International School of Physics "Enrico Fermi"

COURSE 211 –
QUANTUM MIXTURES WITH ULTRA-COLD ATOMS

Giacomo Lamporesi

INO-CNR Trento

CNR-INO
ISTITUTO NAZIONALE DI OTTICA
CONSIGLIO NAZIONALE DELLE RICERCHE

PROVINCIA AUTONOMA
DI TRENTO

Quantum Mixtures

Mixture of ***two*** *(or more) distinguishable* ***constituents***
forming a composite system with ***quantum features***

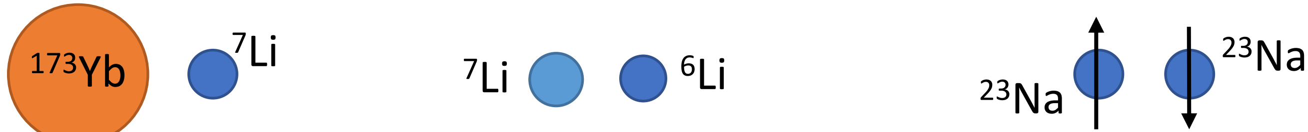
Overview

SPECIES

Species mixture (Inguscio-Grimm-Zaccanti

Isotopic mixture (Salomon

Spin mixture (Dalibard-Lamporesi-Oberthaler-Ueda-Fallani-Stringari



Possible large mass imbalance

Possible statistical mix

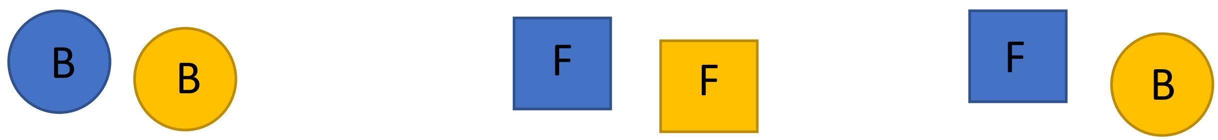
Possible state interconversion (Rabi / Raman coupling)

STATISTCS

Bose-Bose mixture (Inguscio-Dalibard-Lamporesi-Oberthaler-Ueda

Fermi-Fermi mixture (Fallani-Grimm-Zaccanti

Bose-Fermi mixture (Inguscio-Salomon-Grimm

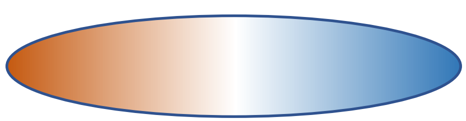


FEW to MANY-BODY

Few-body physics (scattering,Efimov,molecules (Grimm-Inguscio-Zaccanti-Fallani

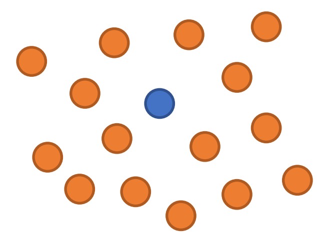
Many-body dynamics (Waves,Droplets,topological structures… (Dalibard-Lamporesi-Oberthaler-Ueda-Santos-Stringari

Impurity (Polarons,Ions,… (Parish-Petrov-Zaccanti-Gerritsma



Balanced

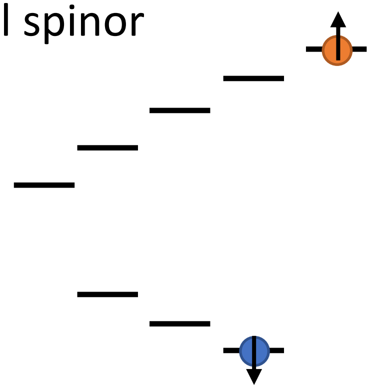
Strongly imbalanced

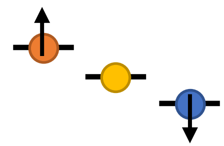


NUMBER of COMPONENTS

Two components (Grimm-Zaccanti-Lamporesi-Dalibard-Oberthaler-Santos-Stringari

Full spinor (Ueda-Dalibard-Oberthaler-Fallani



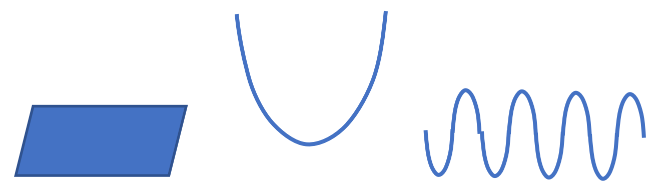


GEOMETRY

Flat potential (Dalibard

Harmonic trap (Lamporesi-Oberthaler

Lattice geometry (Fallani-Törmä



DIMENSIONALITY

3D (Grimm-Santos-Petrov-Zaccanti-Gerritsma

2D (Dalibard

1D (Lamporesi-Oberthaler

0D (Oberthaler-Fallani-Törmä

… and even mix-D (Inguscio

Two-component spin mixtures

Giacomo Lamporesi

INO-CNR Trento

This Lecture (**Part One**)

This Lecture (**Part Two**)

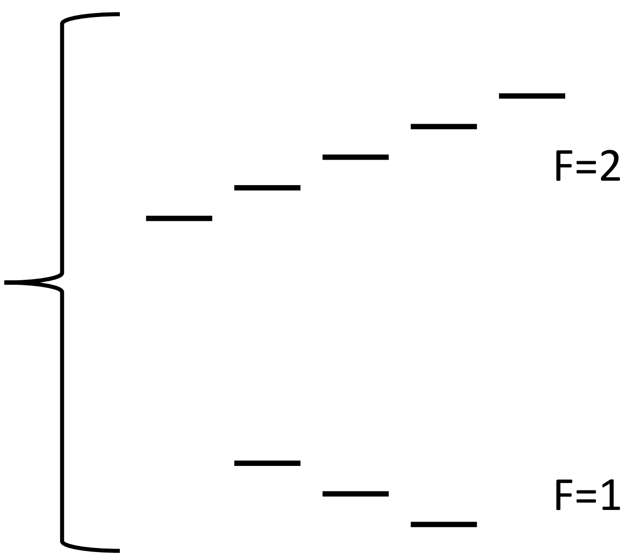
Two-component **spin mixtures** of Sodium atoms (many-body regime)

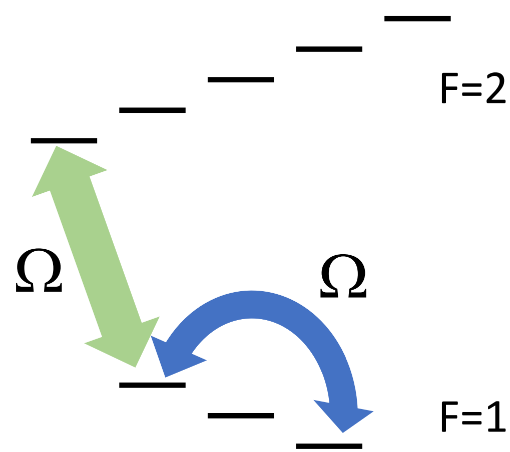
Two-component **spin mixtures** of Sodium atoms (many-body regime)

+ coherent coupling

^{23}Na (bosonic)

Electronic ground state $3^2S_{1/2}$





- Clean system to investigate the ***properties of superfluid mixtures***

- Versatile system to perform ***quantum simulation*** of complex phenomena

SINGLE COMPONENT BEC – introducing relevant quantities

Wavefunction

$$\psi(x,t) = |\psi(x,t)|e^{i\phi(x,t)}$$

GPE

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x,t) + g|\psi(x,t)|^2\right)\psi(x,t) = \mu\psi(x,t)$$

Interaction constant

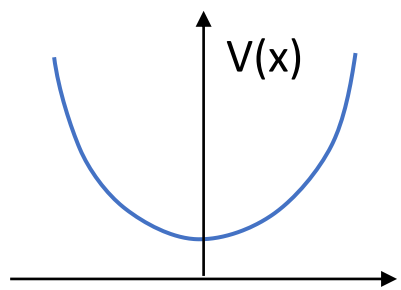
$$g = \frac{4\pi\hbar^2 a}{m}$$

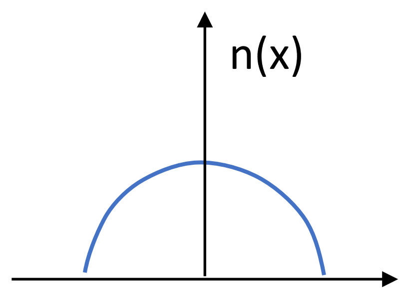
Scattering length

TF approximation
Density profile

$$n(x) = \frac{\mu - V(x)}{g}$$

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

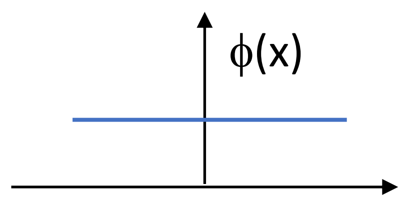




$$R_{TF} = \sqrt{\frac{2\mu}{m\omega^2}}$$

Ground state
Excitations,
velocity and phase

$$\mathbf{v_s}(x) = \frac{\hbar}{m}\nabla\phi(x)$$



SINGLE COMPONENT BEC – introducing relevant quantities

1 comp.

Wavefunction

$$\psi(x,t) = |\psi(x,t)|e^{i\phi(x,t)}$$

GPE

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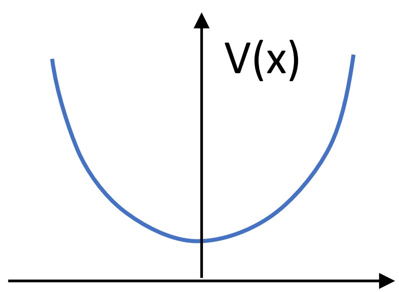
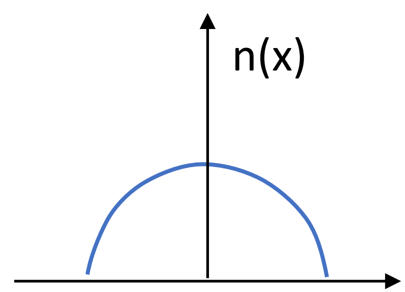
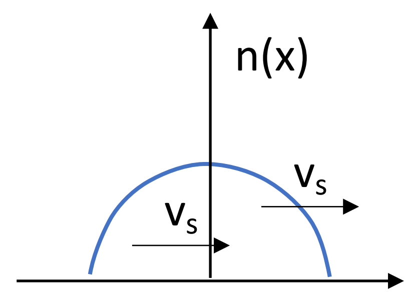
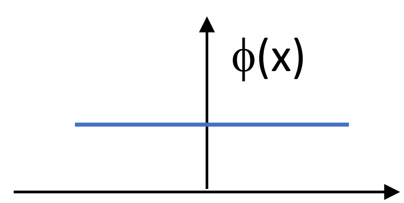


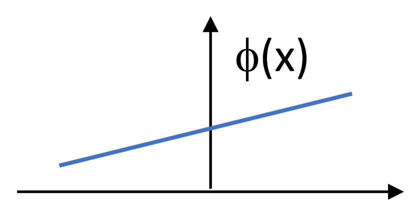




Ground state
Excitations,
velocity and phase

$$\mathbf{v_s}(x) = \frac{\hbar}{m}\nabla\phi(x)$$





ELEMENTARY EXCITATIONS

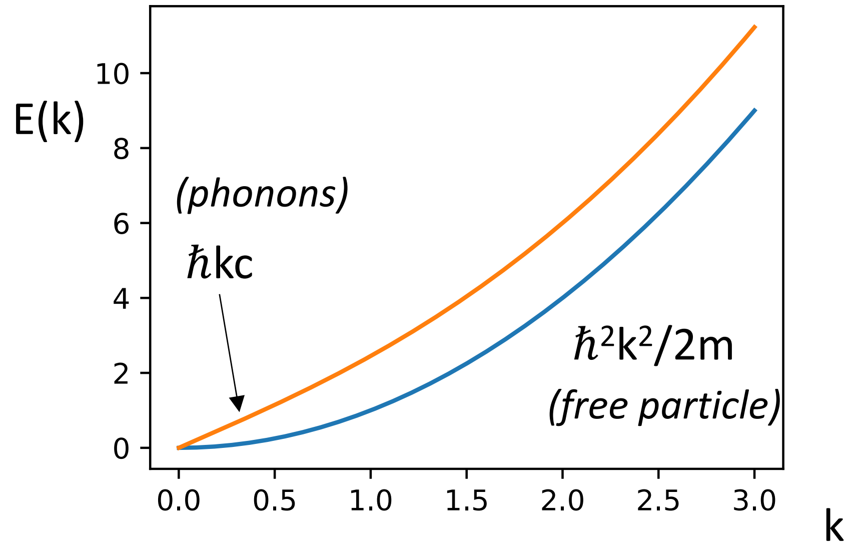
1 comp.

Bogoljubov spectrum

$$E(k) = \hbar\omega(k) = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2\mu\right)}$$

Phonons

$$E(k) \simeq \hbar k \sqrt{\frac{\mu}{m}}$$

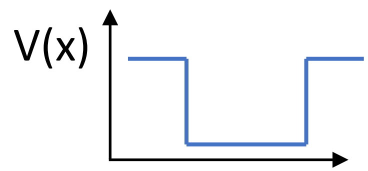


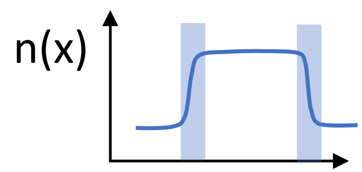
Speed of sound

$$c = \sqrt{\frac{\mu}{m}} = \sqrt{\frac{ng}{m}}$$

Healing length

$$\xi = \frac{1}{\sqrt{8\pi\, n\, a}} = \frac{\hbar}{\sqrt{2m\, n\, g}}$$





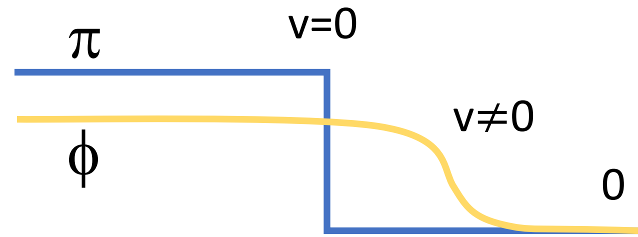
TOPOLOGICAL EXCITATIONS

1 comp.

Solitons

Localized solitary waves (stable in 1D)
Balance between dispersion and nonlinear effects (GPE)

$\phi(x)$



Dark soliton (v=0)
Abrupt phase jump of π
Full density depletion

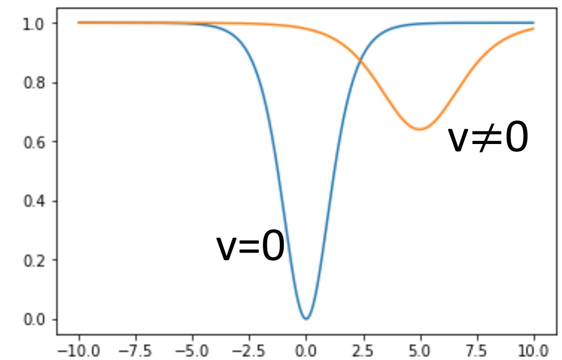
Grey soliton (v=v_{sol})
Phase changes by $\Delta\phi$ in a length Δx
Density depletion Δn

$$\Delta\phi = \frac{-2}{\cos\left(v_{\rm sol}/c\right)}$$

$$\Delta x = \frac{\xi}{\sqrt{1 - v_{\rm sol}^2/c^2}}$$

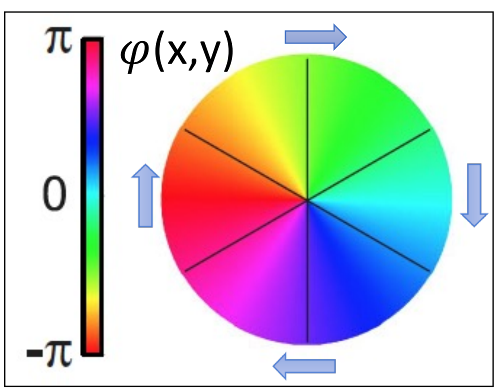
$$\Delta n = n\left(1 - v_{\rm sol}^2/c^2\right)$$

$n(x)$



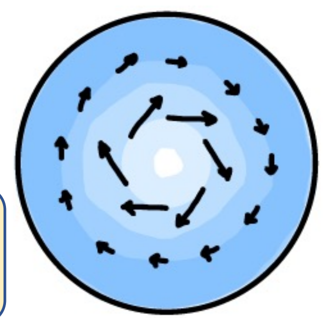
Quantized vortices

Phase winding of 2π around a point
Quantized circulation due to irrotational nature of superfluids

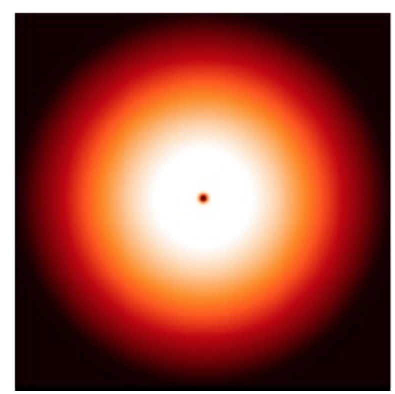


$$\mathbf{v_s}(x) = \frac{\hbar}{m}\nabla\phi(x)$$

$$v \propto \frac{1}{r}$$

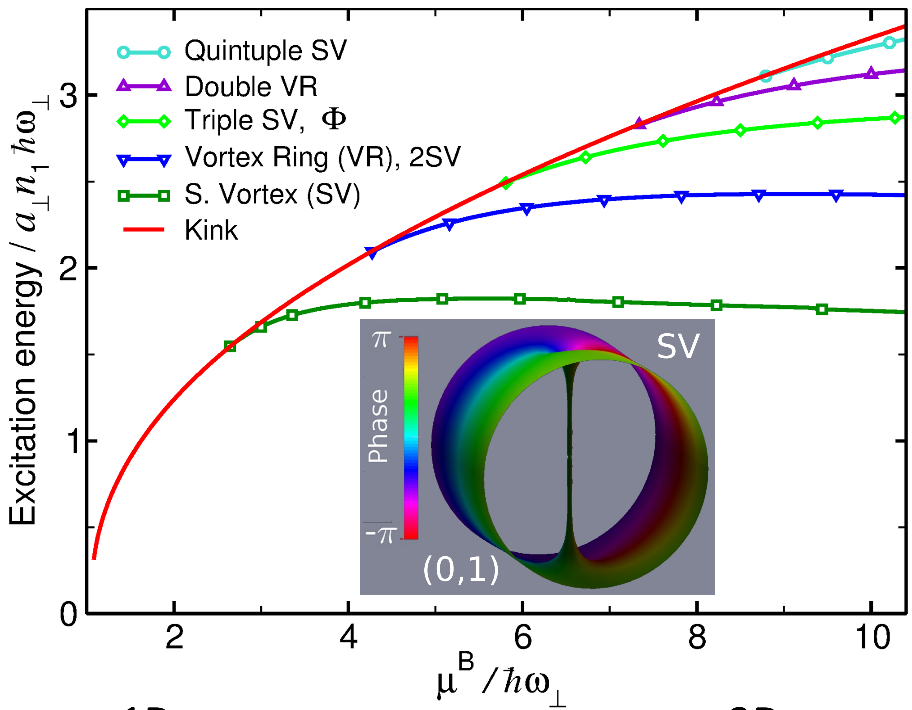


$n(x,y)$

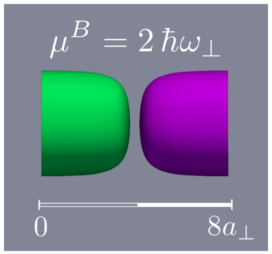


DIMENSIONALITY

1 comp.

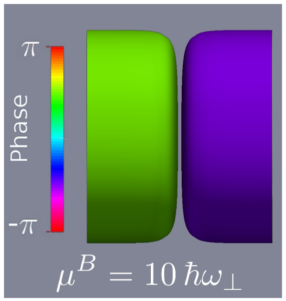


1D



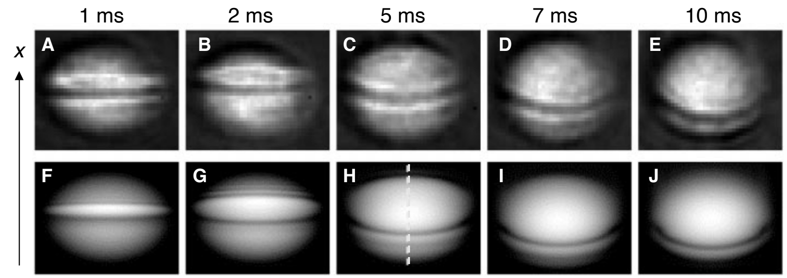
$$\frac{\mu}{\hbar\omega} = \frac{R_\perp}{2\xi}$$

3D

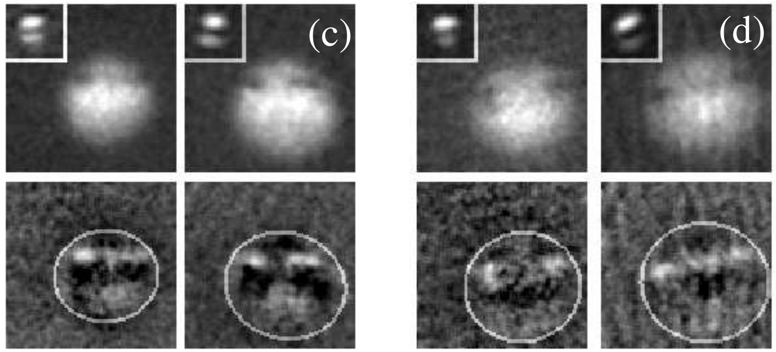


Mateo *et al.*, PRL **115**, 033006 (2015)

Soliton decay in 3D



Denschlag *et al.*, Science **287**, 5450 (2000)



Anderson *et al.*, PRL **86**, 2926 (2001)

TWO-COMPONENT MIXTURE

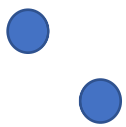
2 comp.

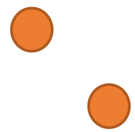
Wavefunctions

$$\psi_a(x,t) = |\psi_a(x,t)|e^{i\phi_a(x,t)}$$

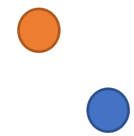
$$\psi_b(x,t) = |\psi_b(x,t)|e^{i\phi_b(x,t)}$$

Intracomponent interactions

g_a 

g_b 

Intercomponent interactions

g_{ab} 

Set of coupled GPEs

$$i\hbar\frac{\partial}{\partial t}\psi_a = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(r,t) + g_a|\psi_a(r,t)|^2 + g_{ab}|\psi_b(r,t)|^2\right)\psi_a$$
$$i\hbar\frac{\partial}{\partial t}\psi_b = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(r,t) + g_b|\psi_b(r,t)|^2 + g_{ab}|\psi_a(r,t)|^2\right)\psi_b$$

$$\delta g = \frac{g_a + g_b}{2} - g_{ab}$$

Spin healing length

$$\xi_s = \frac{\hbar}{\sqrt{mn\,\delta g}}$$

INTERACTIONS

2 comp.

All attractive interactions

g_a g_b g_{ab}

Collapse (Bose nova)

All repulsive interactions

g_a g_b g_{ab}

Stable Mixture

Repulsive interactions

g_a g_b

Attractive interactions

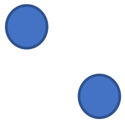
g_{ab}

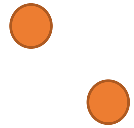
Quantum droplets

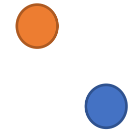
MISCIBILITY

2 comp.

All repulsive interactions

g_a 

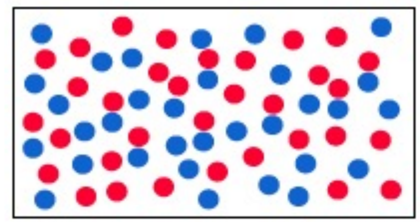
g_b 

g_{ab} 

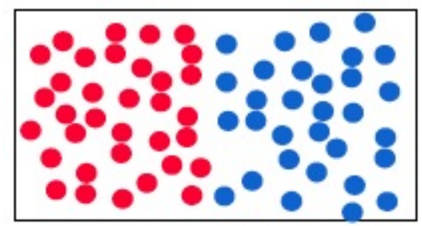
Interplay between intra- and intercomponent interactions

Miscible – Immiscible phase transition

$$E_m = \frac{1}{2} g_a \frac{N_a^2}{V} + \frac{1}{2} g_b \frac{N_b^2}{V} + g_{ab} \frac{N_a N_b}{V}$$



$$E_i = \frac{1}{2} g_a \frac{N_a^2}{V_a} + \frac{1}{2} g_b \frac{N_b^2}{V_b} = \frac{1}{2} g_a \frac{N_a^2}{V} + \frac{1}{2} g_b \frac{N_b^2}{V} + \sqrt{g_a g_b} \frac{N_a N_b}{V}$$

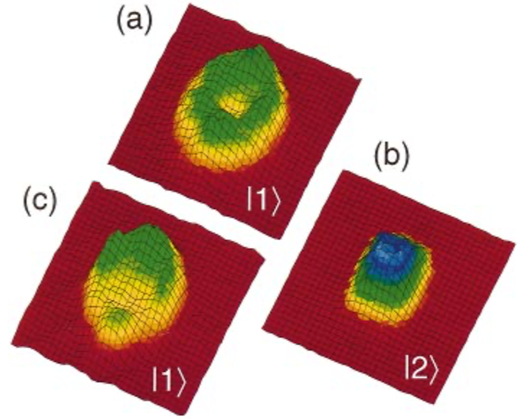


Miscibility condition $g_{ab} < \sqrt{g_a g_b}$

MISCIBILITY and BUOYANCY

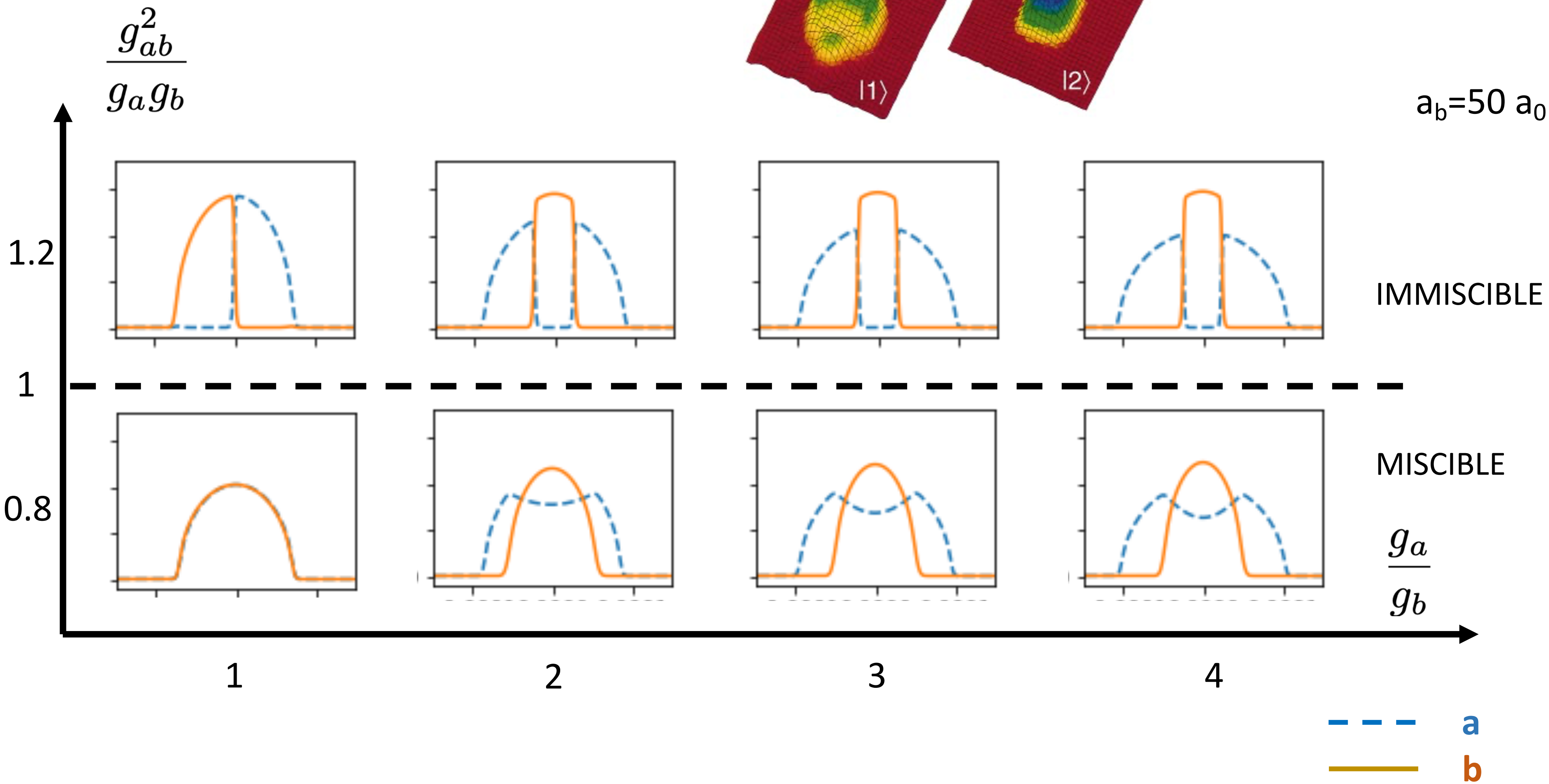
2 comp.

In nonuniform potentials… BUOYANCY



Hall *et al.*, PRL **81**, 1539 (1998)

a_b=50 a_0

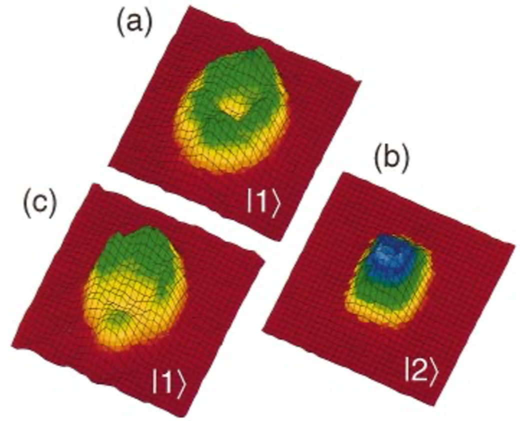


Least repulsive goes in the center
Most repulsive adapts outside

MISCIBILITY and BUOYANCY

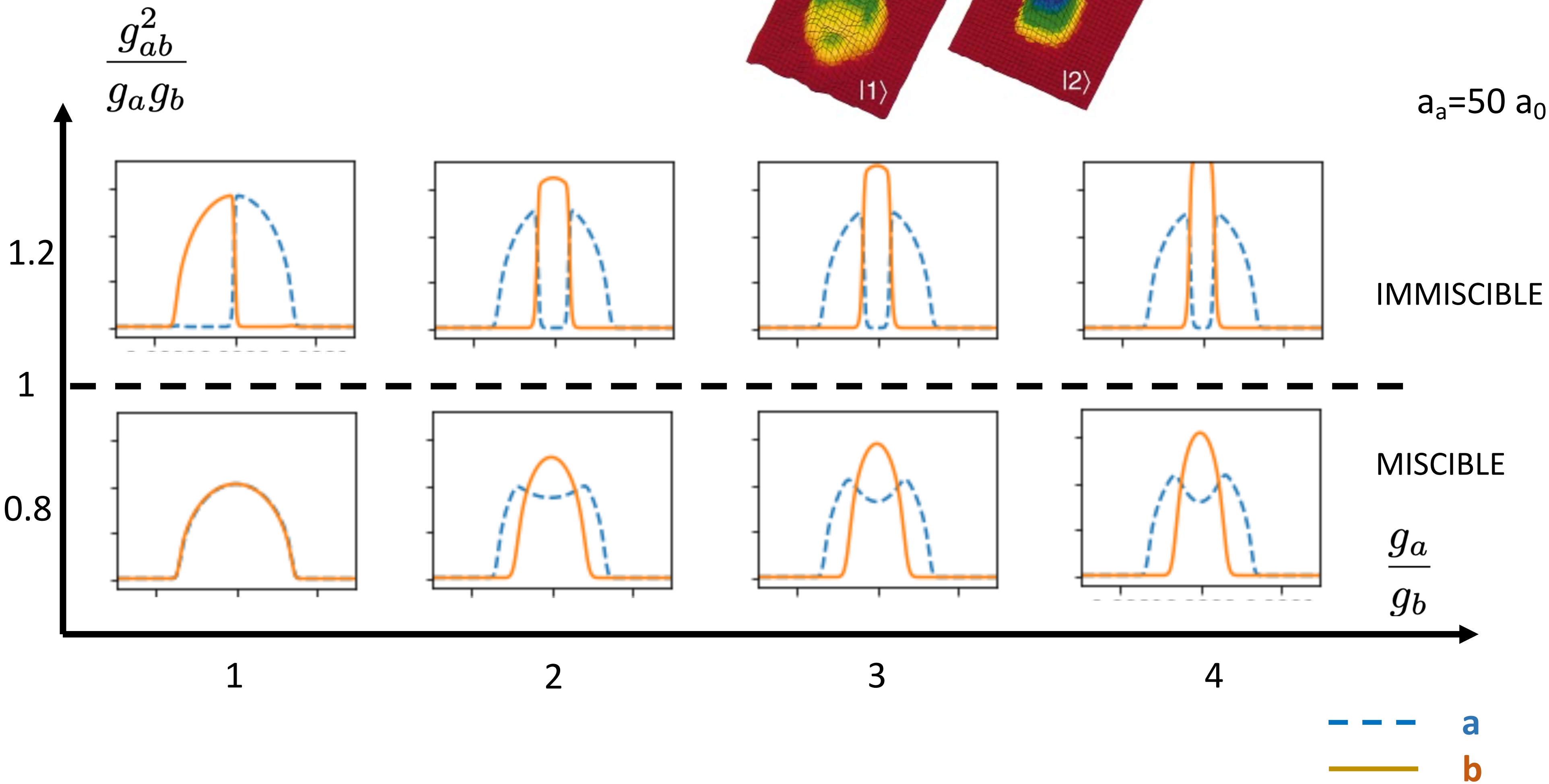
2 comp.

In nonuniform potentials… BUOYANCY



Hall *et al.*, PRL **81**, 1539 (1998)

a_a=50 a_0



Least repulsive goes in the center
Most repulsive adapts outside

MISCIBILITY

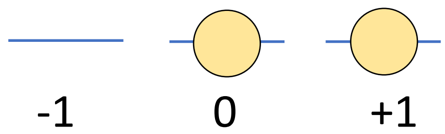
2 comp.

^{87}Rb

$c_2 < 0$

FERROMAGNETIC

F=1



$a_{0,0}$ = 100.9 a_0

$a_{0,1}$ = 100.4 a_0

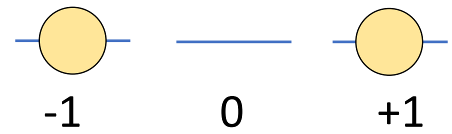
$a_{1,1}$ = 100.4 a_0

NOT SYMMETRIC

^{23}Na

$c_2 > 0$

ANTIFERROMAGNETIC

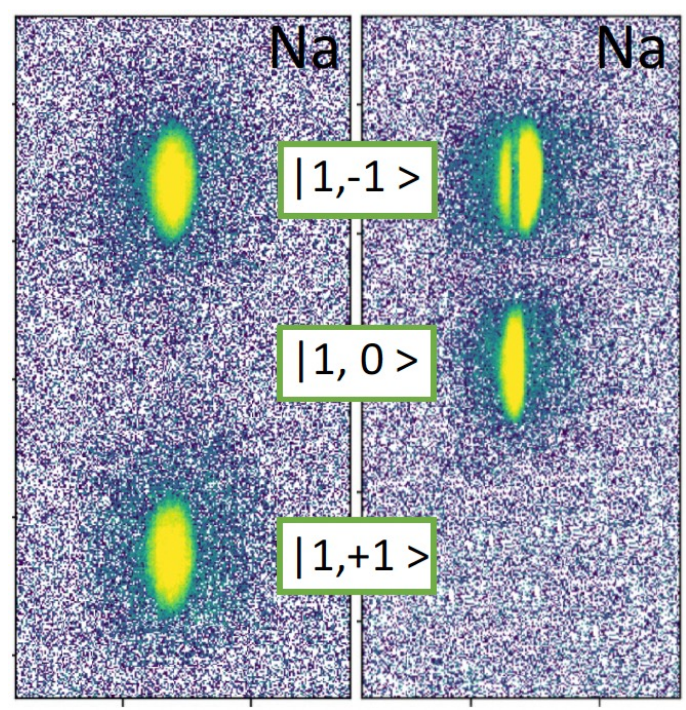


$a_{-1,-1}$ = 54.5 a_0

$a_{1,-1}$ = 50.8 a_0

$a_{1,1}$ = 54.5 a_0

SYMMETRIC



DENSITY AND SPIN MODES

2 comp.

Total DENSITY

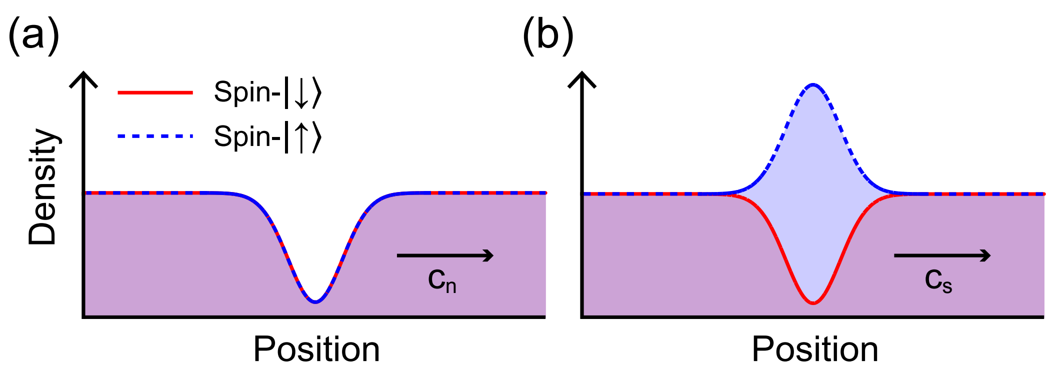
$$n = n_a + n_b$$

$$\Phi = \phi_a + \phi_b$$

SPIN density (magnetization)

$$m = n_a - n_b$$

$$\phi = \phi_a - \phi_b$$



Common mode

Differential mode

DENSITY AND SPIN MODES

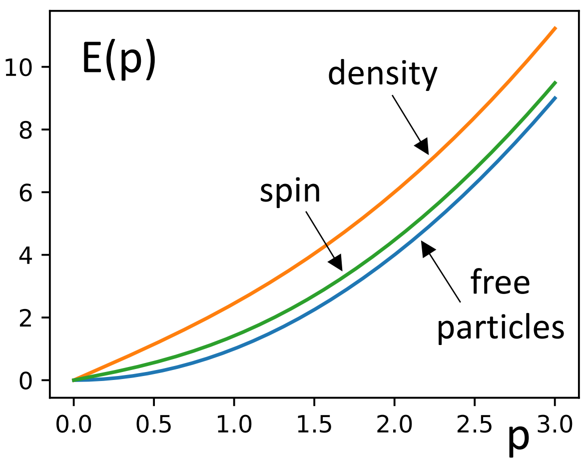
2 comp.

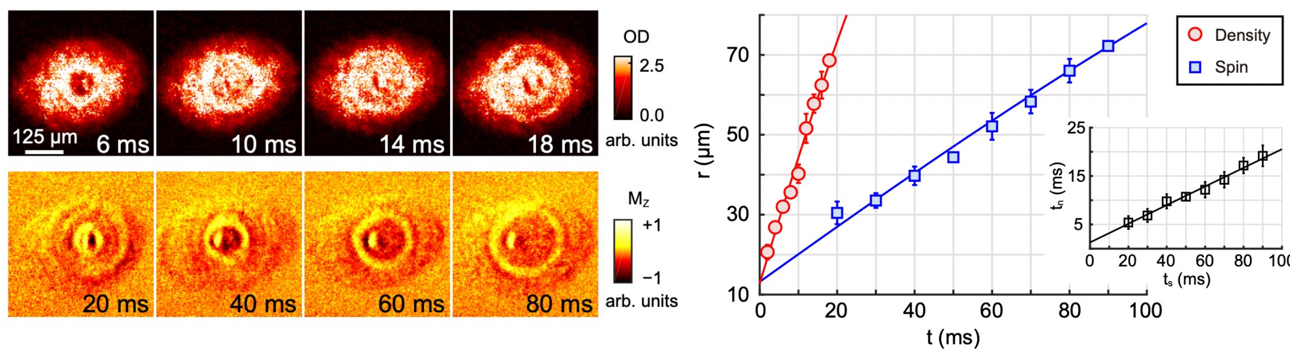
Bogoljubov spectra

$$E_{d,s}(k) = \hbar\omega_{d,s}(k) = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2\mu_{d,s}\right)}$$

Sound speeds

$$c_{d,s}^2 = \frac{\mu_{d,s}}{m} = \frac{g_a n_a + g_b n_b \pm \sqrt{(g_a n_a - g_b n_b)^2 + 4 n_a n_b g_{ab}^2}}{2m}$$





Kim *et al.*, PRA **101**, 061601(R) (2020)

$$c_d^2 = \frac{n}{2m}(g + g_{ab})$$

$$c_s^2 = \frac{n}{2m}(g - g_{ab})$$

(for balanced mixtures and equal intracomponent interactions)

DENSITY AND SPIN PARAMETERS

2 comp.

$$\frac{(g+g_{ab})}{2} \simeq g \qquad \frac{(g-g_{ab})}{2} \simeq \frac{\delta g}{2}$$

Density chemical potential

$$\mu = n\frac{(g+g_{ab})}{2} \longrightarrow \mu \simeq ng$$

Spin chemical potential

$$\mu_s = n\frac{(g-g_{ab})}{2} \longrightarrow \mu \simeq \frac{n\delta g}{2}$$

Density healing length

$$w = \xi = \frac{\hbar}{\sqrt{2mn\,g}}$$

Spin healing length

$$w_s = \xi_s = \frac{\hbar}{\sqrt{2mn\,\delta g/2}}$$

Density speed of sound

$$c = \sqrt{\frac{\mu}{m}} = \sqrt{\frac{n\,g}{m}}$$

Spin speed of sound

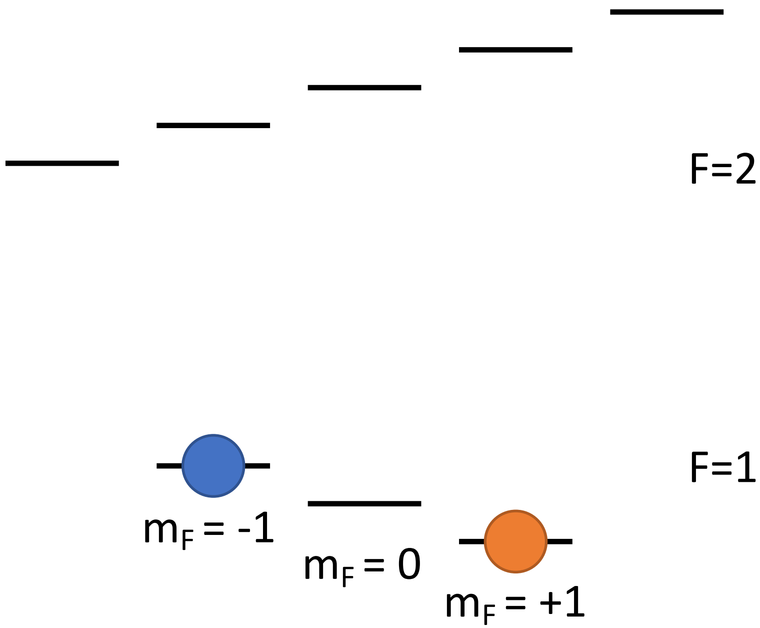
$$c_s = \sqrt{\frac{\mu_s}{m}} = \sqrt{\frac{n}{m}\frac{\delta g}{2}}$$

$$\left(\frac{\delta g}{2g}\right)_{Na} \simeq 3\%$$

SODIUM MISCIBLE MIXTURE

2 comp.

Elongated cigar-shaped optical trap



Symmetric, no buoyancy

$$g_a = g_b = g$$

Miscible mixture

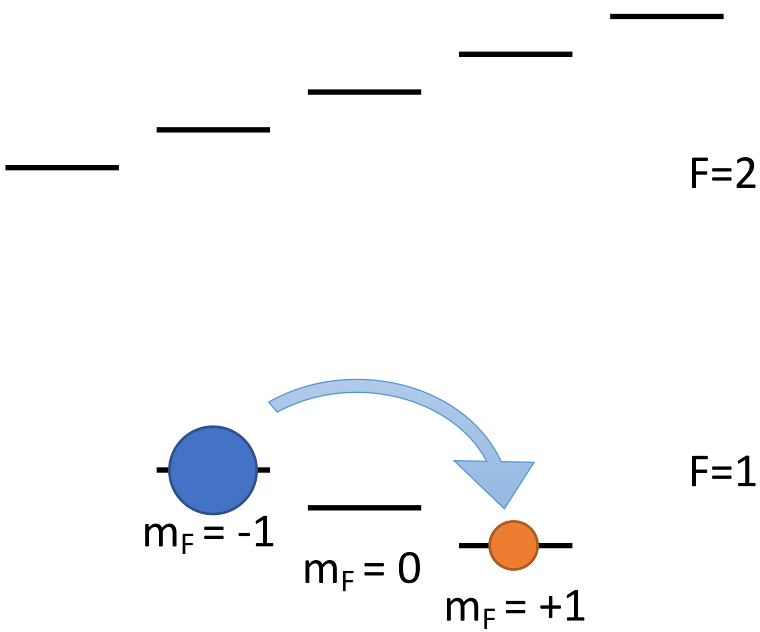
$$g_{ab} < g$$

$$\delta g > 0$$

SODIUM MISCIBLE MIXTURE

2 comp.

Elongated cigar-shaped optical trap



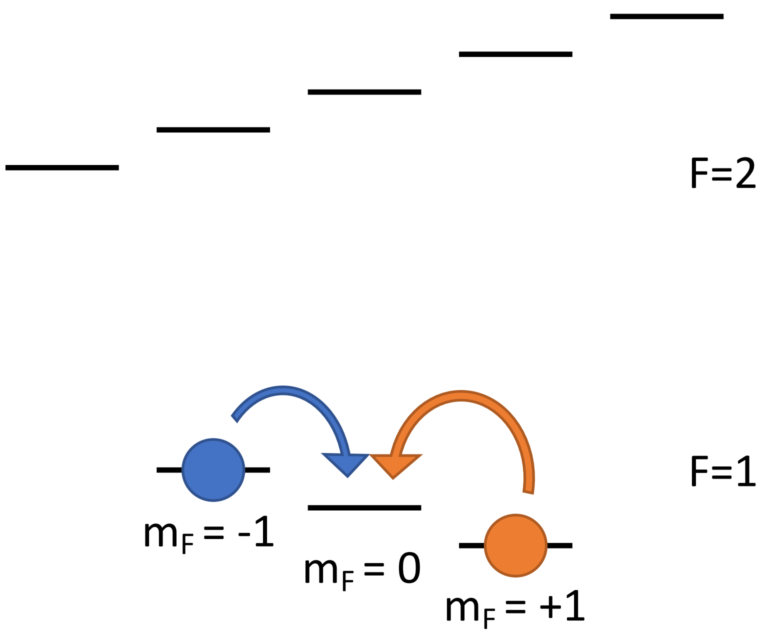
Balanced mixture <u>preparation</u>

- Rabi $\pi/2$ pulse
- Adiabatic Rapid Passage

SODIUM MISCIBLE MIXTURE

2 comp.

Elongated cigar-shaped optical trap



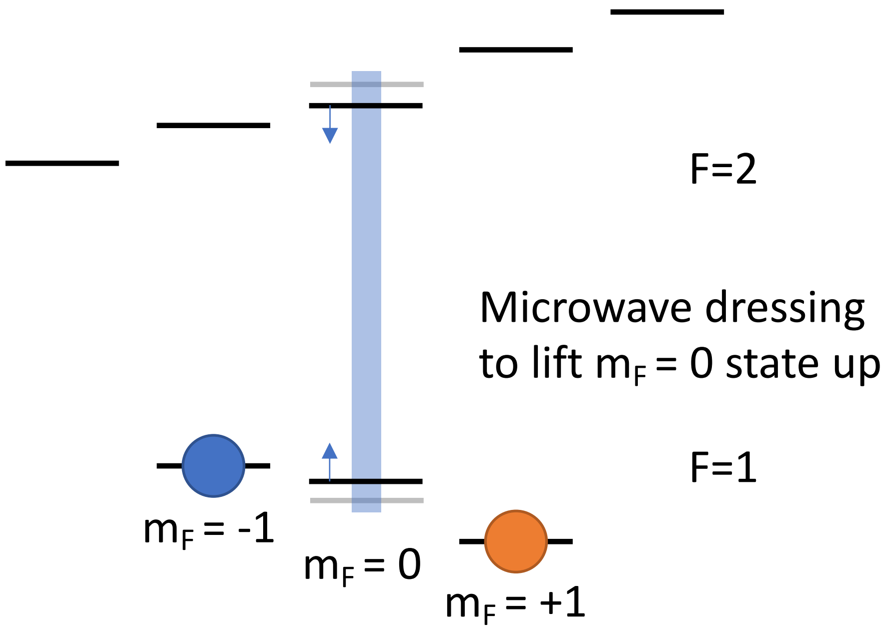
Naturally unstable mixture

<u>Spin relaxation</u>
(energetically favourable to recombine and form two atoms in $m_F = 0$)

SODIUM MISCIBLE MIXTURE

2 comp.

Elongated cigar-shaped trap



Naturally unstable mixture

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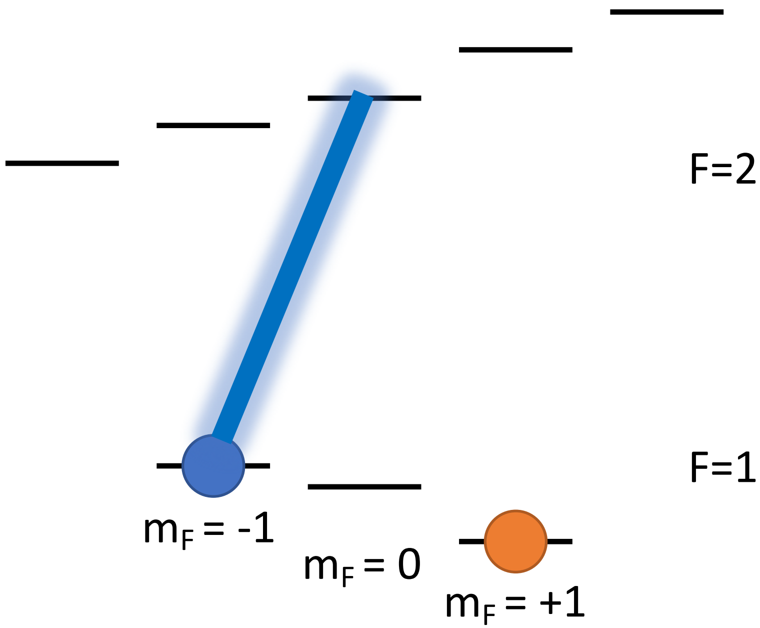
SODIUM MISCIBLE MIXTURE

2 comp.

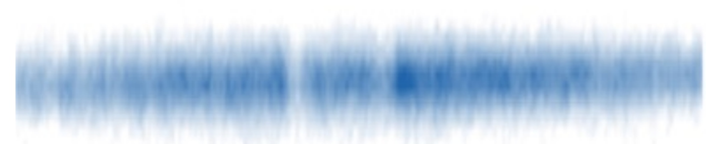
Elongated cigar-shaped optical trap



<u>Imaging</u>

Delayed, but overlapped
Selective transfer to F=2 + abs imaging

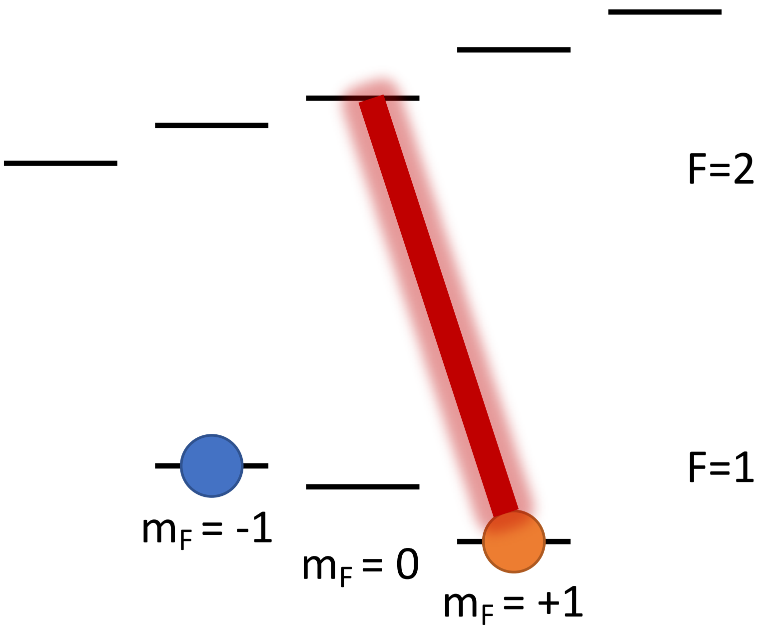
Simultaneous, but spatial separation
Stern Gerlach separation in TOF + abs imaging



SODIUM MISCIBLE MIXTURE

2 comp.

Elongated cigar-shaped optical trap



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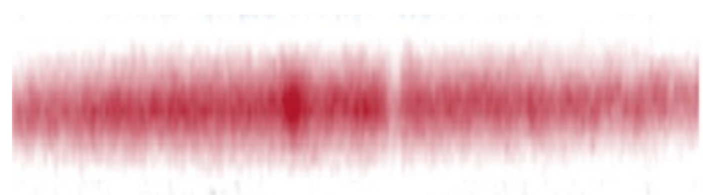
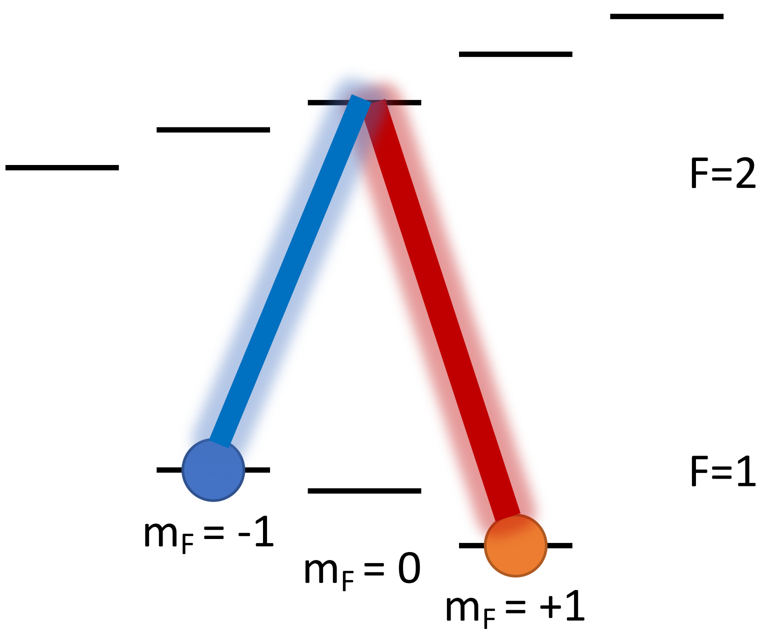


SODIUM MISCIBLE MIXTURE

2 comp.

Elongated cigar-shaped optical trap



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SODIUM MISCIBLE MIXTURE

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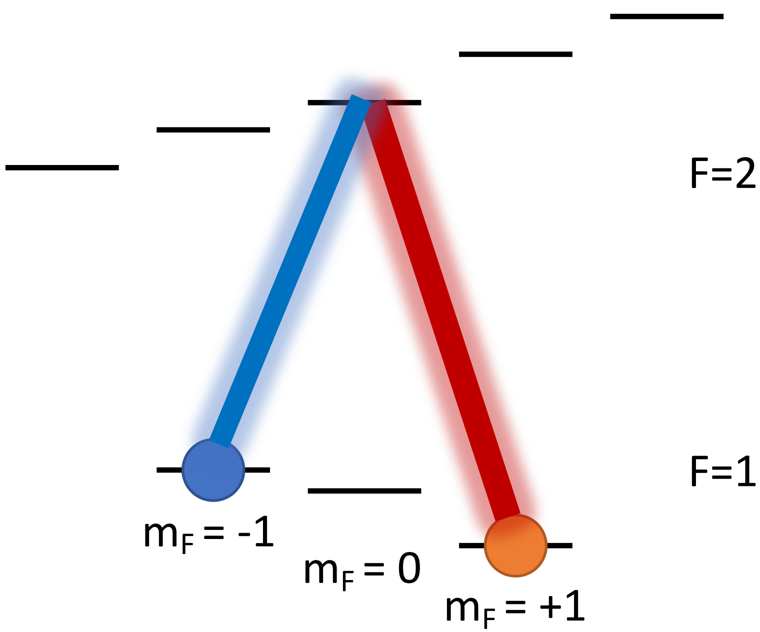

Imaging

Delayed, but overlapped
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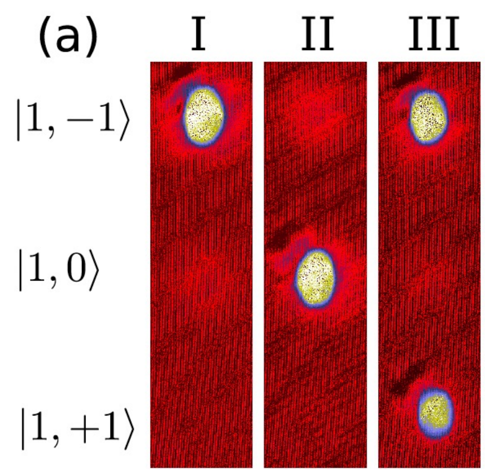
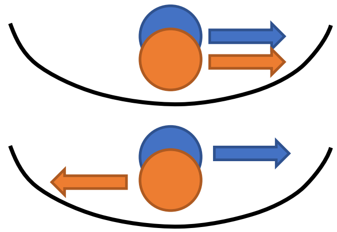


SPIN DIPOLE OSCILLATIONS

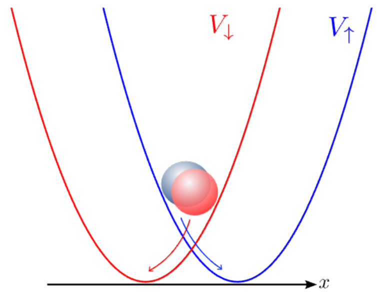
$\Omega = 0$



Dipole mode ω_x

Spin-dipole mode ω_{SD}

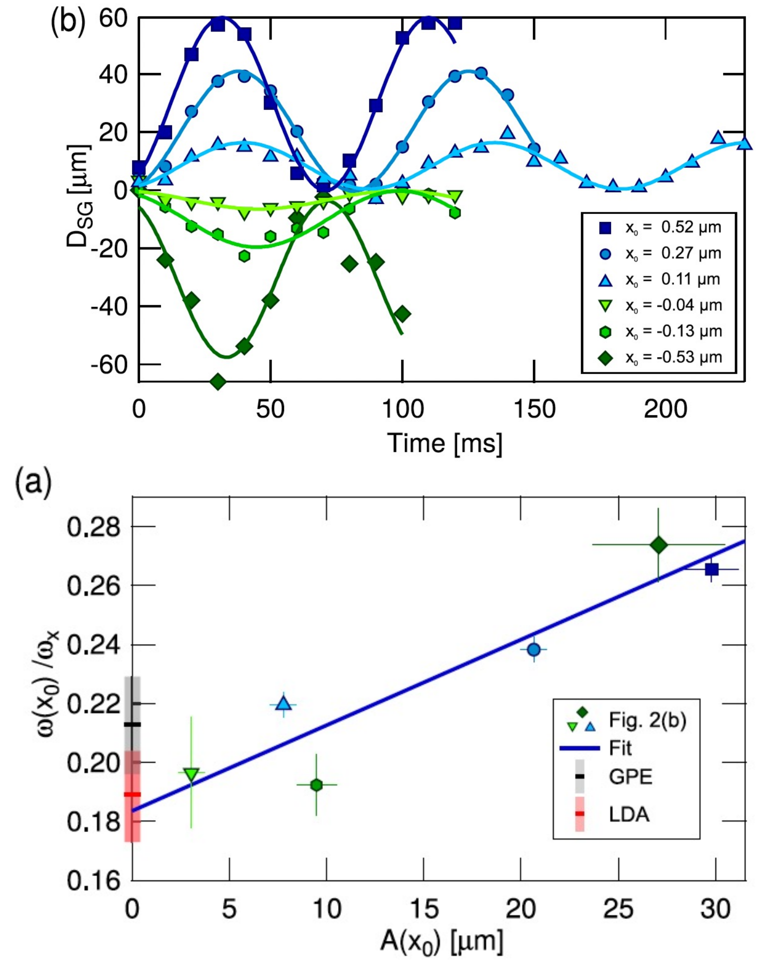
Differential potential
(magnetic field gradient)
To trigger spin-dipole oscillations



For small displacements
Linear response
No excitation of density channel

Through LDA

$$\omega_{SD} = \omega_x \sqrt{\frac{g - g_{ab}}{g + g_{ab}}}$$

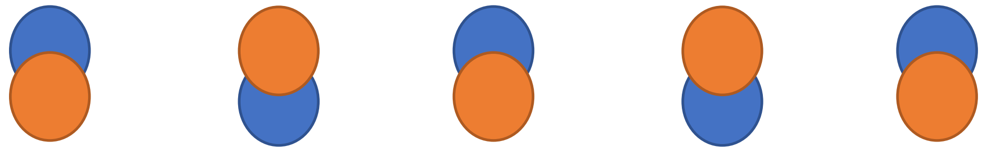


Bienaimé *et al.*, PRA **94**, 063652 (2016)

$\Omega = 0$

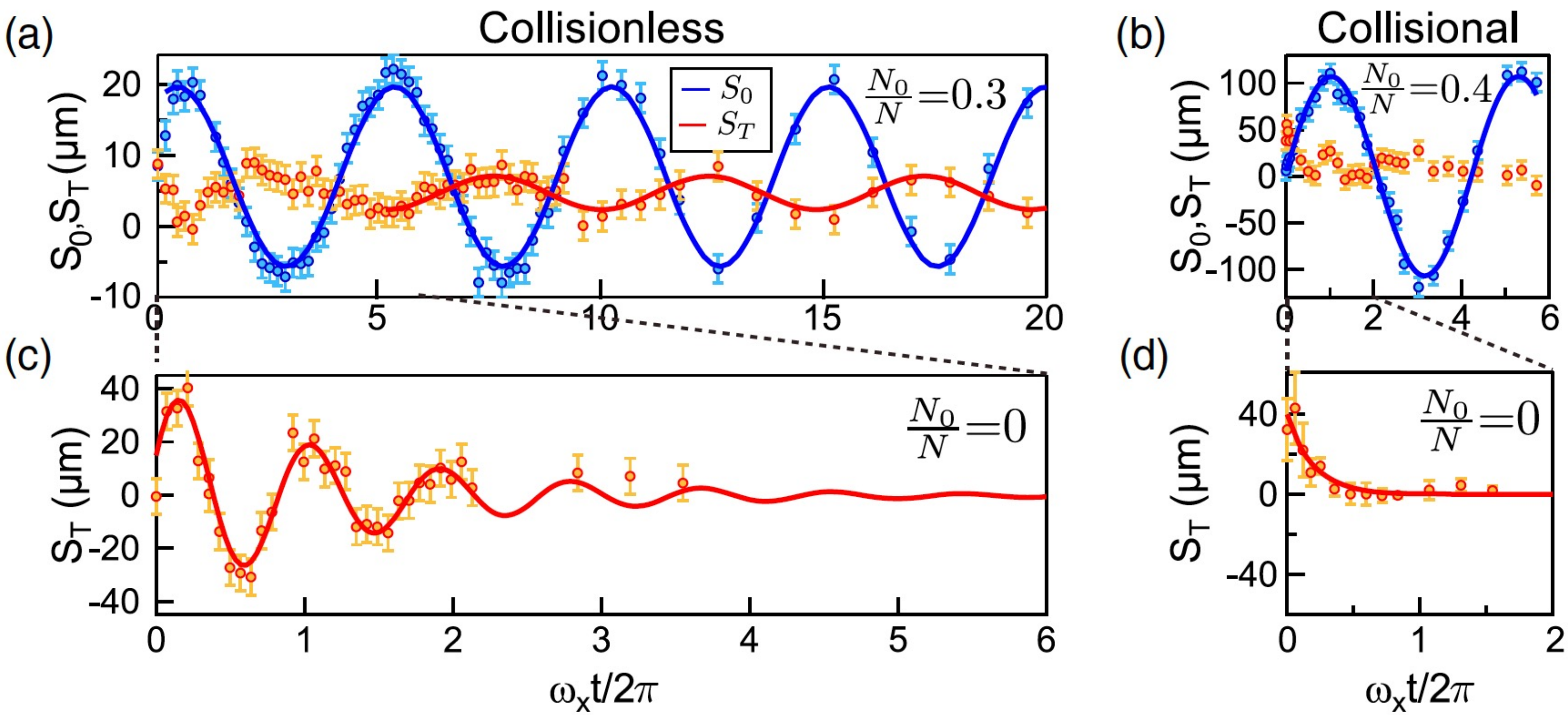
SPIN SUPERFLUIDITY

BECs oscillate (spin-dipole motion) even at finite T

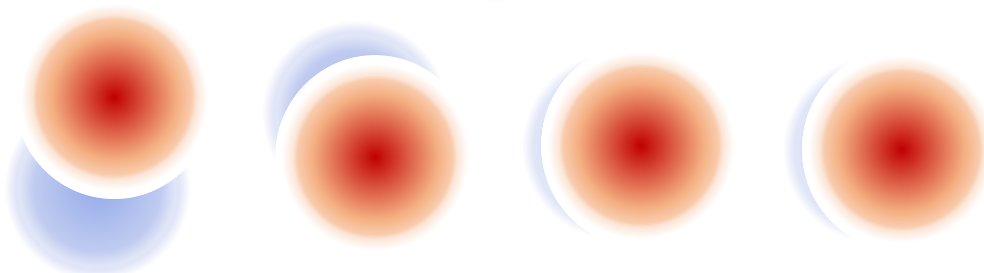


Relative motion between condensed parts

Relative motion between thermal parts



Fava *et al.*, PRL **120**, 170401 (2018)

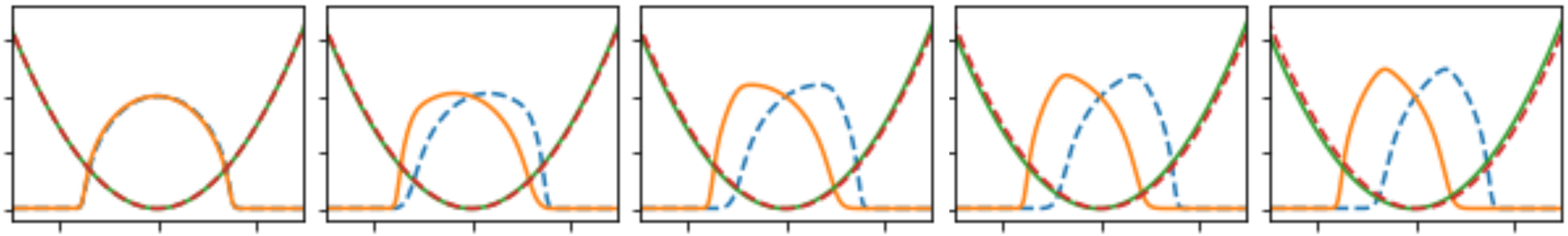


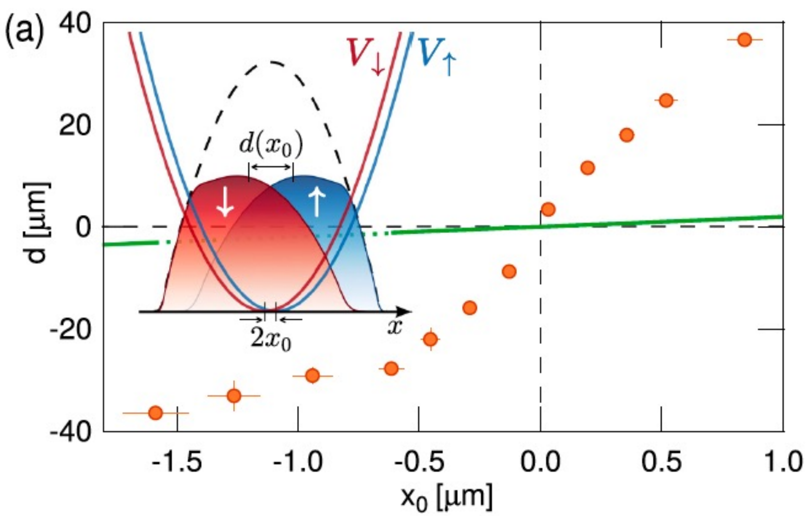
Ω = 0

SPIN POLARIZABILITY

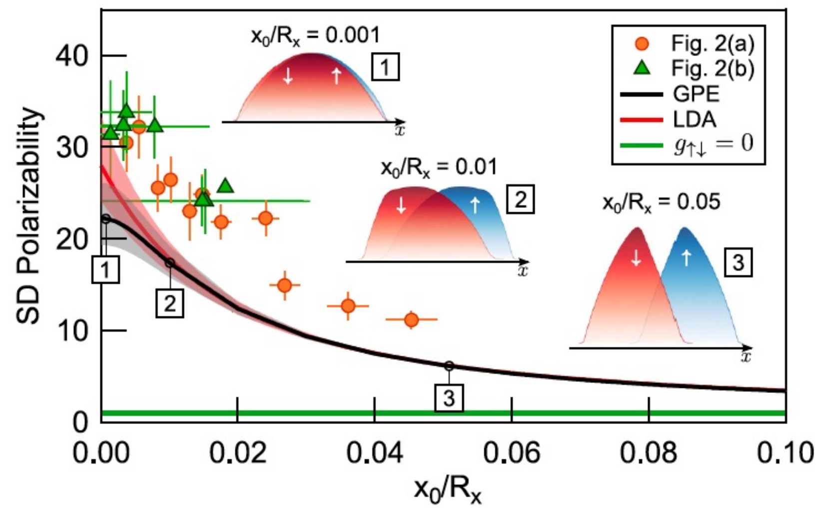
Displacement ($2x_0$) between the minima leads to Displacelent of the CM enhanced by $\mathcal{P}$

$$\mathcal{P} = \frac{d(x_0)}{2x_0}$$





$$\mathcal{P}(0) = \left(\frac{\omega_x}{\omega_{SD}}\right)^2$$



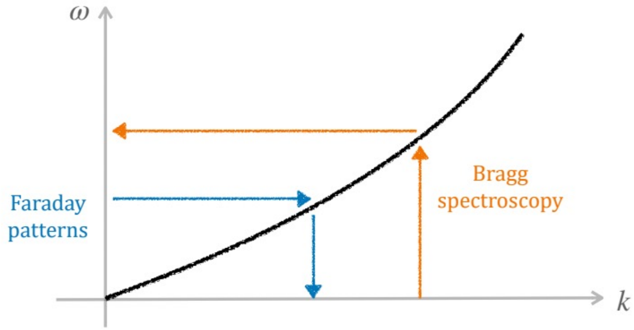
Bienaimé *et al.*, PRA **94**, 063652 (2016)

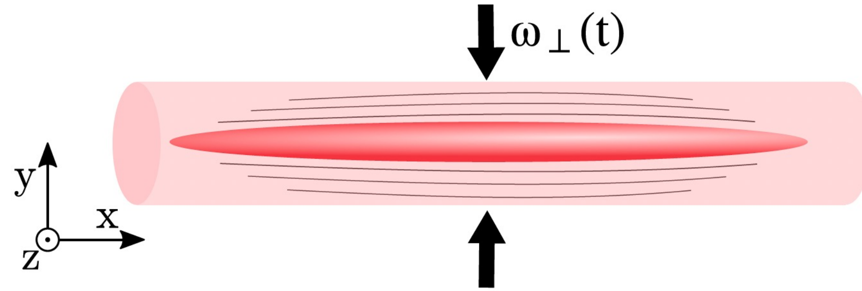
→ POSTER 40 (Spada)

$\Omega = 0$

FARADAY WAVES

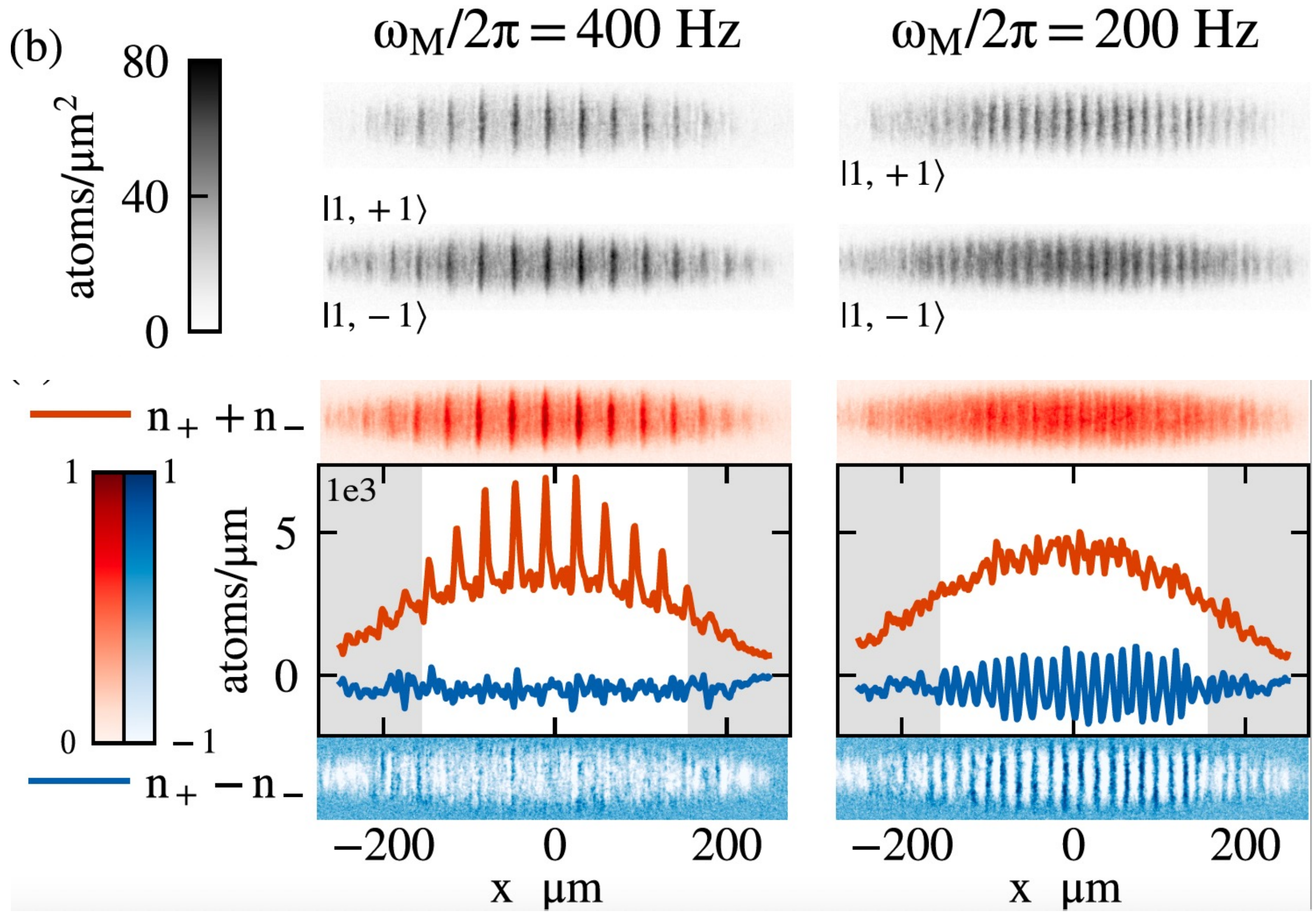
$$E_{d,s}(k) = \hbar\omega_{d,s}(k) = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2mc_{d,s}^2\right)}$$

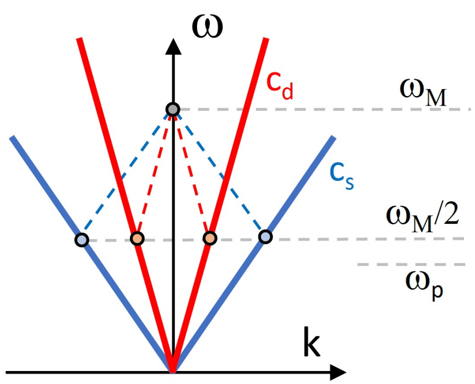




-Modulation of radial compression

-Decay into Faraday waves

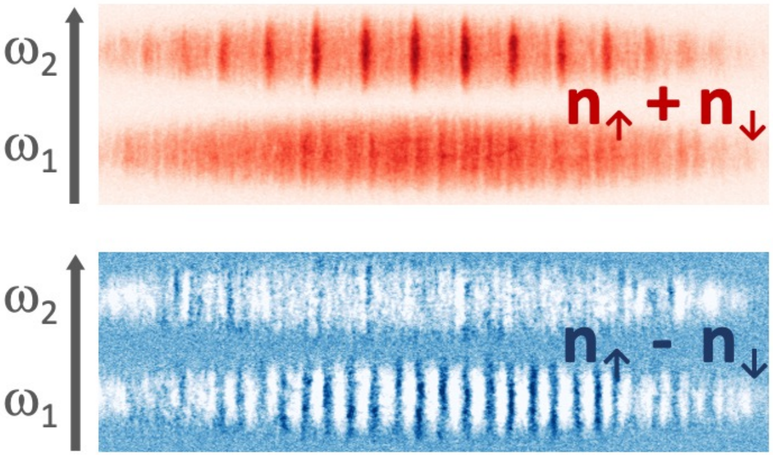




Cominotti *et al.*, PRL **128**, 210401 (2022)

FARADAY WAVES

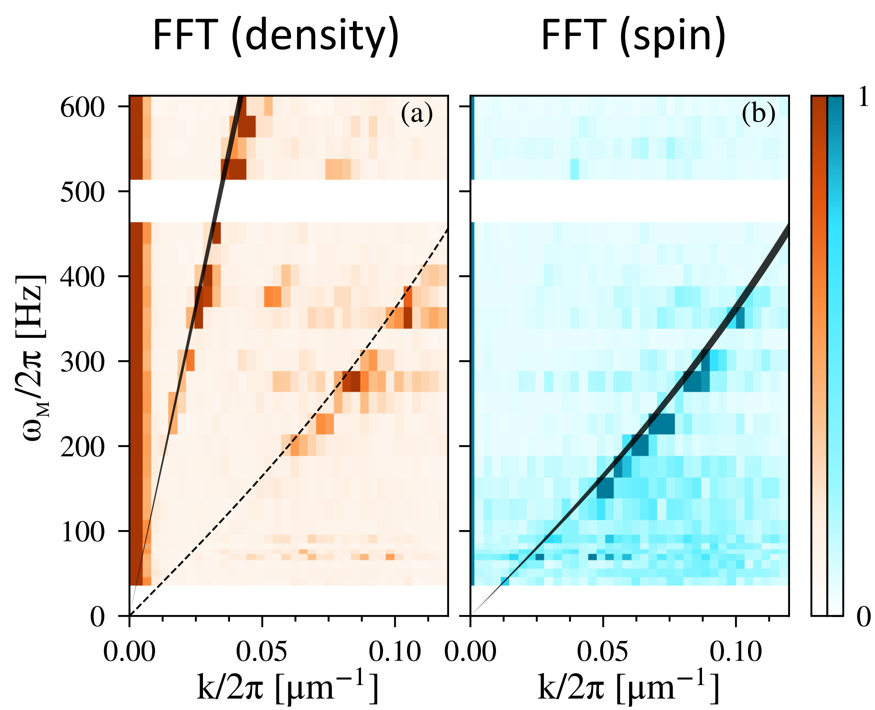
Ω = 0



- Radial integration
- Axial FFT

$$E_{d,s}(k) = \hbar\omega_{d,s}(k) = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2mc_{d,s}^2\right)}$$

Measurement of density and spin dispersion relations

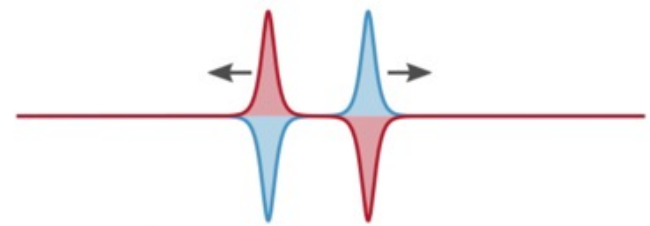


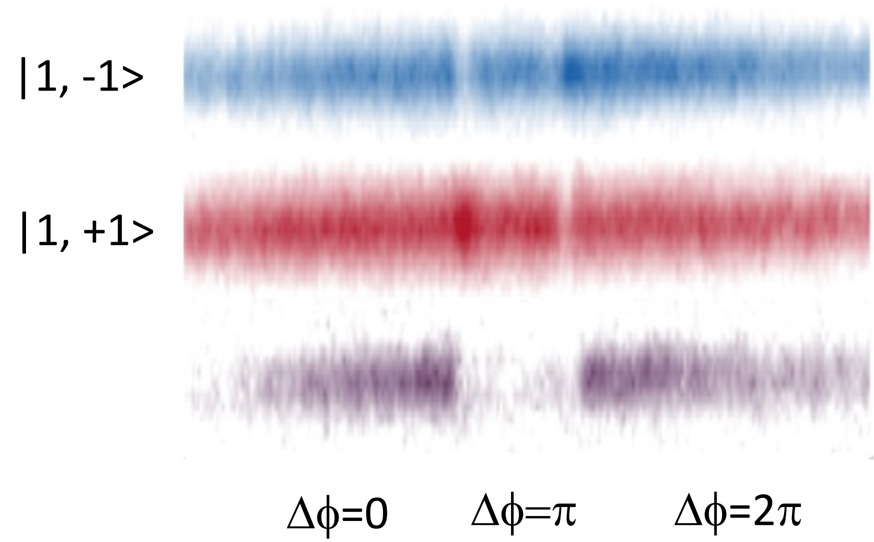
Cominotti *et al.*, PRL **128**, 210401 (2022)

MAGNETIC SOLITONS

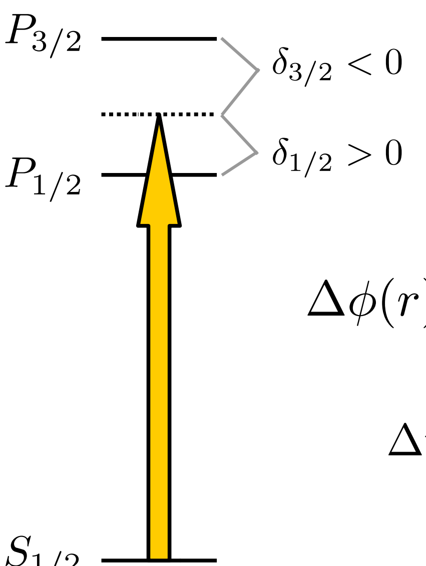
Localized magnetic solitary wave in balanced mixtures (**extension** ~ ξ_s)

π-phase jump across the soliton
(for any velocity)



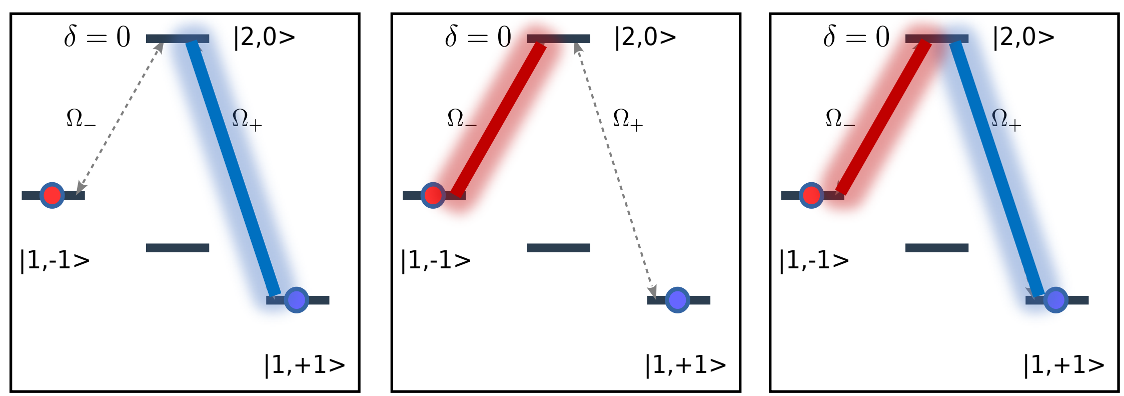


Imprint 2π in the relative phase
+π to the state +1
-π to the state -1



$$\Delta\phi(r) = -U(r)\Delta t/\hbar$$

$$\Delta t \ll \frac{\hbar}{n\,\delta g}$$



Farolfi *et al.*, PRL **125**, 030401 (2020)

$\Omega = 0$

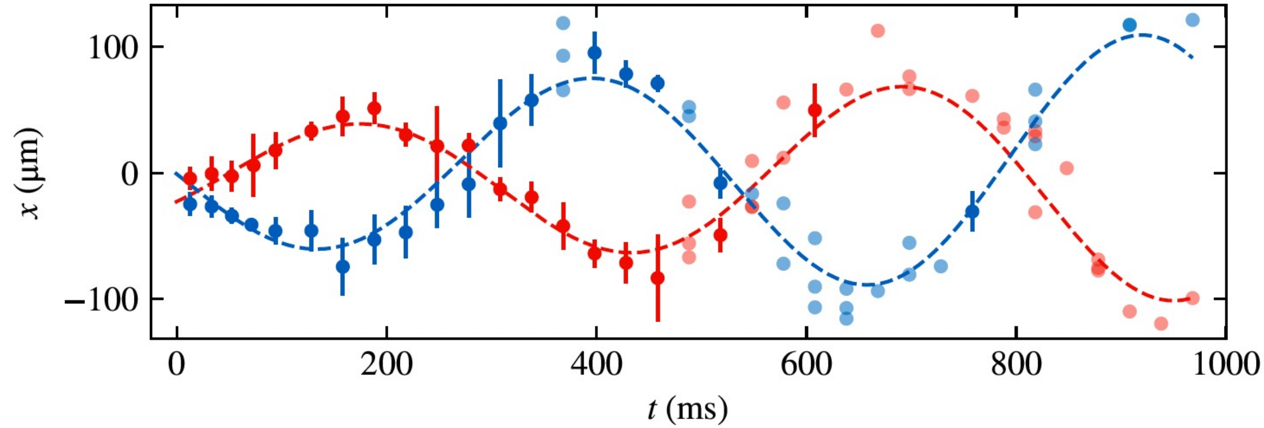
MAGNETIC SOLITONS

Long living and oscillating in the harmonic potential
Enhanced stability vs density solitons ($\xi_s \gg \xi$)

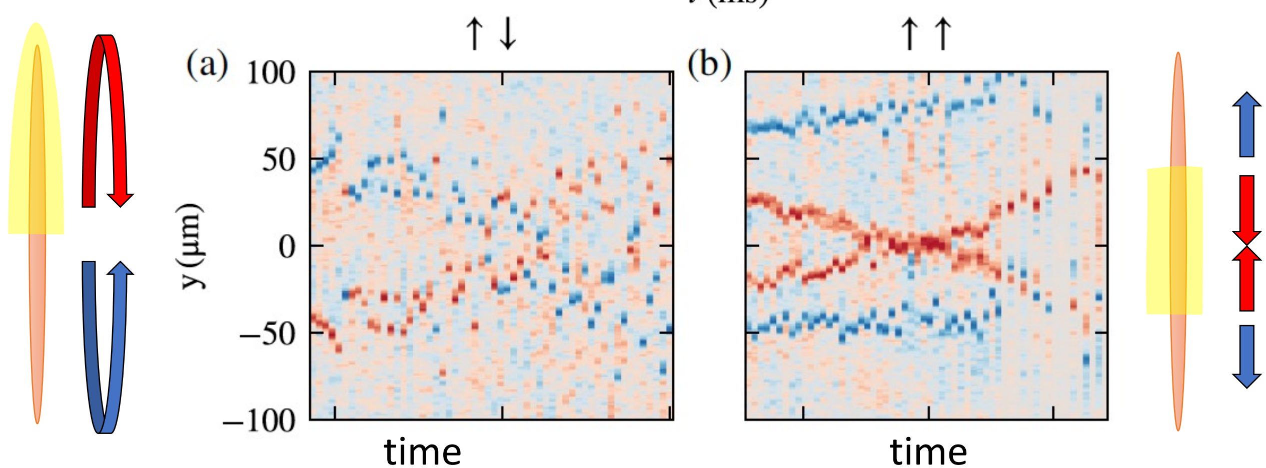
Density	3D dynamics
Spin	1D dynamics

Measurement of the oscillation period of magnetic solitons

$T = 4.7\ T_x$



Two types of collisions (sign dependent)



Farolfi *et al.*, PRL **125**, 030401 (2020)

Also:

Chai *et al.*, PRL **125**, 030402 (2020)
Lannig *et al.*, PRL **125**, 170401 (2020)

→ **POSTER 5 (Bresolin)**