International School of Physics "Enrico Fermi"

COURSE 211 –
QUANTUM MIXTURES WITH ULTRA-COLD ATOMS

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CONSIGLIO NAZIONALE DELLE RICERCHE

PROVINCIA AUTONOMA
DI TRENTO

This Lecture (**Part One**)

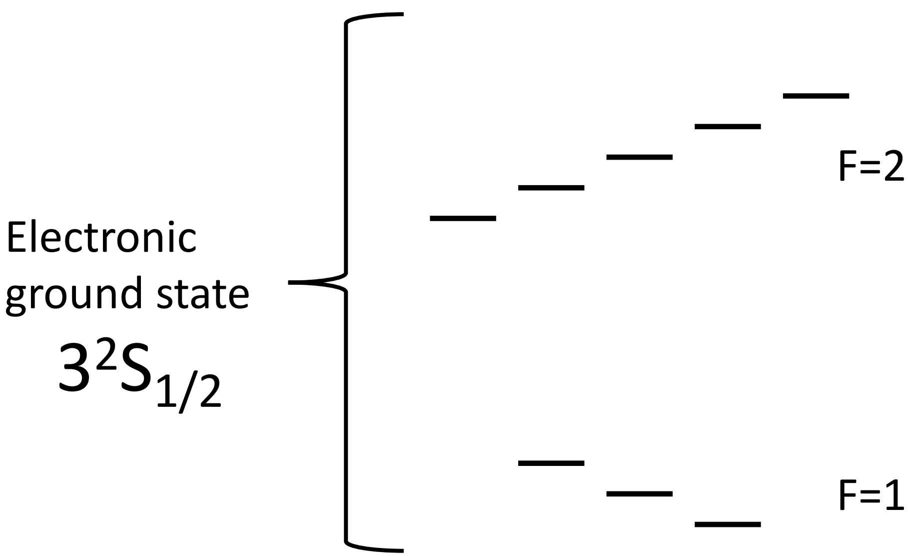
This Lecture (**Part Two**)

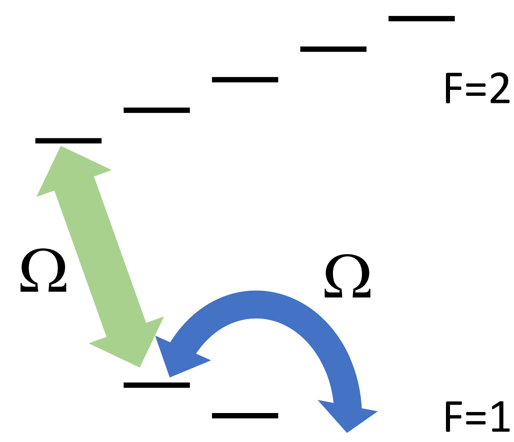
Two-component **spin mixtures** of Sodium atoms (many-body regime)

Two-component **spin mixtures** of Sodium atoms (many-body regime)

+ coherent coupling

^{23}Na (bosonic)





MAGNETIC FIELD STABILITY

$\Omega \neq 0$

Experimental requirement : low magnetic field noise environment

to study the dynamics

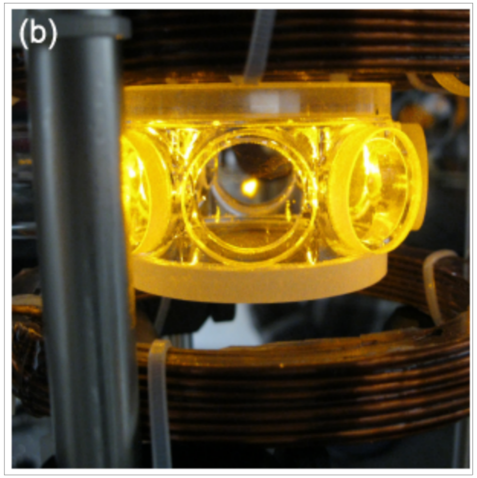
to keep coherence

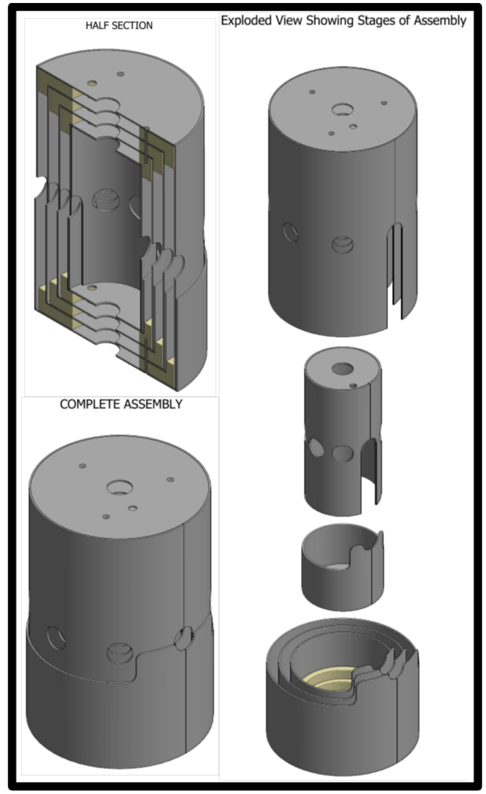
$$g_F \mu_B \Delta B \ll \hbar\Omega < n\delta g \ll ng$$

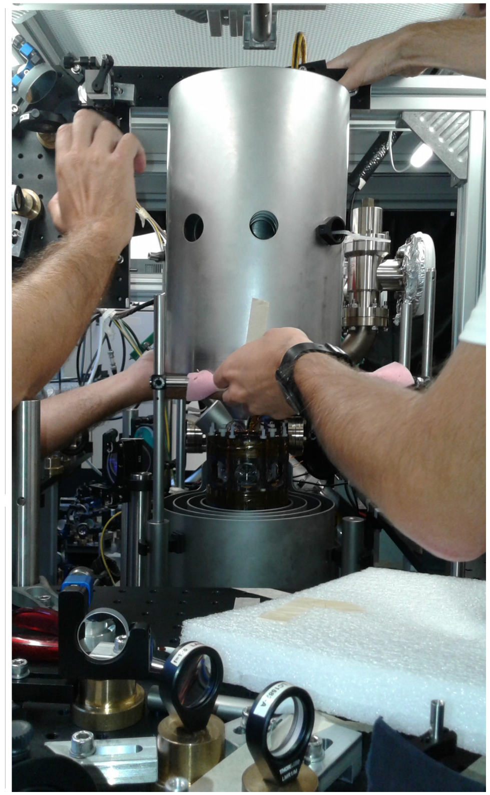
		1 nK 10 nK	15 nK 150 nK
1 Hz 10 Hz	10 Hz 100 Hz	20 Hz 200 Hz	300 Hz 3 kHz
1 μG 10 μG			

MAGNETIC SHIELD

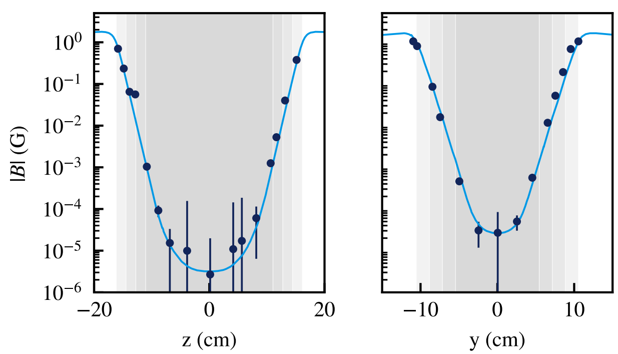
$\Omega \neq 0$

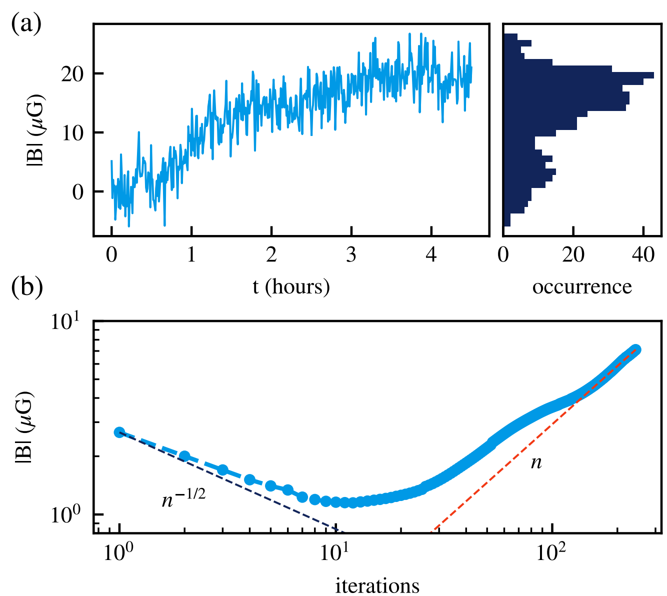






Attenuation: 10^5



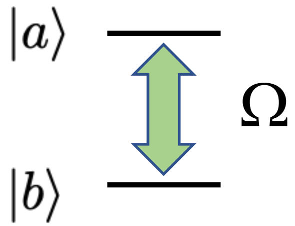


Field stability:
A few μG
over hours

Farolfi *et al.*, Rev. Sci. Instrum. **90**, 115114 (2019)

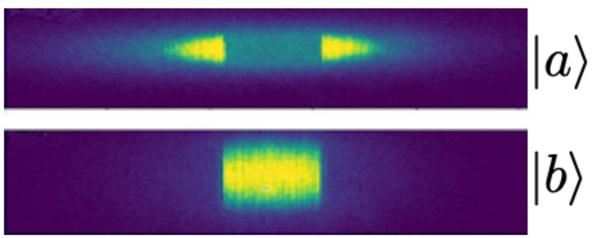
$\Omega \neq 0$

TWO-LEVEL SYSTEM

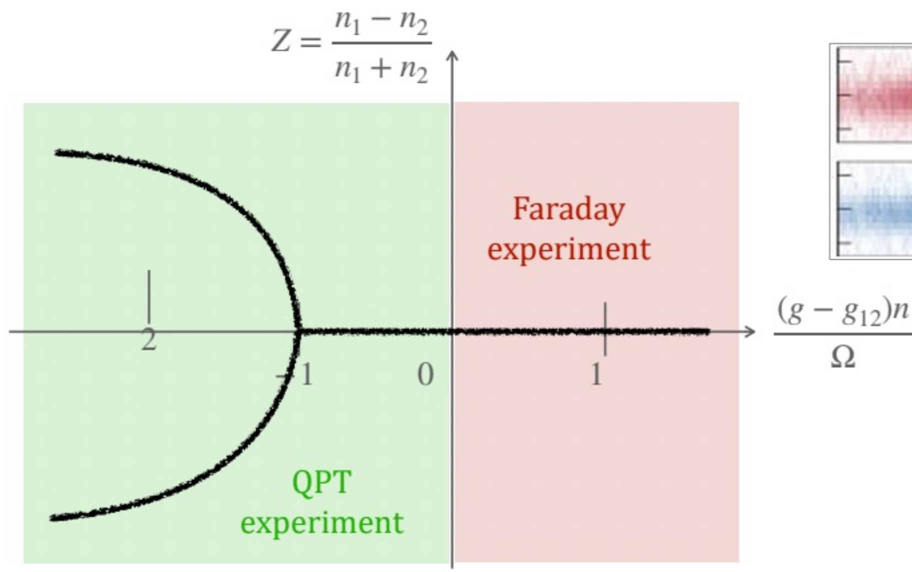


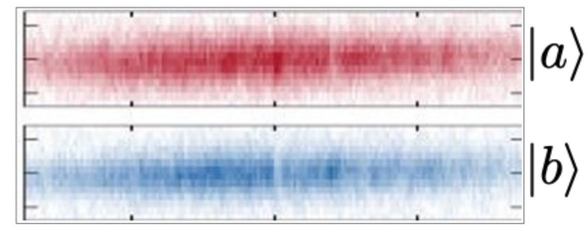
$$i\hbar\frac{\partial}{\partial t}\psi_a = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g_a|\psi_a|^2 + g_{ab}|\psi_b|^2\right)\psi_a - \frac{\hbar\Omega}{2}\psi_b$$
$$i\hbar\frac{\partial}{\partial t}\psi_b = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g_b|\psi_b|^2 + g_{ab}|\psi_a|^2\right)\psi_b - \frac{\hbar\Omega^*}{2}\psi_a$$

Phase diagram

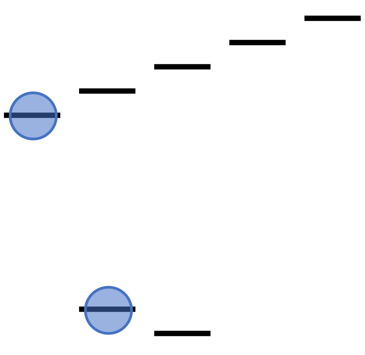


Immiscible





Miscible



Z_2 symmetry breaking

$|2,-2\rangle$

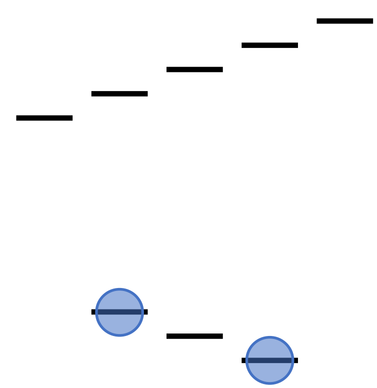
$|1,-1\rangle$

$\boxed{\delta g < 0}$

$|1,-1\rangle$

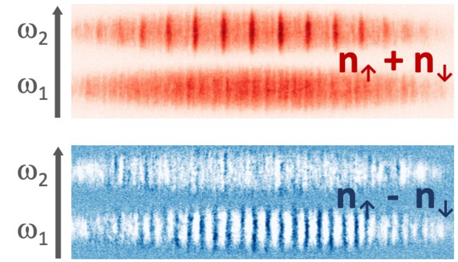
$|1,+1\rangle$

$\boxed{\delta g > 0}$



$\Omega \neq 0$

FARADAY WAVES (**miscible mixture**)

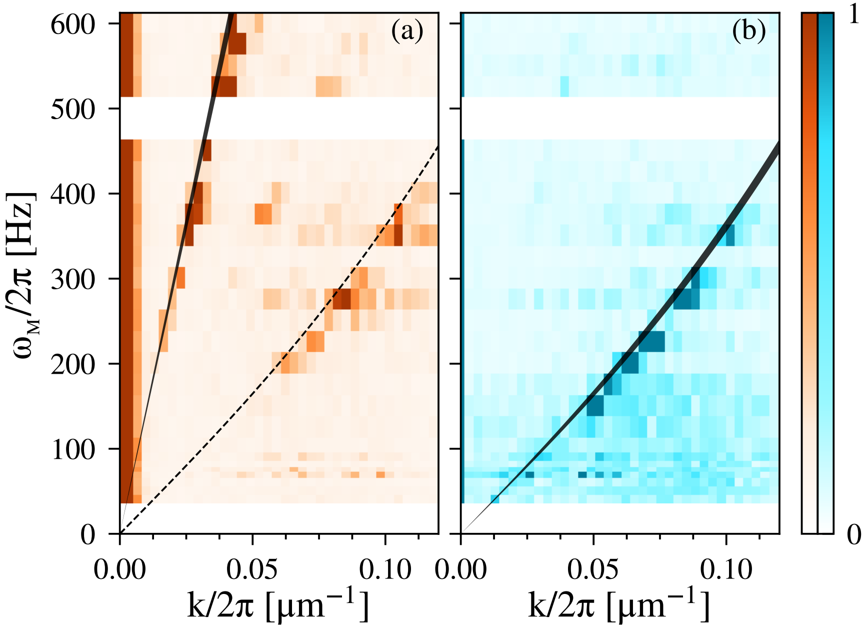


Measurement of density and spin dispersion relations

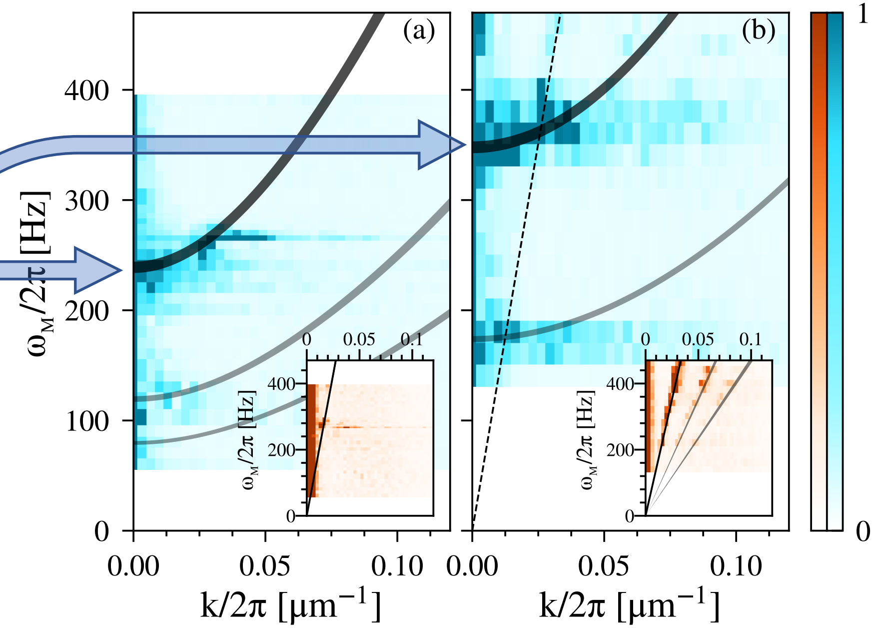
$$E_{d,s}(k) = \hbar\omega_{d,s}(k) = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2\mu_{d,s}\right)}$$

$$E_s(k) = \hbar\omega_s(k) = \sqrt{\left(\frac{\hbar^2 k^2}{2m} + \hbar\Omega\right)\left(\frac{\hbar^2 k^2}{2m} + 2\mu_s + \hbar\Omega\right)}$$

Massless spin excitations



Massive spin excitations



Cominotti *et al.*, PRL **128**, 210401 (2022)

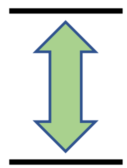
$$\hbar\omega_p = \sqrt{\hbar\Omega(\hbar\Omega + 2\mu_s)}$$

$$\hbar\omega_s = \hbar\omega_p + \frac{\hbar^2 k^2}{2M}$$

TWO-LEVEL SYSTEM - Single atom

Ω ≠ 0

$|a\rangle$



Ω

$|b\rangle$

$$i\hbar\frac{\partial}{\partial t}\psi_a = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g_a|\psi_a|^2 + g_{ab}|\psi_b|^2\right)\psi_a - \frac{\hbar\Omega}{2}\psi_b$$
$$i\hbar\frac{\partial}{\partial t}\psi_b = \left(-\frac{\hbar^2}{2m}\nabla^2 + V + g_b|\psi_b|^2 + g_{ab}|\psi_a|^2\right)\psi_b - \frac{\hbar\Omega^*}{2}\psi_a$$

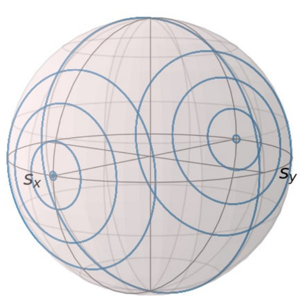
Generic **state** of the system

$$|\psi\rangle = c_a|a\rangle + e^{i\phi}c_b|b\rangle$$

Population difference $\quad s_z = c_a^2 - c_b^2 \qquad |\mathbf{s}| = 1$

Phase difference $\quad \phi$

Spin (state)
Bloch Vector $\quad \mathbf{R} \qquad \mathbf{W} = (\Omega, 0, \Delta)$



Dynamics: Precession about **W** at $\quad \Omega' = \sqrt{\Omega^2 + \Delta^2}$

$$\dot{\mathbf{R}} = \mathbf{W} \times \mathbf{R}$$
$$\dot{\mathbf{R}} \cdot \mathbf{W} = 0$$

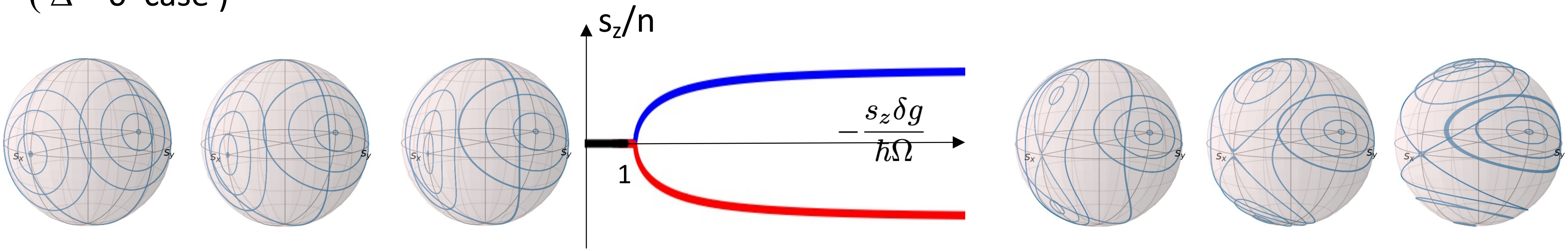
TWO-LEVEL SYSTEM – interacting particles (**immiscible mixture**)

$|\mathbf{s}| = n$

Ω≠0

Single atom	$\mathbf{R}$	$\mathbf{W} = (\Omega, 0, \Delta)$	$\dot{\mathbf{R}} = \mathbf{W} \times \mathbf{R}$
Many-body interacting system	$\mathbf{R}$	$\mathbf{W}_{\text{eff}} = (\Omega, 0, \Delta + s_z \delta g/\hbar)$	$\dot{\mathbf{R}} = \mathbf{W}_{\text{eff}} \times \mathbf{R}$

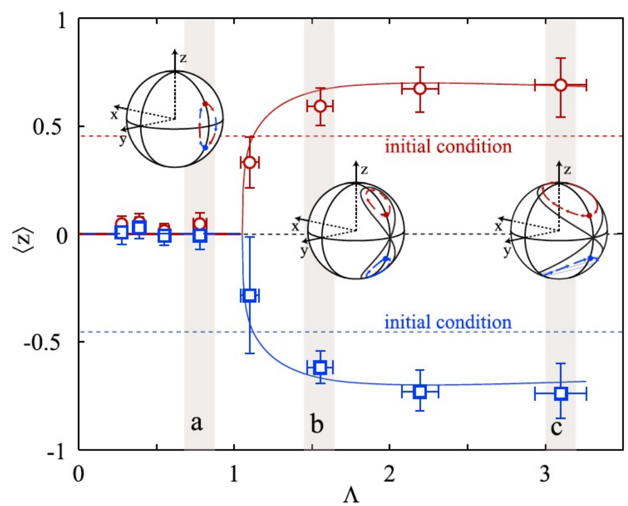
($\Delta = 0$ case)



0   1   2 $-\frac{s_z \delta g}{\hbar \Omega}$

Coupling-driven dynamics   Interaction-driven dynamics

Z_2 symmetry breaking
Bifurcation



Rb immiscible mixture
Single mode (0D, no position dependence)
= uniform density case

$R_{TF} \lesssim \xi_s$

Zibold *et al.*, PRL **105**, 204101 (2010)

MAGNETIC ANALOGUE

$\Omega \neq 0$

$$\mathcal{H} = -\alpha\left(\mathbf{W}_{\text{eff}}(\mathbf{s}) \cdot \mathbf{s}\right)$$

$\mathbf{s}$

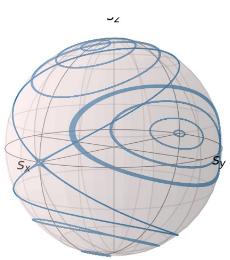
$$\boxed{\dot{\mathbf{s}} = \left(\mathbf{W}_{\text{eff}} \times \mathbf{s}\right)}$$

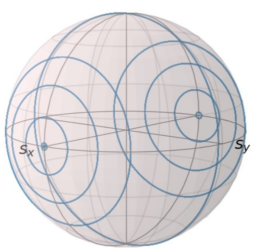
$$\boxed{\mathbf{W}_{\text{eff}} = \left(\Omega, 0, \Delta + s_z \delta g / \hbar\right)}$$

Coupling strength

Coupling detuning

Magnetic (spin) interactions





μ

$$\mathcal{H} = -\left(\mathbf{B}_{\text{eff}} \cdot \mu\right)$$

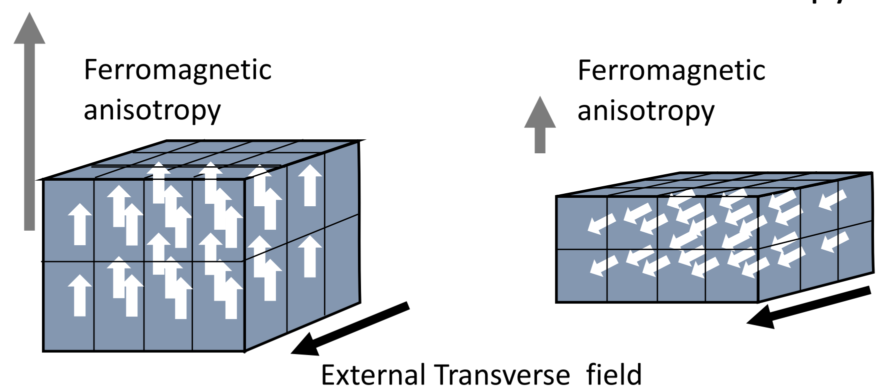
$$\boxed{\dot{\mu} = \beta\left(\mathbf{B}_{\text{eff}} \times \mu\right)}$$

$$\boxed{\mathbf{B}_{\text{eff}} = \left(B_{\perp}, 0, B_z + K\right)}$$

Transverse External field

Longitudinal external field

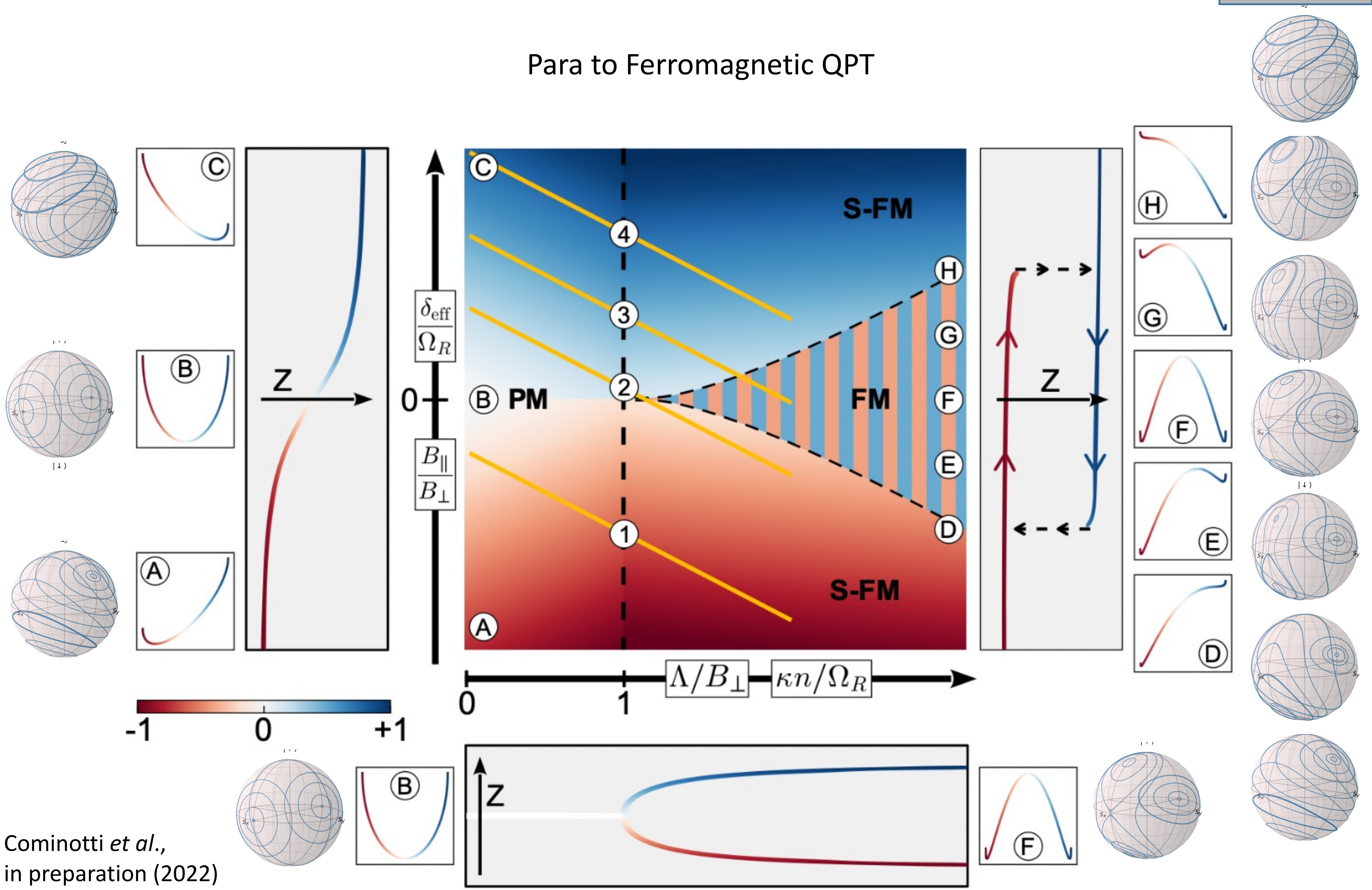
Ferromagnetic anisotropy



PHASE DIAGRAM

$\Omega \neq 0$

Para to Ferromagnetic QPT

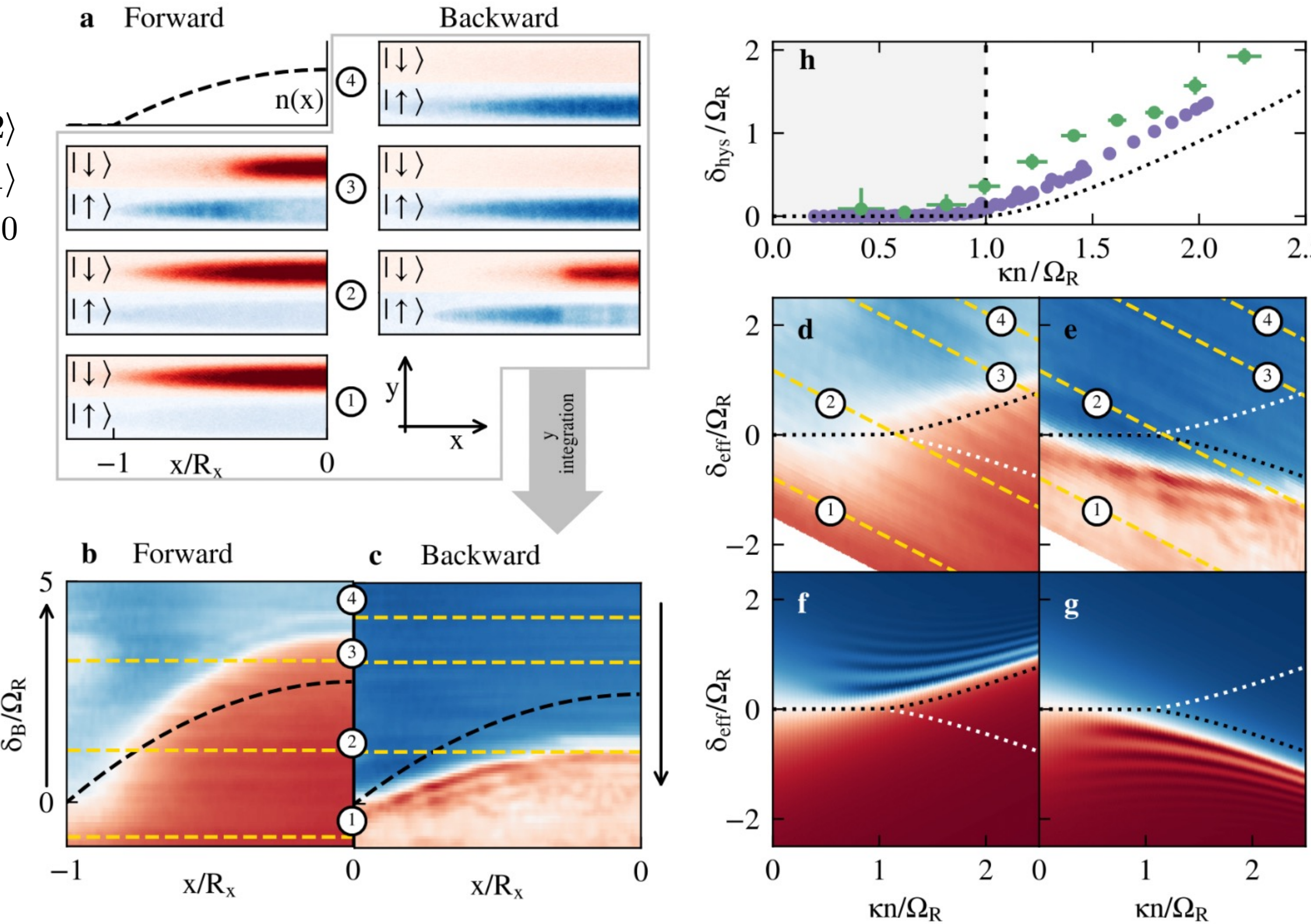


Cominotti *et al.*,
in preparation (2022)

EXPERIMENT Measurement of the **equation of state** of magnetic systems

Ω ≠ 0

Na
$|2,-2\rangle$
$|1,-1\rangle$
$\delta g < 0$



Hysteresis width

Hysteresis observation

Hysteresis simulation

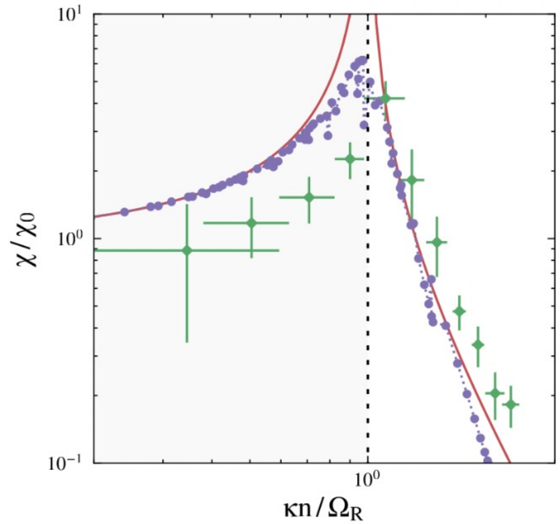
Cominotti *et al.*, in preparation (2022)

DIVERGING QUANTITIES AT THE CRITICAL POINT

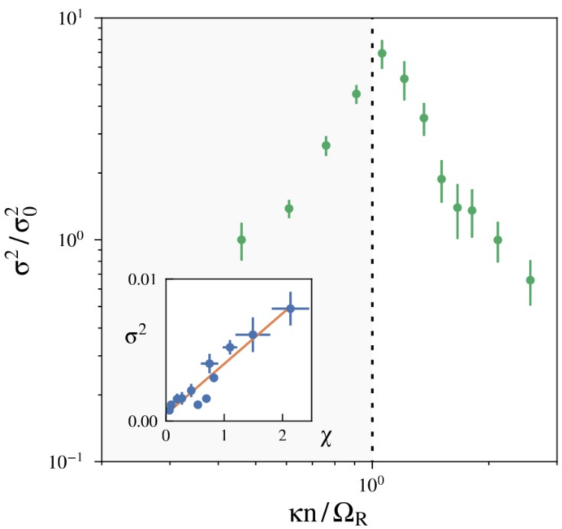
$\Omega \neq 0$

Susceptibility

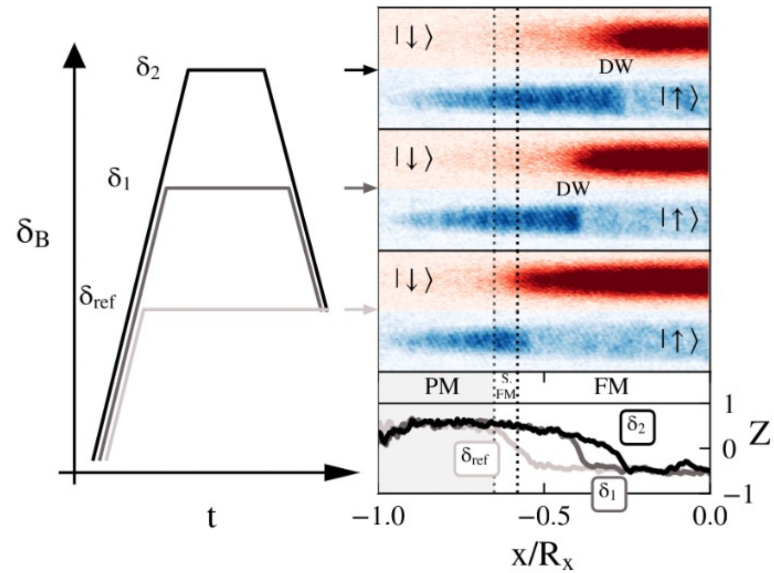
$$\chi = \frac{\Omega_R}{n} \frac{\partial s_z}{\partial \delta_{\text{eff}}}\bigg|_{\delta_{\text{eff}}=0}.$$



Magnetization fluctuations



Domain walls
Between ferromagnetic regions



Cominotti *et al.*, in preparation (2022)

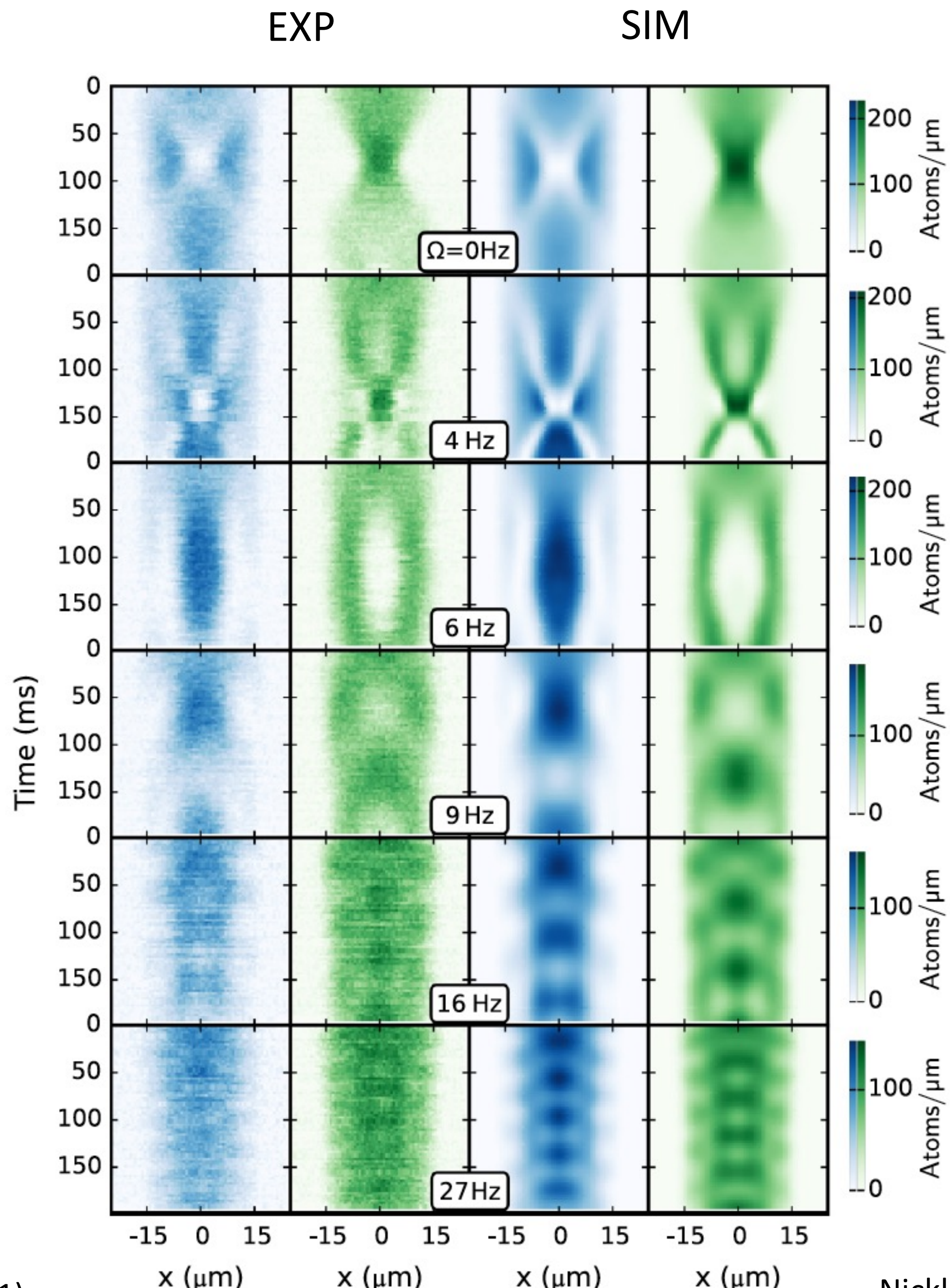
→ POSTER 10 (Cominotti)

OUT-OF-EQUILIBRIUM

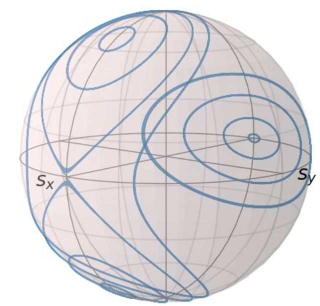
$\Omega \neq 0$

1D extended system

dynamics



Rb
F=2,mF=+1
F=1,mF=-1
dg<0



Nonlinear dressed states

Nicklas *et al.*, PRL **92**, 107193 (2011)

Nicklas *et al.*, PRA **92**, 053614 (2015)

OUT-OF-EQUILIBRIUM

Na

F=1,mF=-1

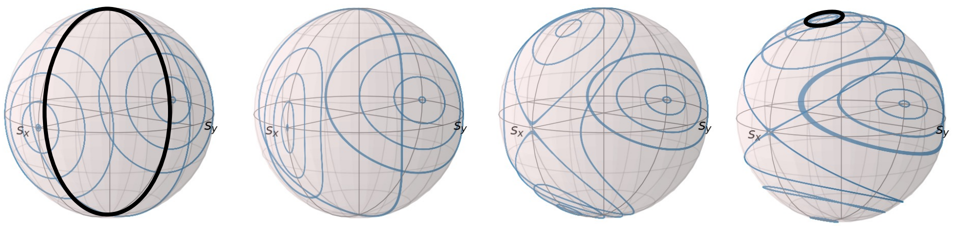
F=1,mF=+1

dg>0

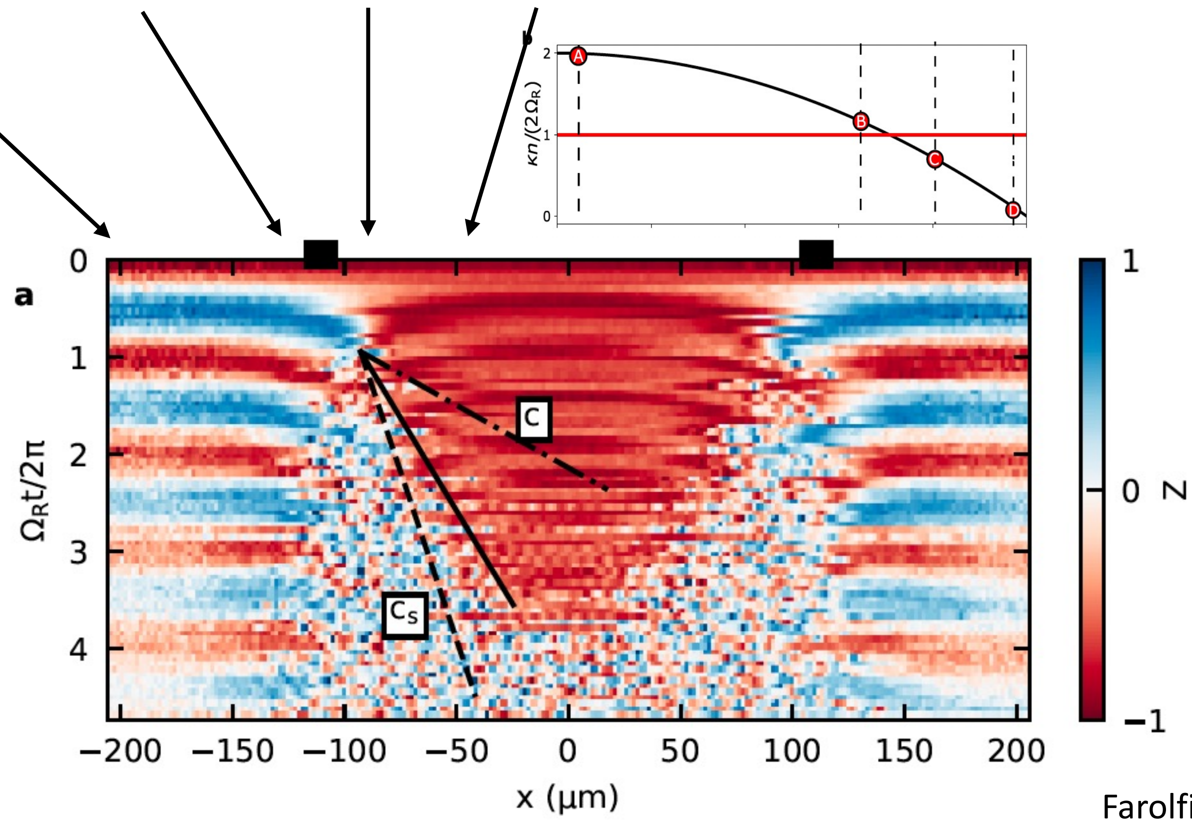
$\Omega \neq 0$

Bifurcation happens to the maximum energy, not to the minimum energy state

Initial state:
far-from-equilibrium (whole system in |-1>)



Different **evolution**
In different regions



DYNAMICAL PHASE TRANSITION

Magnetic heterostructure
Magnetic interfaces

Farolfi *et al.*, Nat. Phys. **17**, 1359 (2021)

$\Omega \neq 0$

QUANTUM TORQUE

$$\mathcal{H} = -\alpha\left(\mathbf{W}(\mathbf{s})\cdot\mathbf{s}\right)$$

Local evolution

$$\dot{\mathbf{s}} = \mathbf{W}(\mathbf{s}) \times \mathbf{s}$$

Adding **external degrees of freedom**:

$$\dot{n} + \partial_x j = 0$$

n satisfies continuity eq.
***s** does not*

$$\dot{\mathbf{s}} + \partial_x \mathbf{j}_\mathbf{s} = \mathbf{W}(\mathbf{s}) \times \mathbf{s}$$

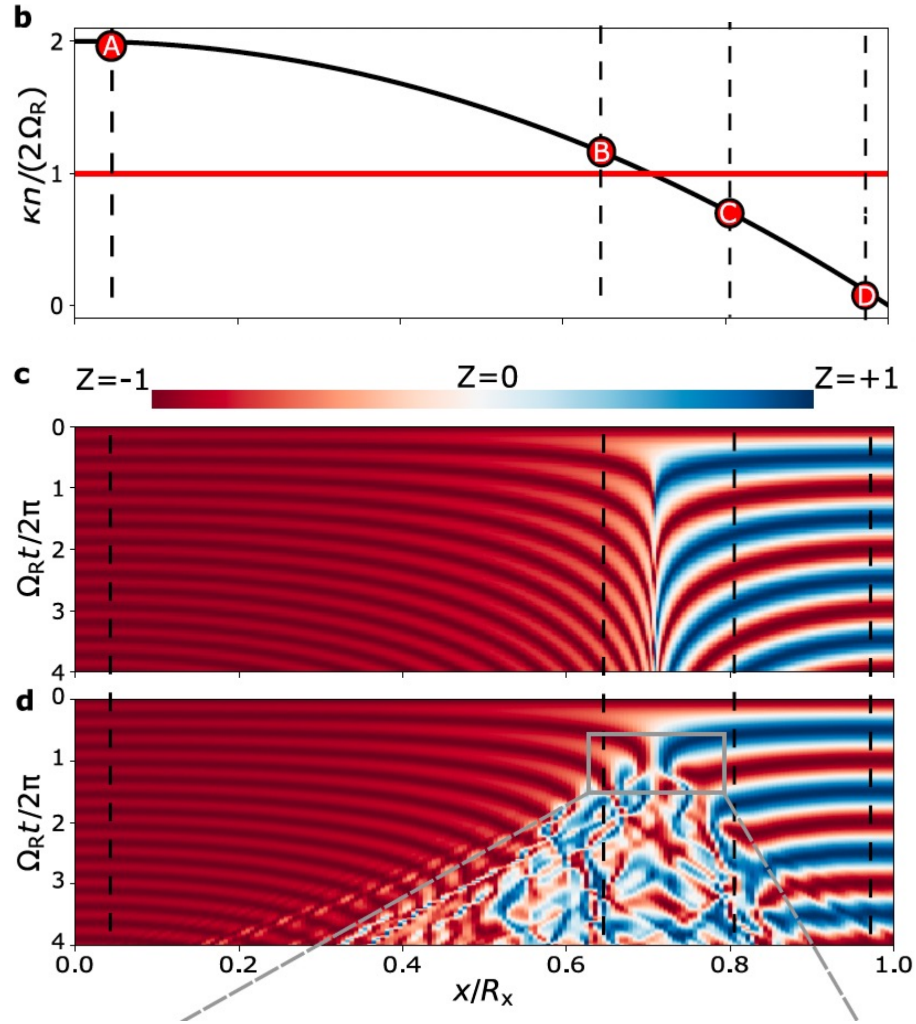
Dissipationless LLE

Landau Lifshitz Equations

Spin current $\mathbf{j}_\mathbf{s} = \underbrace{v\mathbf{s}}_{\text{Spin advection}} + \frac{\hbar}{2mn}\partial_x \mathbf{s} \times \mathbf{s}$

Constant n:

$$\dot{\mathbf{s}} + \underbrace{\partial_x\left(\frac{\hbar}{2mn}\partial_x \mathbf{s}\times\mathbf{s}\right)}_{\text{Quantum torque}} = \mathbf{W}(\mathbf{s}) \times \mathbf{s}$$



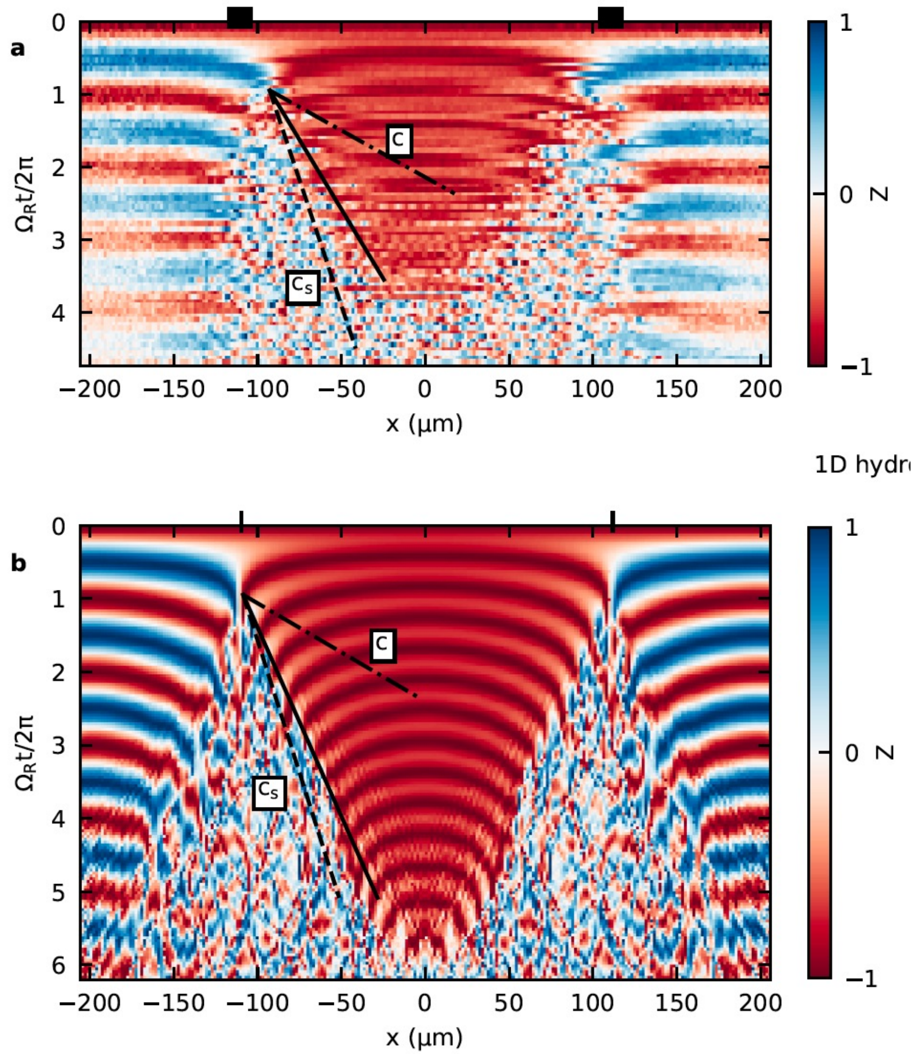
Farolfi *et al.*, Nat. Phys. **17**, 1359 (2021)

QUANTUM TORQUE

$\Omega \neq 0$

Experiment

GPE

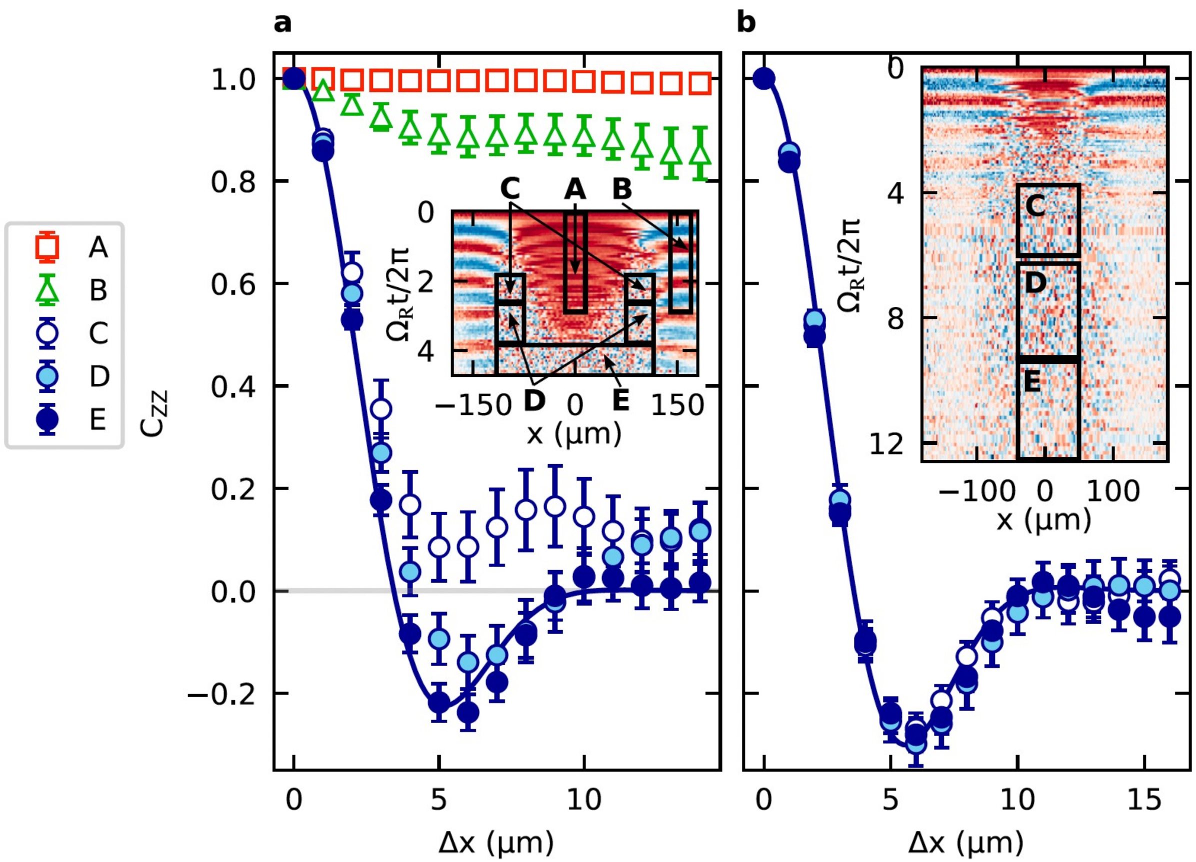


Farolfi *et al.*, Nat. Phys. **17**, 1359 (2021)

MAGNETIZATION CORRELATIONS

$\Omega \neq 0$

Spin waves with anticorrelations
On lengths of a few ξ_s

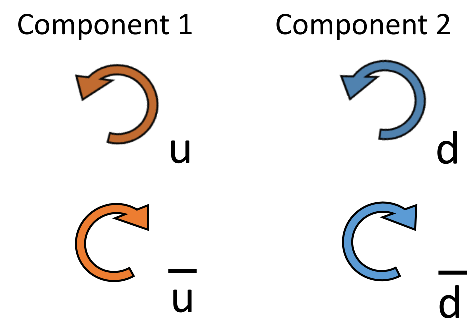


Farolfi *et al.*, Nat. Phys. **17**, 1359 (2021)

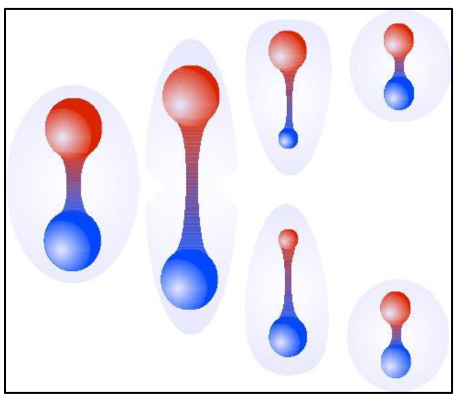
QUARK CONFINEMENT

$\Omega \neq 0$

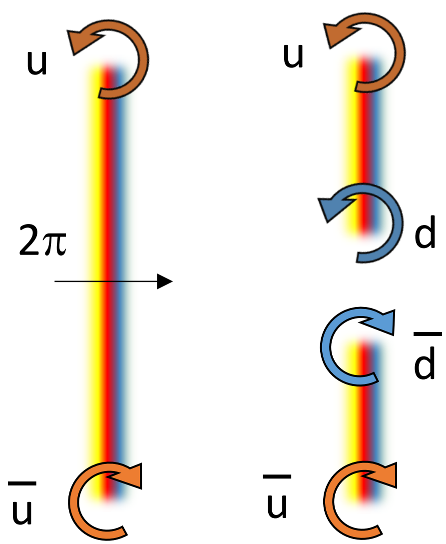
Coherently coupled
Mixture in 2D



Quark confinement



VORTEX CONFINEMENT



Vortex-antivortex
pair

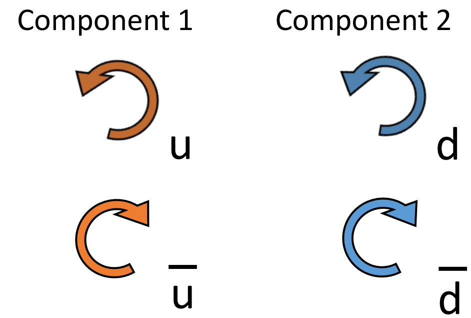
Son and Stephanov, PRA **65**, 063621 (2002)
Eto and Nitta, PRA **97**, 023613 (2018)

Tylutki et al., PRA **93**, 043623 (2016)
Gallemí et al.,PRA **100**, 023607 (2019)

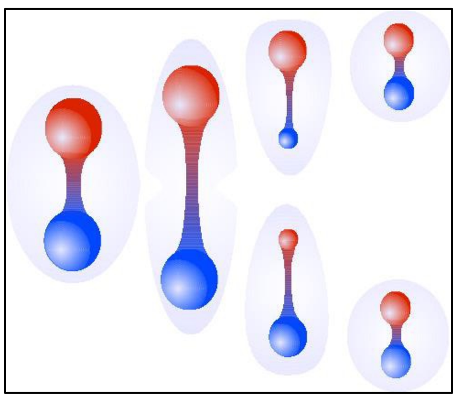
QUARK CONFINEMENT

$\Omega \neq 0$

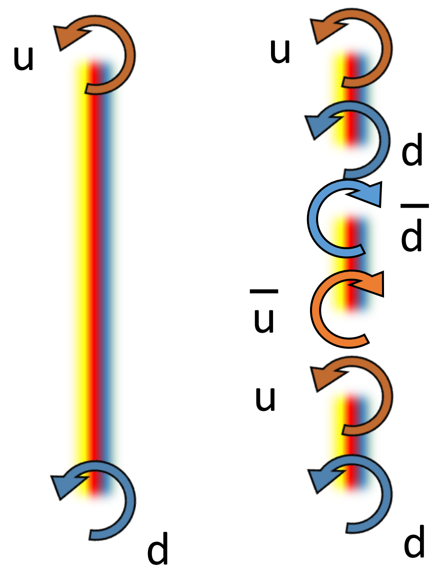
Coherently coupled
Mixture in 2D



Quark confinement



VORTEX CONFINEMENT



Vortex-antivortex
pairs

Son and Stephanov, PRA **65**, 063621 (2002)
Eto and Nitta, PRA **97**, 023613 (2018)

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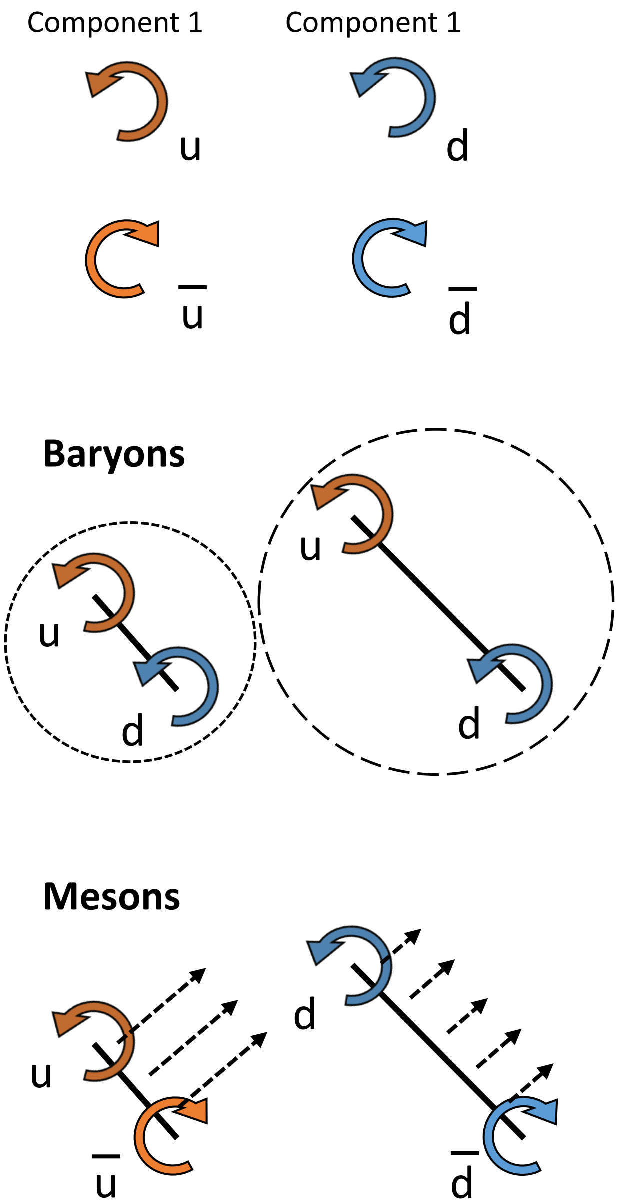
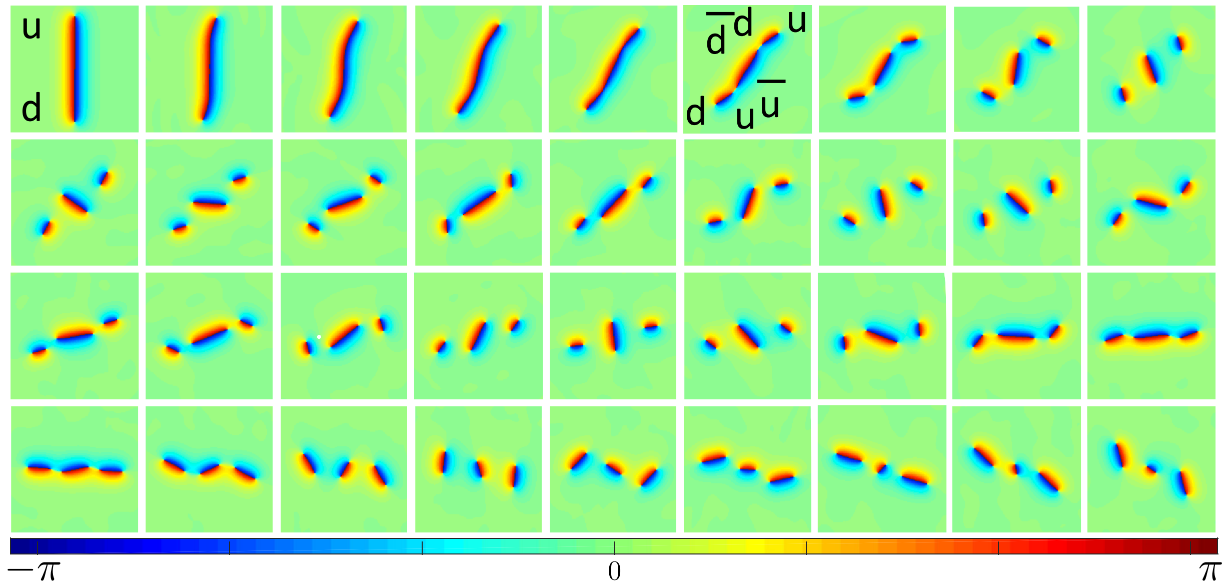
Gallemí et al.,PRA **100**, 023607 (2019)

QUARK CONFINEMENT

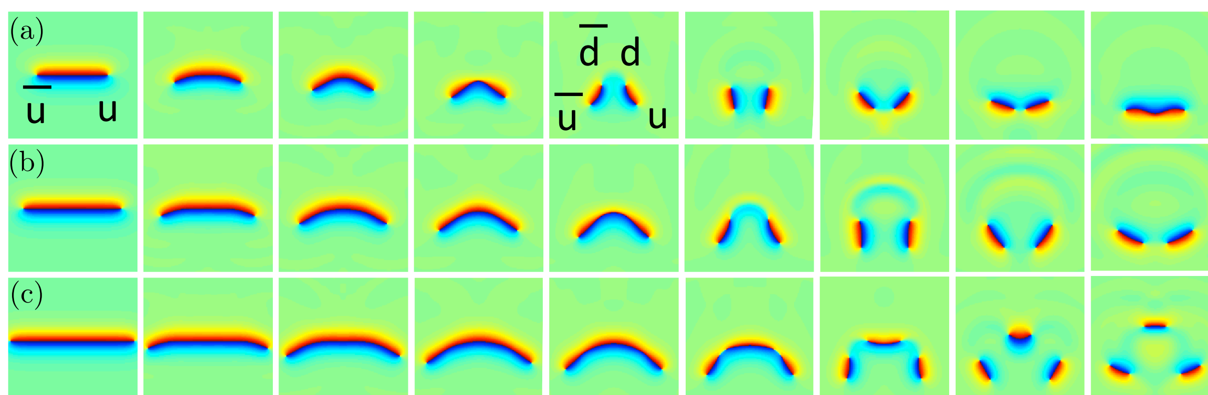
VORTEX CONFINEMENT



Baryon dynamics



Meson dynamics



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Eto and Nitta, PRA **97**, 023613 (2018)
Tylutki et al., PRA **93**, 043623 (2016)
Gallemí et al.,PRA **100**, 023607 (2019)