Varenna School on QUANTUM FLUIDS of LIGHT and MATTER, July 2-7, 2022

The cavity Kerr medium model and the surprising history around it

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Thanks are due to Enrico Brambilla, Alessandra Gatti, Franco Prati

The talk is focused on the equation (LLE)

$$\frac{\partial E(\bar{x},\bar{y},\bar{t})}{\partial \bar{t}} = F - (1+i\alpha)E + i|E|^2 E + i\left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}\right) E$$

L.A. Lugiato and R. Lefever, "Spatial dissipative structures in passive optical systems", Phys. Rev. Lett. **58**, 2209 (1987).

- Special issue on "Theory and Applications of the LLE", European Physical Journal D (2017).
- L.A. Lugiato, F. Prati and M. Brambilla, "Nonlinear Optical systems", Cambridge University Press, 2015, Ch.28.
- L.A. Lugiato, F. Prati, M. Gorodetsky and T. Kippenberg, "From the LLE to microresonator based soliton Kerr frequency combs", Phyl. Trans. Roy. Soc. A 376, 20180113 (2018), in the special issue on "Dissipative Structures in Matter out of equilibrium, from Chemistry, Photonics and Biology", in honour of Ilya Progogine

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- How the model was derived to provide a paradigm for optical pattern formation
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PARADIGMATIC MODEL FOR NONLINEAR CHEMICAL REACTIONS: THE "BRUSSELATOR"

(I Prigogine and R. Lefever, J. Chem. Phys. **48**, 1695 (1968))

TWO EQUATIONS:

$$\frac{\partial X}{\partial \bar{t}} = A - (B+1)X + X^2 Y + D_1 \left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} \right) X$$
$$\frac{\partial Y}{\partial \bar{t}} = BX - X^2 Y + D_2 \left(\frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} \right) Y$$

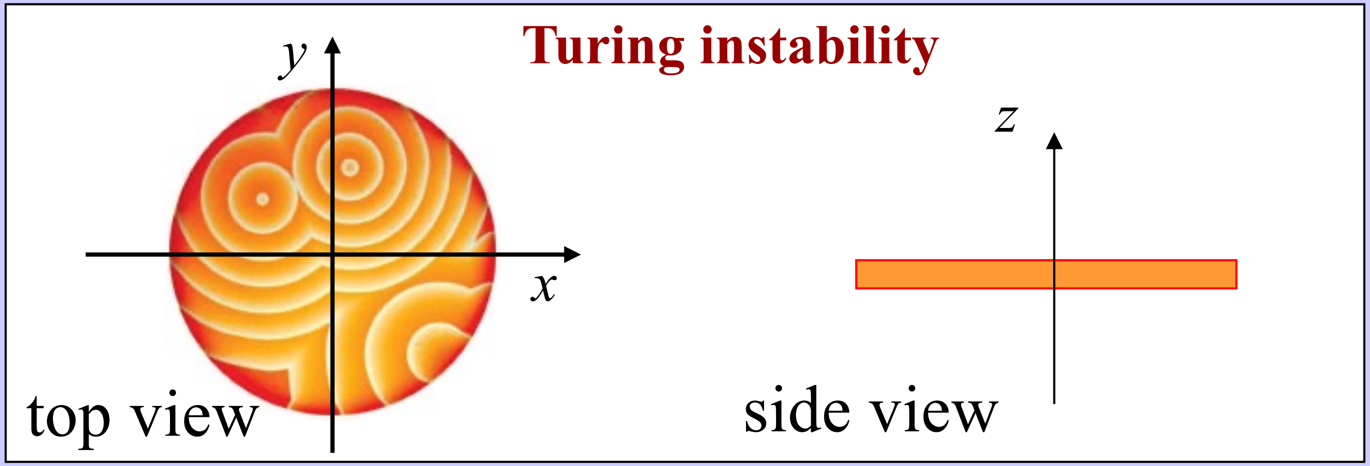
X, Y = normalized concentration of reactants

$\bar{x}, \bar{y}, \bar{t}$ = normalized space-time coordinates

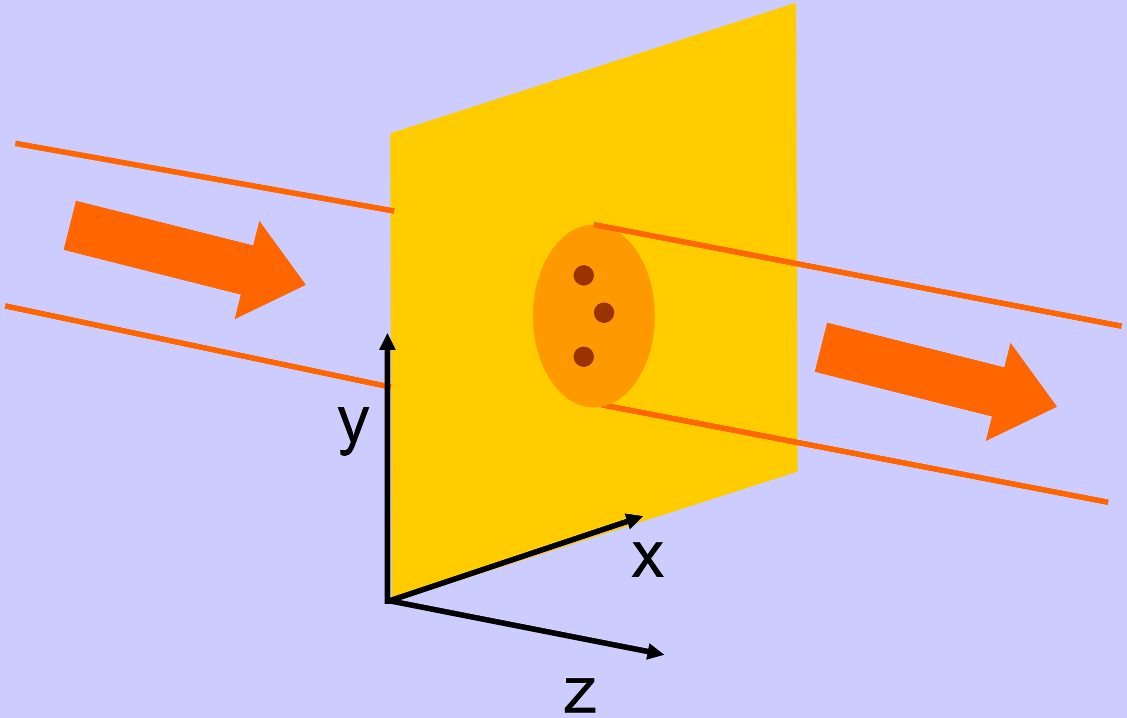
A, B = parameters

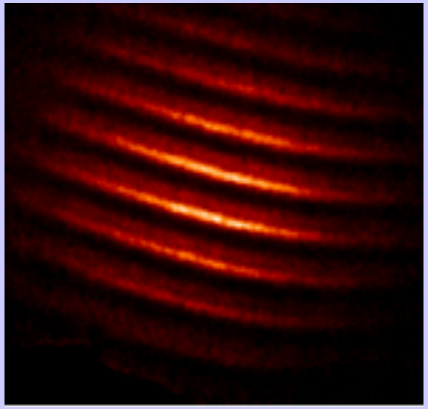
D_1, D_2 = diffusion coefficients

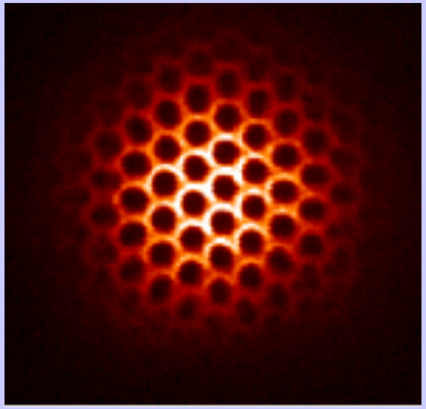
large aspect ratio system

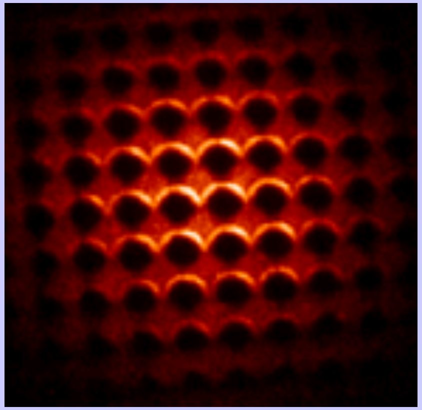


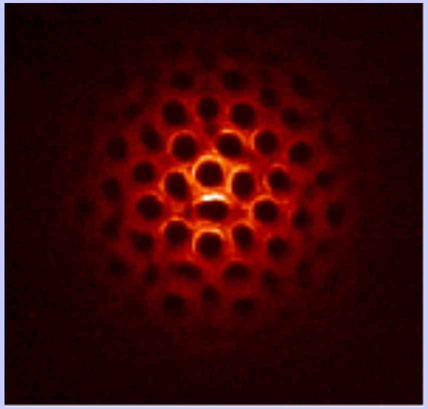
Optical Pattern Formation











IN THE CASE OF OPTICS, THE ROLE OF DIFFUSION (COUPLING THE SPATIAL POINTS (*x,y*)) IS PLAYED BY DIFFRACTION.

If we indicate by $\mathcal{E}$ the linearly polarized electric field, treated as a scalar, we write

$$\mathcal{E}(x,y,z,t) \propto \frac{1}{2}\left[E(x,y,z,t)e^{-i\omega_0\left(t-\frac{z}{c}\right)} + E^*(x,y,z,t)e^{i\omega_0\left(t-\frac{z}{c}\right)}\right]$$

where ω_0 is the reference frequency and E the slowly varying envelope. In the paraxial approximation, diffraction is described by a term proportional to the transverse Laplacian $\boxed{i\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)E(x,y,z,t)}$ which has the same form of diffusion apart from the presence of the imaginary unit.

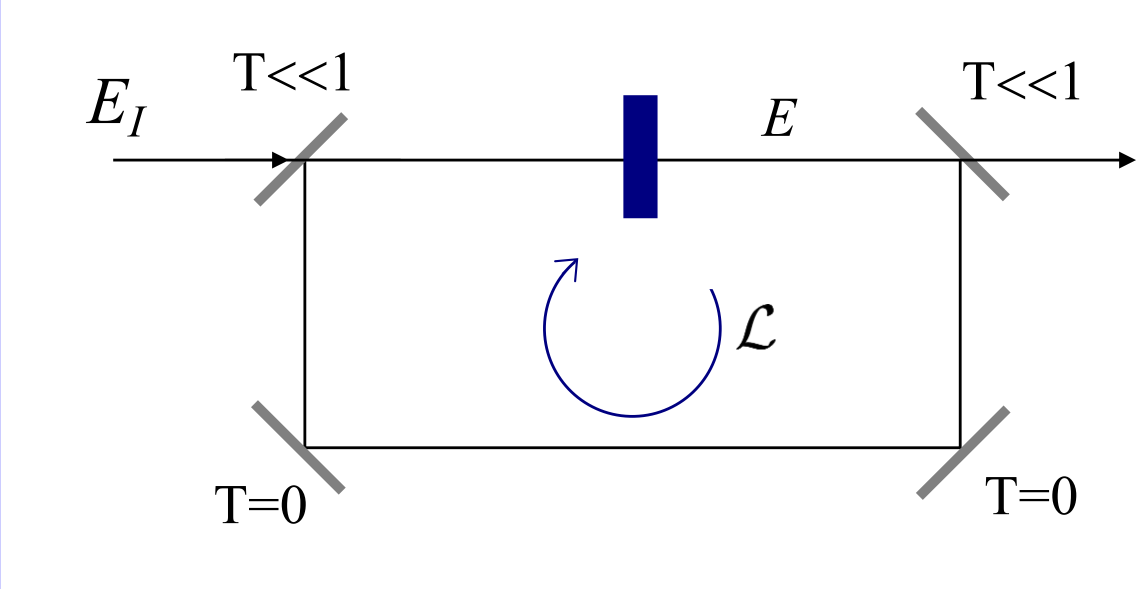
However, the typical configuration of optical systems appears at first sight the contrary of that of a large aspect ratio system, because
- Laser beams are usually well focused and therefore narrow
- In the description of laser beams, propagation in the longitudinal direction *z* is important, therefore in general the coordinate *z* cannot be ignored.

The first point is not a difficulty, because one can consider broad section laser beams. The second point is, instead, a major difficulty in view of the goal of constructing a model with the same level of simplicity as the Brusselator. As a matter of fact, the models initially used to describe transverse optical patterns (e.g. Moloney and Gibbs, Phys.Rev. Lett. **48,** 1607 (1982)) were substantially more complicated.

However, previous work on the analysis of nonlinear optical cavities neglecting transverse effects (i.e. neglecting diffraction) had shown that there are conditions in which the electric field envelope is basically uniform along the cavity, so that the variation of E along z can be neglected in the stationary state, which allows to get rid of the variable z. Such conditions are defined by the "high-Q limit" (also called "mean field limit" or "uniform field limit" in the literature) introduced in Bonifacio and Lugiato, Nuovo Cimento **21**, 505 (1978).

Qualitatively, the high-Q limit prescribes that:

1) The medium is located into a cavity with mirrors of low transmission



ring cavity

2) In a single pass through the medium, the field envelope undergoes a small variation. There are two possibilities: i) the medium is thin

ii) the nonlinearity is weak

Combining points 1) and 2) one has that, since the field envelope goes through the medium several times before going out of the cavity because of the small transmissivity, it undergoes a significant variation in time.

CHOICE OF THE NONLINEARITY

The aim is to construct a model in which the only variable is the field envelope *E,* which is complex, so that the model involves two equations like the Brusselator.
Hence the model must not involve equations for material variables.
The choice of the nonlinearity follows a **criterium of simplicity .**

Quadratic nonlinearities are not appropriate, because they involve two envelopes, one for the fundamental and one for the harmonic.
The simplest choice is that of a cubic nonlinearity, i.e. the **Kerr non-linearity** proportional to

$$i\chi^{(3)}E|E|^2$$

In conclusion, the model includes the nonlinear term, the diffraction term and three terms which arise from the presence of the cavity

$$\frac{\partial E}{\partial \bar{t}} = \overset{3}{F} \; \overset{4}{-E} \; \overset{5}{-i\alpha E} \; \overset{1}{+i|E|^2 E} \; \overset{2}{+i\left(\frac{\partial^2 E}{\partial \bar{x}^2} + \frac{\partial^2 E}{\partial \bar{y}^2}\right)}$$

1 = Nonlinearity (self-focusing)

2 = Diffraction

3 = F = stationary, uniform, monochromatic input field

4 = Field damping with damping rate $\kappa = \dfrac{cT}{\mathcal{L}}$

T= mirror transmissivity, $\mathcal{L}$ = ring cavity length

5 = Cavity detuning $\alpha = \dfrac{\omega_c - \omega_0}{\kappa}$

ω_c = cavity frequency nearest to ω_0

$\bar{t} = \kappa t, \quad \bar{x} = \dfrac{x}{x_T}, \; \bar{y} = \dfrac{y}{x_T}, \quad x_T \propto \dfrac{\sqrt{\lambda \mathcal{L}}}{T}, \quad \lambda = \dfrac{2\pi c}{\omega_0}$ wavelength

F and E are appropriately normalized to reduce parameters to the minimum

An advantage with respect to the Brusselator is that the Kerr medium model is realistic, whereas a realistic model for nonlinear chemical reactions involves many more than two equations (see Nicolis, Introduction to nonlinear science, Cambridge University Press, 1995).

HOMOGENEOUS STATIONARY SOLUTIONS

The homogeneous $\left(\frac{\partial^2 E}{\partial \bar{x}^2} + \frac{\partial^2 E}{\partial \bar{y}^2} = 0\right)$ and stationary $\left(\frac{\partial E}{\partial \bar{t}} = 0\right)$ solution obeys the equation

$$F = E_{st}\left[1 + i(|E_{st}|^2 - \alpha)\right]$$

so that one obtains (Gibbs, McCall, Venkatesan, Phys. Rev. Lett **36**, 1135 (1976))

$$F^2 = |E_{st}|^2\left[1 + (\alpha - |E_{st}|^2)^2\right]$$

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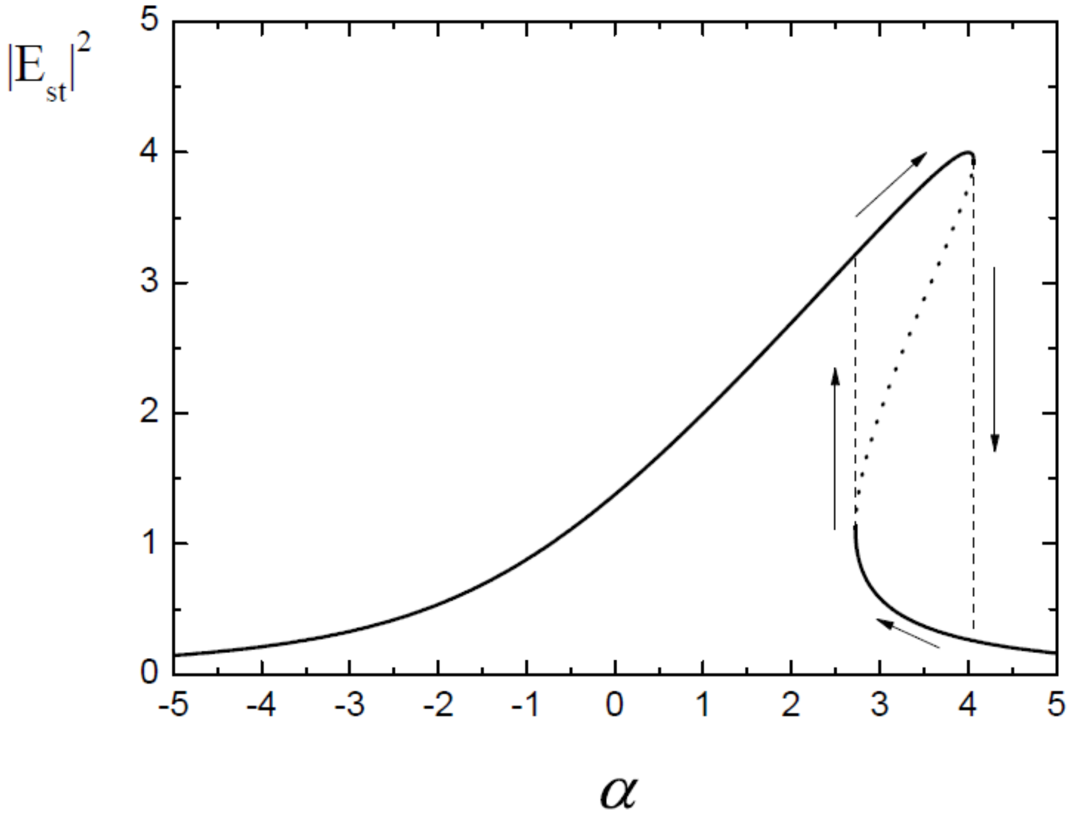
OPTICAL BISTABILITY



$F^2=4$

The linear stability analysis of the homogeneous stationary solutions showed the existence of a **spatial modulational instability**. An analytical calculation performed in 1D demonstrated that the pattern which develops beyond the instability threshold is stable under appropriate parametric conditions.

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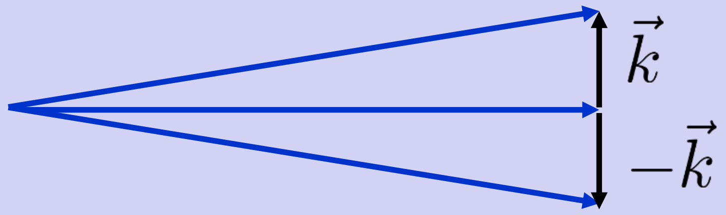
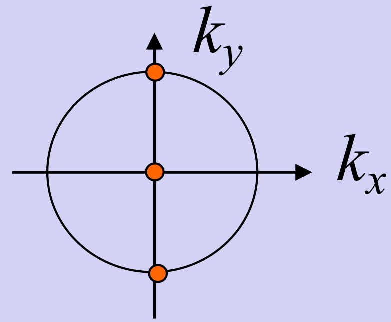
The **Kerr nonlinearity** corresponds to the process of **four-wave mixing**. A possibility is that two photons which propagate in the longitudinal direction ($\vec{k}=0$) are absorbed, and two photons which propagate symmetrically (transverse wave vectors $\vec{k}, -\vec{k}$) are emitted.





$$E = E_{st}\, e^{i\vec{0}\cdot\vec{x}} + \sigma\, e^{i\phi_+}\, e^{i\vec{k}\cdot\vec{x}} + \sigma\, e^{i\phi_-}\, e^{-i\vec{k}\cdot\vec{x}}$$

Turing pattern

$$= E_{st}\, e^{i\vec{0}\cdot\vec{x}} + 2\sigma\, \cos\left[\vec{k}\cdot\vec{x} + (\phi_+ - \phi_-)\right]\, e^{\frac{i}{2}(\phi_+ + \phi_-)}$$

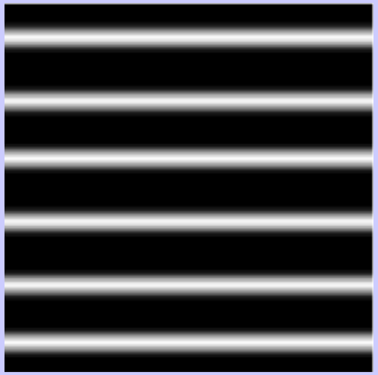
Due to the rotational symmetry any rotated version of the figure is possible.

1D case (*y* only)

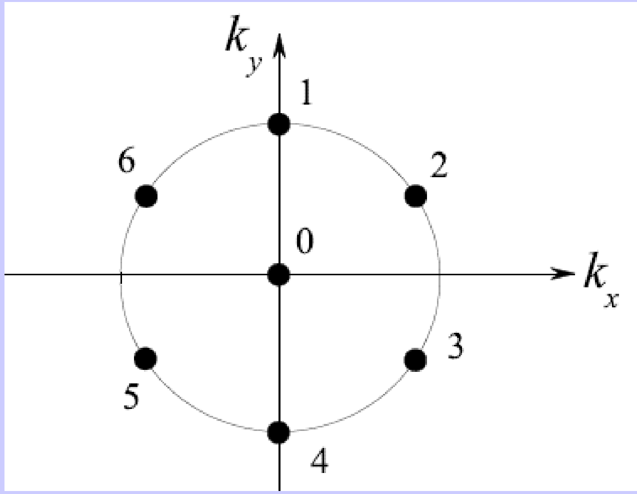
$$E = E_{st} + 2\sigma\, \cos\left[ky + (\phi_+ - \phi_-)\right]\, e^{\frac{i}{2}(\phi_+ - \phi_-)}$$

A remark of paramount importance is that the two photons, emitted in tilted directions, are in a state of **quantum entanglement** (they are entangled in energy and momentum). This fact is fundamental for the **quantum aspects of optical patterns** and, more in general, for the field of **quantum imaging** (see **Gatti, E. Brambilla, Lugiato, in Progress in Optics Vol LI**)

2D case (x,y) $\Rightarrow$ roll patterns

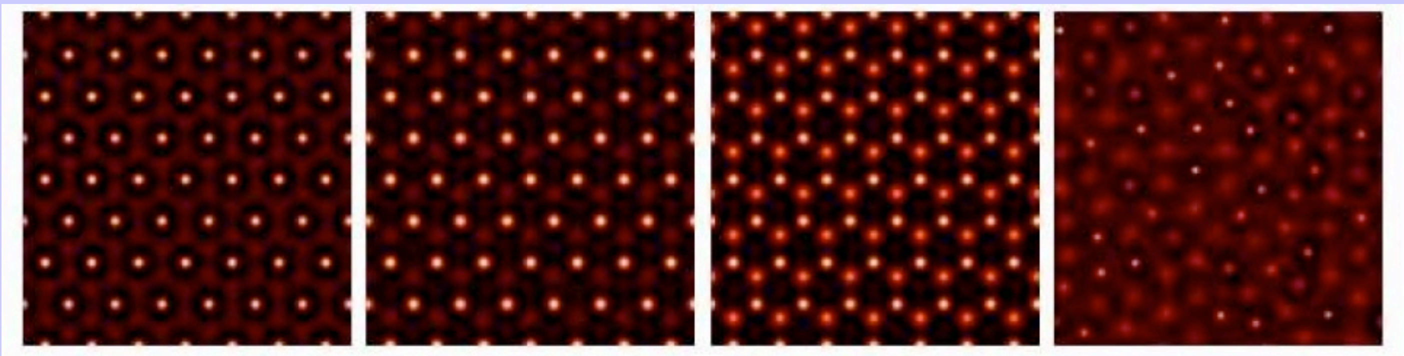


However, in 2D the roll pattern is destabilized



A second four-wave mixing process creates two additional photons (2 and 6) from 0 and 1, and the pair of 3 and 5 from 0 and 4, which gives a **hexagonal structure.**

D. Gomila and P. Colet, Phys. Rev. E **76,** 16217 (2007) analyzed the hexagonal patterns which arise in the model.



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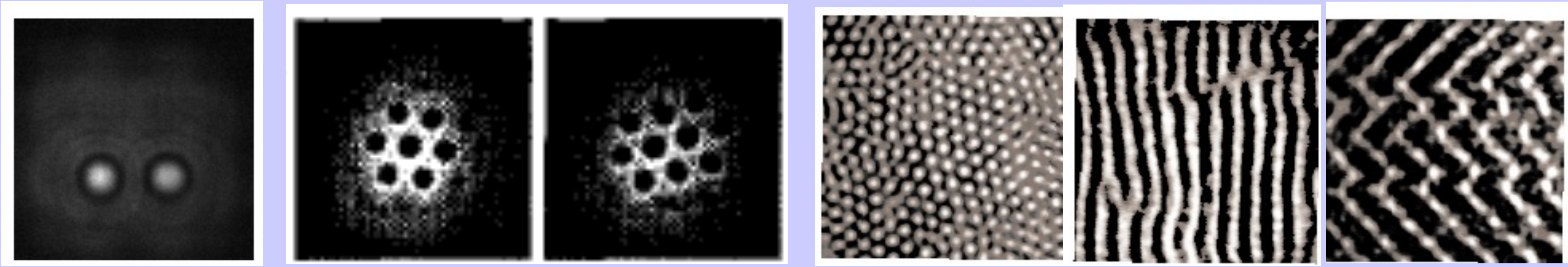
Global and *localized structures in optical systems*

Highly correlated configurations (**global structures**) or independent, isolated intensity peaks (**localized structures**) appear in the transverse field profile.

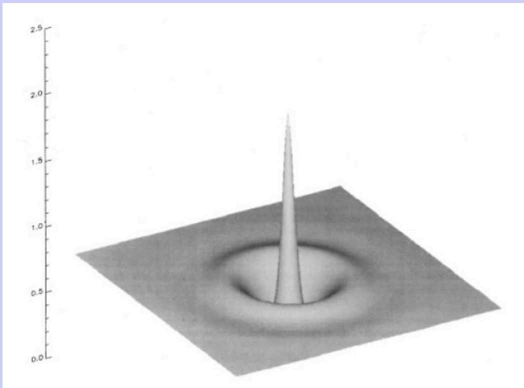
Experiments (Muenster university)



(a) (b) (c)

(a) Localized structures (M. Tlidi, P. Mandel and R. Lefever, Phys. Rev. Lett. **73,** 64 (1994))

(b) global structures in sodium vapour; (c) global structures in LCV.



Kerr cavity soliton

Scroggie, Firth, McDonald, Tlidi, Lefever, Lugiato, Chaos, Solitons and Fractals 4, 1323 (1994)

Firth, Harkness, Lord, McSloy, Gomila, Colet, J. Opt. Soc. Am. B 19,747 (2002)

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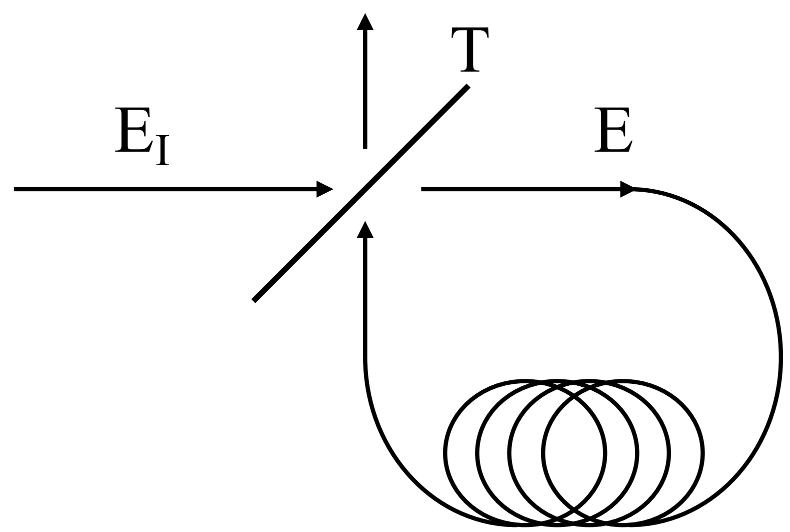
THE TEMPORAL/LONGITUDINAL VERSION OF THE LLE

(M. Haelterman, S. Trillo, S. Wabnitz, Opt. Commun. 91, 401 (1992))
They were inspired by the analogy between two kinds of Hamiltonian solitons:

- Temporal solitons, described by a nonlinear Schroedinger equation with frequency (chromatic) dispersion
- Spatial solitons, described by a nonlinear Schroedinger equation with diffraction (transverse Laplacian)

and extended this analogy to the dissipative case of an optical cavity

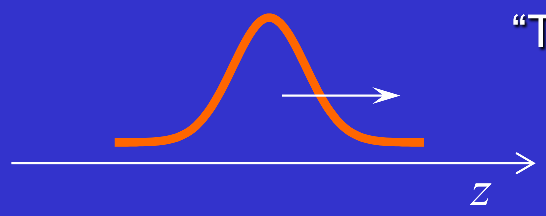
They considered a nonlinear fiber loop with an input/output mirror. In the practical realization the mirror is replaced by input and output fiber couplers



SOLITONS IN PROPAGATION PROBLEMS

Solitons are localized waves that propagate (in nonlinear media) without change of form

Temporal Solitons: no dispersion broadening



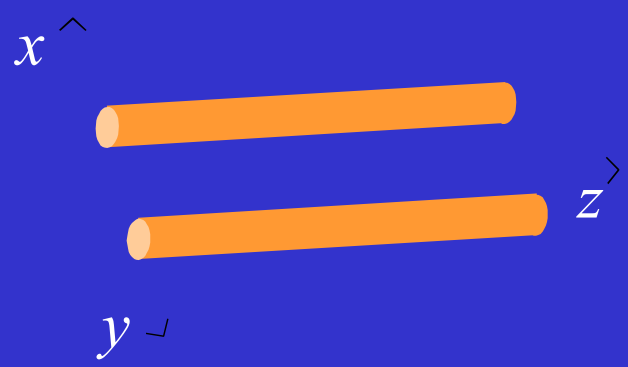
"Temporal" NLSE:

$$i\frac{\partial u}{\partial z} + |u|^2 u + \frac{\partial^2 u}{\partial \tau^2} = 0$$

propagation

dispersion

Spatial Solitons: no diffraction broadening



"Spatial" NLSE:

$$i\frac{\partial u}{\partial z} + |u|^2 u + \frac{\partial^2 u}{\partial x^2} = 0 \qquad \boxed{\text{1D}}$$

diffraction

$$i\frac{\partial u}{\partial z} + |u|^2 u + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \boxed{\text{2D}}$$

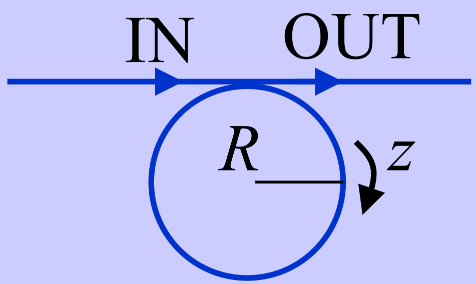
HTW arrived at the equation

$$(*) \qquad \frac{\partial E}{\partial \bar{t}} = F - E - i\alpha E + i|E|^2 E + i\frac{\partial^2 E}{\partial \bar{z}^2}$$

where

$\bar{t} = \kappa t$

$\bar{z}$ normalized longitudinal variable z (normalization includes dispersion)



CIRCULAR CAVITY

Once Eq. (*) has been solved with periodic boundary condition in z, in the solution $E(\bar{z}, \bar{t})$, z must be replaced by $z - v_g t$, which means that <u>the spatial pattern rotates along the cavity</u> with the group velocity v_g.

Eq. (*) is equivalent to the LLE in 1D. Instead of the transverse variables x and y, there is the longitudinal variable z.

TWO REMARKS

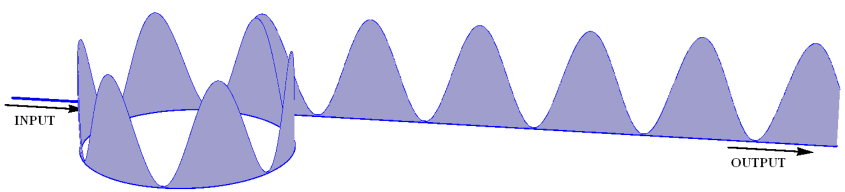
1) A model, equivalent to the temporal/longitudinal version of the LLE introduced by HWT, was derived from the Maxwell-Bloch equations in

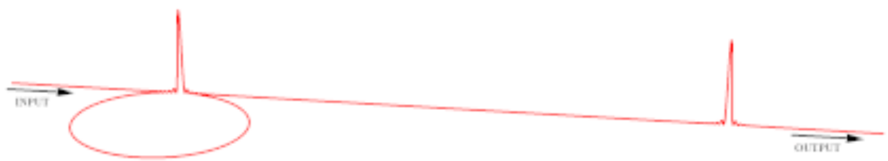
BRAMBILLA, CASTELLI, GATTI, LUGIATO, PRATI in SUSSP Proceedings 41, 115 (1993)

The complete derivation is found in SAME AUTHORS, Eur. Phys. J. D 71, 84 (2017)

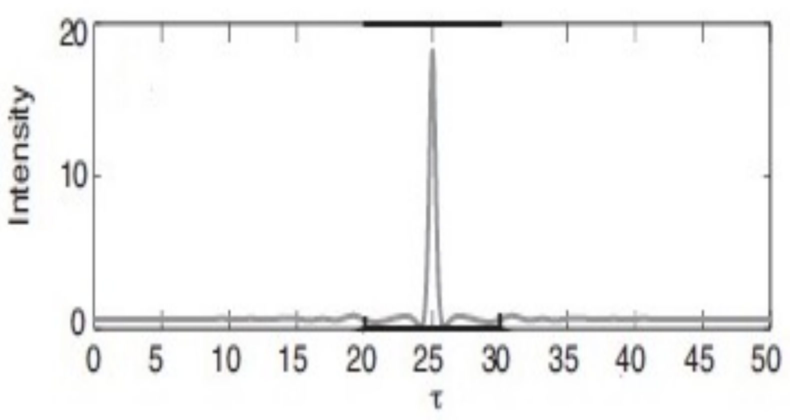
2) The pattern forming instability of the temporal/longitudinal LLE is a special case of the MULTIMODE INSTABILITY OF OPTICAL BISTABILITY

(BONIFACIO, LUGIATO, Lett. Nuovo Cimento 21, 510 (1978)
BONIFACIO, GRONCHI, LUGIATO, Opt. Commun. 30, 129 (1979))





TEMPORAL/LONGITUDINAL KERR CAVITY SOLITONS



- A temporal cavity soliton is a narrow pulse that circulates indefinitely along the fiber cavity without deformations, apart from fluctuations, with a period equal to the cavity roundtrip time

- It sits over the pedestal of a stable uniform stationary solution
- It is excited by injecting into the cavity an address pulse that adds to the driving field

First experimental observation
Leo, Coen, Kockaert, Gorza, Emplit, and Haelterman, Nature Photonics 4, 471 (2010)

380 m silica optical fiber
CW driving field, 1551 nm
cavity roundtrip time 1.85 μs
cavity soliton width ~ 4 ps
anomalous dispersion

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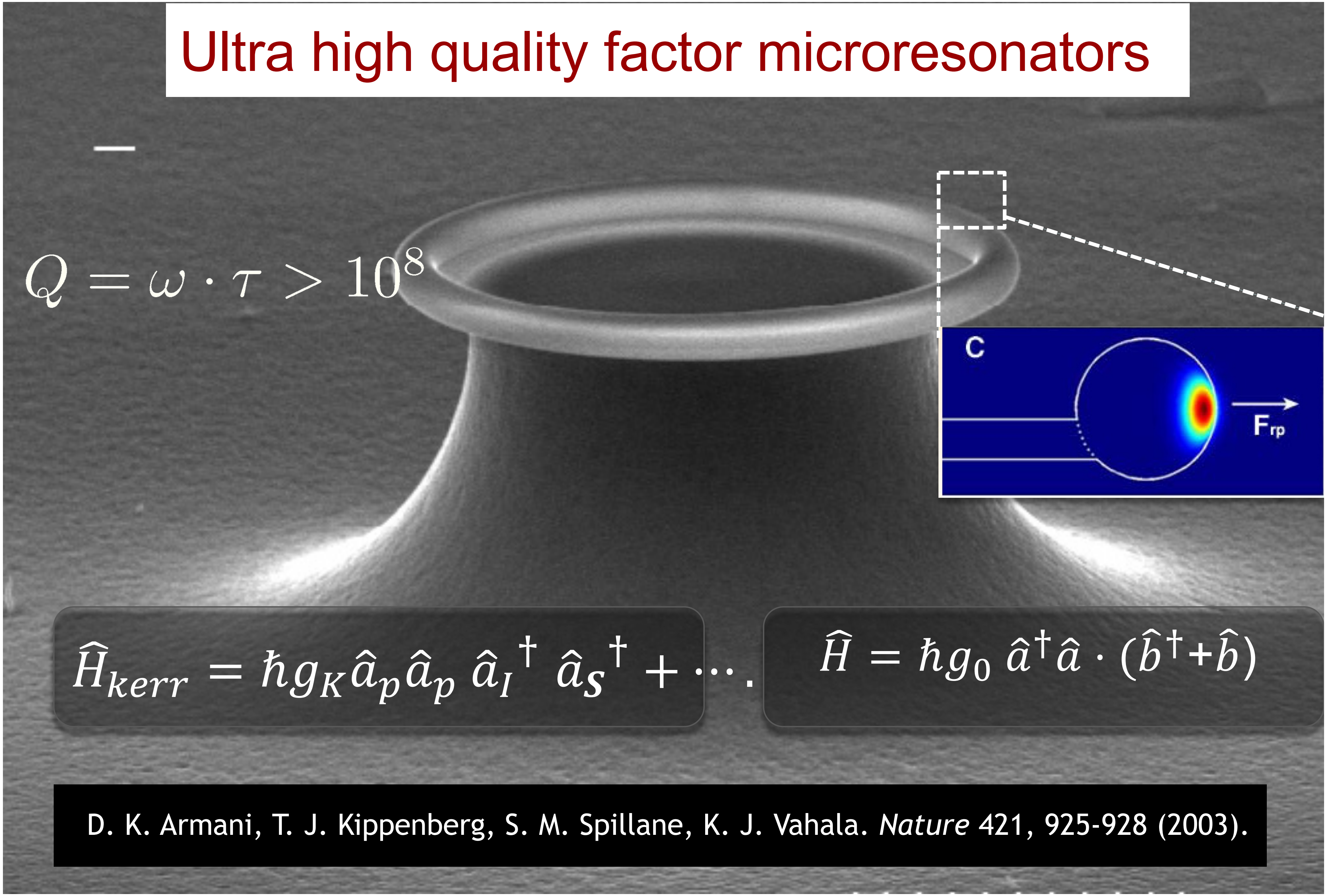
In the spectral domain the periodical train of pulses which arises from a Turing pattern corresponds to a comb of frequencies

BROADBAND KERR FREQUENCY COMBS

Cornerstone experiment:
Del'Haye, Schliesser, Arcizet, Wilken, Holzwarth, Kippenberg,
Nature 450, 1214 (2007) in a high-Q microresonator with a Kerr medium

THE HIGH-Q LIMIT BECOMES REALITY

Ultra high quality factor microresonators



$$Q = \omega \cdot \tau > 10^8$$

$$\hat{H}_{kerr} = \hbar g_K \hat{a}_p \hat{a}_p \, \hat{a}_I{}^{\dagger} \, \hat{a}_S{}^{\dagger} + \cdots .$$

$$\hat{H} = \hbar g_0 \, \hat{a}^{\dagger} \hat{a} \cdot (\hat{b}^{\dagger} + \hat{b})$$

D. K. Armani, T. J. Kippenberg, S. M. Spillane, K. J. Vahala. *Nature* 421, 925-928 (2003).

Discovery of microresonator frequency combs

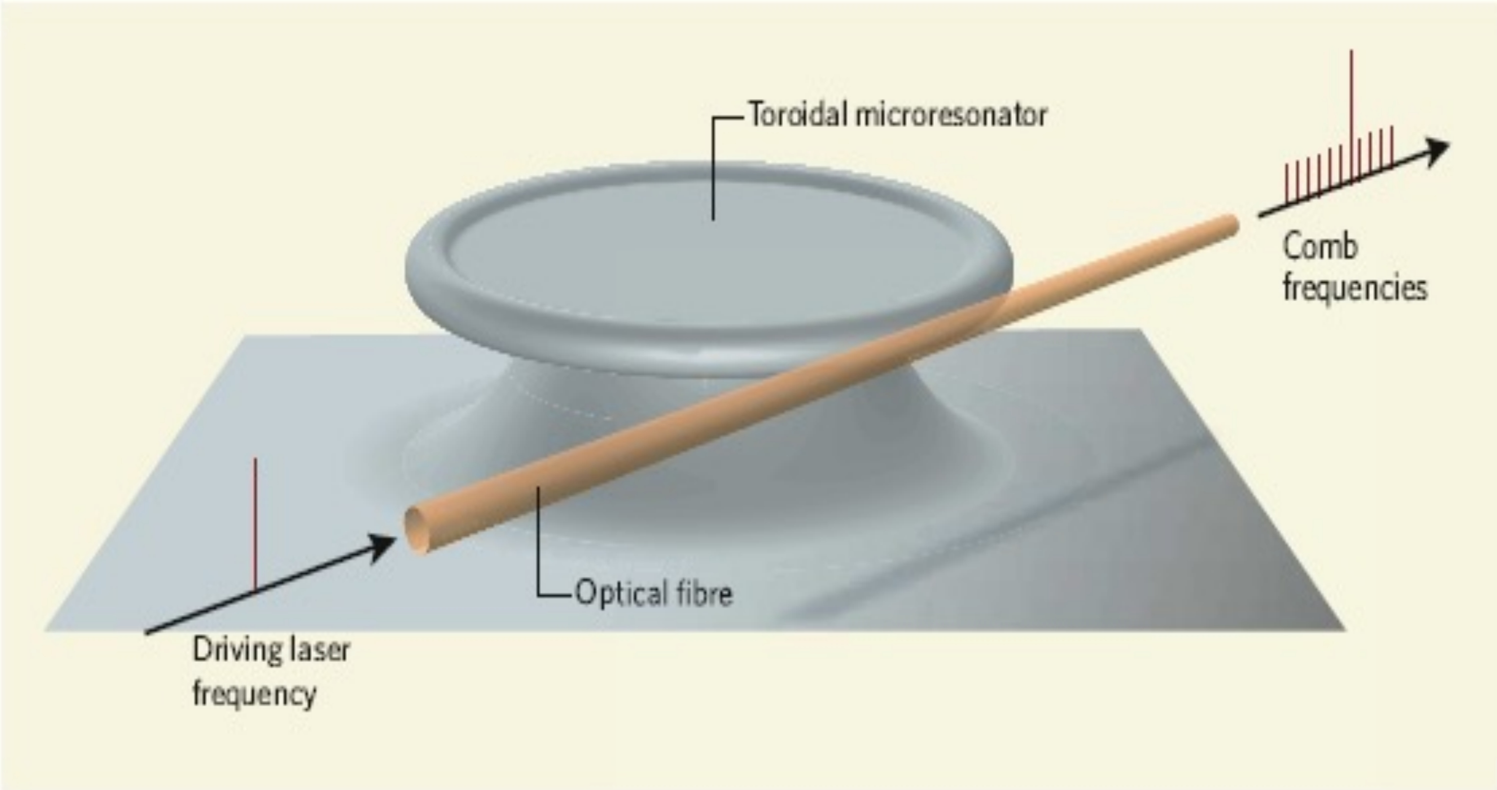
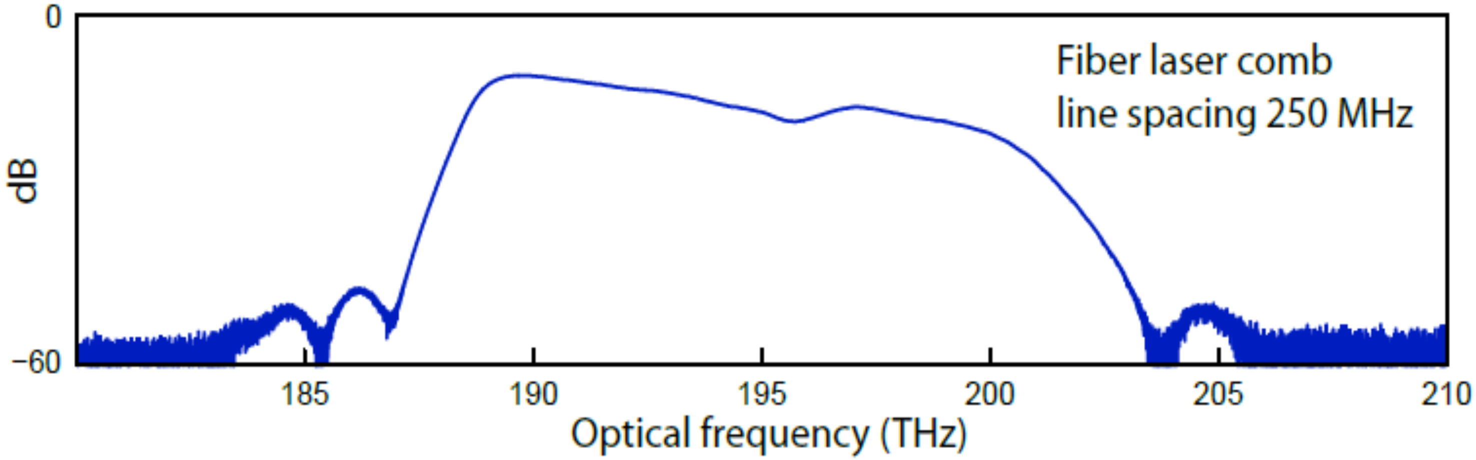


Image credit: S. Cundiff News&Views, Nature, Dec. 20, 2007

Del Haye,Schliesser,Wilkins, Holzwarth,Kippenberg, *Nature*, 2007
EU & US Patent application “Optical Comb Generator using Microresonators”
TJ Kippenberg, et al., *Science* 2011

Microresonator based frequency combs

a



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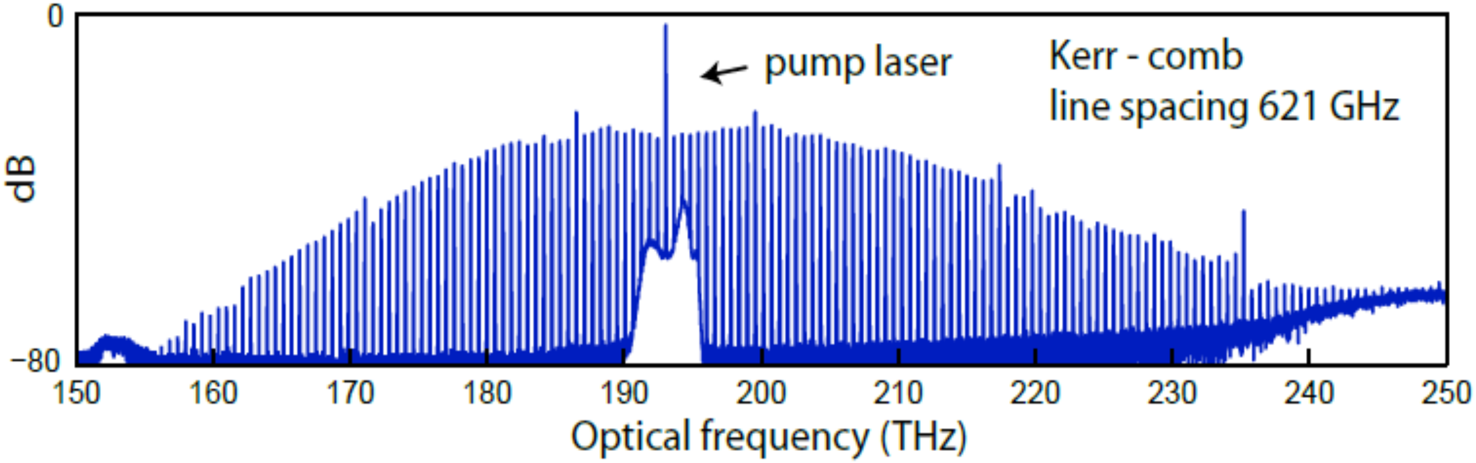

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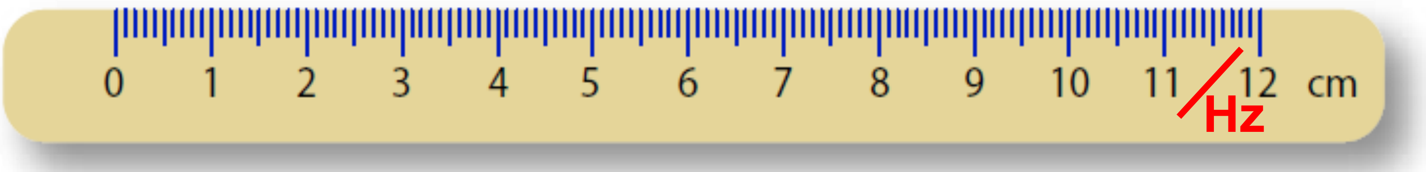
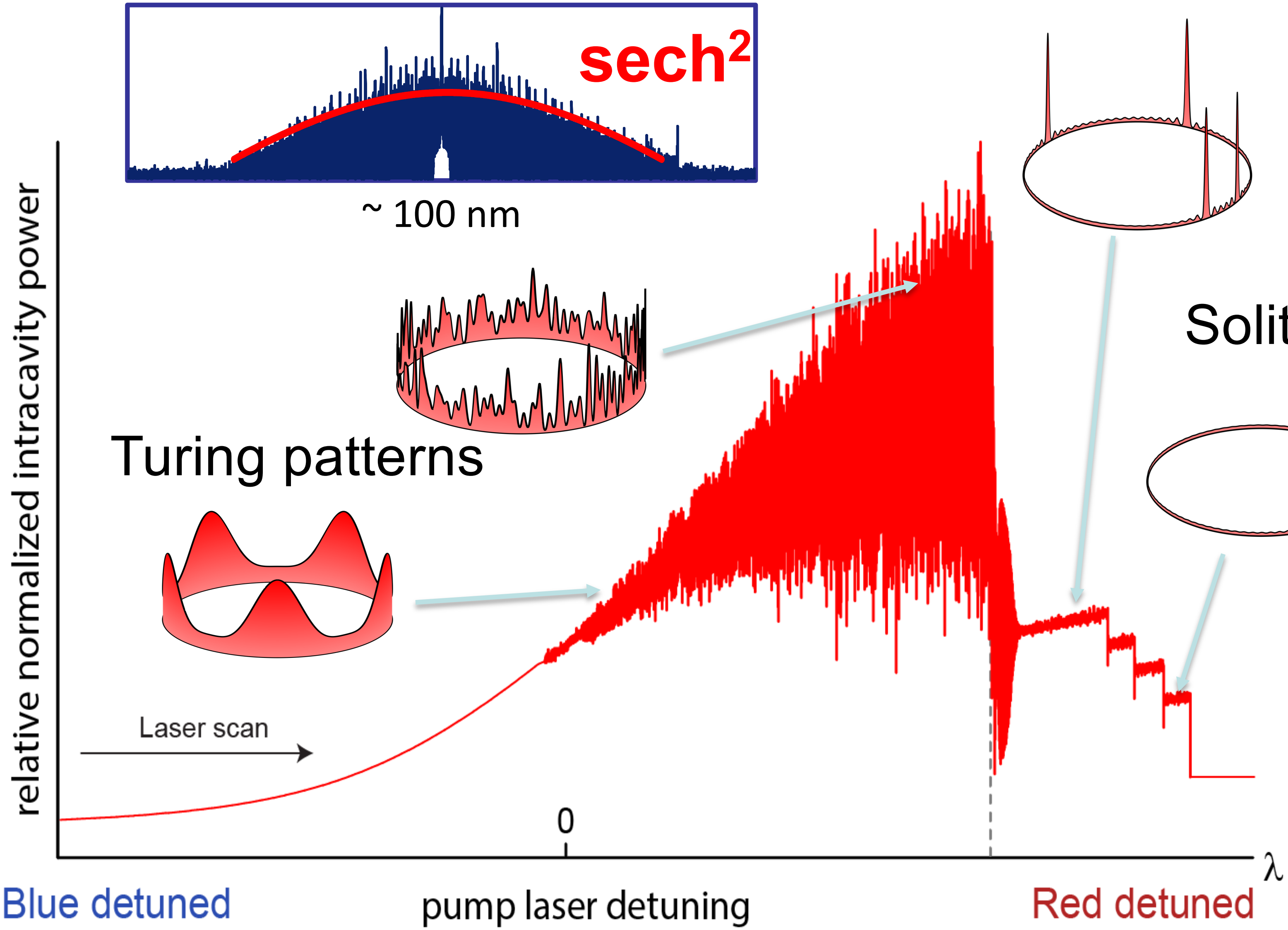


Dissipative Kerr solitons



Adapted from: T. Herr et al., Nat. Photonics, 2013.

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CONNECTION BETWEEN THE LLE AND KERR FREQUENCY COMBS

Kerr frequency combs were discovered without being aware of the LLE.

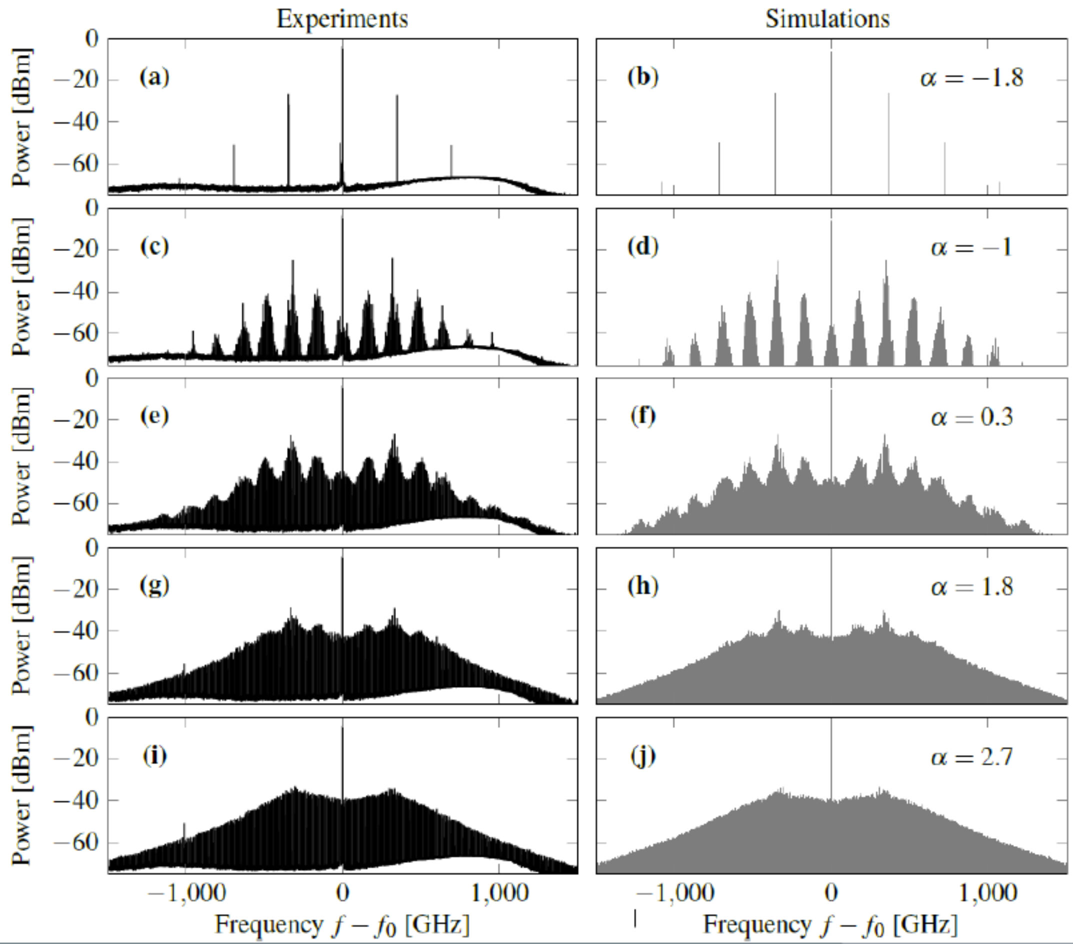
The connection was pointed out later

- MATSKO, SAVCHENKOV, LIANG, ILCHENKO, SEIDEL, MALEKI, Opt. Lett. <u>36</u>, 2845 (2011)
- CHEMBO, MENYUK, Phys. Rev. A <u>87</u>, 053852 (2013)
- HERR, BRASCH, JOST, WANG, KONDRATIEV, GORODETSKY, KIPPENBERG, Nat. Photon. 2013, 343
- COEN, RANDLE, SYLVESTRE, ERKINTALO, Opt. Lett. 38, 37 (2013)

Courtesy Y.K. Chembo

Lugiato-Lefever equation for Kerr combs

Excellent agreement between experiments and simulations



Agreement over 80 dB of dynamical range, and >300 modes

femto-st SCIENCES & TECHNOLOGIES

APPLICATIONS

Investigations on Kerr fequency combs have been applied to numerous areas, including

- coherent telecommunications
- spectroscopy
- atomic clocks
- laser ranging
- astrophysical spectrometer calibration

- See e.g. these reviews

  - Kippenberg, Holtzwarth, Diddams, Microresonator-based optical frequency combs, Science 332, 555 (2011)

  - Chembo, Kerr optical frequancy combs: theory, applications and perspectives, Nanophotonics 5, 214 (2016)

- The LLE has been generalized to the case of Fabry-Perot cavity in Cole, Gatti, Papp, Prati, Lugiato, Phys, Rev. A 98, 013831 (2018)

Menu

- How the model was derived to provide a paradigm for optical pattern formation
- Connection with optical bistability
- Turing patterns.
- Kerr cavity solitons in the spatial domain
- The temporal/longitudinal version of the LLE
  Temporal / longitudinal Kerr cavity solitons in fiber cavities
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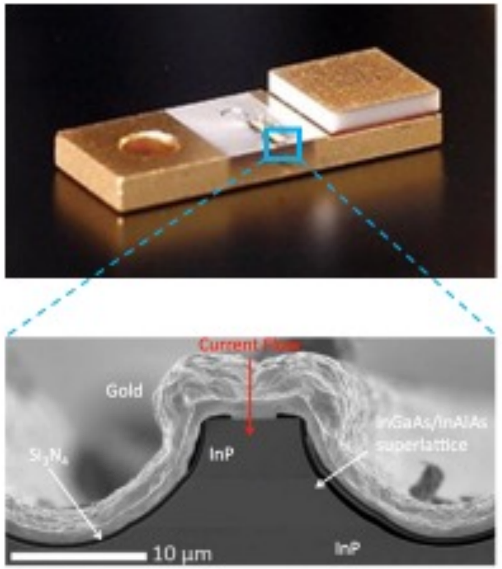
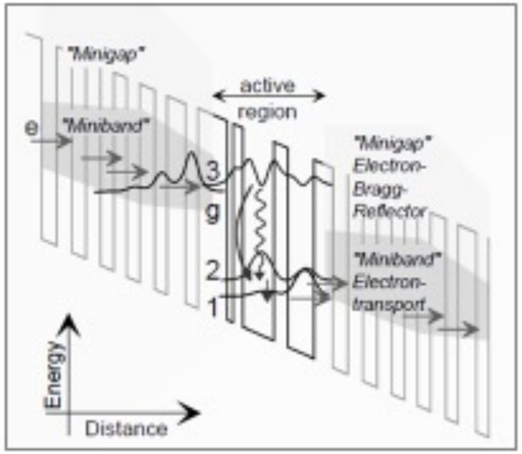
- **Frequency combs in quantum cascade lasers**
- Quantum aspects of optical patterns
- Conclusions

Quantum cascade laser





- **Intersubband**: wavelength determined by layer thickness, not by the bandgap of the material!
- QCLs can be designed to **emit from 3 to 300 µm**

J. Faist, F. Capasso, D. L. Sivco, C. Sirtori, A. L. Hutchinson, A. Y. Cho, Science **264**, 553 (1994)

In their paper **“Frequency combs induced by phase turbulence”, Nature 582, 560 (2020)**, Piccardo, Capasso et al. derive a complex Ginzburg-Landau equation (CGLE) for the ring quantum cascade laser near threshold

$$\frac{\partial E}{\partial \bar{t}} = E - (1 - ic_1)E|E| + (1 + ic_2)\frac{\partial^2 E}{\partial \bar{z}^2}$$

This equation has basically the same form of an equation formulated by

Lugiato, Oldano, Narducci, J. Opt. Soc. Am. B **5**, 879 (1988).

$$(*) \qquad \frac{\partial E}{\partial t} = -E(1 - i\Delta)(|E|^2 - r) + ia\frac{\partial^2 E}{\partial x^2}$$

as the laser counterpart of the LLE, formulated for optical bistability.

As in the original LLE, *x* is a transverse variable. Therefore equation (*) plays, with respect to the quantum cascade laser, a role analogous to that of the LLE in fiber cavities and in microcavities.

Therefore the LLE works for both microresonators without population inversion and for ring quantum cascade lasers near threshold.

PHYSICAL REVIEW LETTERS **126,** 173903 (2021)

Unifying Frequency Combs in Active and Passive Cavities: Temporal Solitons in Externally Driven Ring Lasers

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[5]*Dipartimento di Scienza e Alta Tecnologia, Università dell'Insubria, 22100 Como, Italy*
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(Received 8 October 2020; accepted 29 March 2021; published 30 April 2021)

Frequency combs have become a prominent research area in optics. Of particular interest as integrated comb technology are chip-scale sources, such as semiconductor lasers and microresonators, which consist of resonators embedding a nonlinear medium either with or without population inversion. Such active and passive cavities were so far treated distinctly. Here we propose a formal unification by introducing a general equation that describes both types of cavities. The equation also captures the physics of a hybrid device—a semiconductor ring laser with an external optical drive—in which we show the existence of temporal solitons, previously identified only in microresonators, thanks to symmetry breaking and self-localization phenomena typical of spatially extended dissipative systems.

DOI: 10.1103/PhysRevLett.126.173903

Formulation of a GENERALIZED LLE that describes frequency combs in both passive systems and quantum cascade lasers near threshold

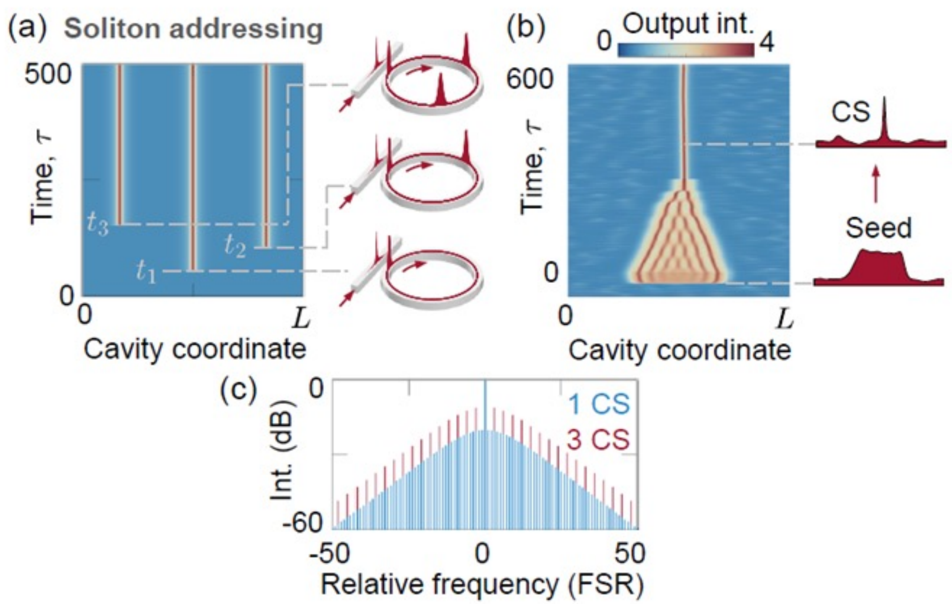

FIG. S2. **Spectral shaping by external writing of temporal solitons.** (a) Spatio-temporal plot showing the evolution of the intracavity pattern in a ring QCL as multiple cavity solitons (CSs) are sequentially excited by the injection of pulses at times t_1, t_2 and t_3. (b) Excitation of a single CS with a wide pulse seed, approximately 20 times wider than the CS. (c) Frequency comb spectra taken at the times t_1 and t_3 in (a), corresponding to the excitation of a single CS and three equidistant CSs, respectively.

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A QUANTUM EFFECT

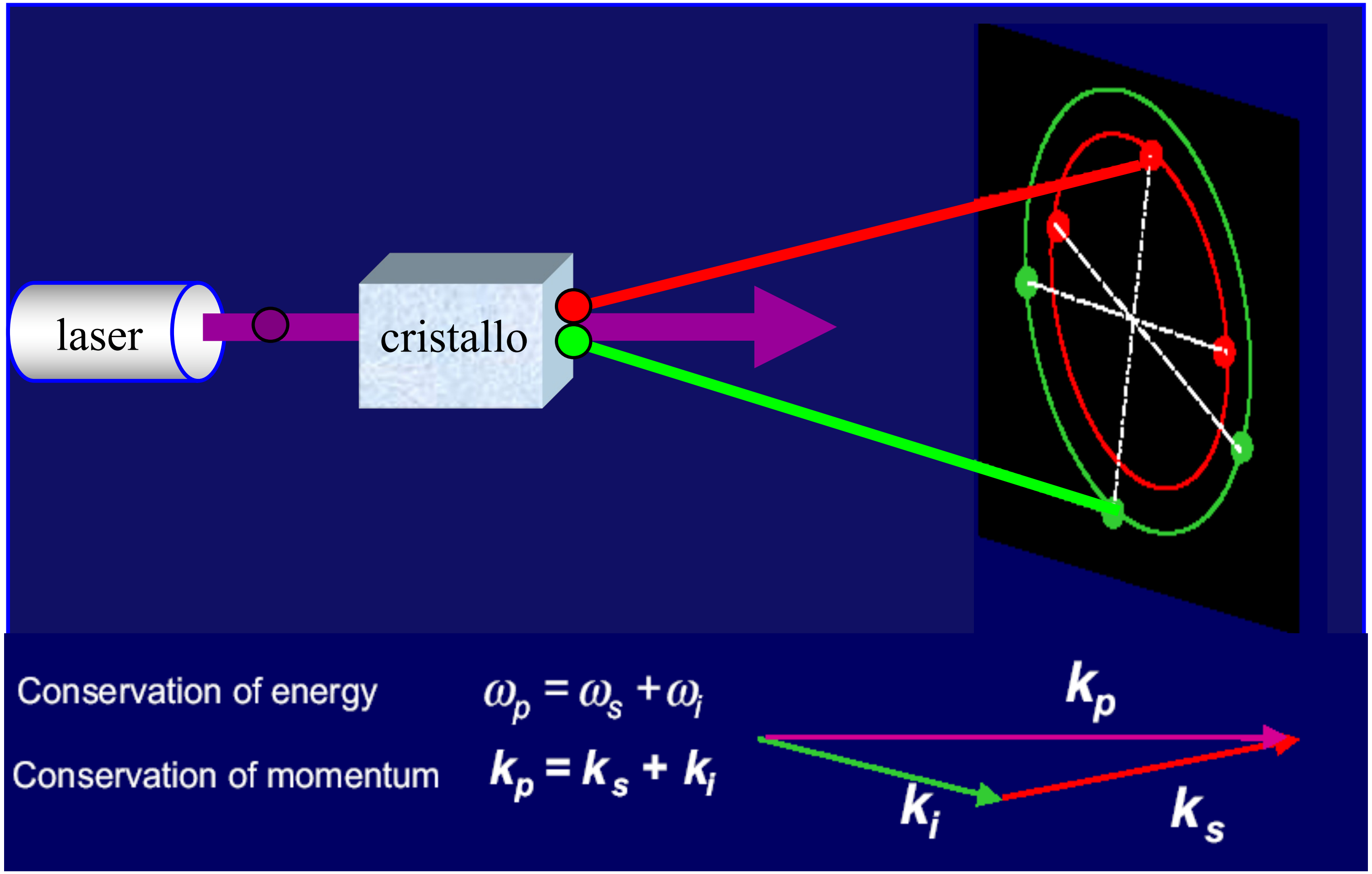
The intensity difference between the two symmetrically emitted beams in four-wave mixing exhibits fluctuations that lie below the shot-noise level, which is a quantum effect (**Castelli Lugiato, PRL 68,328 (1992)**).

This effect has been experimentally confirmed (**Boyer, Marino and Lett, Phys.Rev.Lett.100, 143601 (2008)**) .

See also a paper in which they measured the intensity difference between two symmetrical modes in a Kerr frequency comb (**Dutt, Luke, Manipatruni, Gaeta and Lipson, Phys.Rev.Appl. 3, 044005 (2015)**) - These authors called this effect "on-chip squeezing".

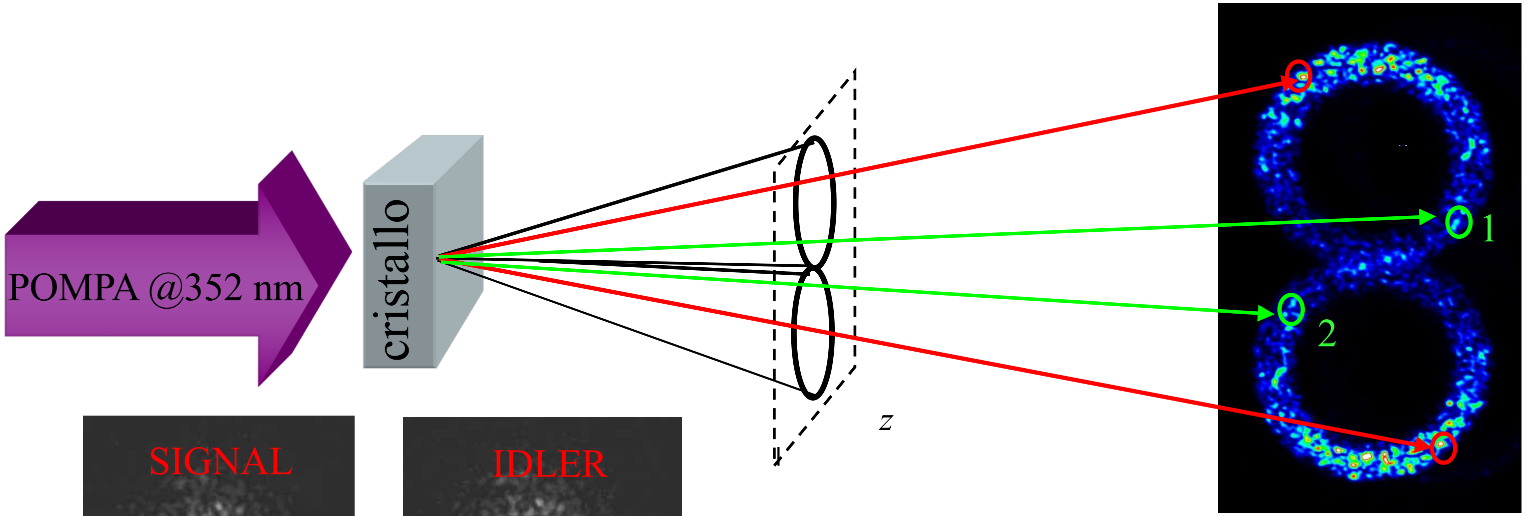
II) Quantum correlated states of light

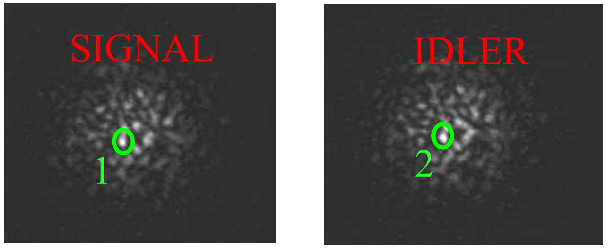
Example: twin photons generated in a crystal with nonlinear optical properties



The two- emitted photons are quantum correlated: by measuring energy and momentum of the signal photon one can infer energy and momentum of the idler photon even without measuring them.

Position-momentum entanglement of twin photons





Position correlation: position x of photon 1 determined from a measurement of the position of the photon 2

Momentum correlation : direction of propagation of photon 1 determined from a measurement of the direction of propagation of photon 2

Simultaneous presence of correlation in both position and momentum of the two photons→ Entangled (nonseparable) state, similar to the original EPR (Einstein-Podolsky Rosen , 1935) state.

Exp. test: Howell, Bennink, Bentley and Boyd, PRL 92 210403, 2004;
Theory: Brambilla, Gatti, Bache, Lugiato Phys .Rev.A 69, 023802 (2004)

QUANTUM IMAGING

This field exploits the quantum nature of light and the natural parallelism of optical signals to devise novel technique for optical imaging and for parallel information processing at the quantum level.

Gatti, Brambilla, Lugiato, Quantum Imaging, Progress in Optics, Vol. 51, p.251, 2008

CONCLUSIONS

- INTRINSIC CONNECTION BETWEEN THE LLE AND KERR FREQUENCY COMBS /
  LEO, COHEN, KOCKAERT, GORZA, EMPLIT, HAELTERMAN,
  Nature Photonics 4, 471 (2010)
  LUGIATO, PRATI, GORODETSKY, KIPPENBERG,
  Phil. Trans. Roy. Soc. A 376, 20180113 (2018)

- THIS EXAMPLE SHOWS THAT SOMETIMES THE EVOLUTION IN SCIENCE IS NOT LINEAR BUT COMPLEX. SCIENCE IS ESSENTIALLY A COLLECTIVE PROCESS.

- THE CRITERION OF SIMPLICITY, FOLLOWED IN THE DERIVATION OF THE LLE, TURNED OUT TO BE VISIONARY. THIS FACT IS QUITE INTERESTING.

- KERR FREQUENCY COMBS IN QUANTUM CASCADE LASER AND THE LASER COUNTERPART OF THE LLE.

- UNIFIED TREATMENT OF KFC IN PASSIVE AND ACTIVE SYSTEMS: GENERALIZED LLE

- NOTEWORTHY EPR ASPECTS EMERGE IN OPTICAL PATTERN FORMATION

Reprinted from Journal of the Optical Society of America B, Vol. 5, page 879, May 1988

Cooperative frequency locking and stationary spatial structures in lasers

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Received October 21, 1987; accepted January 14, 1988

We investigate the spontaneous emergence of transverse patterns in lasers by using both the standard two-level model and the so-called cubic approximation, which is generally valid in the threshold regions. The stationary intensity configurations fall into two distinct classes. The first includes solutions of the single-mode type with the frequency and spatial structure of one of the transverse resonances. The solutions of the second group involve the simultaneous oscillation of several cavity modes, operating in such a way as to produce a stationary intensity profile. The stationary character of these multimode configurations emerges from the fact that the transverse modes of the resonator lock onto a common frequency during the nonlinear transient. We call this phenomenon cooperative frequency locking.

1. INTRODUCTION

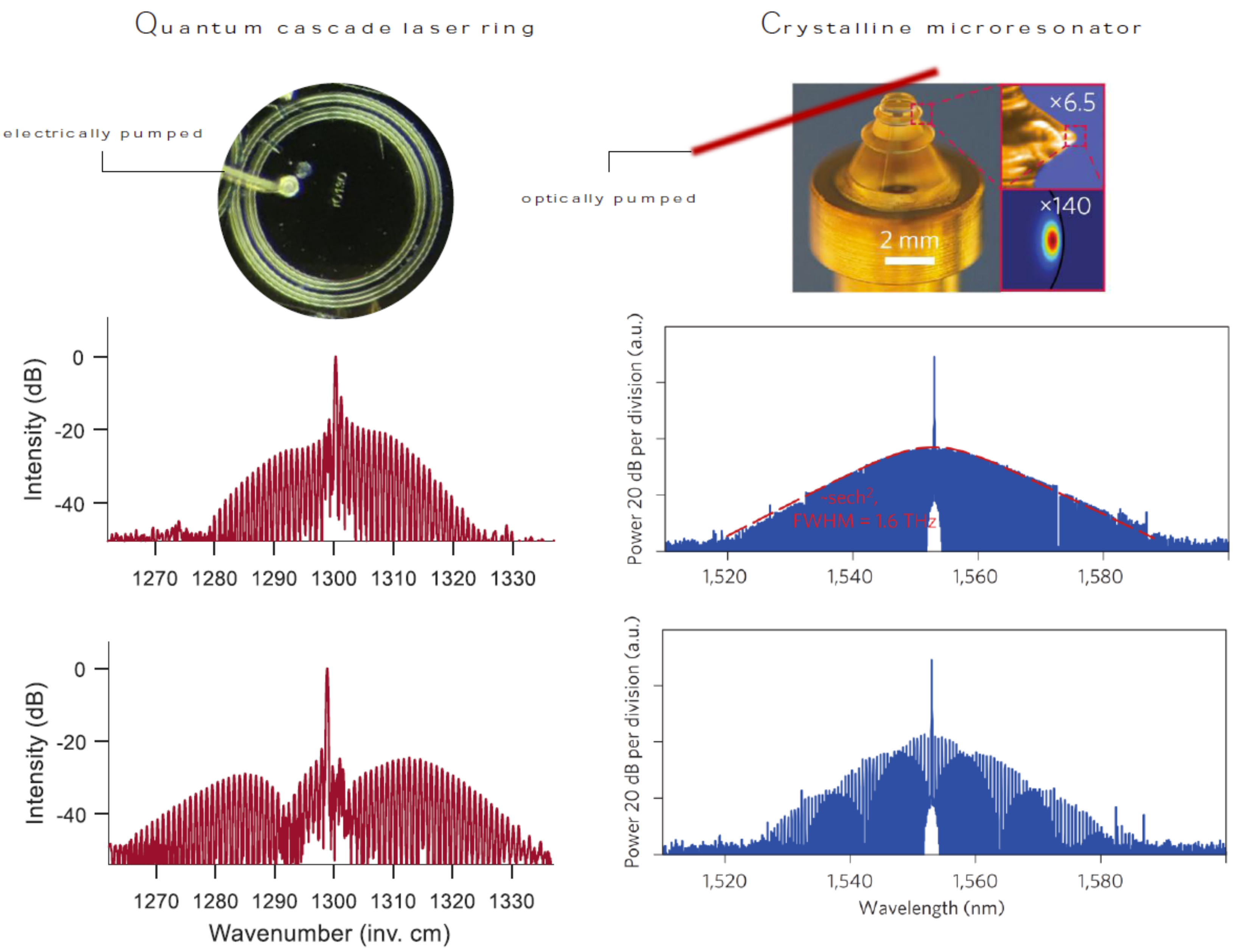
The spontaneous formation of stationary spatial structures in homogeneous systems has been the object of extensive investigations in such fields as nonlinear chemical reactions and developmental biology.[1–3] Here the instabilities that are responsible for the emergence of spatial patterns arise from a diffusive mechanism and are usually referred to as Turing instabilities.[4]

Optical systems are much more widely known for their propensity to produce temporal structures in the form of spontaneous oscillations of the regular or chaotic type.[5,6] Only recently has a Turing instability been discovered[7–10] in an optical model. Here, the resulting stationary pattern is produced by the interplay between diffraction and nonlinear coupling and not by a diffusion process. The optical arrangement found to produce these interesting new effects can be described as follows. A passive medium is contained in an optical ring or Fabry–Perot cavity fitted with an additional pair of lateral mirrors that act as a waveguide for the radiation field. With an appropriate selection of the state of polarization of the incident field, injected along the z direction, the electric field in the resonator acquires a uniform nal mode and the nearest transverse resonances be of the order of the cavity linewidth. This situation creates a competition between transverse and longitudinal modes. The end result is the loss of stability of the spatially homogeneous stationary solution. At the same time the input field imposes its oscillation frequency on the competing modes, so that the eventual stationary state displays no temporal intensity modulation.

Our aim in this paper is twofold. First, we extend the description of the spatial pattern formation to the case of an active system, such as a homogeneously broadened laser with detuning between the atomic transition frequency and the longitudinal cavity modes. Second, we show that in the case of the laser the occurrence of the spatial patterns is accompanied by a new phenomenon, which we propose to call cooperative frequency locking.

The novelty here resides in the fact that a typical laser system operates either in a single or in a multimode configuration and that in the multimode case the output intensity undergoes oscillations caused by the interference among the competing modes. The stationary spatial structure described in this paper corresponds to a different type of multimode operation in which the coexisting modes select coop-

Courtesy F. Capasso and M. Piccardo



Harvard and TU Wien, unpublished

Kippenberg group, Nat. Photonics 8, 145 (2014)

Conference "Microresonator frequency combs, Fundamentals and applications" 6-8 April 2020, Bath, UK

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