

$$\varphi_{12}(t+\tau) = \varphi_{12}(t) + (\omega_{d1} - \omega_{d2})\tau$$

Because going around the plaquette returns to the same initial site energy $\omega_1 \to \omega_2 \to \omega_3 \to \omega_4 \to \omega_1$,

The phase acquired is gauge (time-translation) invariant:

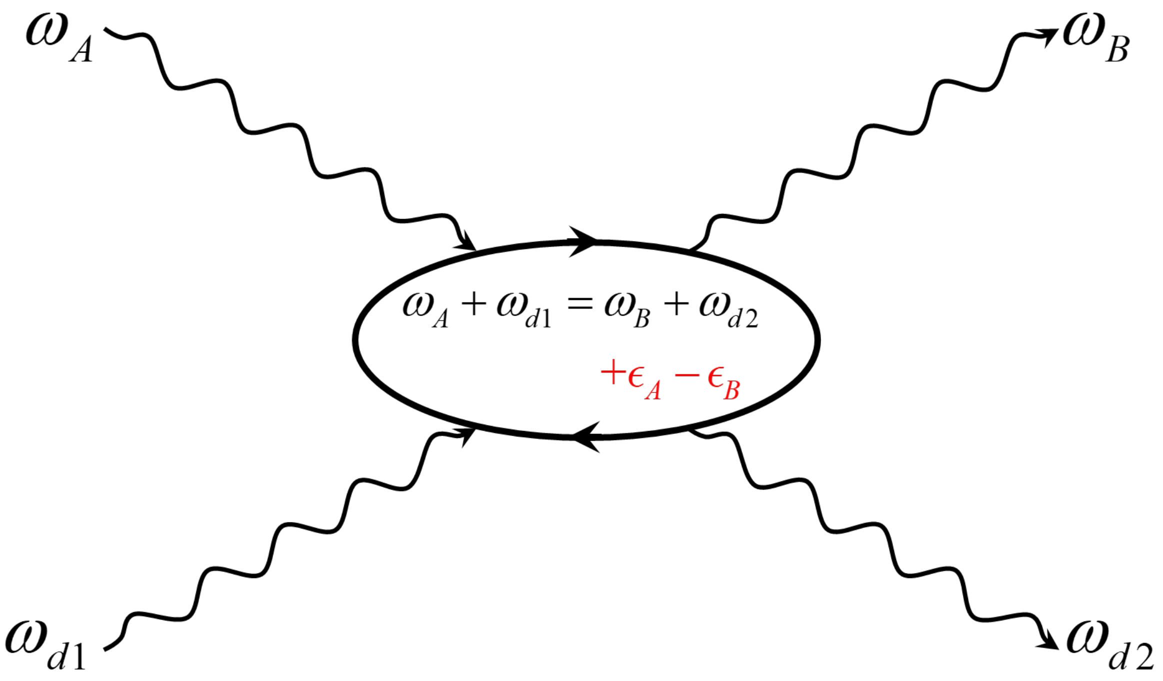
$$\phi_{1234} = \varphi_{12} + \varphi_{23} + \varphi_{34} + \varphi_{41} = \mathrm{Arg}[J_{12}J_{23}J_{34}J_{41}]$$

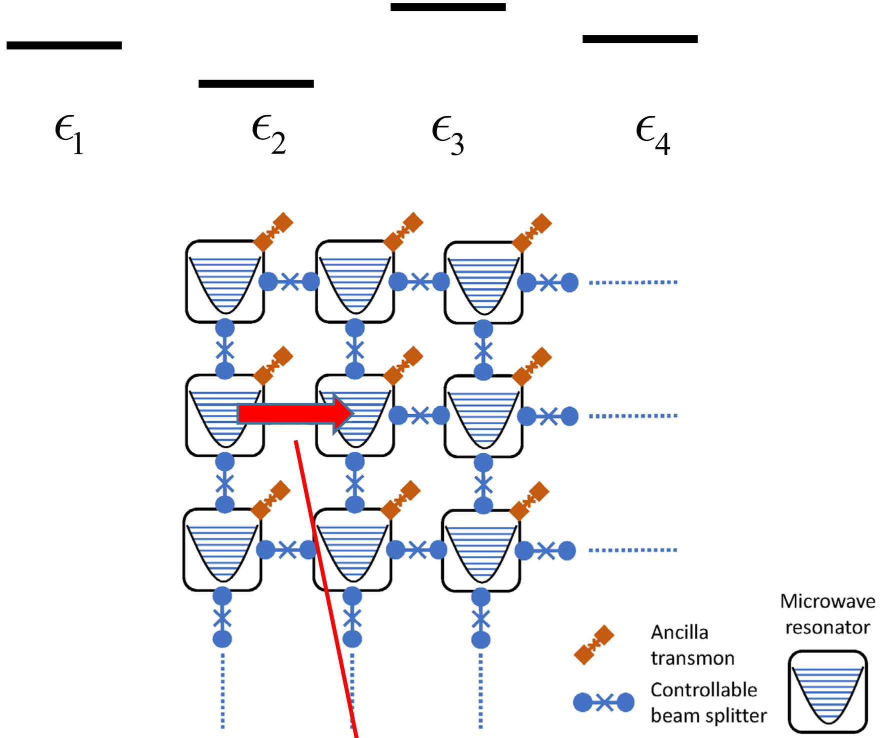
Ordinary gauge Invariance = charge conservation: $\phi_{1234} = \frac{q}{\hbar}\oint d\vec{r}\cdot\vec{A}(\vec{r}) = \frac{q}{\hbar}\Phi$

How do we create the randomly disordered site energy terms?

$$\boxed{H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k}$$

Fully in-operando programmable site energies.





Detuning the pump drives means the photon cannot resonantly hop from one cavity to the next: $\omega_{d1} - \omega_{d2} = \omega_B - \omega_A + \epsilon_A - \epsilon_B$.

Analogy to electrodynamics:

$$\vec{E} = -\vec{\nabla} V + \frac{\partial \vec{A}}{\partial t}$$

$$H = H_J + H_V + H_U$$

☑ $$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$ boson hopping

☑ $$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$ randomly disordered site energies

? $$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$ Hubbard *U* boson repulsion

The quadratic terms in the Hamiltonian are now fully programmable.

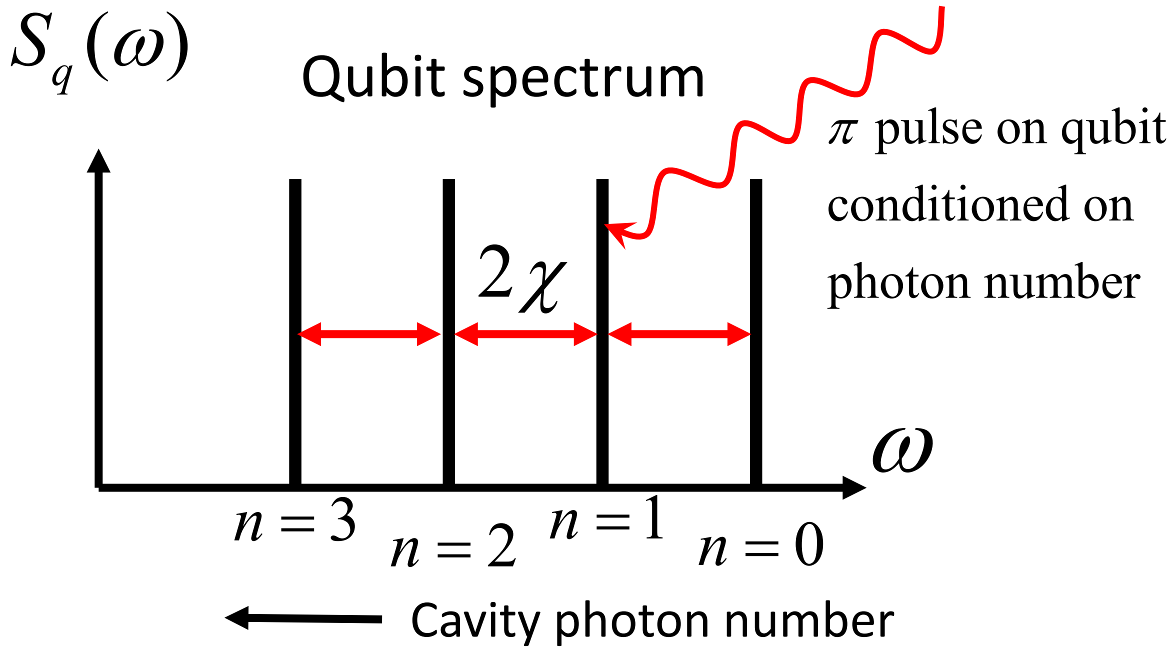
How do we program the boson-boson repulsive interaction term?

Synthesizing the cavity Hubbard *U* interaction using the cavity-qubit dispersive coupling.

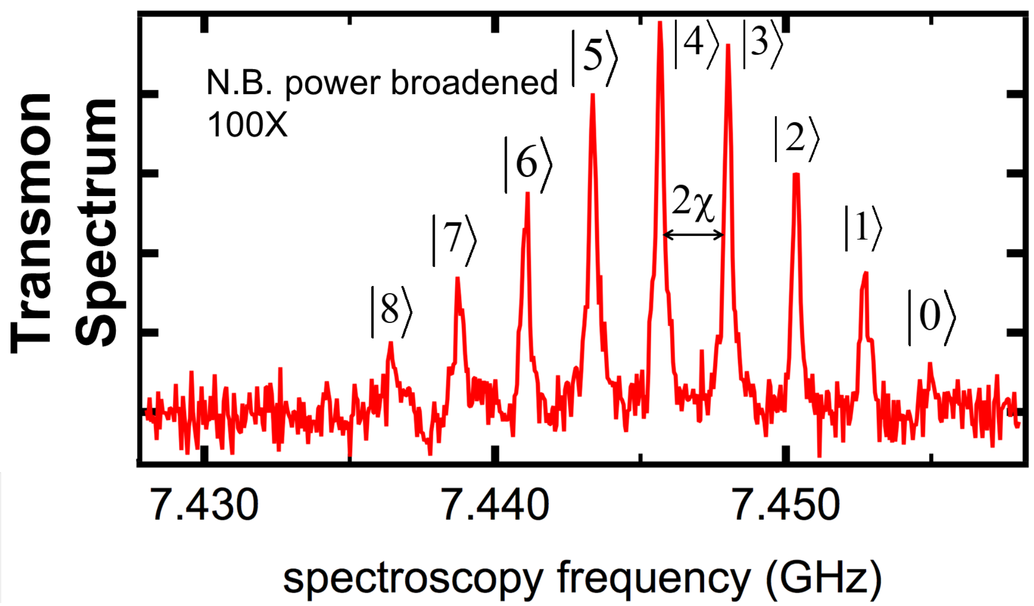
Qubit-Cavity Strong Dispersive Coupling

$$\omega_c \neq \omega_q$$

$$\boxed{H = \omega_c a^\dagger a + \frac{\omega_q}{2}\sigma^z + \chi \sigma^z a^\dagger a}$$



Microwaves are particles!!



SNAP-gate Instruction Set

Unconditional Displacement Gate

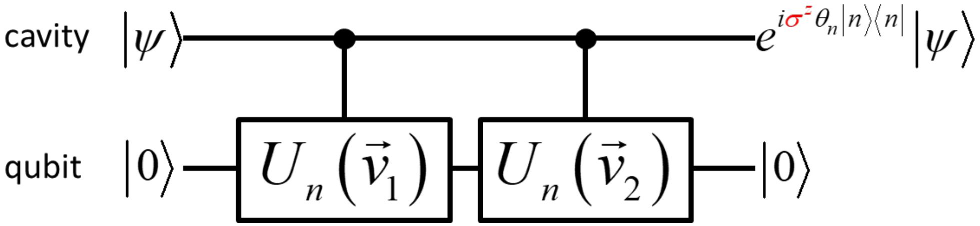
$$D[\alpha] \equiv e^{\alpha a^\dagger - \alpha^* a}$$

$$U_{\mathrm{SNAP}}(\vec{\theta})\left[\sum_m \Psi_m |m\rangle\right] = \left[\sum_m e^{i\theta_m}\Psi_m |m\rangle\right]$$

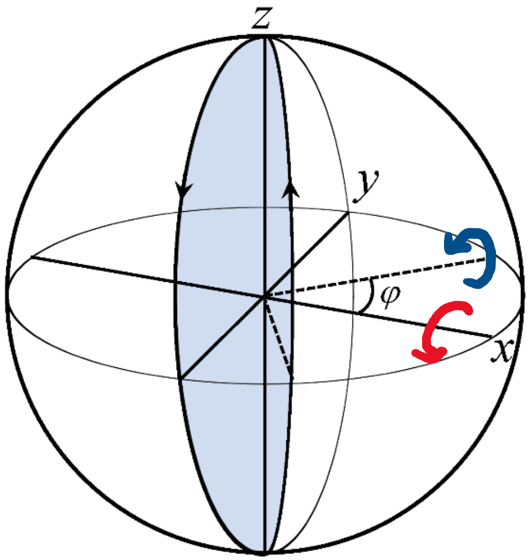
Provable universal control.

Krastanov et al., *Phys. Rev. A* **92**, 040303(R) (2015)

Heeres et al., *Phys. Rev. Lett.* **115**, 137002 (2015)



$$\boxed{U_n(\vec{v}) = e^{-\frac{i}{2}\left(v_x\sigma^x + v_y\sigma^y\right)|n\rangle\langle n|}}$$



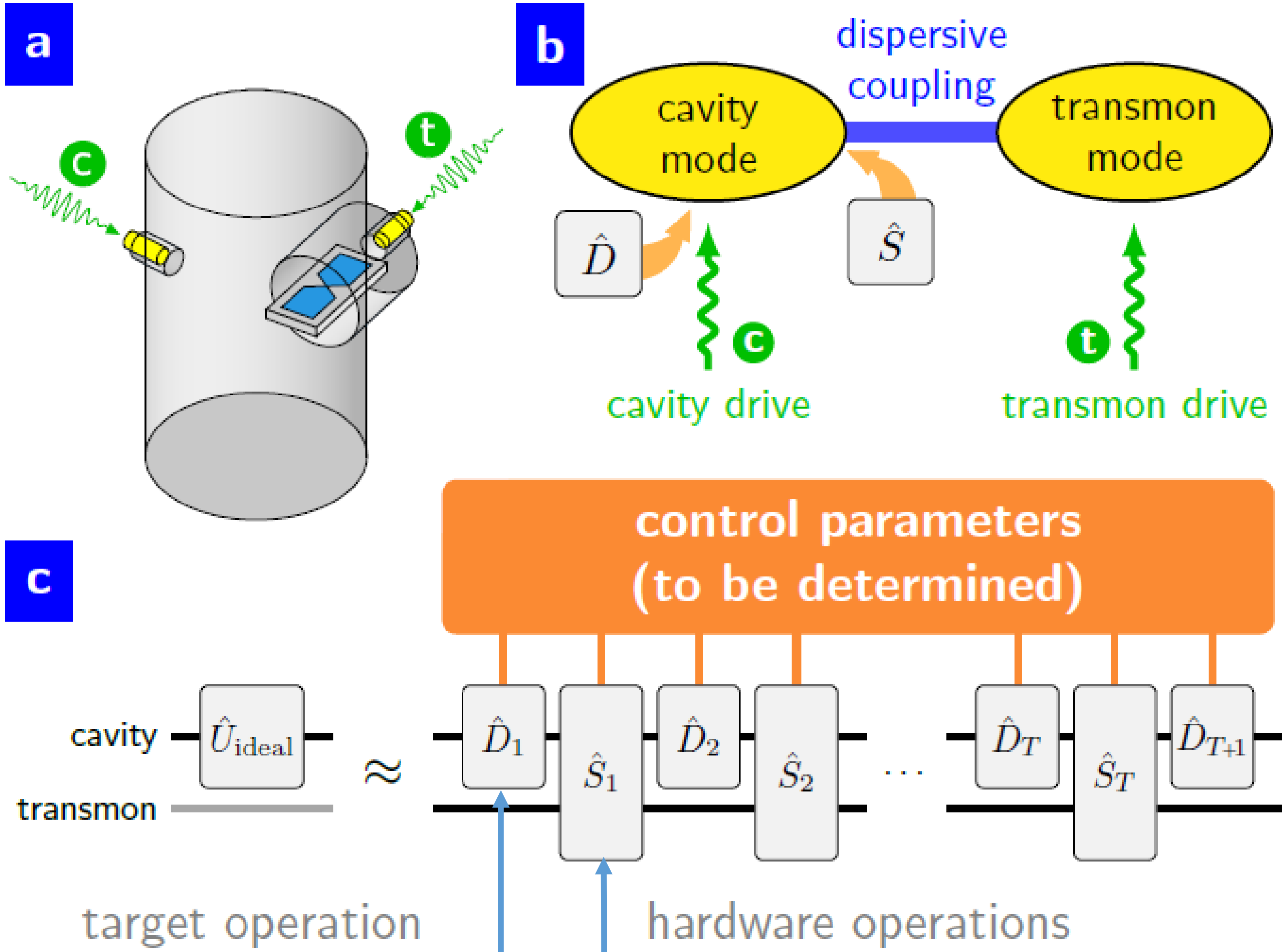
Use dispersive coupling of qubit to cavity to apply separate independent geometric phases to each photon Fock state.

$$U_{\mathrm{SNAP}}(\vec{\theta}) \equiv e^{i\sigma^z \sum_{n=0}^{n_{\max}} \theta_n \hat{P}_n}$$

$$\hat{P}_n = |n\rangle\langle n|$$

$$\vec{\theta} = (\theta_0, \theta_1, \ldots, \theta_{n_{\max}})$$

SNAP instruction set is extremely efficient

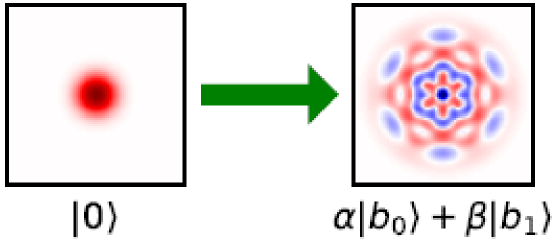


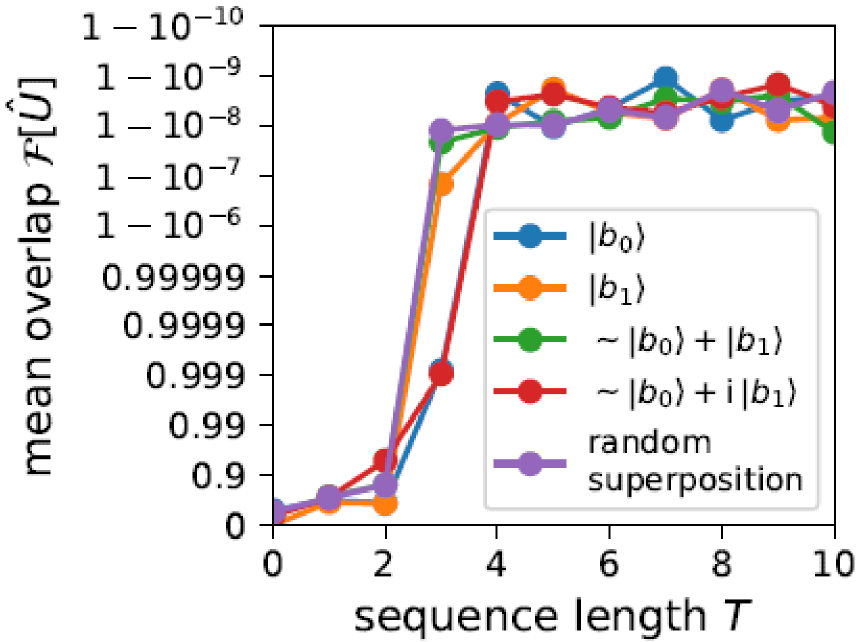
Cavity displacement gate

SNAP gate

Binomial QEC Code State Prep.

$$|b_0\rangle = \frac{|0\rangle + \sqrt{3}|6\rangle}{2} \qquad |b_1\rangle = \frac{\sqrt{3}|3\rangle + |9\rangle}{2}$$





'Efficient cavity control with SNAP gates,' Fösel et al., arXiv:2004.14256

Programming the Hubbard boson repulsion

$$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$

On each site *k:*

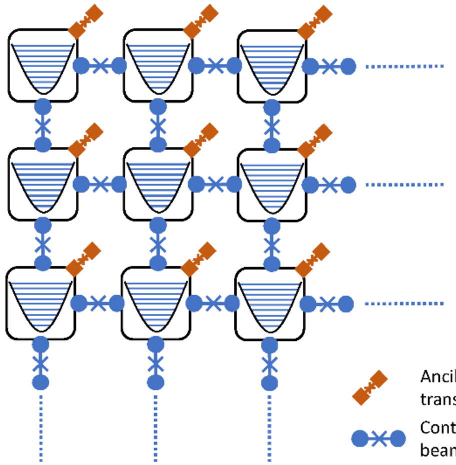
$$e^{-iH_U t} = e^{-iUt\hat{n}_k(\hat{n}_k - 1)} = U_{\mathrm{SNAP}}(\vec{\theta})$$

$$\theta_n = -Ut[n(n-1)]$$

$$U_{\mathrm{SNAP}}(\vec{\theta}) \equiv e^{i\sigma^z \sum_{n=0}^{n_{\max}} \theta_n \hat{P}_n}$$

$$\hat{P}_n = |n\rangle\langle n|$$

$$\vec{\theta} = (\theta_0, \theta_1, \ldots, \theta_{n_{\max}})$$



All required technology has been experimentally demonstrated, but not yet at scale.

$$H = H_J + H_V + H_U$$

$$H_J \equiv \sum_{\langle ij \rangle} \left\{ J_{ij} b_i^\dagger b_j + J_{ij}^* b_j^\dagger b_i \right\}$$ boson hopping

$$H_V \equiv \sum_k \epsilon_k b_k^\dagger b_k$$ randomly disordered site energies

$$H_U \equiv U \sum_k b_k^\dagger b_k^\dagger b_k b_k$$ Hubbard *U* boson repulsion

Experimental status with microwave cavities:

Two sites ☑

Two sites ☑

One site ☑

These microwave bosons are not strictly conserved.

It is possible to create an engineered quantum bath that will gently (adiabatically) replace missing bosons as long there is an excitation gap.

But for the FQHE:

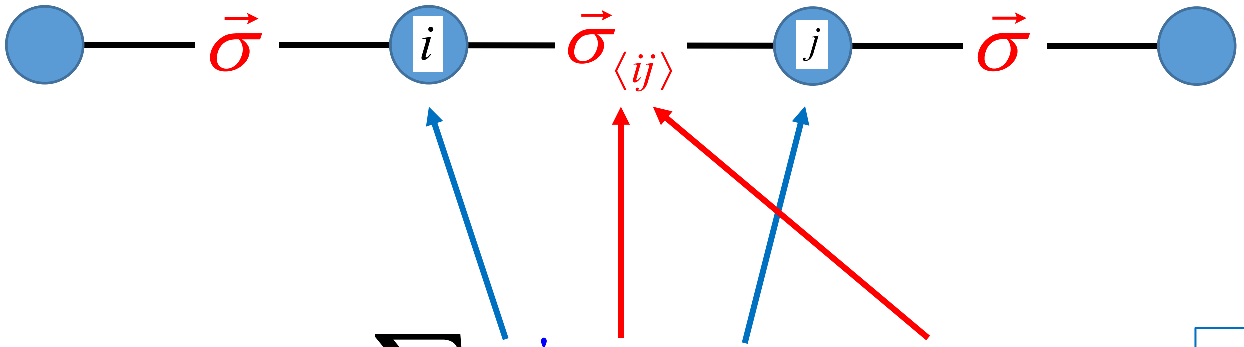
Novel slow dynamics if the ‘hole’ (missing photon) fractionalizes into two ‘charge’-1/2 quasiholes:

Autonomous stabilization of photonic Laughlin states through angular momentum potentials,
R. O. Umucalılar, J. Simon and I. Carusotto, Phys. Rev. A **104**, 023704 (2021).

Stabilizing the Laughlin state of light: dynamics of hole fractionalization, Kurilovich et al., arXiv:2111.01157.

Another target application: $\mathbb{Z}_2$ lattice gauge theory for bosons hopping on a lattice

bosons on lattice sites, $\mathbb{Z}_2$ gauge fields on links



$$H = -J\sum_{\langle ij\rangle} a_i^\dagger \sigma^z_{\langle ij\rangle} a_j - \lambda \sigma^x_{\langle ij\rangle}$$

Physical intuition: each boson hop changes the sign of $\sigma^x_{\langle ij\rangle}$

<u>Dynamical</u> gauge field:

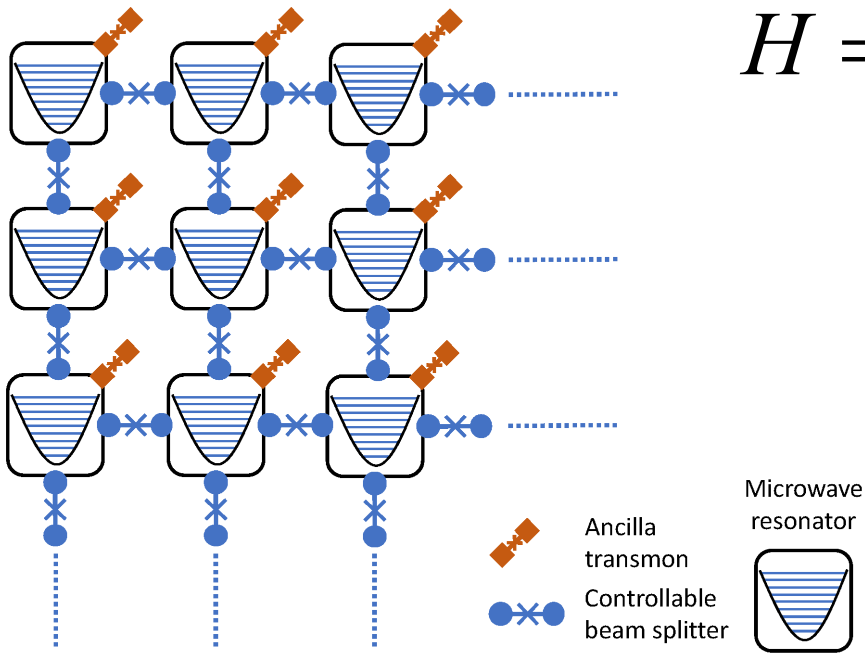
$\sigma^z =$ vector potential

$\sigma^x =$ conjugate electric field

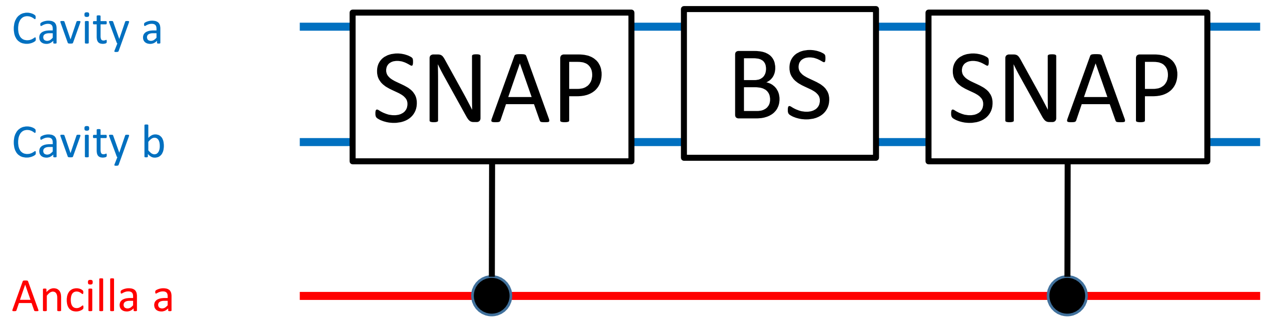
Gauss Law Constraint $\quad G_j \equiv \prod_{i\in\langle ij\rangle} \sigma^x_{\langle i,j\rangle} e^{i\pi a_j^\dagger a_j}$

$$[H, G_j] = 0$$

Realization of $\mathbb{Z}_2$ lattice gauge theory for bosons with SNAP ISA



$$H = \boxed{-J\sum_{\langle ij\rangle} a_i^\dagger \sigma^z_{\langle ij\rangle} a_j} - \lambda\sigma^x_{\langle ij\rangle}$$



$$U(\vec{\theta})e^{Jt[a_i^\dagger a_j - a_j^\dagger a_i]}U^\dagger(\vec{\theta}) = e^{iJt[a_i^\dagger \sigma^z_{\langle ij\rangle} a_j + a_j^\dagger \sigma^z_{\langle ij\rangle} a_i]}$$

Physical intuition: each boson hop changes the sign of $\sigma^x_{\langle ij\rangle}$

$$U(\vec{\theta}) = \text{SNAP} = e^{i\frac{\pi}{2}a_i^\dagger a_i \sigma^z_{\langle ij\rangle}}$$ [controlled parity gate]

[In 1D only need ancillae connected to 1 cavity.]

C²QA ISA (theory) collaboration

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U. Toronto & PNNL

Tim Stavenger
PNNL

Chris Kang
U. Washington

Eleanor Crane
UCL

Micheline Solely
Yale

Kevin Smith
Yale

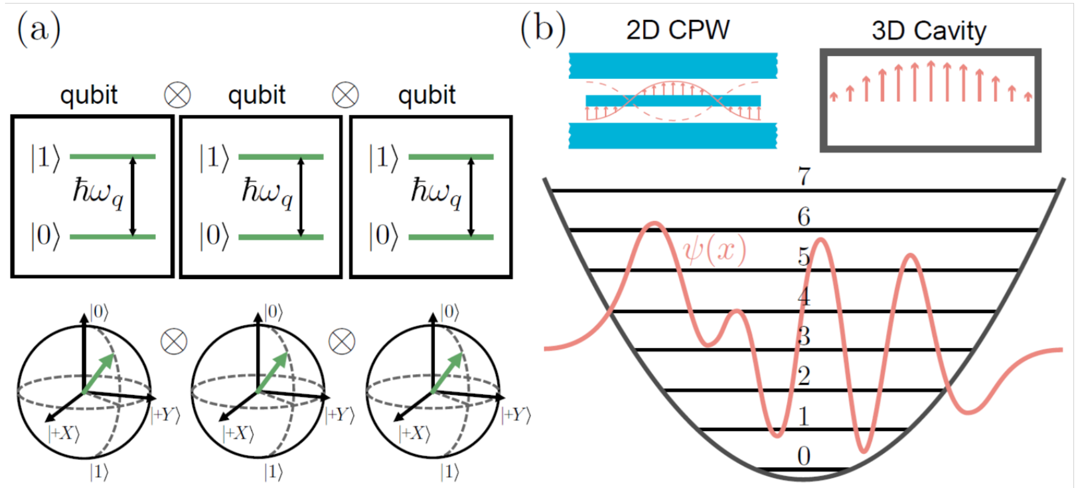
+ Ike Chuang (MIT)
+ Ali Javadi (IBM)

+ Alec Eickbusch and Devoret Lab

+Michael DeMarco
Teague Tomesh
Lena Funke
Stefan Kuehn

Discrete variable
(transmon qubits)

Continuous variable
(microwave or mechanical oscillators)



$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + a_3|011\rangle + a_4|100\rangle + a_5|101\rangle + a_6|110\rangle + a_7|111\rangle$$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle + a_5|5\rangle + a_6|6\rangle + a_7|7\rangle$$

Boson Fock
(photon number)
states

- Instruction Set Architecture for hybrid qubit/oscillator systems
- Qiskit extension to oscillators
  - Represent $\Lambda = 2^n$ levels of oscillator with a register of $n = \log_2 \Lambda$ qubits
  - Access ISA and Wigner tomography toolkit within Qiskit