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Adventures in Phase Space: Why Bosons are Better than Qubits for Quantum Error Correction

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Take-home message:

Quantum error correction

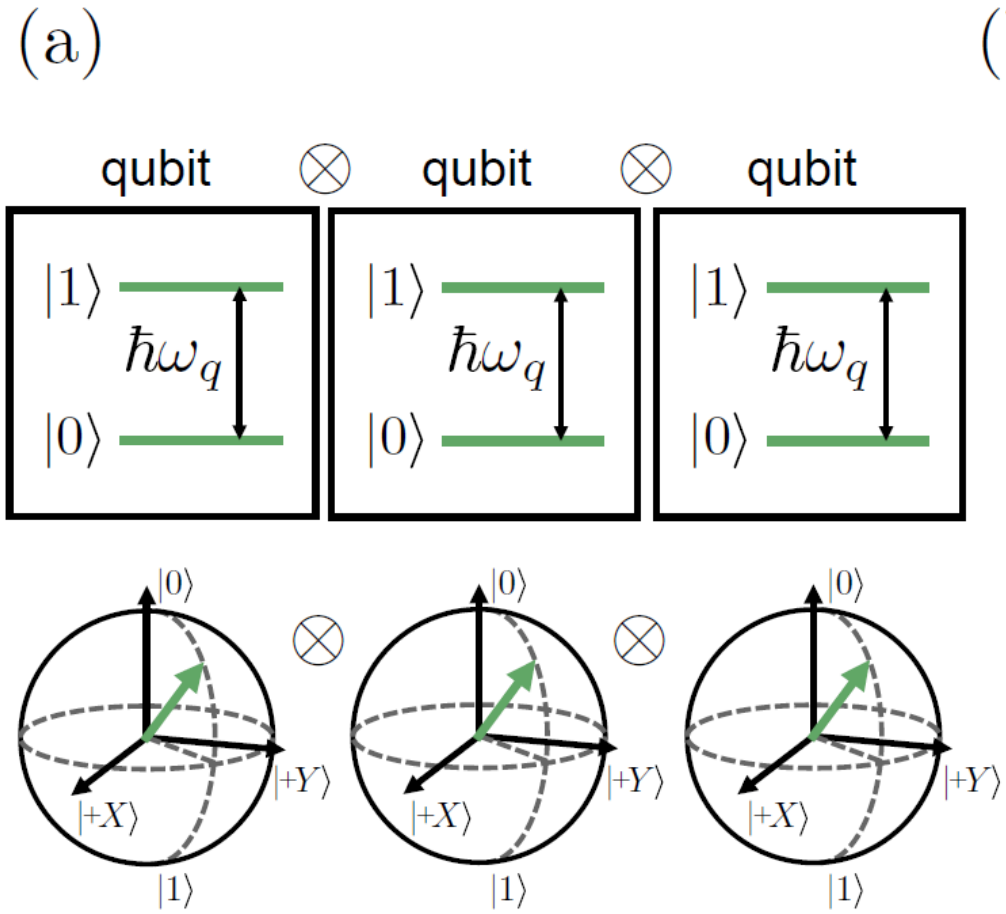
&

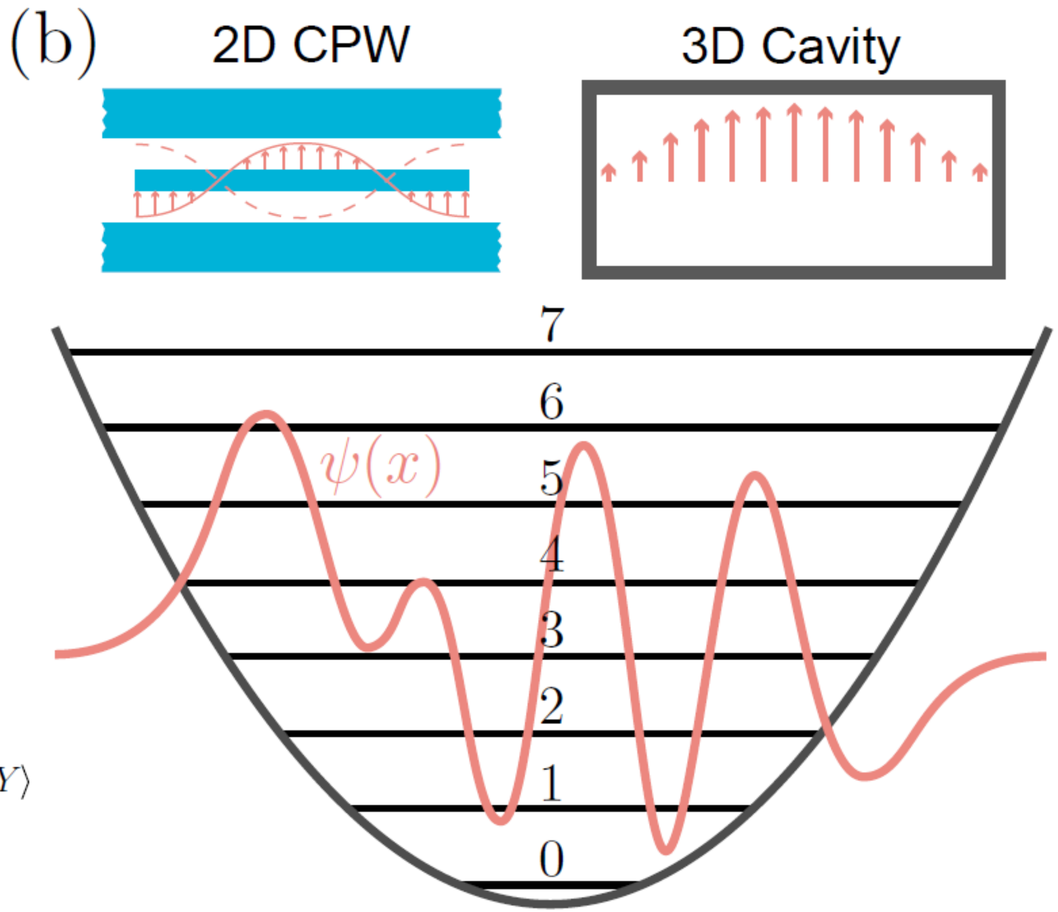
Quantum simulations of physical models containing bosons

are both vastly more efficient on hardware containing ‘native’ bosons

Discrete variable
(transmon qubits)

Continuous variable
(microwave or mechanical oscillators)





Boson Fock
(photon number)
states

$$|\psi\rangle = a_0|000\rangle + a_1|001\rangle + a_2|010\rangle + a_3|011\rangle + a_4|100\rangle + a_5|101\rangle + a_6|110\rangle + a_7|111\rangle$$

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + a_3|3\rangle + a_4|4\rangle + a_5|5\rangle + a_6|6\rangle + a_7|7\rangle$$

The Quantum Error Correction Problem

I am going to give you an unknown quantum state.

If you measure it, it will change randomly due to state collapse (‘back action’).

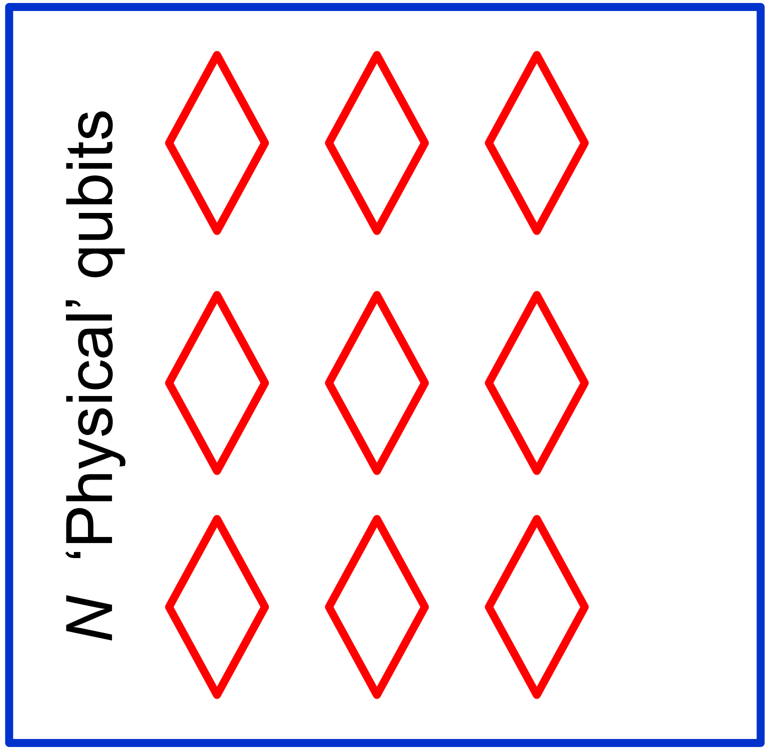
If it develops an error, please fix it.

***Mirable dictu*: It can be done!**

- No-go theorem for error correction in classical analog computation.
- Quantum machines have both analog and digital features.
- Rules of the QEC game:
  - Noise demon has *universal* computational power using arbitrary K-local (bounded Pauli weight) gates (e.g. 1- and 2-qubit (continuous) gates).
  - Noise demon has bounded speed (we hope).
  - You have *less* computational power—only non-universal Clifford gates and measurements.
- You can win!
  (If you are faster than the demon and don't make too many mistakes yourself)

Quantum Error Correction for an unknown state requires storing the quantum information *non-locally* in (non-classical) *correlations* over multiple physical qubits.

‘Logical’ qubit

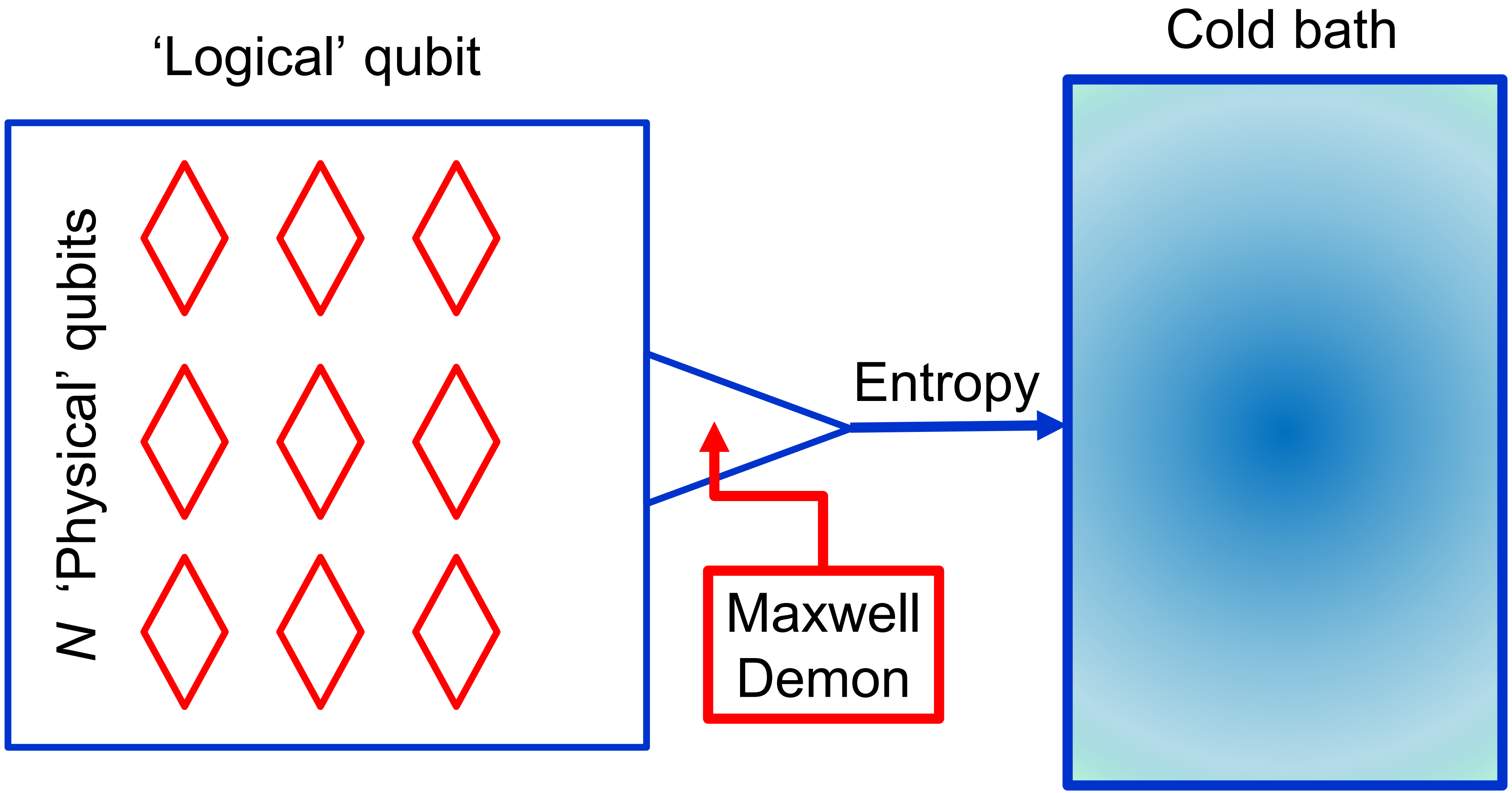


Non-locality: No single physical qubit can “know” the state of the logical qubit.

Special multi-qubit measurements can tell you about errors without telling you the logical state in which the error occurred.

<ins>Miracle</ins>: Quantum errors are analog (i.e. continuous). Measured errors are discrete (i.e. digital). State collapse is our friend!

Quantum Error Correction



N qubits have errors *N* times faster. Maxwell demon must overcome this factor of *N* – *and not introduce errors of its own! (or at least not uncorrectable errors)*

Definition of "better" (QEC Gain)

Average channel fidelity

M. Nielsen, Phys. Lett. A (2002)

["Channel" = CPTP map]

$$\overline{\mathcal{F}}[\mathcal{E}] = \int d\psi \langle\psi|\mathcal{E}(|\psi\rangle\langle\psi|)|\psi\rangle$$

Short time expansion

$$\overline{\mathcal{F}}(\delta t) = 1 - \frac{1}{2}\gamma_{\mathcal{E}}\,\delta t$$

Amplitude damping + dephasing

$$\gamma_{\mathcal{E}} = \frac{\gamma_1 + 2\gamma_2}{3}$$

Pauli channel

$$\gamma_{\mathcal{E}} = \frac{\gamma_X + \gamma_Y + \gamma_Z}{3}$$

<u>QEC gain</u> $G = \frac{\min_i[\gamma_{\mathcal{E}}^{(i)}]}{\gamma_L}$ [Min over all uncorrectable encodings]

"Break-even" $G = 1$

courtesy V. Sivak

Stabilizer Codes

N qubits have 2^N states. Define a 2D logical code subspace: $\mathcal{C} = \text{span}\ \{|0_L\rangle, |1_L\rangle\}$

and logical operators

$$X_L = |0_L\rangle\langle 1_L| + |1_L\rangle\langle 0_L|, \quad Z_L = |0_L\rangle\langle 0_L| - |1_L\rangle\langle 1_L|, \quad Y_L = +iX_L Z_L$$

using $N-1$ stabilizers $\{S_j; j = 1, \ldots, N-1\}$ and imposing $N-1$ constraints

$$\mathrm{S}_j |\psi_{\text{code}}\rangle = (+1)|\psi_{\text{code}}\rangle, \forall j.$$

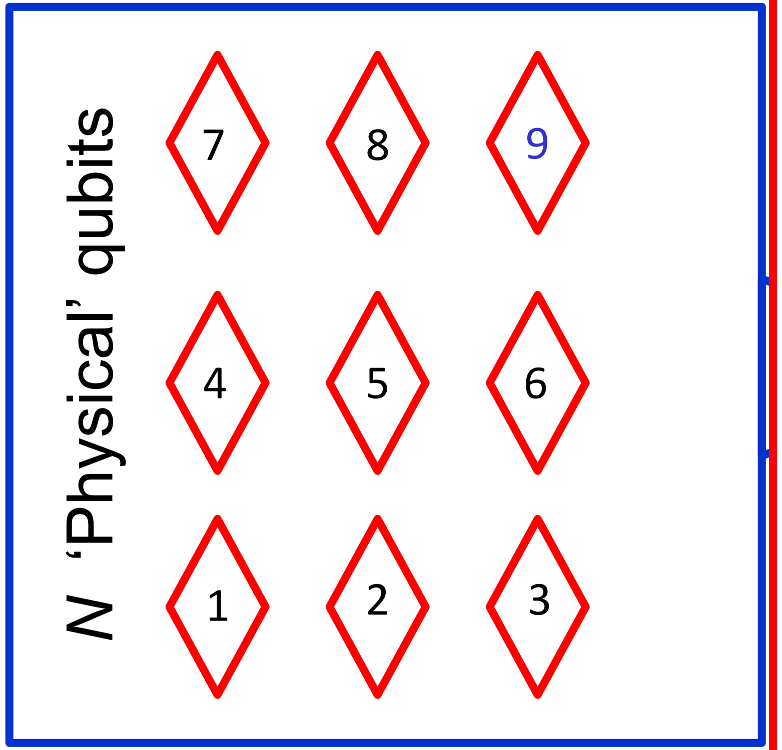
Stabilizers are mutually commuting and commute with logical operators.

[So can be measured simultaneously and without affecting logical state.]

Stabilizers anti-commute with physical errors so measurement of stabilizers give error syndromes that collapse the error state without collapsing the logical state.

Example stabilizer code

‘Logical’ qubit



9 qubit Shor code can correct 1 error: X,Y, or Z

3 types of errors x 9 locations = 27 possible error states + (no-error state)

Code requires 8 stabilizer measurements

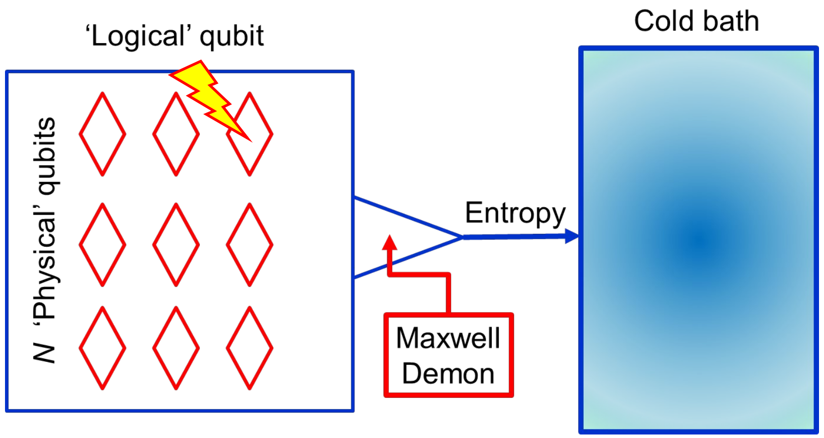
- Z_1Z_2, Z_2Z_3, Z_4Z_5, Z_5Z_6, Z_7Z_8, Z_8Z_9

→ Detect bit flip errors

- $X_1X_2X_3X_4X_5X_6$, $X_4X_5X_6X_7X_8X_9$

→ Detect phase flip errors

Very difficult multi-qubit measurements!
[N.B. cannot measure Z_1, Z_2 separately and multiply results! Need *joint* measurements.]



Idea:
Don't use material objects as qubits.

Use microwave photon states stored in high-Q superconducting resonators.

Cat code (first to exceed break-even):
Ofek, et al., *Nature* **536**, 441–445 (2016)

Binomial Code:
Michael et al., *Phys. Rev. X* **6**, 031006 (2016)
Hu et al., *Nature Physics* **15**, 503 (2019)

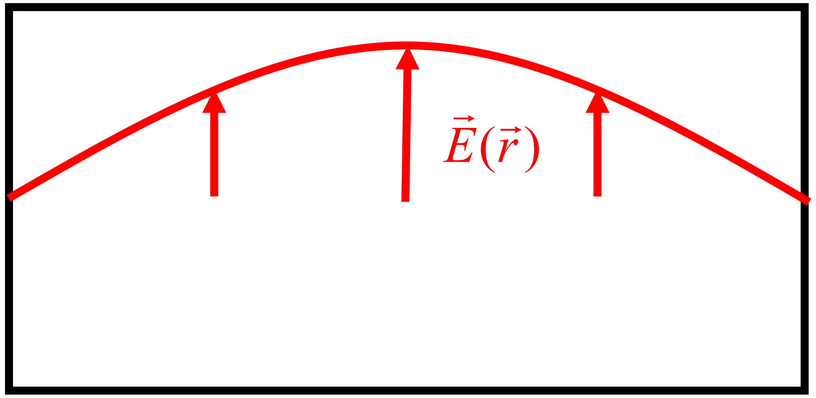
Autonomous Code (T4C truncated cat):
Gertler et al., *Nature* **590**, 243 (2021)

GKP Codes:
Campagne-Ibarcq et al. *Nature* **584**, 368 (2020)

de Neeve et al., *Nature Physics* **18**, 296 (2022)

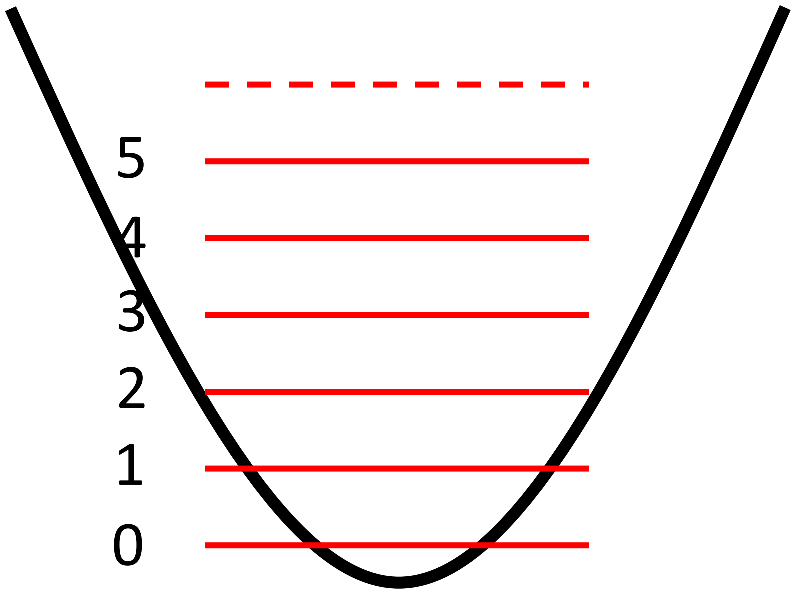
Royer et al., *Phys. Rev. Lett.* **125**, 260509 (2020)
PRX Quantum **3,** 010335 (2022)

Bosonic code reviews:
W. Cai et al., arXiv:2010.08699
A. Joshi et al., arXiv:2008.13471



Single-mode microwave resonators (harmonic oscillators) are empty boxes (vacuum surrounded by superconducting walls)

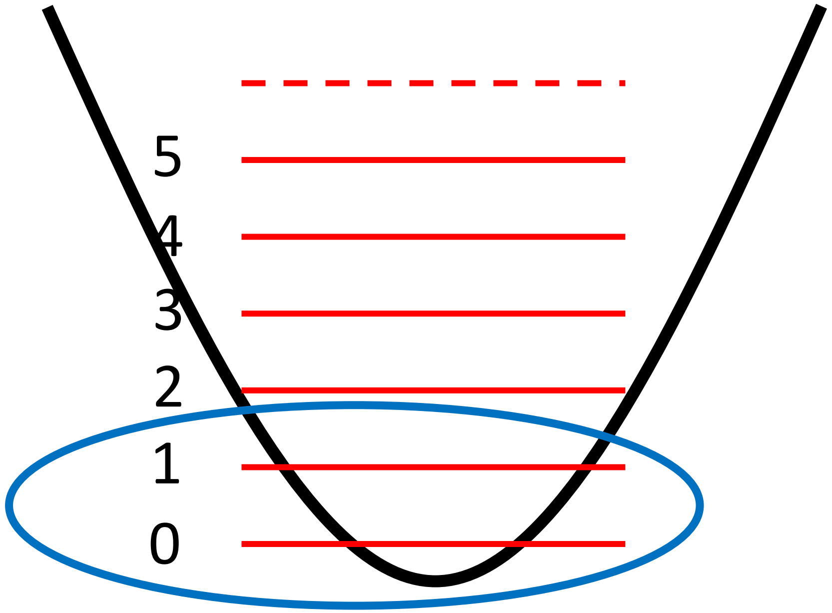
$$\boxed{H = \hbar\omega a^{\dagger} a = \hbar\omega \hat{n}}$$



“Hardware Efficiency”

Oscillators have many quantum levels so can replace multiple physical qubits without adding more ‘moving parts.’

Bosonic Quantum Error Correction Codes



Harmonic oscillator has an infinite number of states. A qubit has only two states.

We need to pick out two orthogonal states to act as 'logical code words' to hold one qubit's worth of (protected) information.

$$\frac{dE}{dt} = -\kappa E \Rightarrow \frac{d\langle \hat{n} \rangle}{dt} = -\kappa \langle \hat{n} \rangle$$

Simplest code: $|0_L\rangle = |0\rangle \quad |1_L\rangle = |1\rangle$

Has smallest possible number of photons and therefore longest lifetime.

But <u>not</u> error correctable after photon loss: $\alpha|0\rangle + \beta|1\rangle \rightarrow |0\rangle$

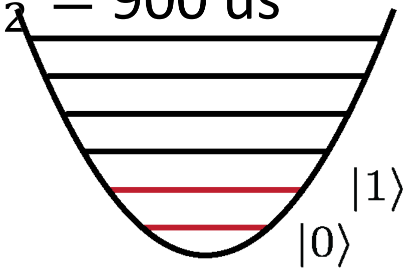
This is what we have to beat to reach break-even.

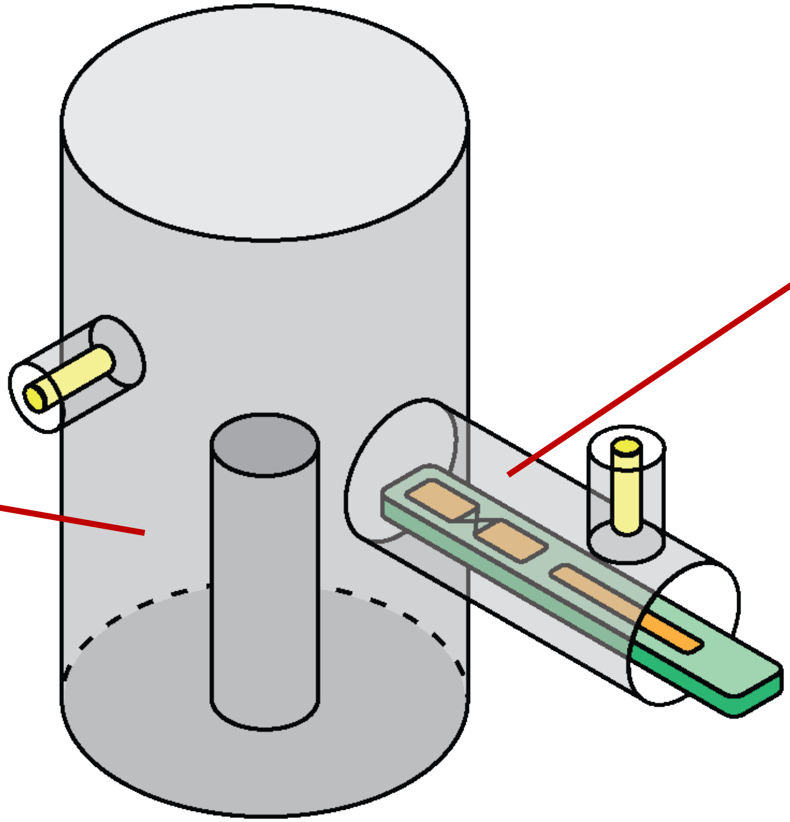
Experimental physics question

Can we leverage active quantum error correction to create a "logical qubit" better than all constituent "physical qubits"?

Physical Qubit #2: Fock
|0>,|1>

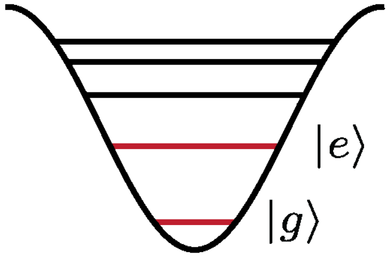
T_1 = 550 us
T_2 = 900 us





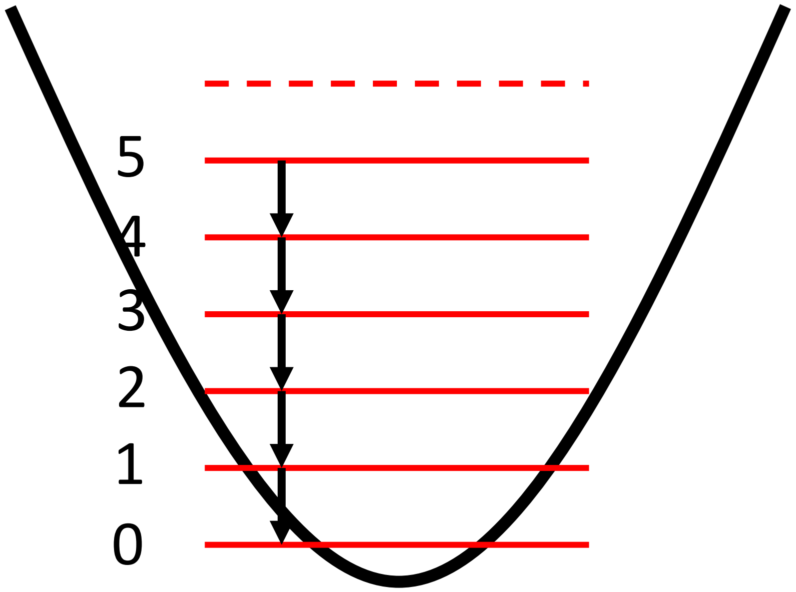
Physical Qubit #1: Transmon

T_1 = 110 us
T_{2e} = 130 us



Transferring QI from transmon to cavity strongly increases lifetime but does NOT constitute QEC "Gain." No QEC yet.

courtesy V. Sivak



Single-mode weakly damped oscillators have a very simple error model: photon loss

$$|\psi\rangle = \alpha|0_L\rangle + \beta|1_L\rangle$$

Use 'code words' with definite photon number parity (e.g. even)

Only a single mode and only <u>one</u> kind of error—photon loss – NOT *3N* errors as for qubits.

Measurement of parity does not tell us the photon number so stabilizer commutes with logical operators.

Only need one simple code 'stabilizer,' Photon number parity:

$$\boxed{\hat{P} = (-1)^{\hat{n}}}$$

Photon loss error flips the parity:

$$\boxed{\hat{P} a \hat{P} = -a}$$

Example code:

$$|0_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}}$$

$$|1_L\rangle = |2\rangle$$

Easy to QND measure with high fidelity (unlike in ordinary quantum optics)

Parity stabilizer measurements 99.8% QND. L. Sun et al., *Nature* **511**, 444 (2014)

Simplest bosonic code example: 'binomial code' uses only 5 photon states 0-4 ($\ln_2$ 5 bits)
Phys. Rev. X 6, 031006 (2016) to correct errors to first order in $\epsilon = \kappa\,\delta t$.

Logical code words	error words
(even parity)	(odd parity)
$\lvert 0_L \rangle = \dfrac{\lvert 0\rangle + \lvert 4\rangle}{\sqrt{2}}$	$a\lvert 0_L\rangle = \sqrt{2}\lvert 3\rangle$
$\lvert 1_L \rangle = \lvert 2\rangle$	$a\lvert 1_L\rangle = \sqrt{2}\lvert 1\rangle$

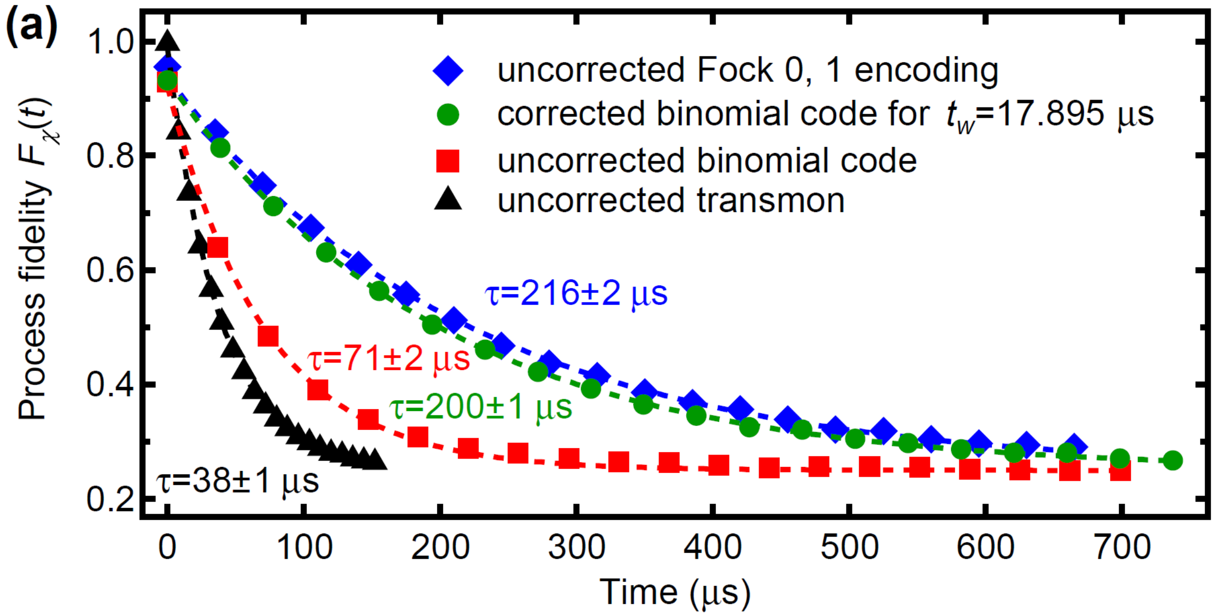
$\langle a^\dagger a\rangle = 2$

Loss is 4x larger!

Break-even for bosonic codes is defined as beating the best uncorrectable bosonic code (0,1) photon Fock encoding:

$\lvert\psi\rangle = \alpha\lvert 0\rangle + \beta\lvert 1\rangle$ $\langle a^\dagger a\rangle = 1/2$

Reaches 92% of break even
Luyan Sun group (Tsinghua)
Nature Phys. **15**, 503 (2019)



First code to (slightly) exceed break even: Schrödinger Cat Code

$$|\Psi\rangle = \psi_0 |0_L\rangle + \psi_1 |1_L\rangle$$

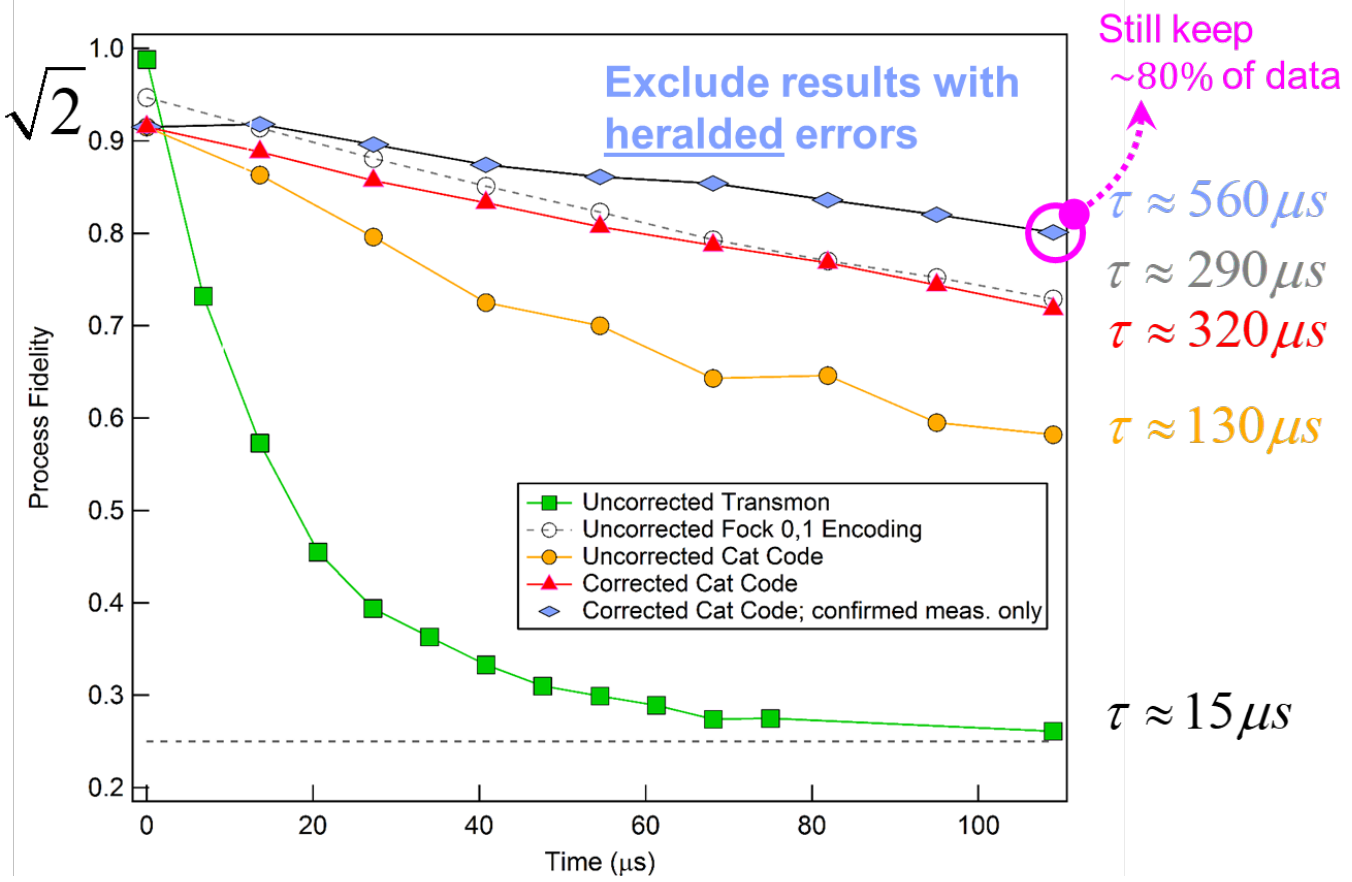
$$|0_L\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|1_L\rangle = |i\alpha\rangle + |-i\alpha\rangle$$

$$\alpha = \sqrt{2}$$

$|0_L\rangle$ $|1_L\rangle$ $|X_L\rangle$

Store a ***qubit*** as a ***superposition*** of two cats of same ***parity***



QEC Gain G:
1.1x break even (unheralded)
1.75x break even (heralded)

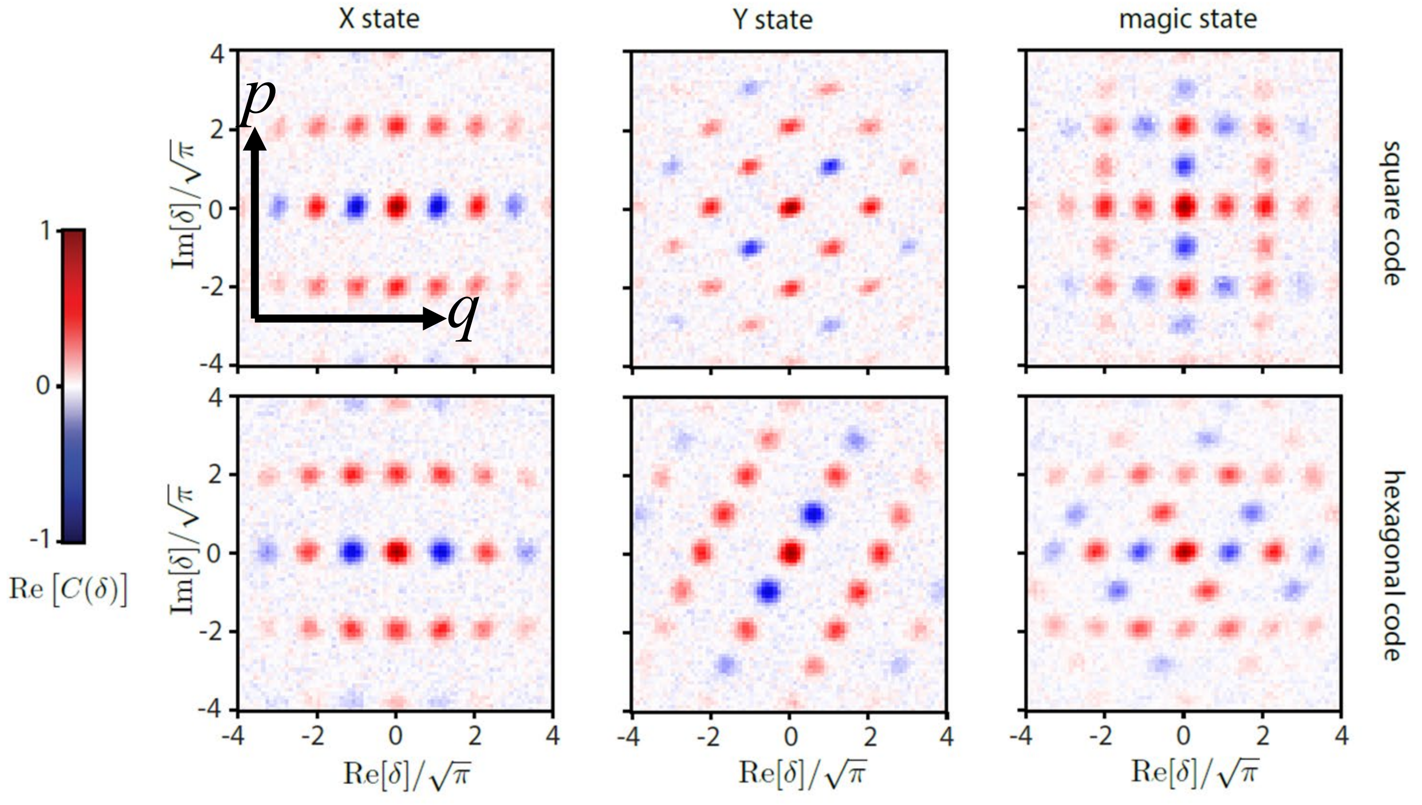
Theory: Leghtas, Mirrahimi, et al., *PRL* **111**, 120501 (2013)
Experiment: Ofek et al. *Nature* **536**, 441 (2016)

'GKP code': Coherent state lattice in phase space ("cat in 35 places at once")

C. Flühmann et al. (Home group) *Nature* **566**, 513 (2019) (state preparation)
P. Campagne-Ibarcq et al. (Devoret group) *Nature* **584**, 368 (2020) (QEC for X,Y,Z errors near break even)
de Neeve et al. (Home group) de Neeve et al. (J. Home group), *Nature Physics* **18**, 296 (2022)
Royer, Singh et al. (Girvin group) *Phys. Rev. Lett.* **125**, 260509 (2020); *PRX Quantum* **3**, 010335 (2022)

Phase space map of oscillator states using Characteristic Function = FT of Wigner function

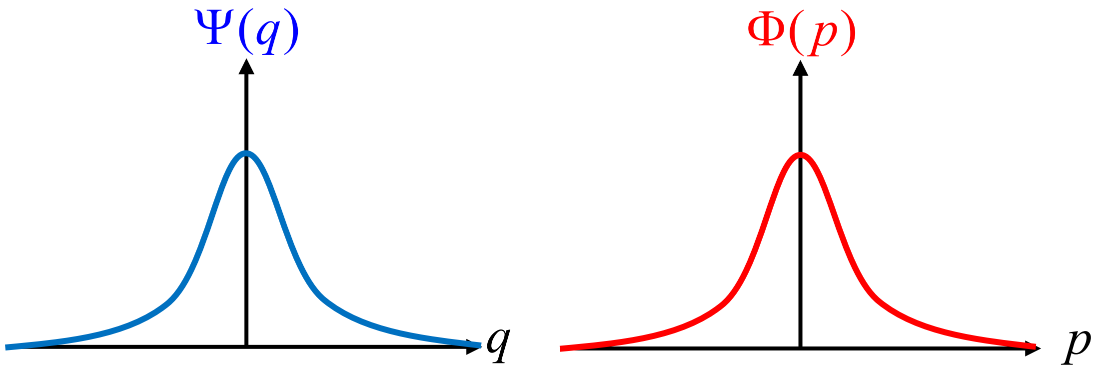
Stabilizers, errors, Clifford gates are all simple displacements!

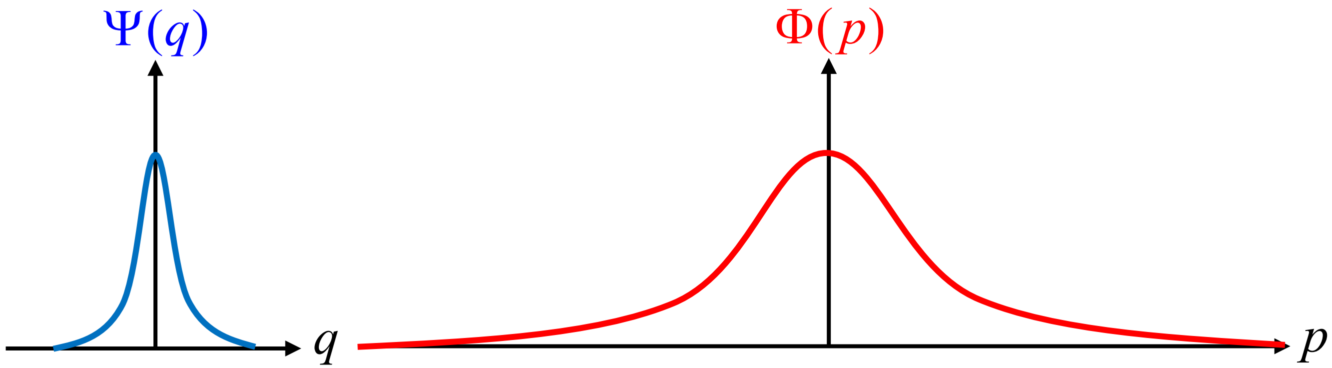


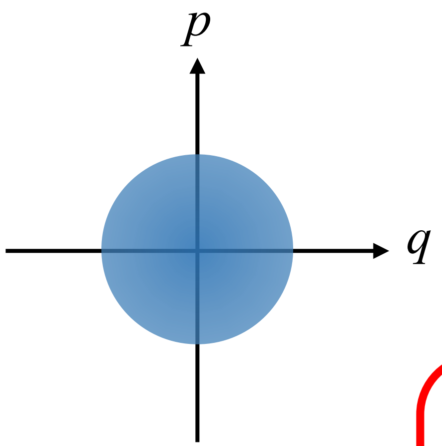
Coherent (vacuum) state of an oscillator

Squeezed state of an oscillator

Understanding phase space….

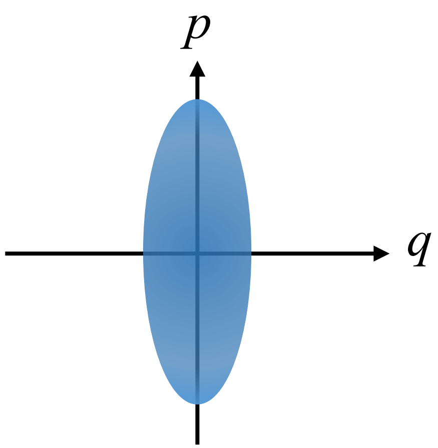






Heisenberg Uncertainty

$$[\hat{q},\hat{p}]=i \;\Rightarrow\; \Delta q \Delta p \geq \frac{1}{2}$$

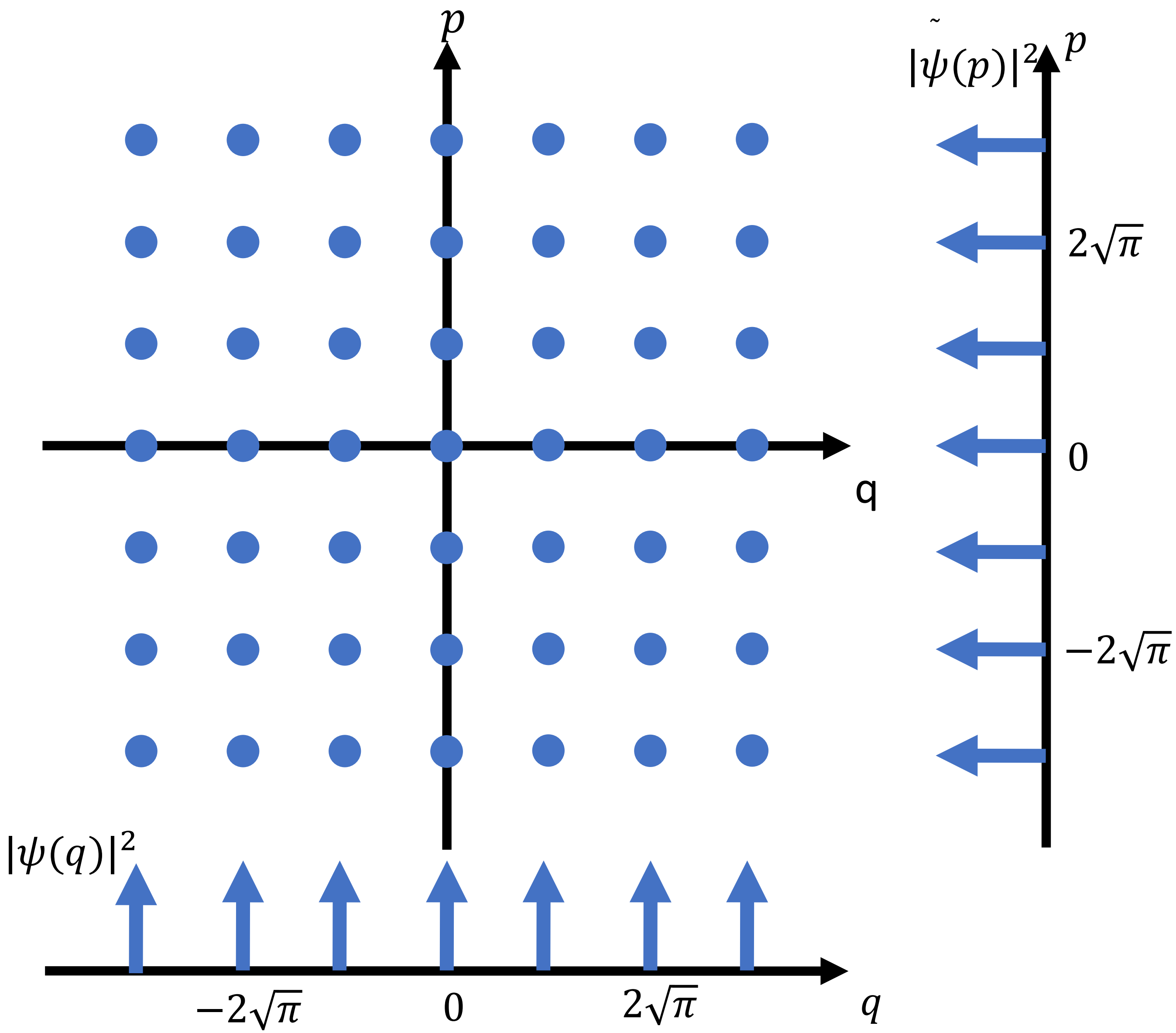


Phase space seems to be 'incompressible'

One state per area $h = 2\pi$

Note: squeezing can be achieved by simply measuring the position of the oscillator with uncertainty less than the zero-point motion.

But recall that a crystal lattice produces sharp Bragg peaks in x-ray diffraction.



Gottesman, Kitaev and Preskill, Phys. Rev. A 64, 012310 (2001)

Proposed encoding a logical qubit in oscillator 'grid' states.

How can the points in this phase space grid be smaller than the minimum uncertainty wave packet?

They seem to be squeezed in both position AND momentum!?

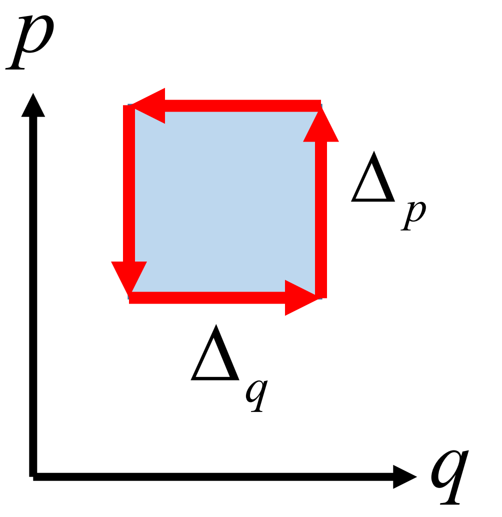
This is possible for special choices of lattice unit cell areas.

$[\hat{q},\hat{p}]=+i \quad \Rightarrow$ translations in phase space do not commute

$$\mathcal{D}(\Delta_q)\Psi(q)=e^{-i\Delta_q\hat{p}}\Psi(q)=\Psi(q-\Delta_q)$$

$$\mathcal{D}(i\Delta_p)\Psi(q)=e^{i\Delta_p q}\Psi(q)$$

$$\mathcal{D}(\Delta_q)\mathcal{D}(i\Delta_p)=\underbrace{e^{i\Delta_p\Delta_q}}_{\text{area}}\mathcal{D}(i\Delta_p)\mathcal{D}(\Delta_q)$$



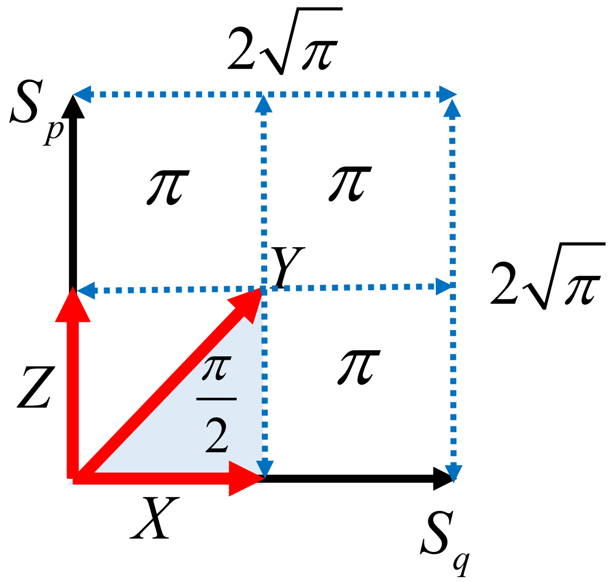
Harmonic Oscillator
Phase Space

Inside the code space:
X,Y,Z translations obey
Pauli group

GKP code space is stabilized by special translations that <u>do</u> commute

$$\boxed{\begin{aligned} S_p &= e^{i2\sqrt{\pi}\hat{q}} \\ S_q &= e^{i2\sqrt{\pi}\hat{p}} \end{aligned}}$$

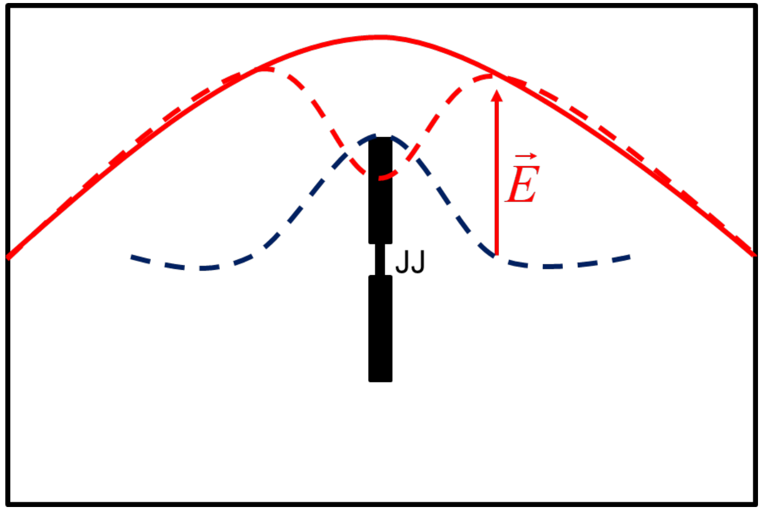
$$\boxed{\begin{aligned} S_qS_p &= e^{i4\pi}S_pS_q \\ S|0_\mathrm{L}\rangle &= (+1)|0_\mathrm{L}\rangle \\ S|1_\mathrm{L}\rangle &= (+1)|1_\mathrm{L}\rangle \end{aligned}}$$



Area $4\pi =$ 2 states

$$S_p = S_q = 1$$
$$X^2 = S_q = 1$$
$$Z^2 = S_p = 1$$
$$ZX = e^{i\pi}XZ = -XZ$$
$$ZX = e^{i\pi/2}Y = iY$$

Exploring phase space with displacements of the oscillator controlled by the ancilla qubit



Strong-dispersive coupling

$$H = \omega_c a^\dagger a + \frac{\omega_q}{2}\sigma^z + \chi\,\sigma^z\, a^\dagger a$$

Qubit state

Cavity displacement

Key ancilla-controlled cavity operation:

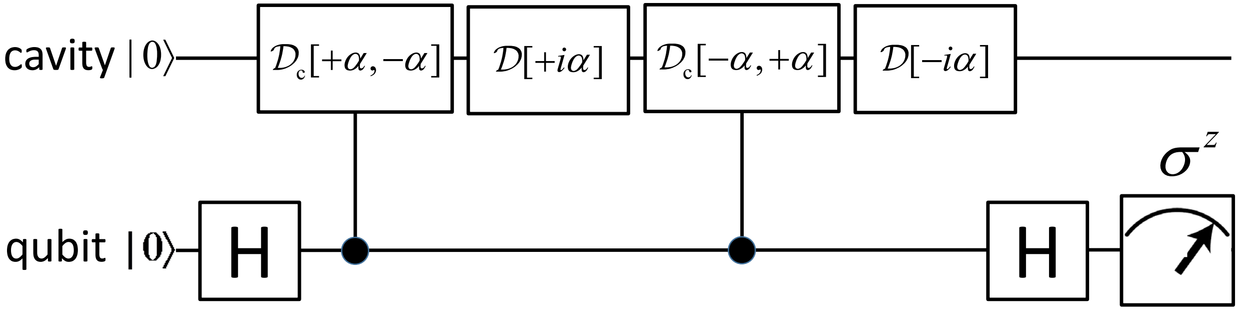
Controlled Displacement gate: $\mathcal{D}_c[\alpha] = e^{\sigma^z[\alpha a^\dagger - \alpha^* a]} = |0\rangle\langle 0|\mathcal{D}[+\alpha] + |1\rangle\langle 1|\mathcal{D}[-\alpha]$

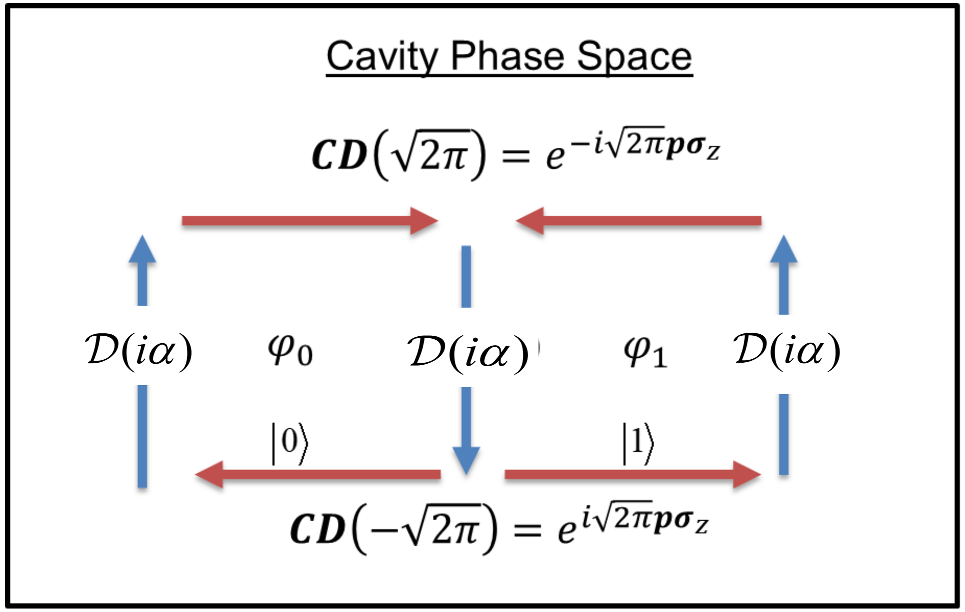
σ^z is for ancilla qubit

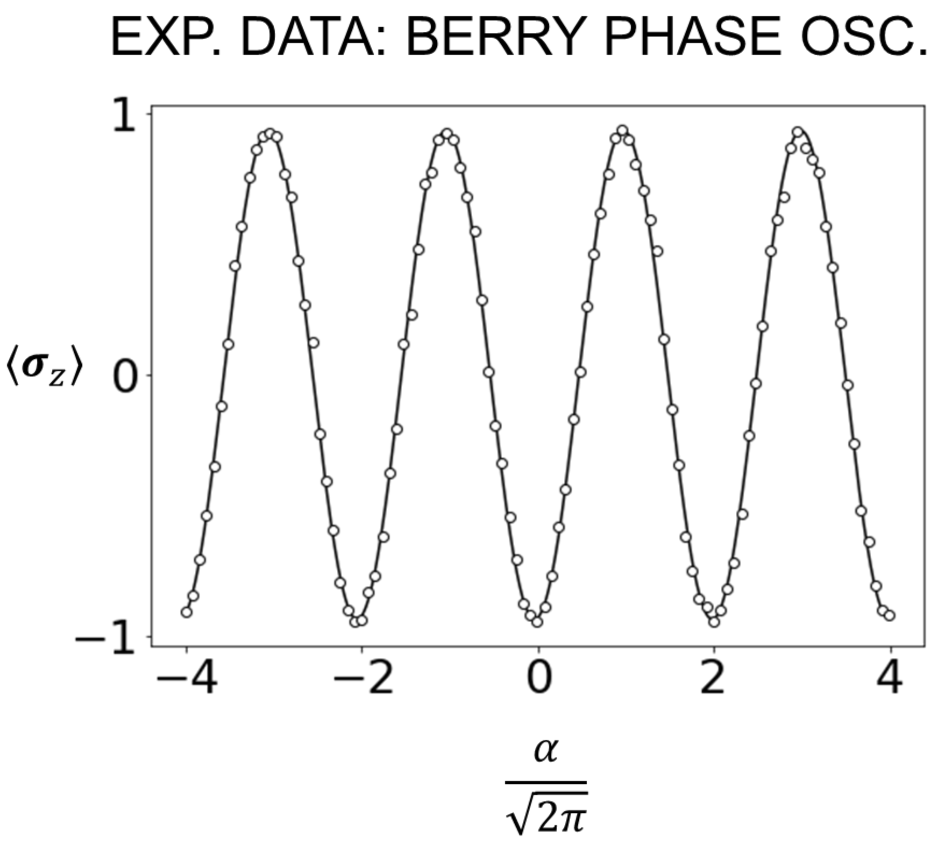
not to be confused with X,Y,Z logical Pauli's for the cavity GKP code

Experimental Calibration of Controlled Displacements Non-Commutativity (Devoret Group)

Conditional Unitary phase kickback measurement:







Campagne-Ibarcq, Eickbusch, Touzard, et al *Nature* **584**, 368 (2020)

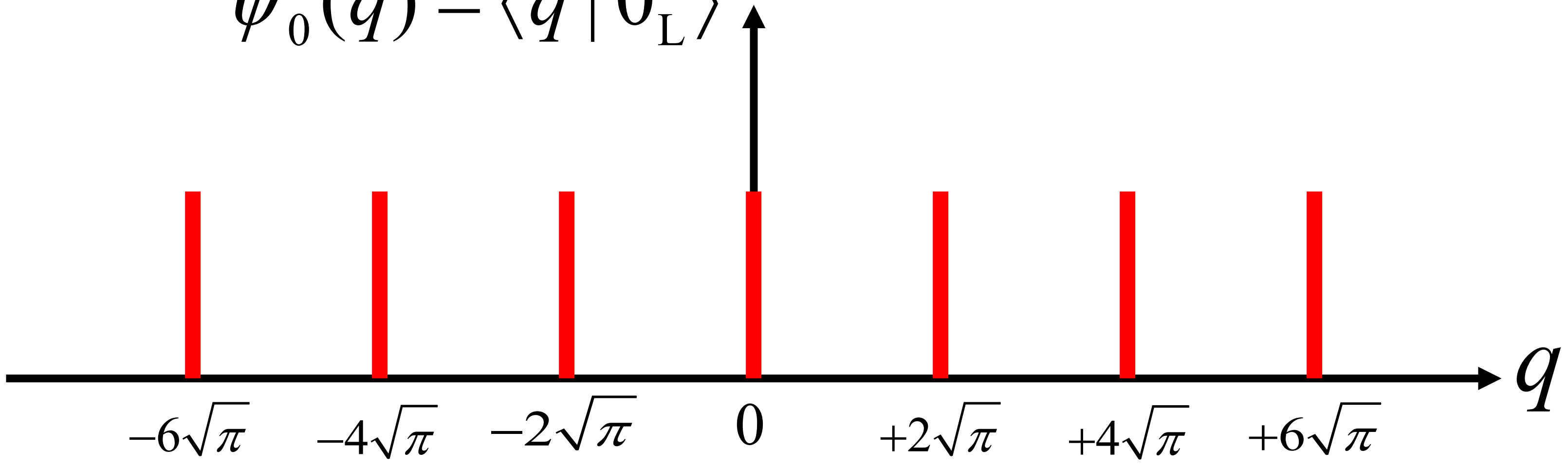
geometric phase acquired over 1 cycle:

$$|0\rangle + |1\rangle \to e^{-i\varphi_0}|0\rangle + e^{i\varphi_1}|1\rangle$$

Bosonic QEC with (idealized) GKP states of an oscillator

Idealized GKP wave functions

$$\psi_0(q) = \langle q \,|\, 0_{\mathrm{L}} \rangle$$



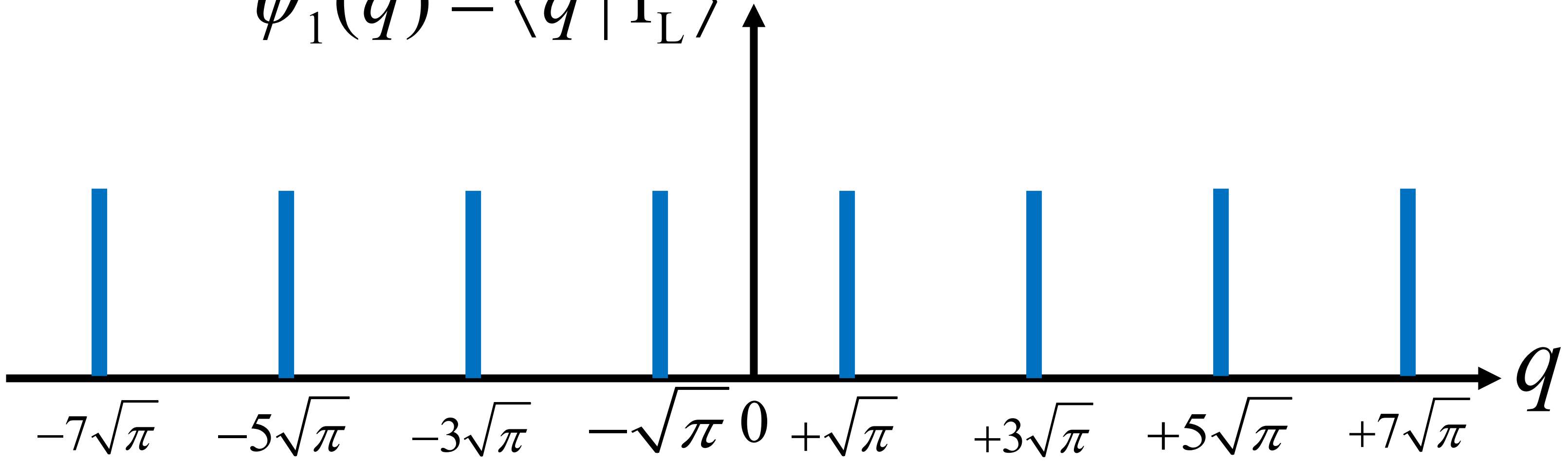
Stabilizers define code space:

$$S_p \,|\, 0_{\mathrm{L}} \rangle = S_q \,|\, 0_{\mathrm{L}} \rangle = (+1) |\, 0_{\mathrm{L}} \rangle$$

$$S_p \,|\, 1_{\mathrm{L}} \rangle = S_q \,|\, 1_{\mathrm{L}} \rangle = (+1) |\, 1_{\mathrm{L}} \rangle$$

$$\boxed{\begin{aligned} S_p &= e^{i2\sqrt{\pi}\hat{q}} \\ S_q &= e^{i2\sqrt{\pi}\hat{p}} \end{aligned}}$$

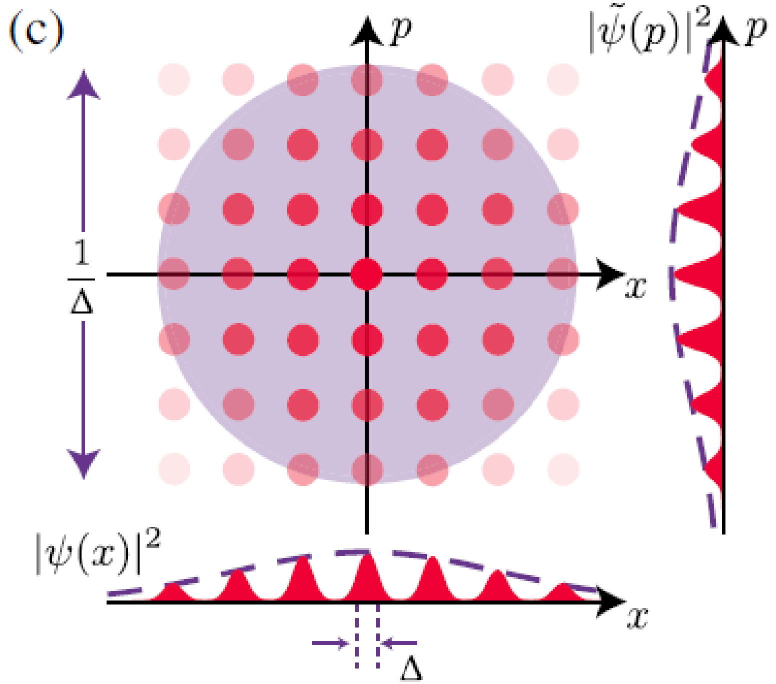
$$\psi_1(q) = \langle q \,|\, 1_{\mathrm{L}} \rangle$$

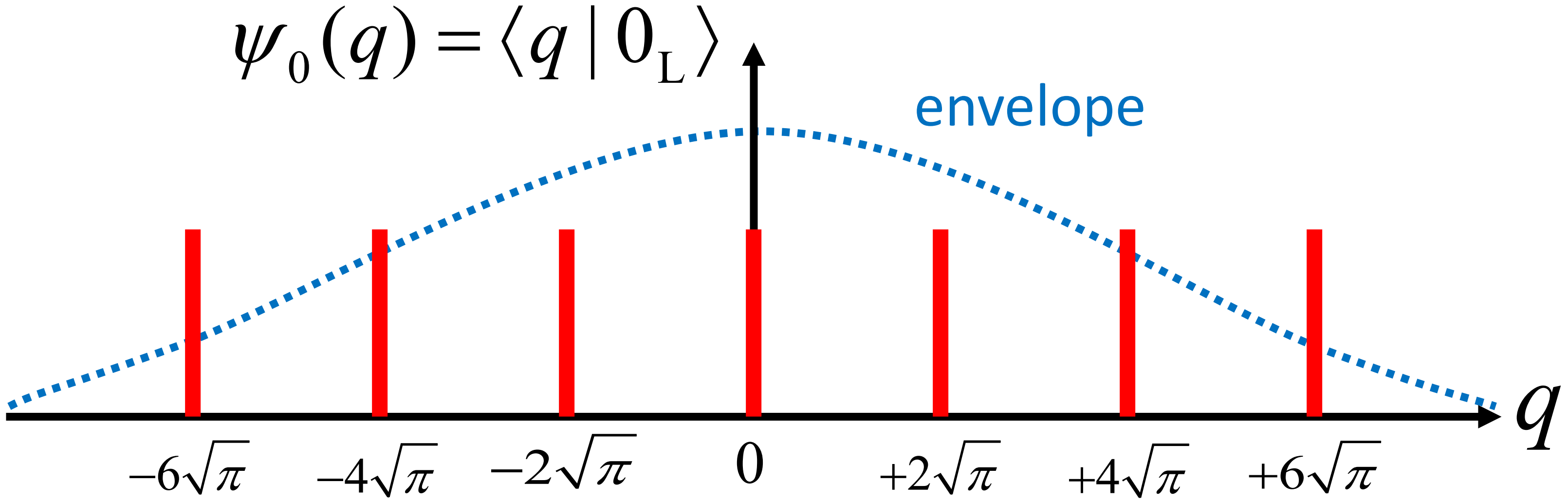


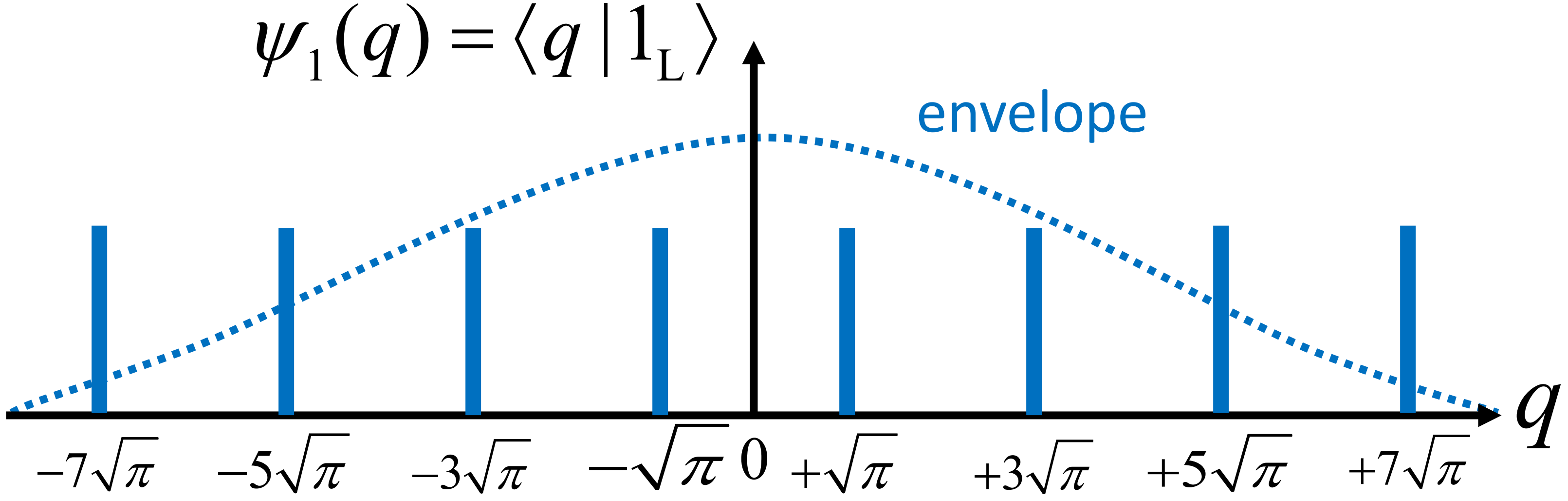
Bosonic QEC with (finite-energy) GKP states of an oscillator

envelope

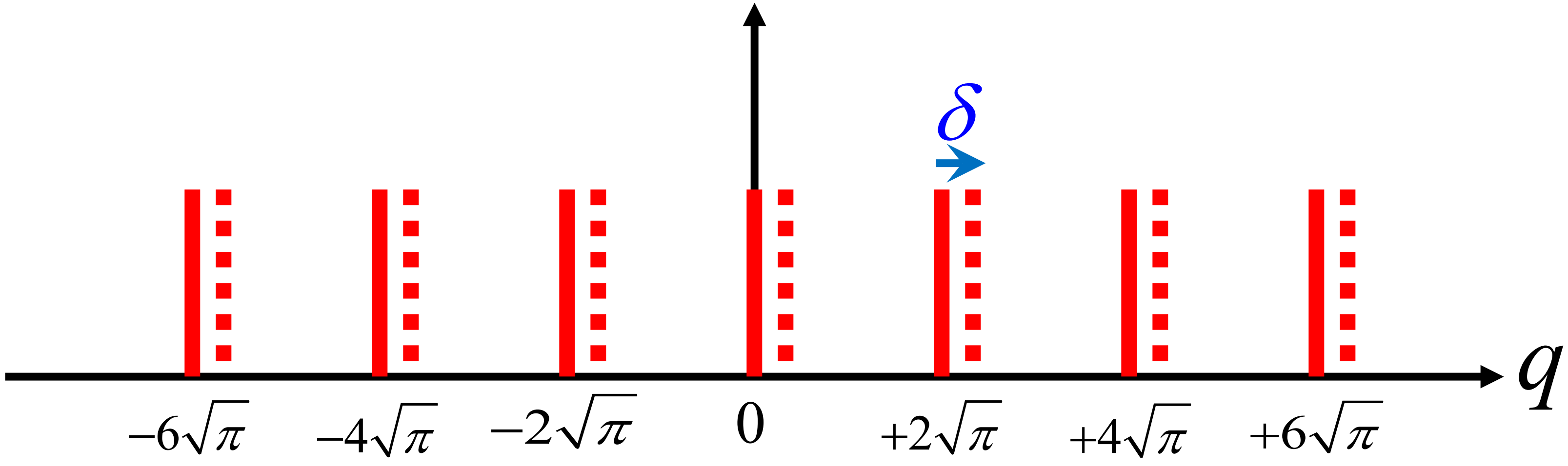
$$|\tilde{0}_L\rangle \sim e^{-\epsilon\hat{n}}|0_L\rangle$$







Bosonic QEC with GKP states of an oscillator



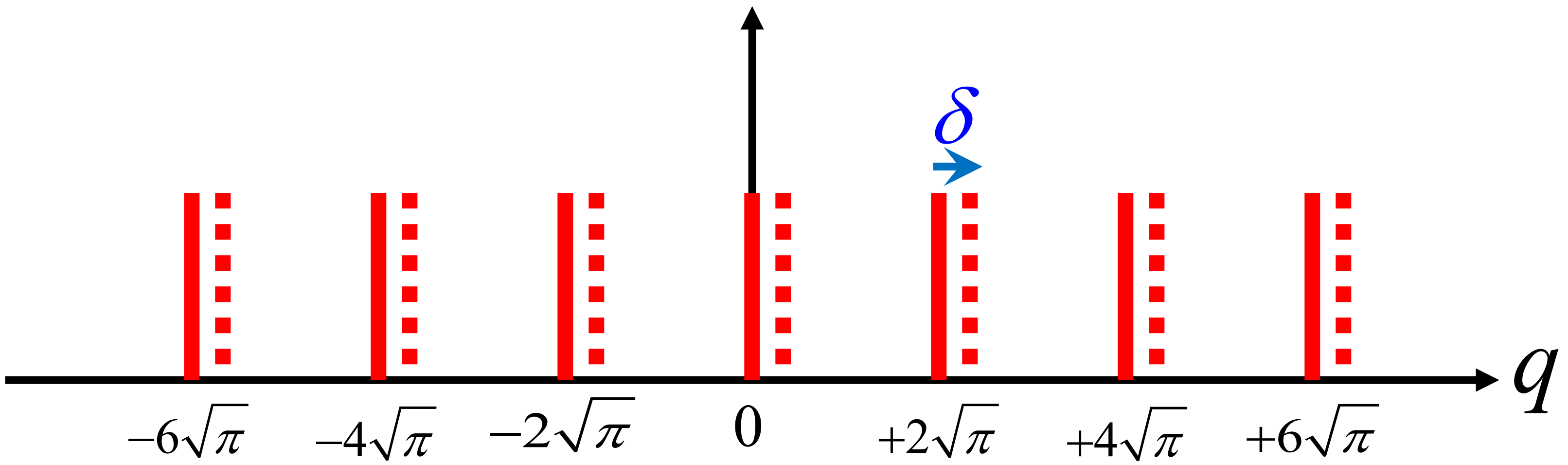
Code space is stabilized by:

$$\boxed{\begin{aligned} S_p &= e^{i2\sqrt{\pi}\hat{q}} \\ S_q &= e^{i2\sqrt{\pi}\hat{p}} \end{aligned}}$$

$$S_p \left|\Psi_\delta\right\rangle = e^{i2\sqrt{\pi}\delta} \left|\Psi_\delta\right\rangle$$

Continuous stabilizer eigenvalue on the unit circle in the complex plane.

N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum.



Code space is stabilized by:

$$\boxed{\begin{aligned} S_p &= e^{i2\sqrt{\pi}\hat{q}} \\ S_q &= e^{i2\sqrt{\pi}\hat{p}} \end{aligned}}$$

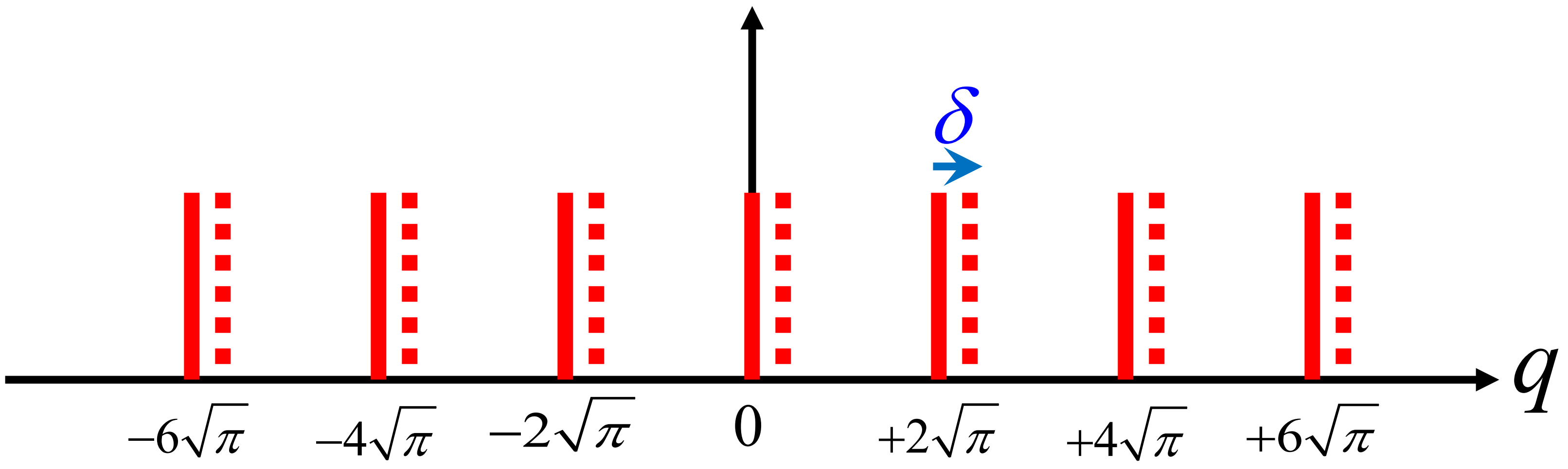
$$S_p \left|\Psi_\delta\right\rangle = e^{i2\sqrt{\pi}\delta}\left|\Psi_\delta\right\rangle$$

Continuous stabilizer eigenvalue on the unit circle in the complex plane.

N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum.

ONLY 2 STABILIZERS NEEDED TO REDUCE INFINITE STATE SPACE DOWN TO 2 LOGICAL STATES!

Bosonic QEC with GKP states of an oscillator



Code space is stabilized by:

$$\boxed{\begin{aligned} S_p &= e^{i2\sqrt{\pi}\hat{q}} \\ S_q &= e^{i2\sqrt{\pi}\hat{p}} \end{aligned}}$$

N.B. Unlike ordinary qubit stabilizers, these have a continuum of eigenvalues on the unit circle corresponding to continuous drift of position or momentum.

Stabilization against drift errors in *position q*

Measure stabilizer to detect error:

$$\mathrm{Im}\langle S_p\rangle = \langle \sin[2\sqrt{\pi}\hat{q}]\rangle$$

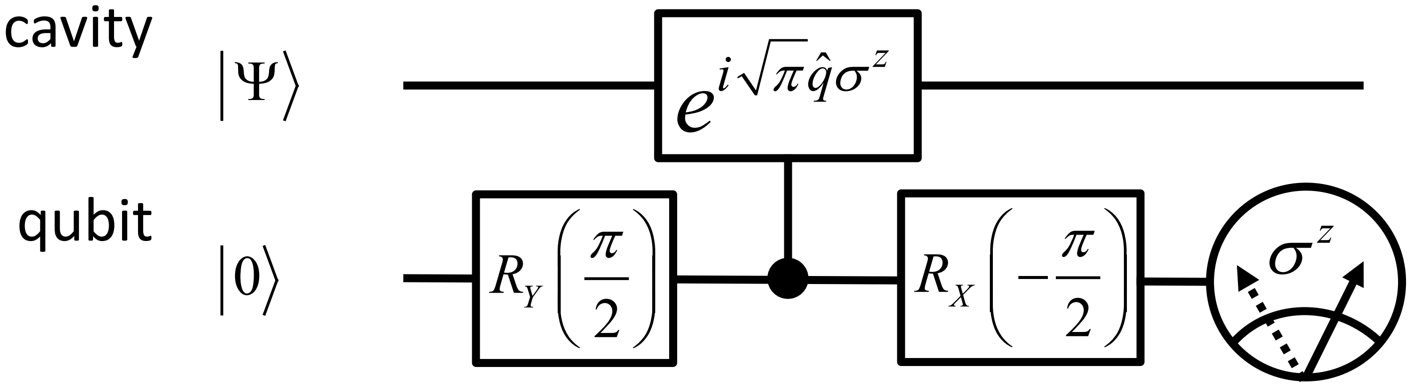
$$= \int dq \sin[2\sqrt{\pi}q]\,|\psi(q)|^2 = \sin[2\sqrt{\pi}\delta]$$

and feedback to correct.

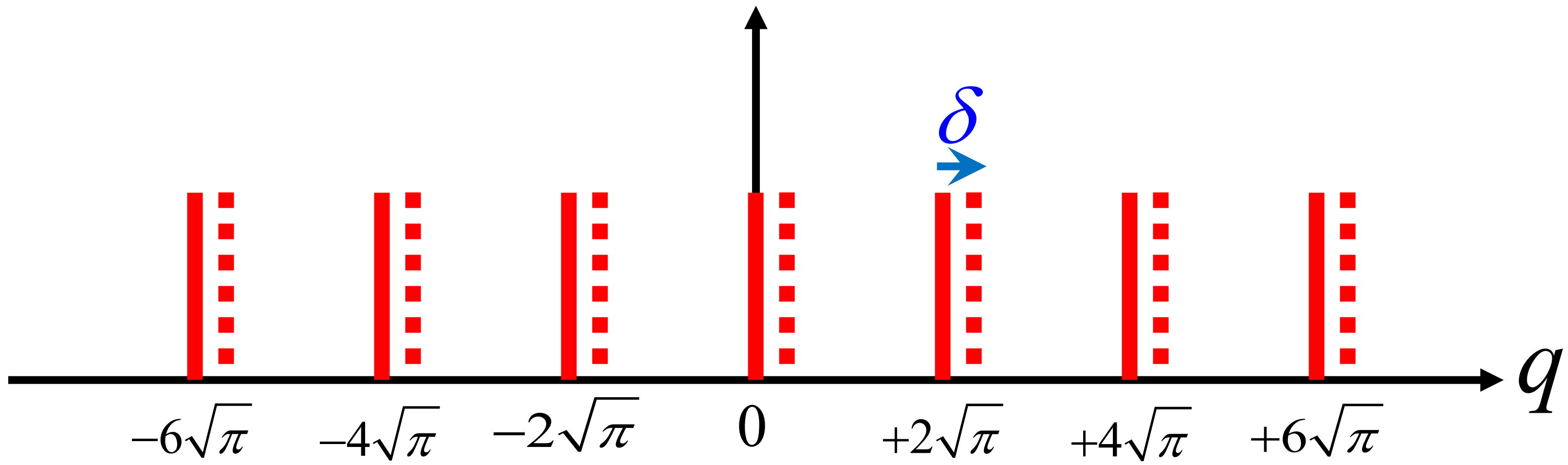
Use 'Bang-Bang' controller to measure stabilizer with one-bit resolution (quantum phase estimation)

$$S_p = e^{i2\sqrt{\pi}\hat{q}}$$

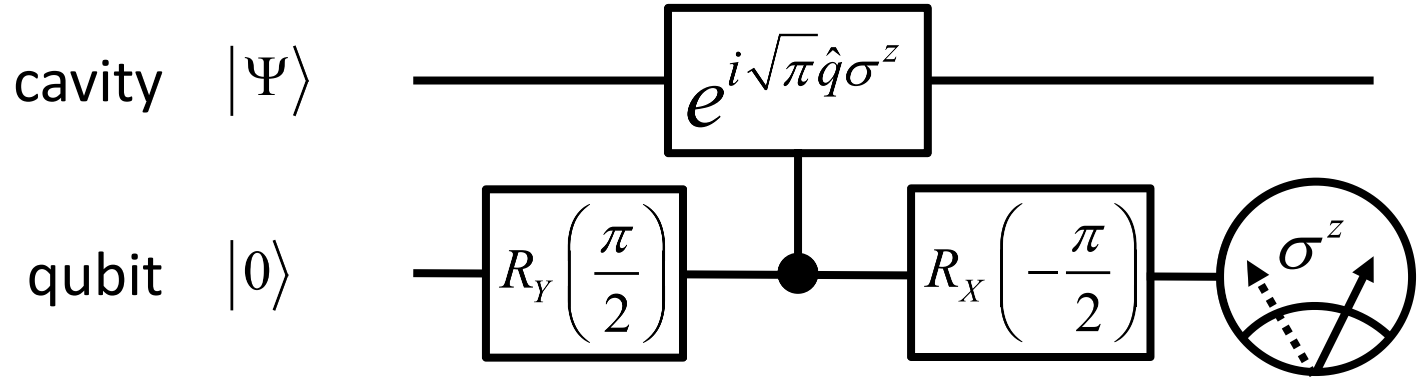
Measuring stabilizer using phase kickback from <u>conditional</u> displacement operation



$$\boxed{\langle\sigma^z\rangle = \mathrm{Im}\langle S_p\rangle = \langle \sin 2\sqrt{\pi}\hat{q}\rangle = \langle \sin 2\sqrt{\pi}\delta\rangle}$$

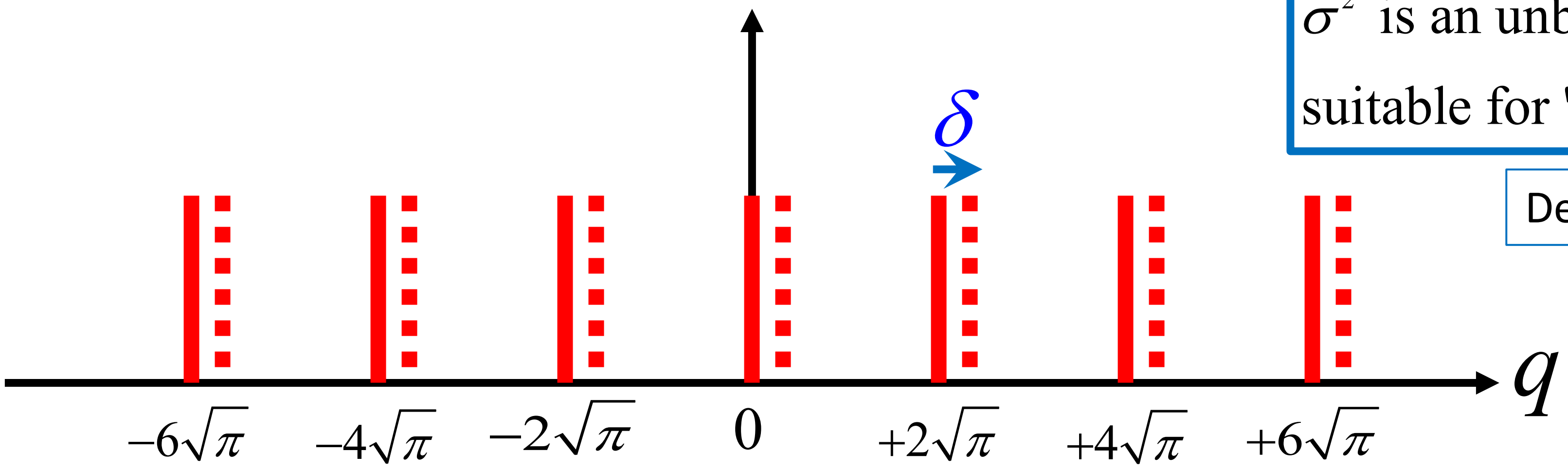


Measuring stabilizer using phase kickback from conditional displacement operation



$$\langle\sigma^z\rangle = \mathrm{Im}\langle S_p\rangle = \langle \sin 2\sqrt{\pi}\hat{q}\rangle = \langle \sin 2\sqrt{\pi}\delta\rangle$$

σ^z is an unbiased single-bit estimator of $\mathrm{Im}\, S_p$ suitable for 'bang-bang' feedback stabilization



Devoret group: *Nature* **584**, 368 (2020)

Quantum state tomography

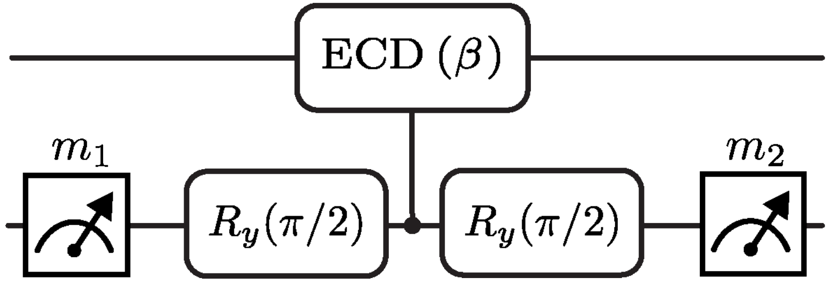
Characteristic function for qubits: $$C(k) = \mathrm{Tr}[\rho W_k], \quad W_k \in \{I, X, Y, Z\}^{\otimes n}$$

A. Eickbusch, **V. Sivak** *et al.,* arXiv:2111.06414

courtesy V. Sivak

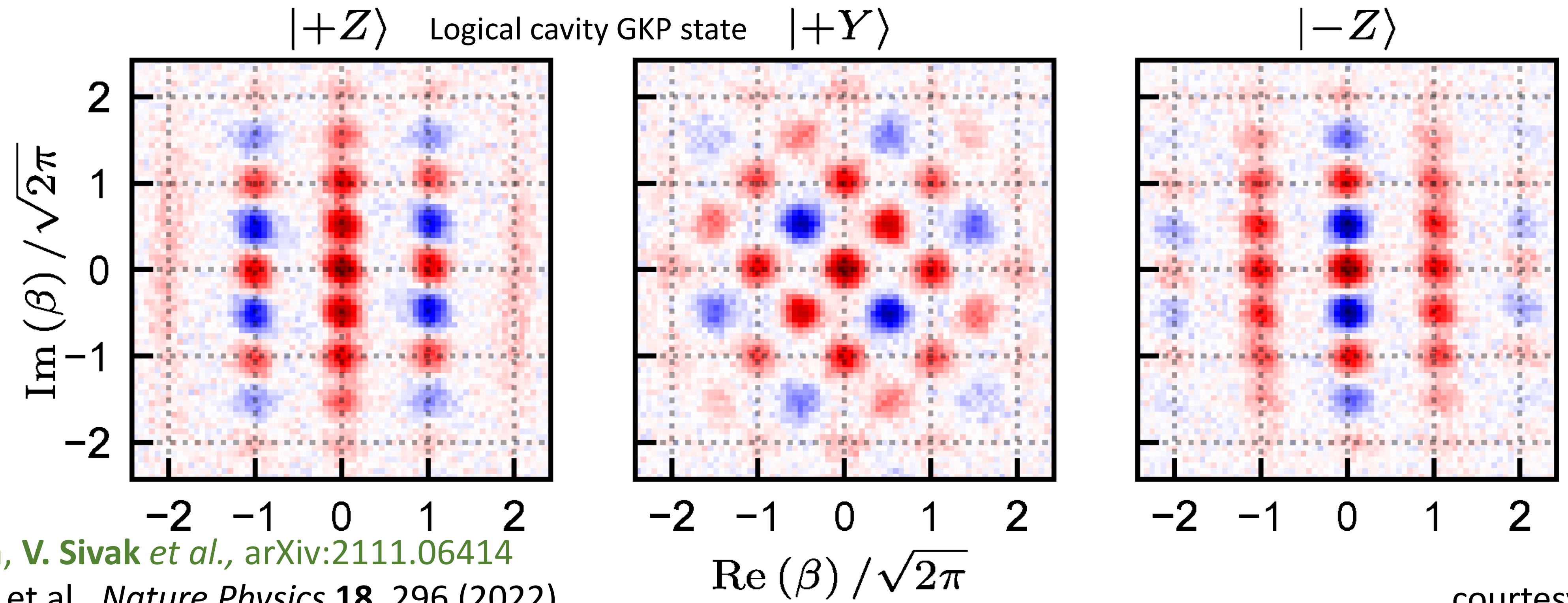
Quantum state tomography

Characteristic function for oscillators: $C(\beta) = \mathrm{Tr}[\rho\, D(\beta)], \quad D(\beta) = \exp(\beta a^\dagger - \beta^* a)$



Fidelity

$\mathcal{F} = 0.8 - 0.85$



A. Eickbusch, **V. Sivak** *et al.*, arXiv:2111.06414

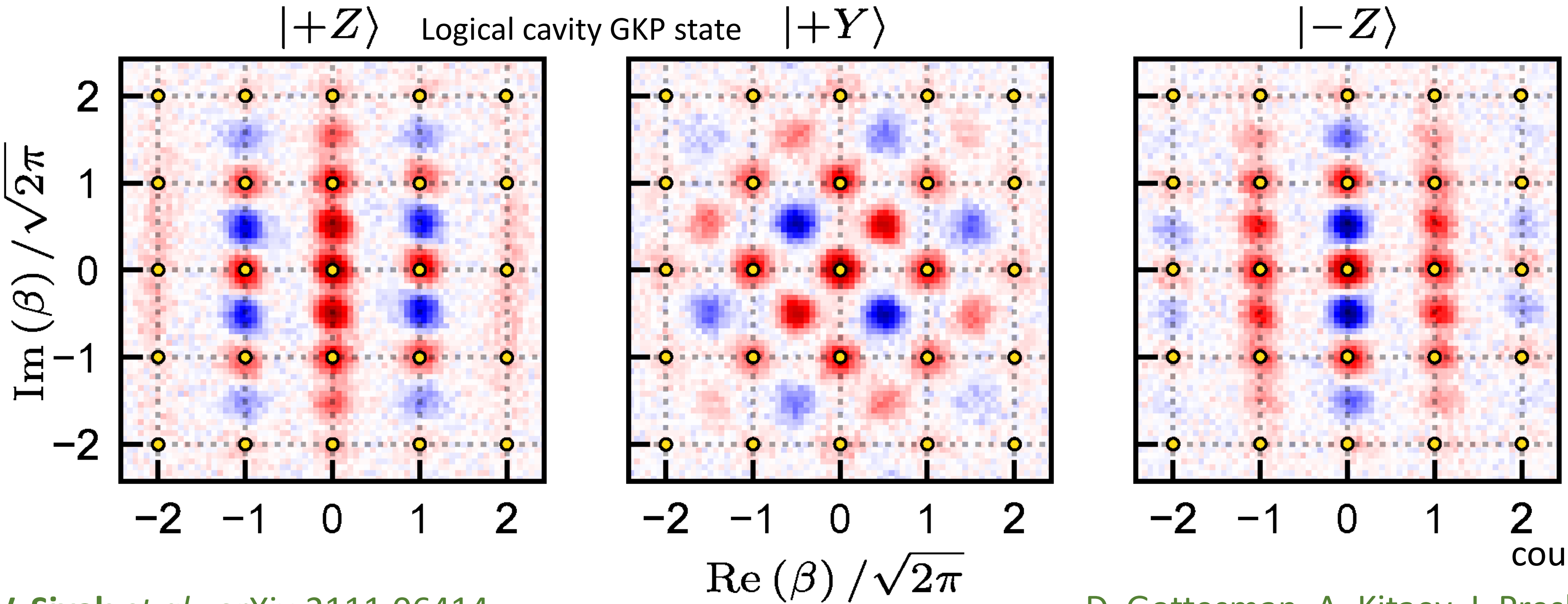
B. de Neeve et al., *Nature Physics* **18**, 296 (2022)

courtesy V. Sivak

Gottesman-Kitaev-Preskill code

Characteristic function: $C(\beta) = \mathrm{Tr}[\rho\, D(\beta)], \quad D(\beta) = \exp(\beta a^\dagger - \beta^* a)$

GKP code stabilizers: $S_X = D(\sqrt{2\pi}\,), \quad S_Z = D(\sqrt{2\pi} i\,)$



courtesy V. Sivak

A. Eickbusch, **V. Sivak** *et al.*, arXiv:2111.06414

D. Gottesman, A. Kitaev, J. Preskill (PRA, 2001)

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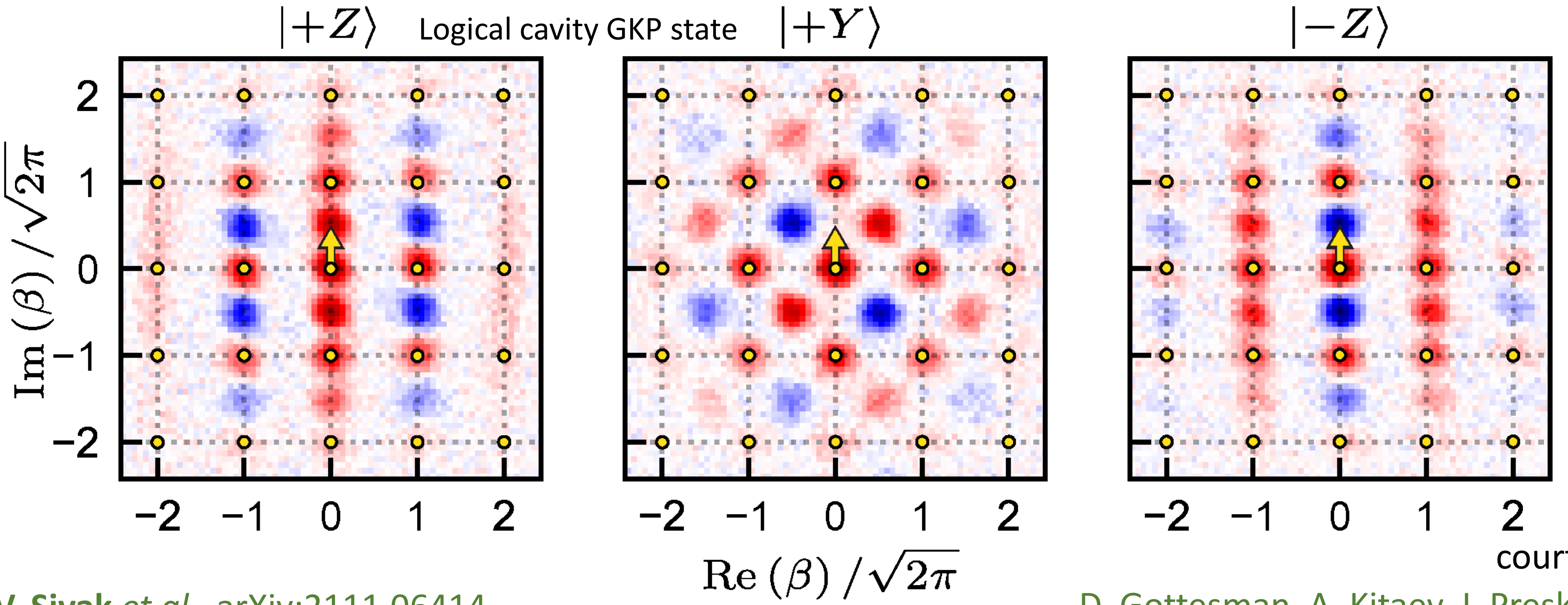
GKP code Pauli operators: $X = D(\sqrt{\pi/2}\,), \quad Z = D(\sqrt{\pi/2}i\,)$



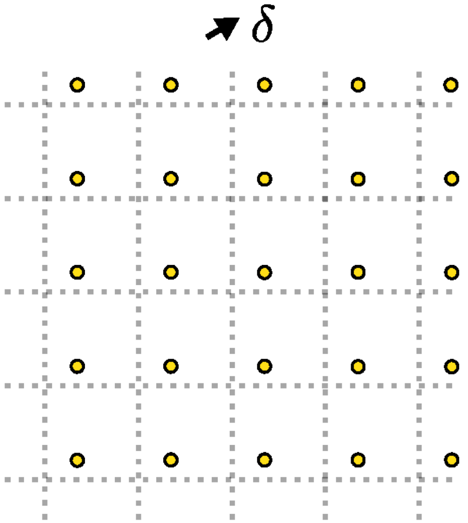
courtesy V. Sivak

A. Eickbusch, **V. Sivak** *et al.,* arXiv:2111.06414

D. Gottesman, A. Kitaev, J. Preskill (PRA, 2001)

Small-Big-Small (SBS) protocol (autonomous and tuned for finite-energy approximate GKP states)

Small displacement error: $|\psi_\delta\rangle = D(\delta)\,|\psi\rangle$ Still a grid state!

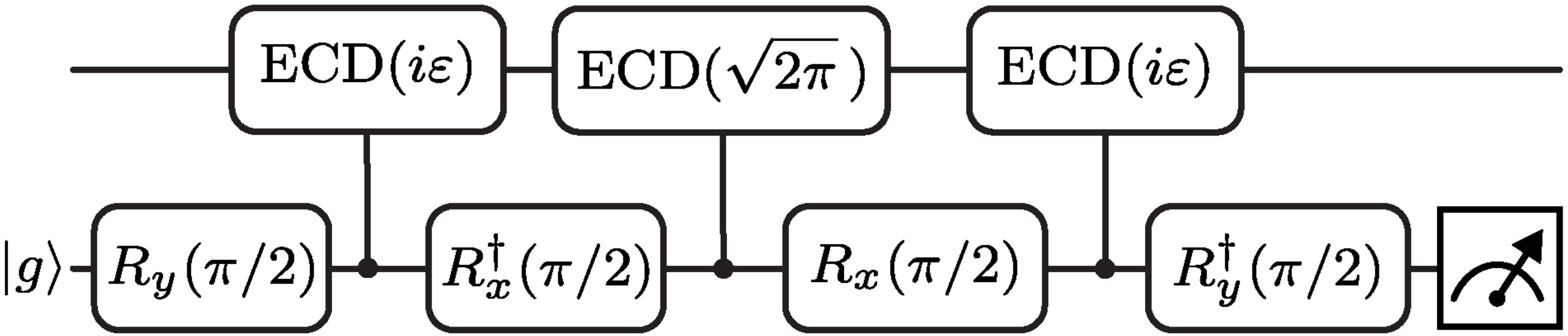


$$S_Z\,|\psi_\delta\rangle = e^{+2i\sqrt{2\pi}\mathrm{Re}[\delta]}|\psi_\delta\rangle$$
$$S_X\,|\psi_\delta\rangle = e^{-2i\sqrt{2\pi}\mathrm{Im}[\delta]}|\psi_\delta\rangle$$

Stabilizer phase estimation

Envelope pre-distortion

Displacement error correction

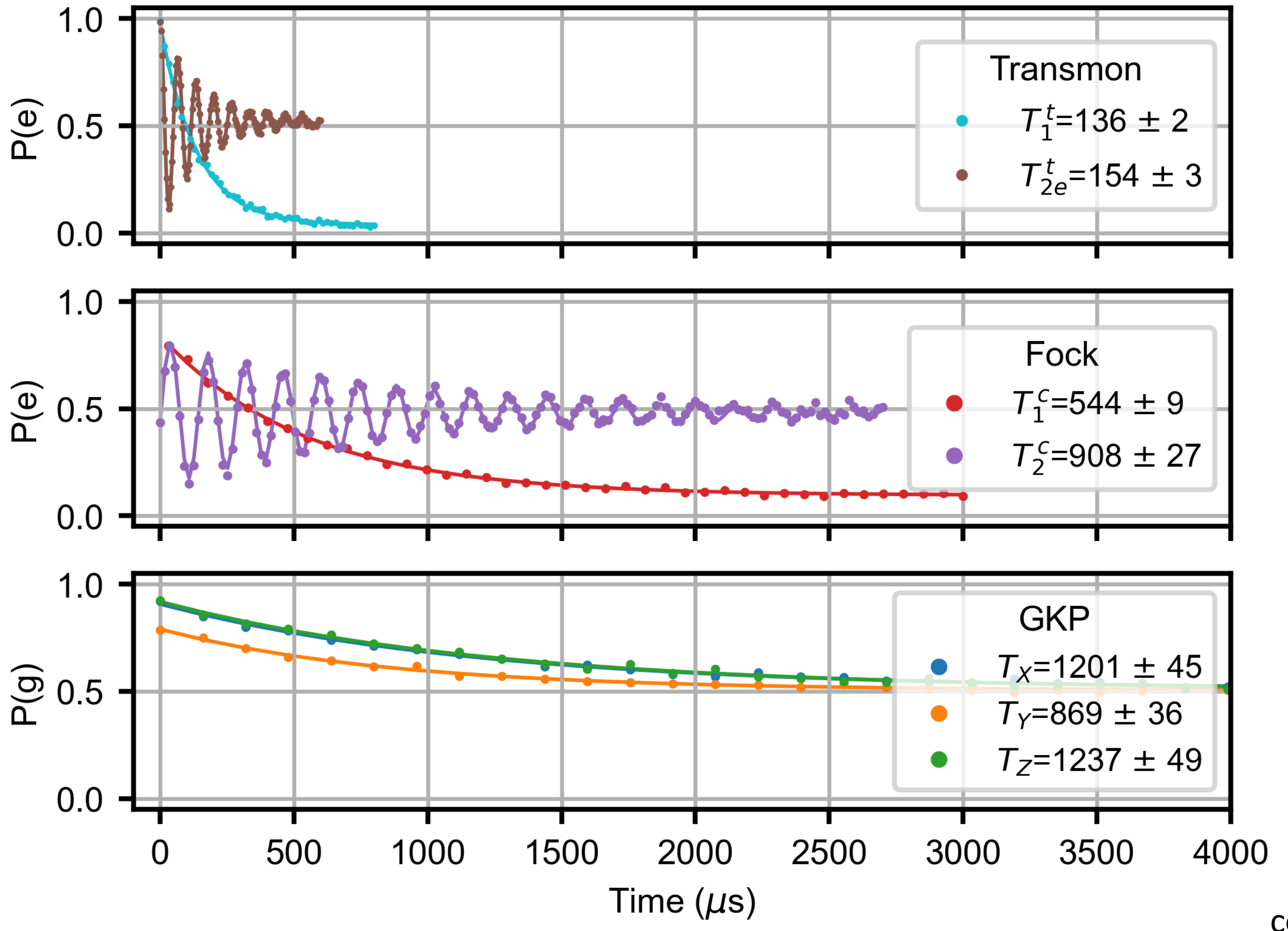


Ancilla reset

B. Royer *et al.*, (PRL, 2020); B. de Neeve *et al.*, (Nature Physics (2022); B. Terhal *et al.*, (PRA, 2016); P. Campagne-Ibarcq *et al.*, (Nature, 2020).

courtesy V. Sivak

QEC gain 1.45 ± 0.04



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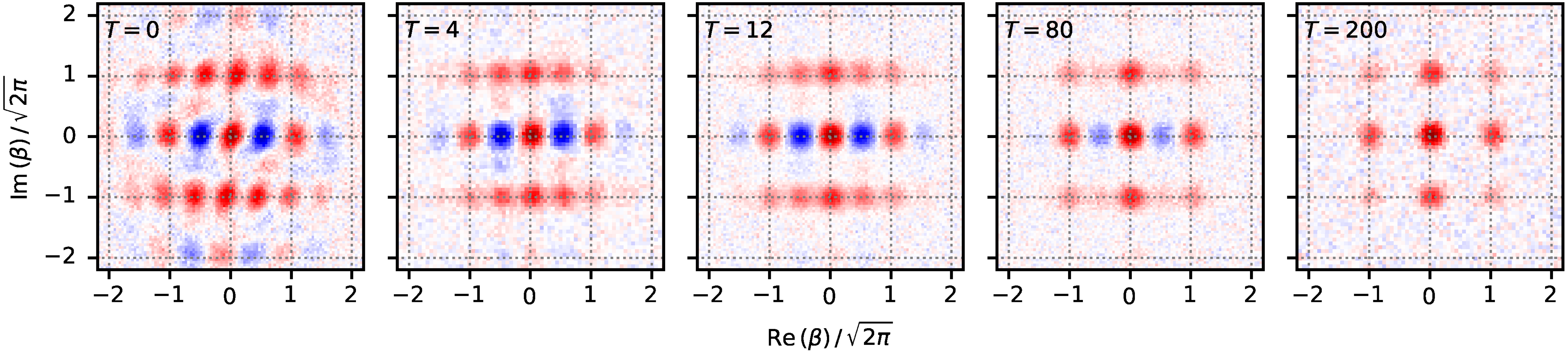
QEC in action

[T = number of rounds of QEC]



- Envelope too big
- Distorted

- Clip to the grid
- Resize envelope

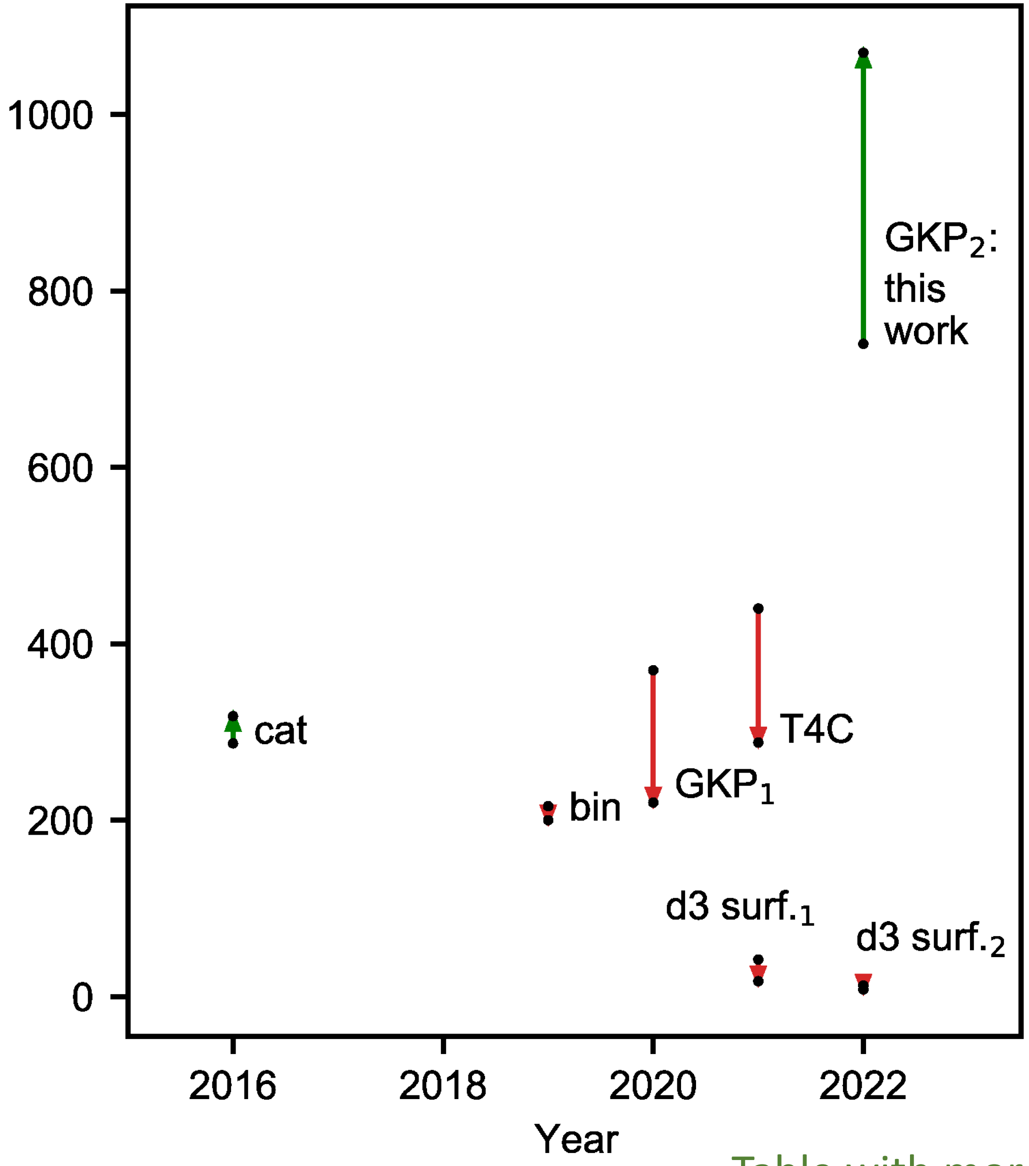
- Remove all deformations

- Depolarization in logical manifold

- Fully mixed logical state

courtesy V. Sivak

Where do we stand? Qubit codes vs. Bosonic codes



	G	$1/\gamma_L$	$1/\min_i[\gamma_{\mathcal{E}}^{(i)}]$
4-legged cat code N. Ofek *et al.*, (Nature, 2016)	1.11	318 µs	287 µs
Binomial code L. Hu *et al.*, (Nature Physics, 2019)	0.93	200 µs	216 µs
GKP code (1) P. Campagne-Ibarcq *et al.*, (Nature, 2020)	0.59	220 µs	≥370 µs
T4C code J. Gertler *et al.*, (Nature, 2021)	0.65	288 µs	440 µs
D3 surface code (1) S. Krinner *et al*		17.5 µs	42.2 µs
D3 surface Y. Zhao *et al.*, (		8.1 µs	12.4 µs
GKP code (2) This work	1.45	1070 µs	740 µs

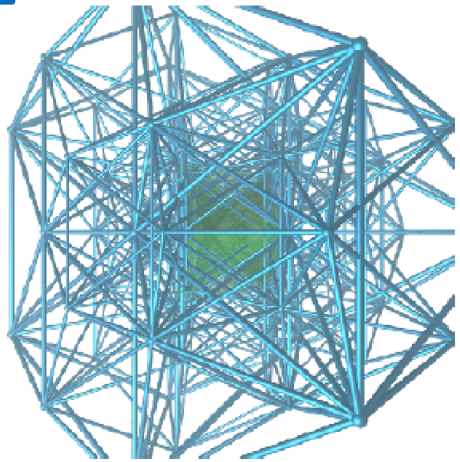
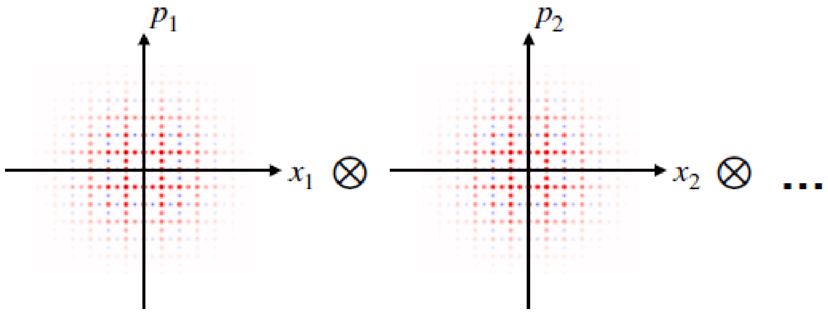
de Neeve et al.,
Nature Physics **18**, 296 (2022)
GKP Gain difficult to estimate

courtesy V. Sivak

Table with more QEC exps.: see SM of Google Quantum AI (Nature, 2021)

Royer et al.,
PRX Quantum **3**, 010335 (2022)
'Tesseract' code highly robust against ancilla errors.

Multimode GKP Code

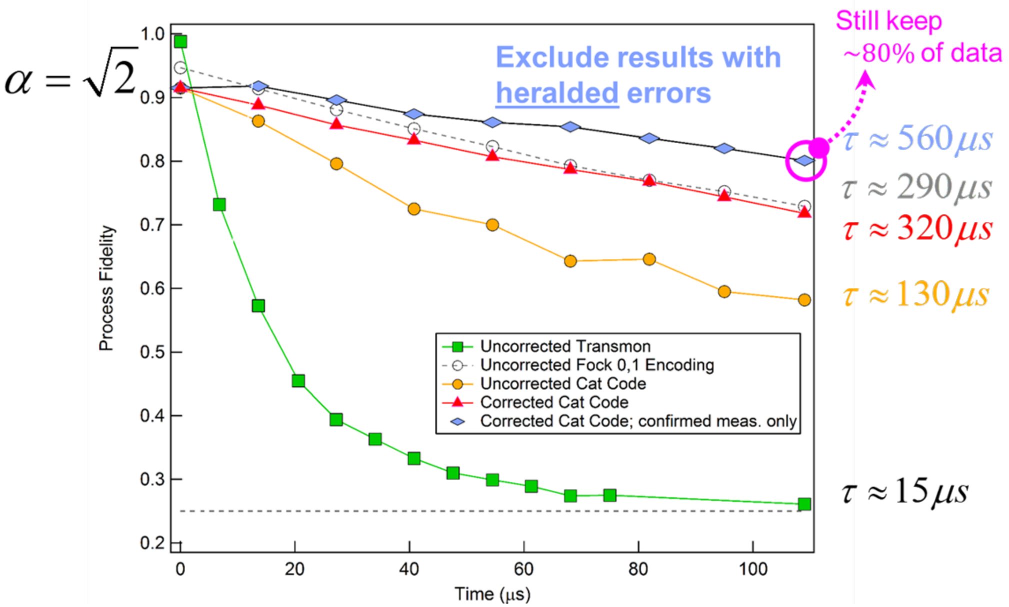
	Lattice in $\mathbb{R}^{2m}$	Quantum code in *m* oscillators
		$\hat{\mathbf{x}} = (\hat{x}_1\ \hat{x}_2 \ldots \hat{p}_1\ \hat{p}_2 \ldots)$ 
Lattice Generators	$\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{2m}$	$\hat{T}(\mathbf{s}_1), \hat{T}(\mathbf{s}_2), \ldots, \hat{T}(\mathbf{s}_{2m})$
Stabilizers commute	$\mathbf{s}_i \Omega \mathbf{s}_j \in \mathbb{Z}$	$[\hat{T}(\mathbf{s}_i), \hat{T}(\mathbf{s}_j)] = 0$
Encode a qubit	Unit cell: Volume of 2	Unit cell: Volume of 2 x $(2\pi)^m$

$$\hat{T}(\mathbf{v}) = e^{i\sqrt{2\pi}\hat{\mathbf{x}}\Omega\mathbf{v}}$$

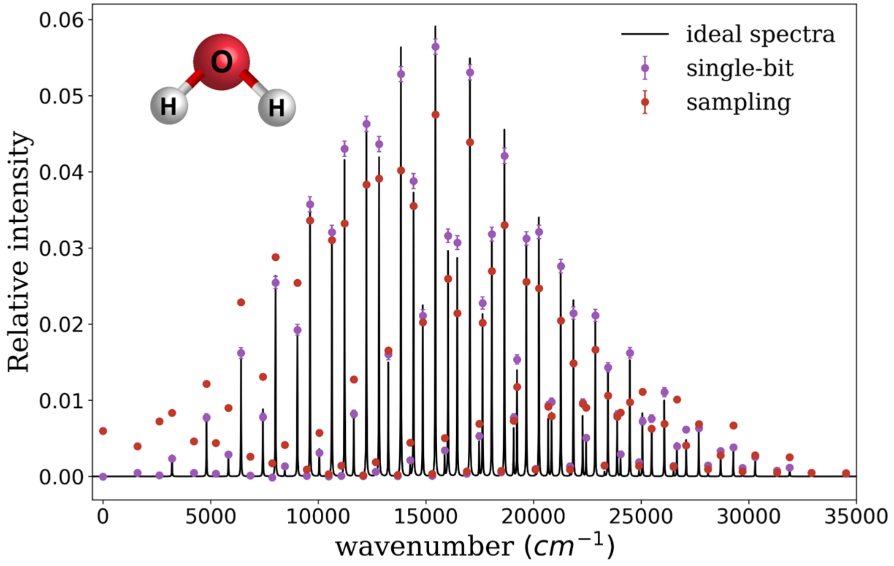
$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

Original proposal : Gottesman, Kitaev and Preskill, Phys. Rev. A **64**, 012310 (2001)

Take-home message:



Molecular Vibrational Spectra via Boson Sampling
Phys. Rev. X **10**, 021060 (2020)



Quantum error correction

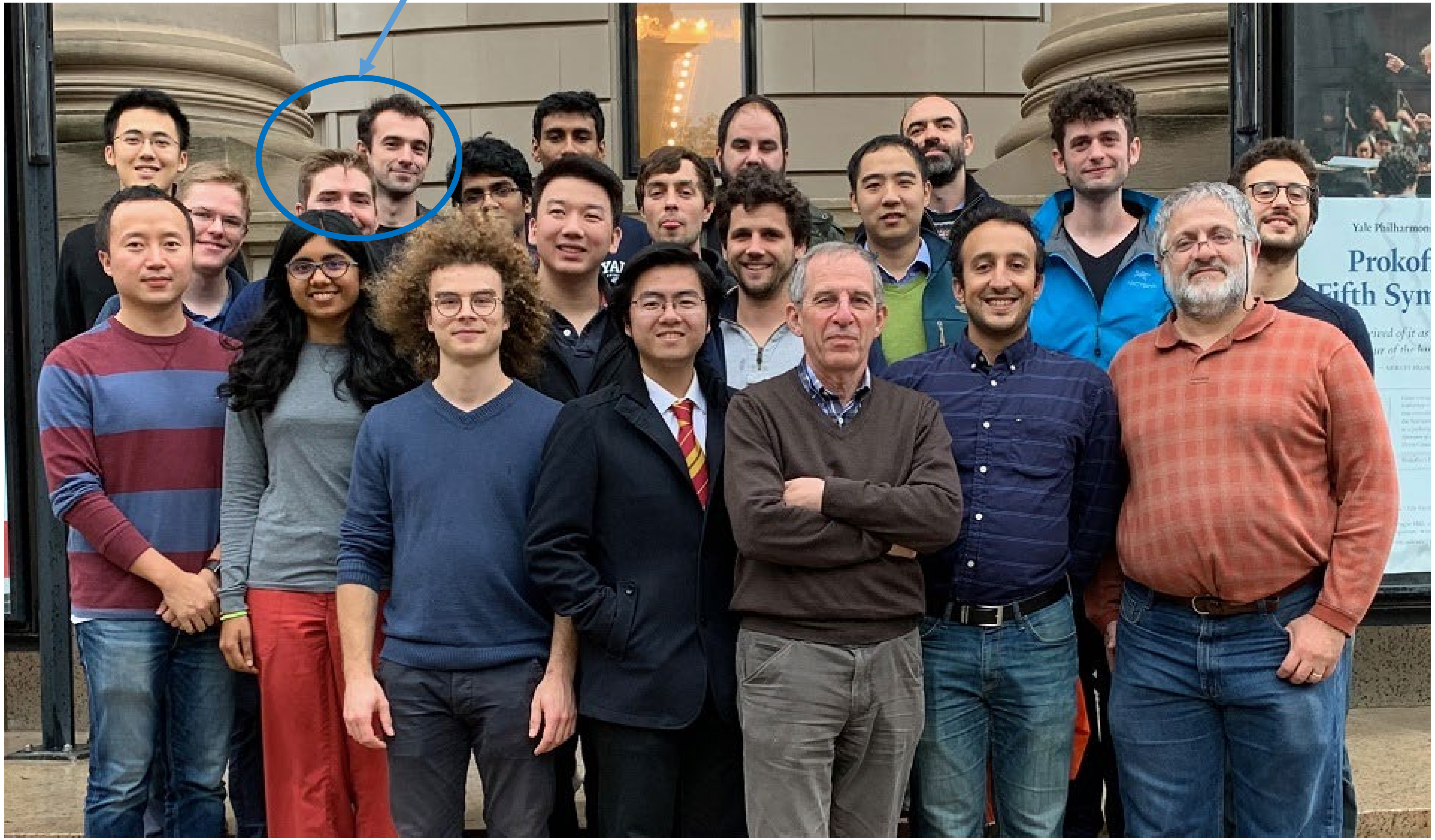
&

Quantum simulations of physical models containing bosons

are both vastly more efficient on hardware containing ‘native’ bosons

Devoret Lab

Alec Eickbusch & Vlad Sivak

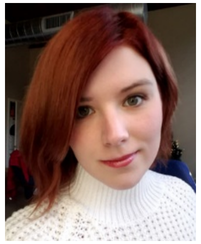


Schoelkopf Lab


B. Lester

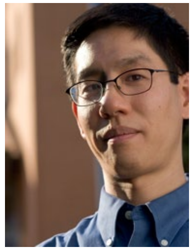

Y. Y. Gao


Y. Zhang


J. Freeze


V. Batista


P. Vaccaro


I. Chuang


L. Jiang


S. Girvin

U.S. DEPARTMENT OF ENERGY

QuantumInstitute.yale.edu

& many others!