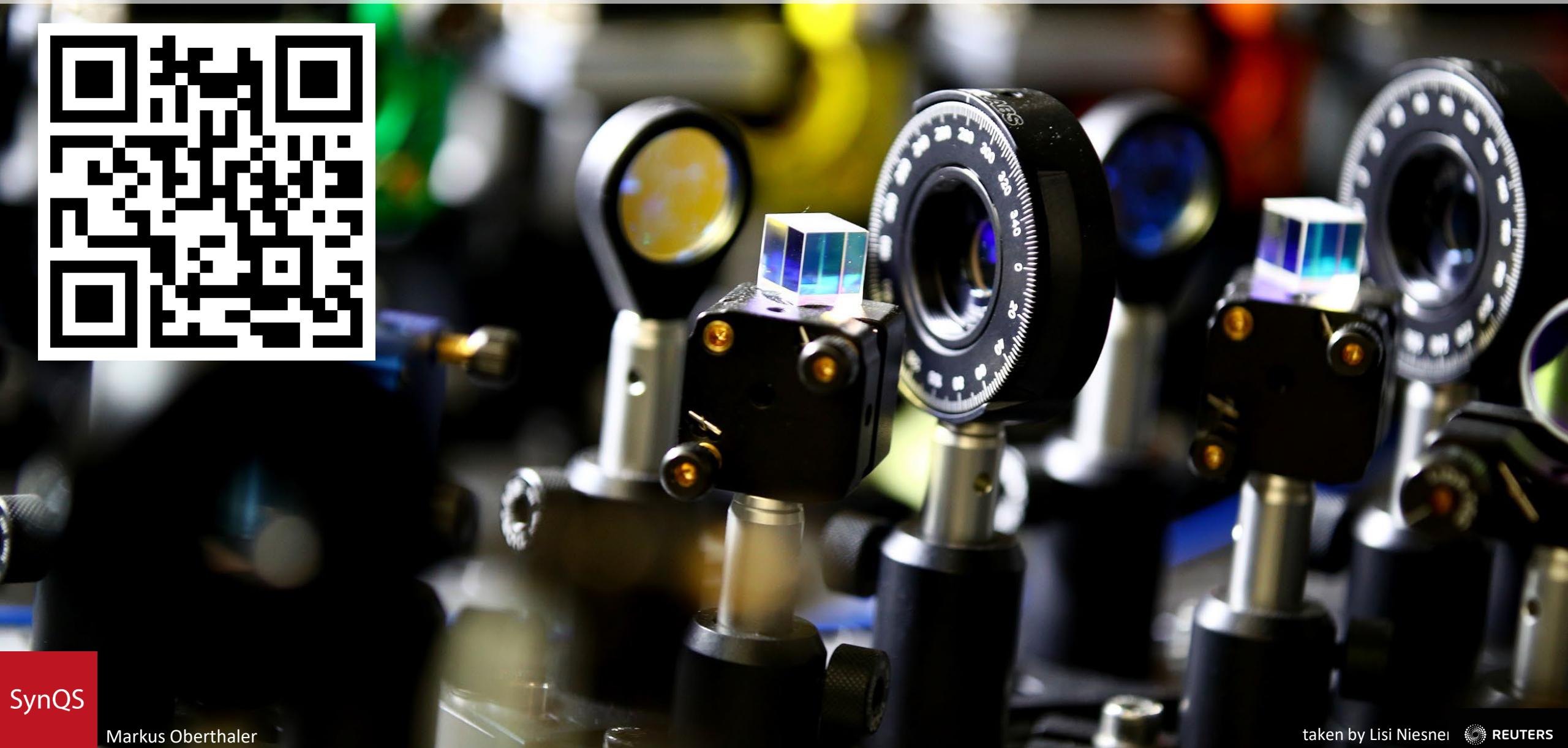


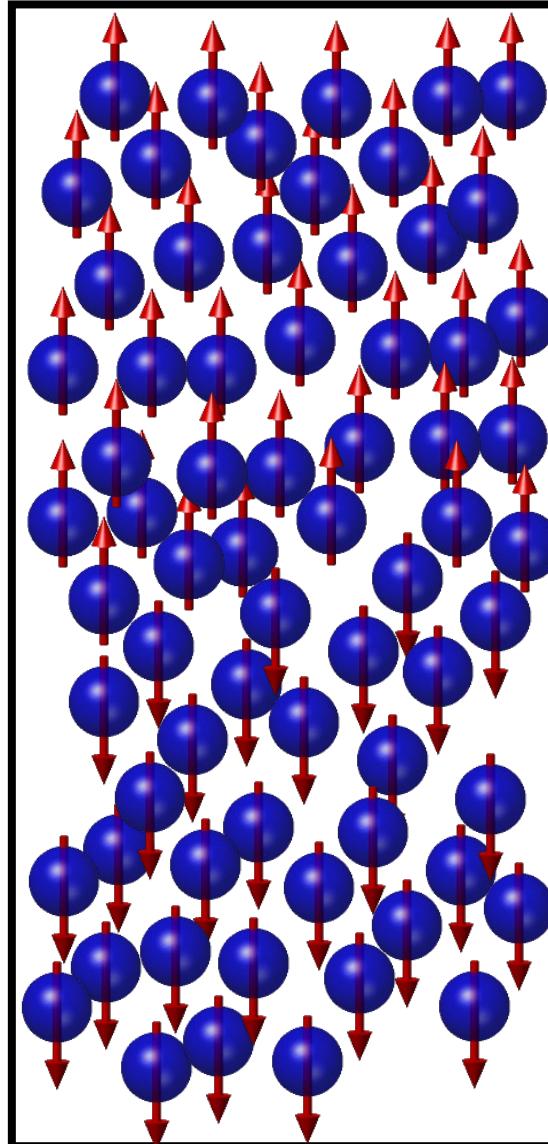
(Quantum)Mixtures @ Synthetic Quantum Systems/Heidelberg University



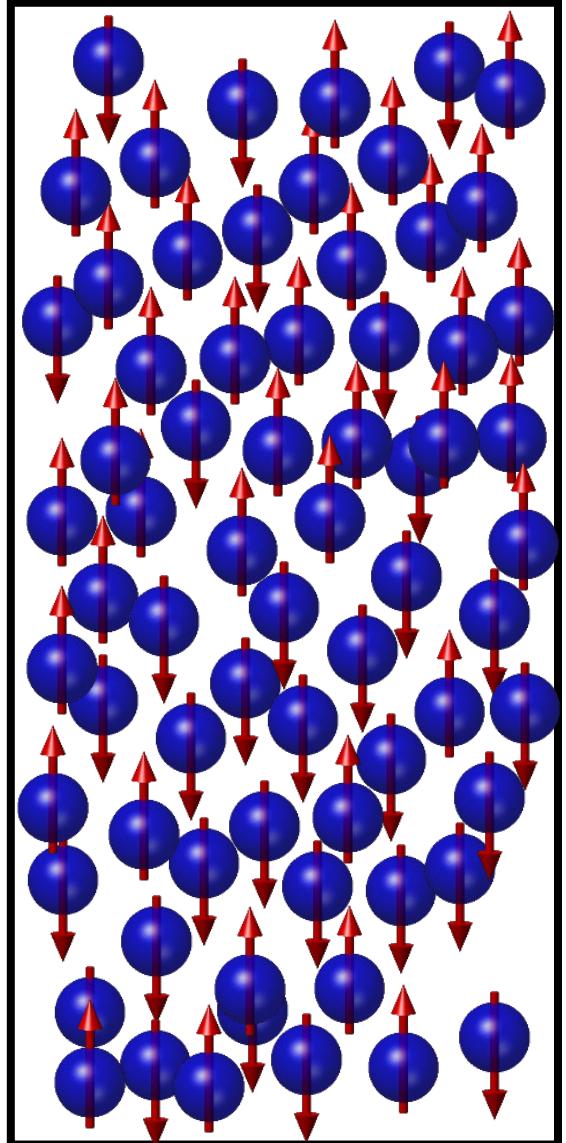
Mixtures

$$\Phi^{\dagger} \begin{matrix} \uparrow \\ \downarrow \end{matrix}$$

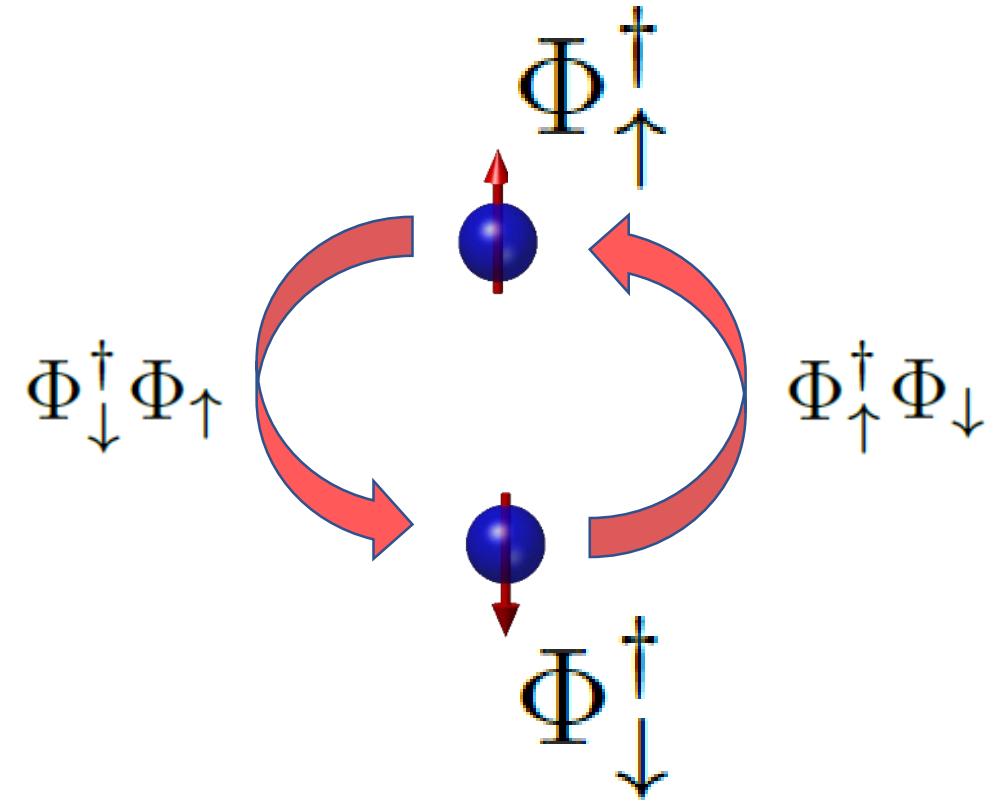
phase separation - immiscible



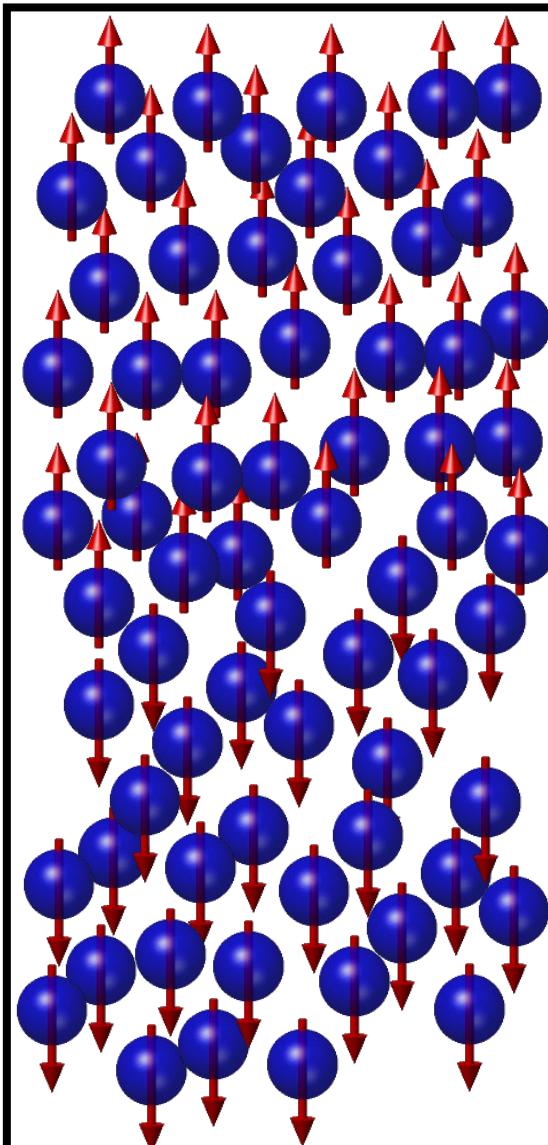
coexistence - miscible



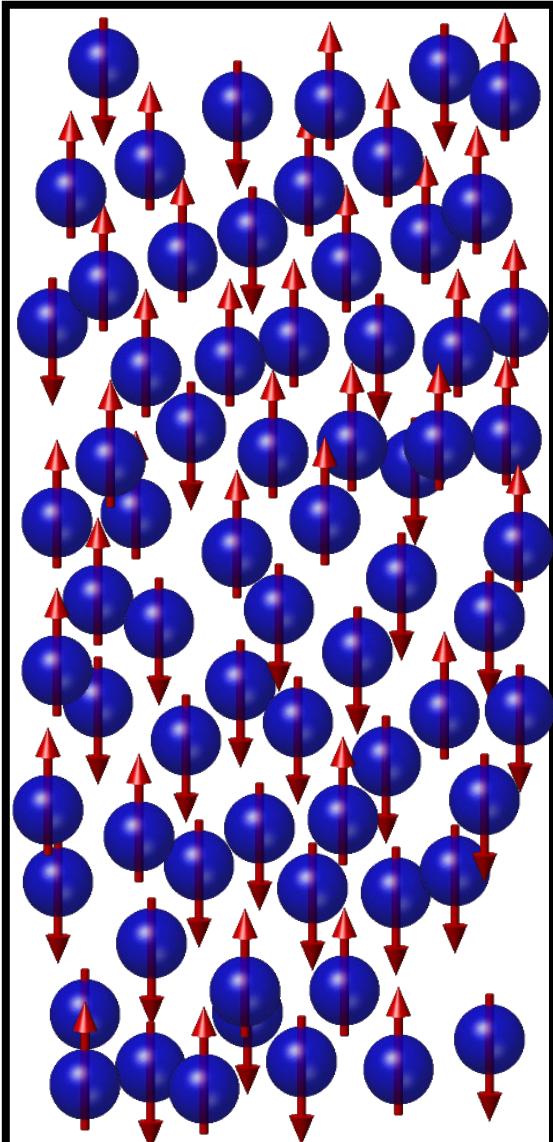
Coherent Mixtures



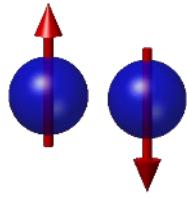
phase separation - immiscible



coexistence - miscible



Coherent mixtures with increasing complexity

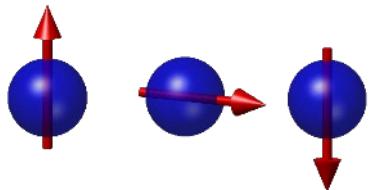


pseudo spin $\frac{1}{2}$

one dimensional

miscible to immiscible transition via linear coupling

,critical' scaling after a quench towards a quantum critical point



spin 1

non-local entanglement

EPR correlation from 0d to 1d

one dimensional

POVM readout

universal dynamics

topological excitations such as solitons

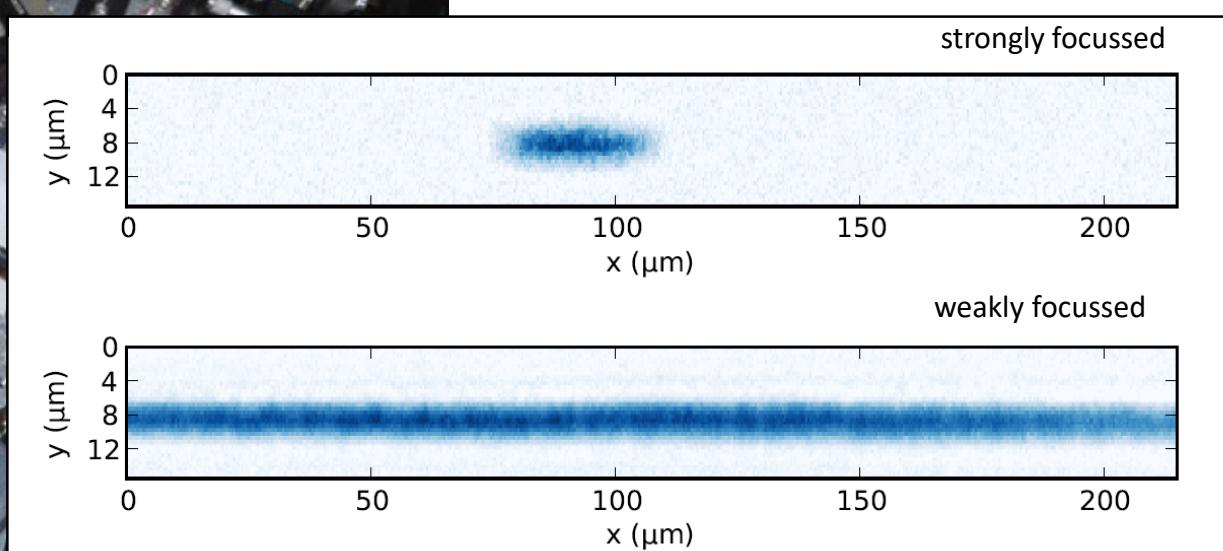
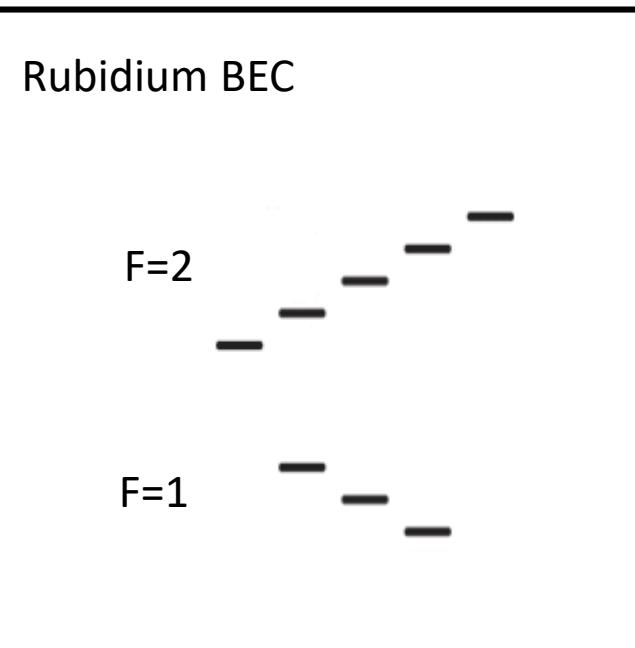
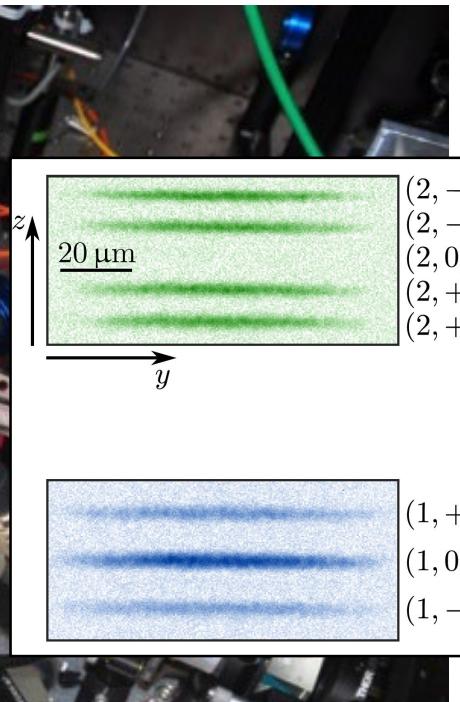
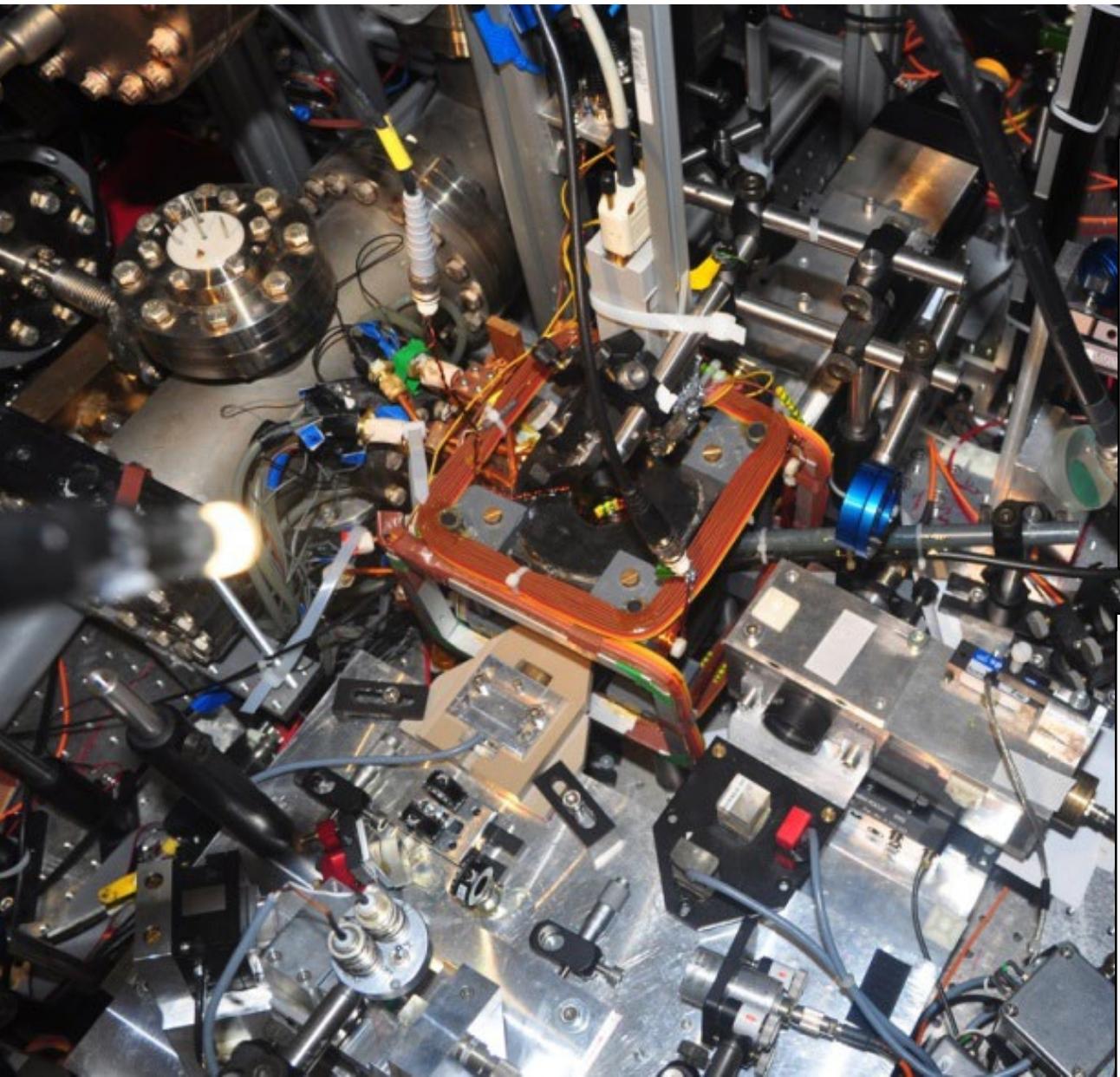
Question

Experimental
details

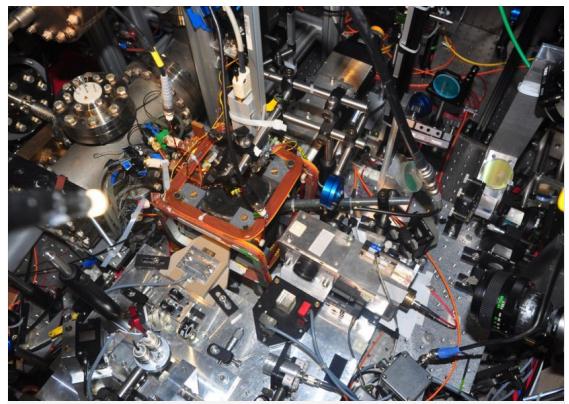
Please
bear with me

Vacuum since 2000, with one day break

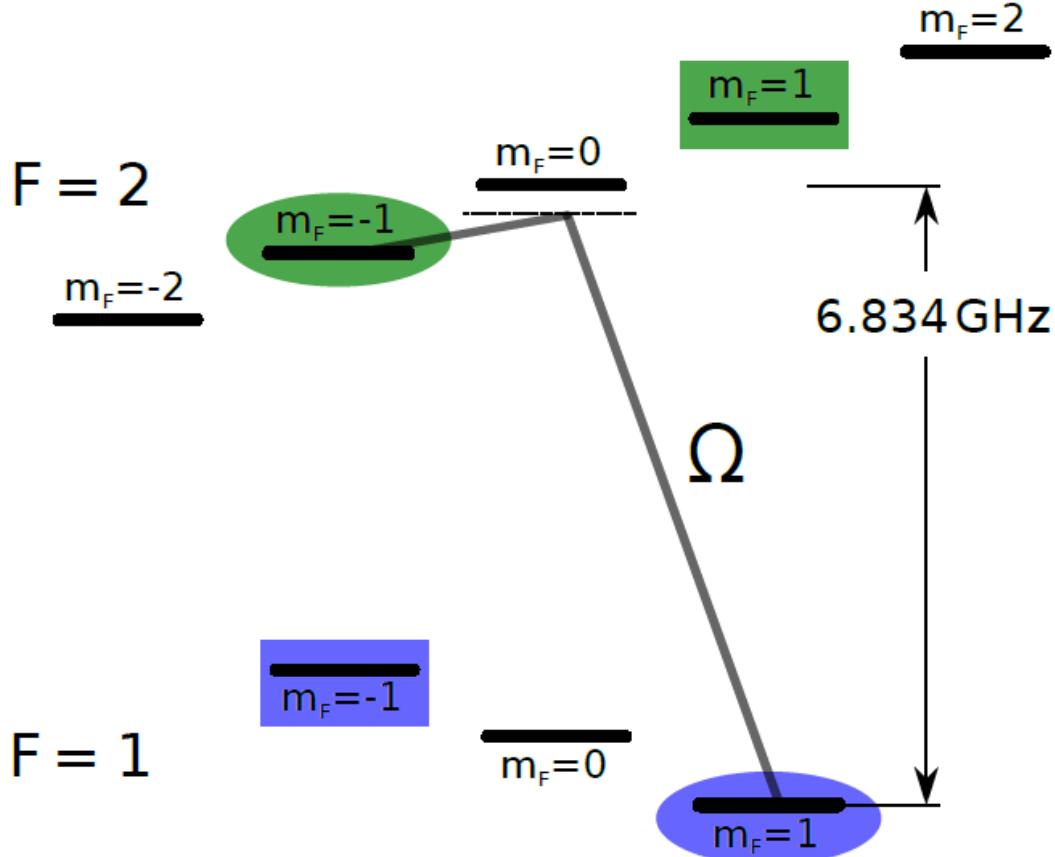
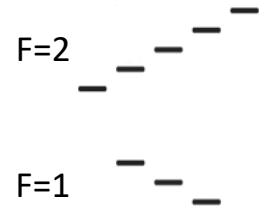
Experimental details



The system



Rubidium BEC

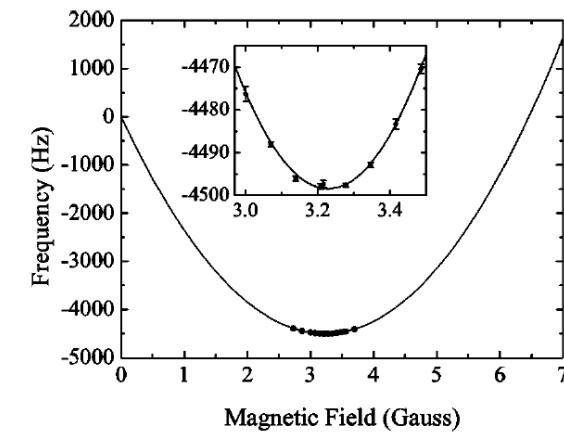


Two mixtures with special properties

$|2\rangle$

$|1\rangle$

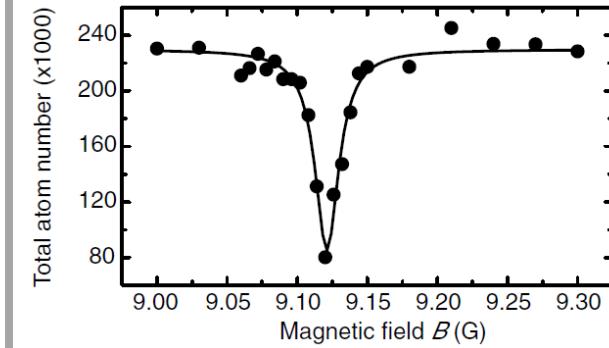
magic field: 3.23 G



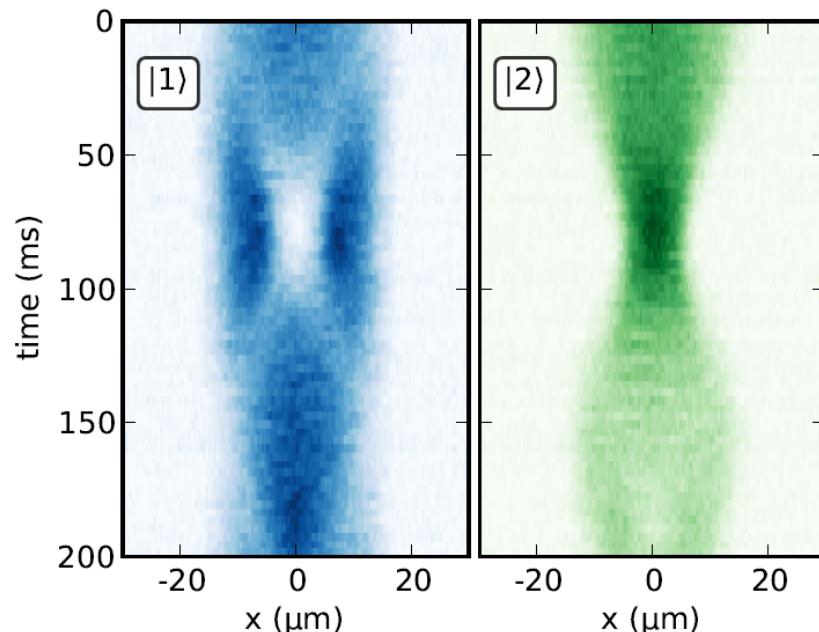
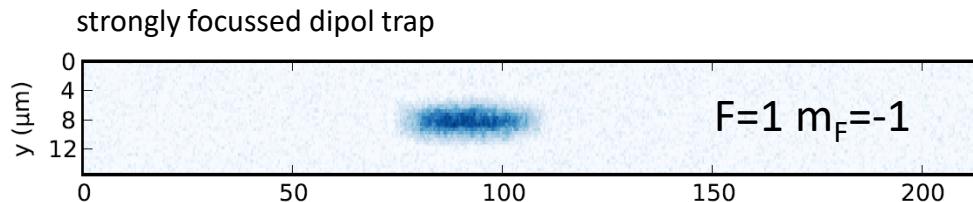
$|2\rangle$

$|1\rangle$

Feshbach resonance:
9.09 G but 20mG width



PRL 92, 160406 (2004)
PRA 69, 032705 (2004)

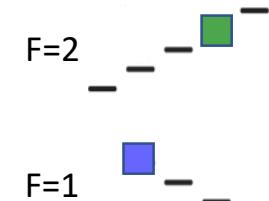


Question

$|1\rangle$

$|2\rangle$

What happens if the superposition @ t=0 is generated?



$$\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 = \sum_{i=1,2} \int dx \hat{\Psi}_i^\dagger \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \hat{\Psi}_i$$

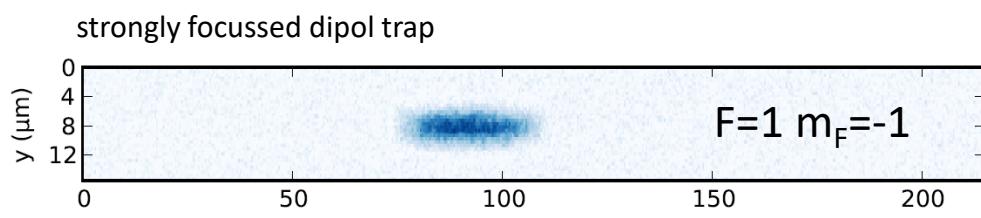
$$\hat{H}_{\text{int}} = \frac{1}{2} \sum_{i,j=1,2} g_{ij} \int dx \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i \quad g_{ij} = \frac{4\pi\hbar^2 a_{ij}}{m}$$

Rubidium is special:

$$a_{11} : a_{22} : a_{12} = 100.44 a_B : 95.47 a_B : 97.7 a_B$$

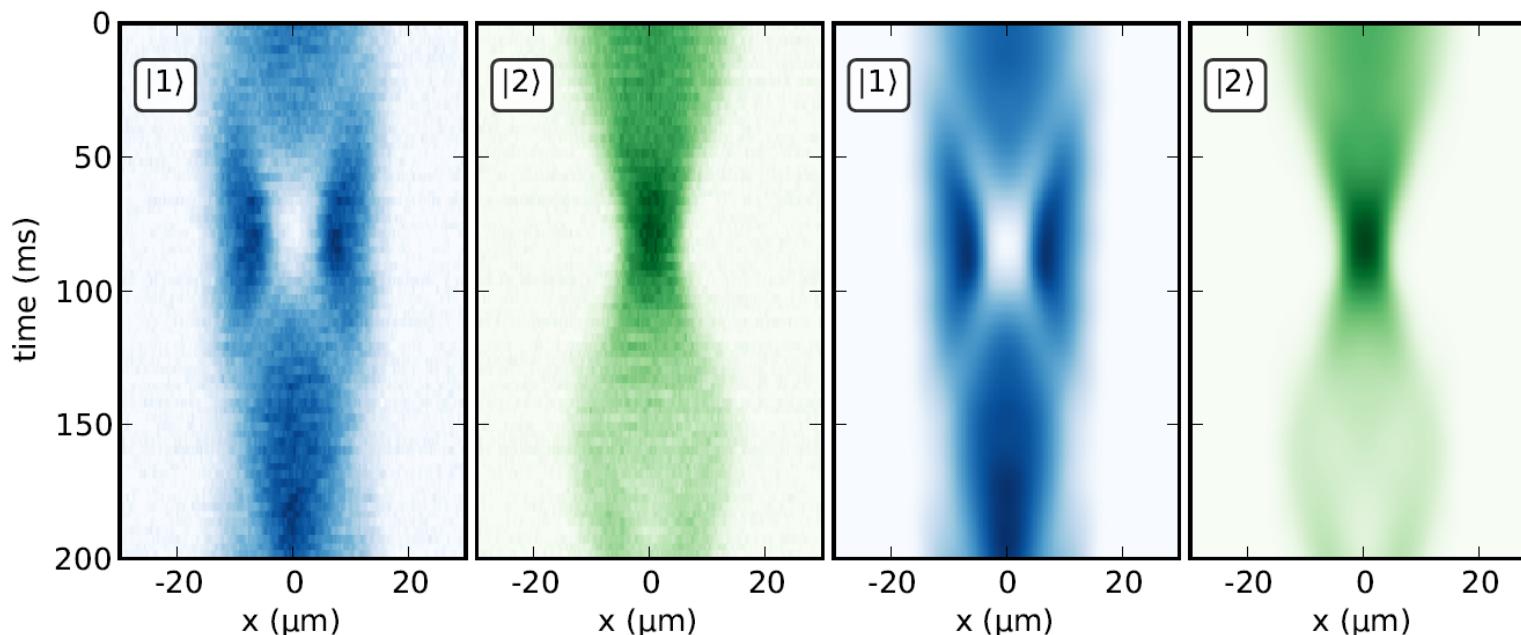
$$i\hbar \frac{\partial}{\partial t} \psi_1 = \left[-\frac{\hbar^2}{2m} \nabla^2 + V + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2 \right] \psi_1$$

$$i\hbar \frac{\partial}{\partial t} \psi_2 = \left[-\frac{\hbar^2}{2m} \nabla^2 + V + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2 \right] \psi_2$$



$$i\hbar \frac{\partial}{\partial t} \psi_1 = \left[-\frac{\hbar^2}{2m} \nabla^2 + V + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2 \right] \psi_1$$

$$i\hbar \frac{\partial}{\partial t} \psi_2 = \left[-\frac{\hbar^2}{2m} \nabla^2 + V + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2 \right] \psi_2$$



Component $F=2$ has smaller
repulsive interaction than $F=1$!

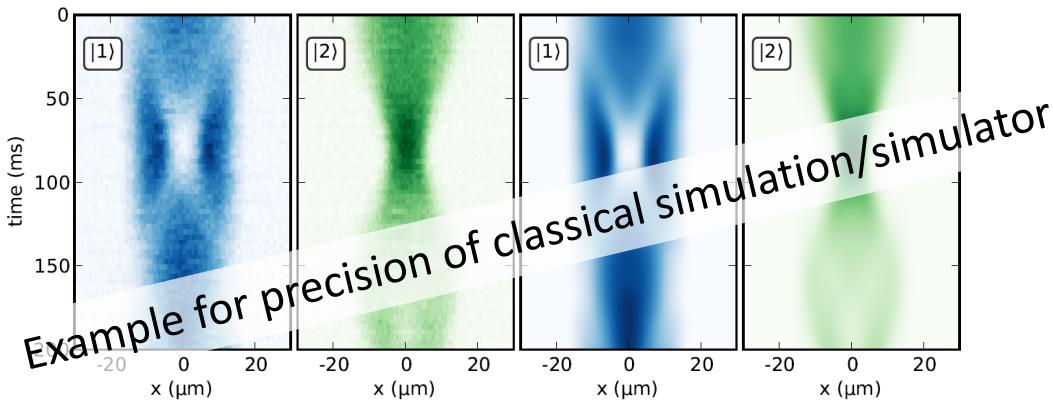
Potential separation – no phase separation!

Phase Separation of Bose-Einstein Condensates

E. Timmermans*

Institute for Atomic and Molecular Physics, Harvard-Smithsonian Center for Astrophysics,
60 Garden Street, Cambridge, Massachusetts 02138

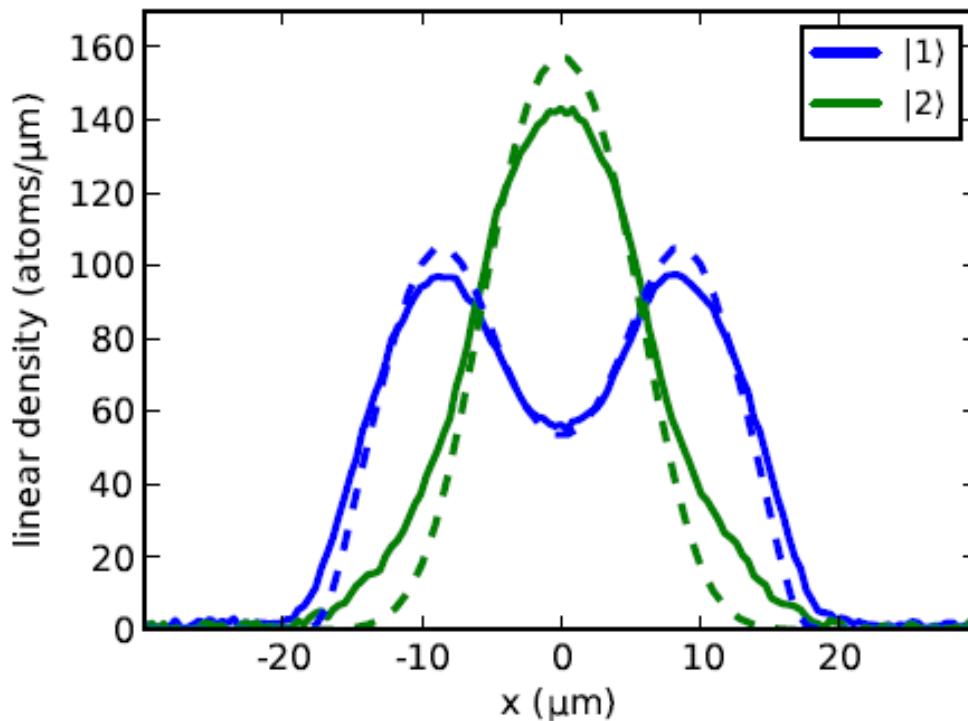
(Received 26 August 1997; revised manuscript received 10 September 1998)



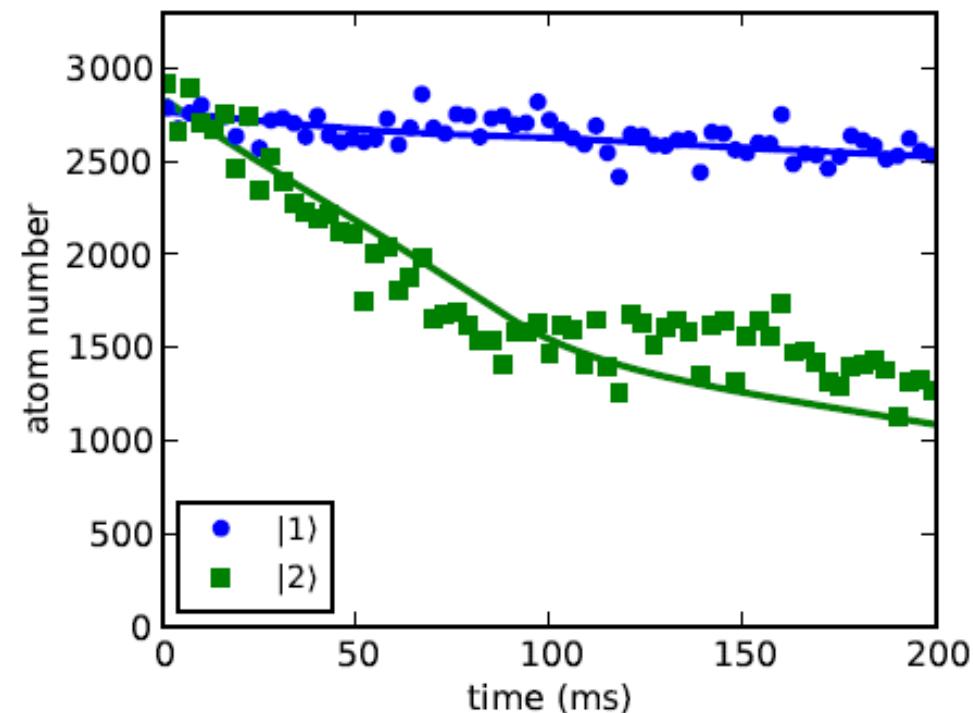
$$i\hbar \frac{\partial}{\partial t} \psi_1 = \left[-\frac{\hbar^2}{2m} \nabla^2 + V + g_{11} |\psi_1|^2 + g_{12} |\psi_2|^2 \right] \psi_1$$

$$i\hbar \frac{\partial}{\partial t} \psi_2 = \left[-\frac{\hbar^2}{2m} \nabla^2 + V + g_{22} |\psi_2|^2 + g_{12} |\psi_1|^2 \right] \psi_2$$

time averaged experiment –
stationary theory solution --



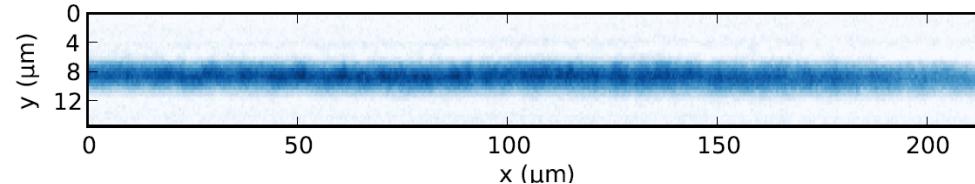
Losses are well understood and
have to be taken into account.



Rubidium BEC

weakly focussed dipol trap

$F=1 m_F=-1$

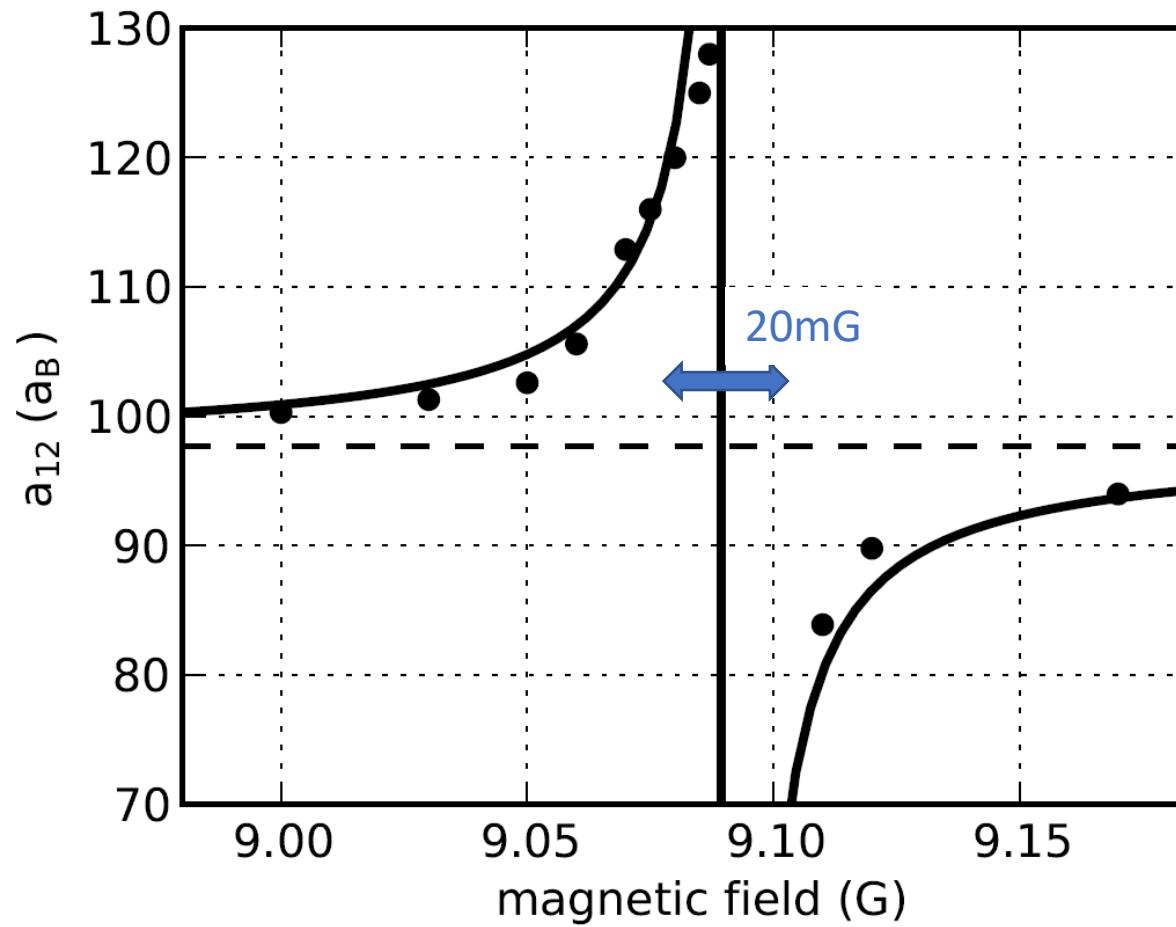
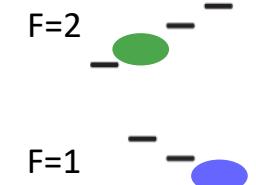


Question

$|1\rangle$

$|2\rangle$

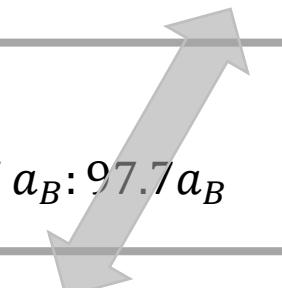
What happens if the superposition @ $t=0$ is generated?



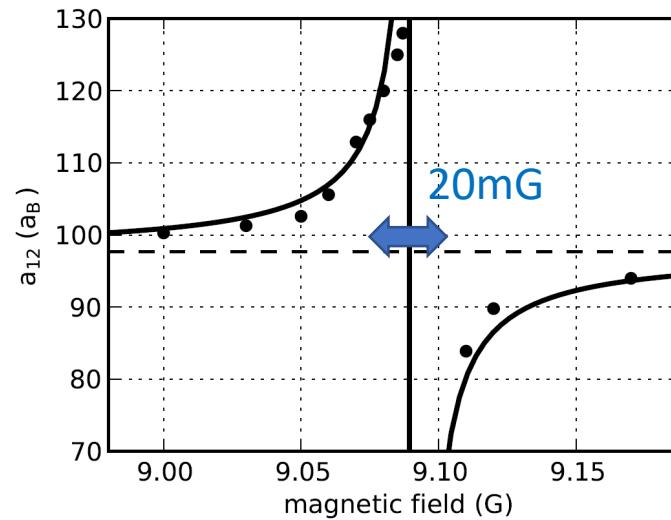
Rubidium is special:

$$a_{11} : a_{22} : a_{12} = 100.44 a_B : 95.47 a_B : 97.7 a_B$$

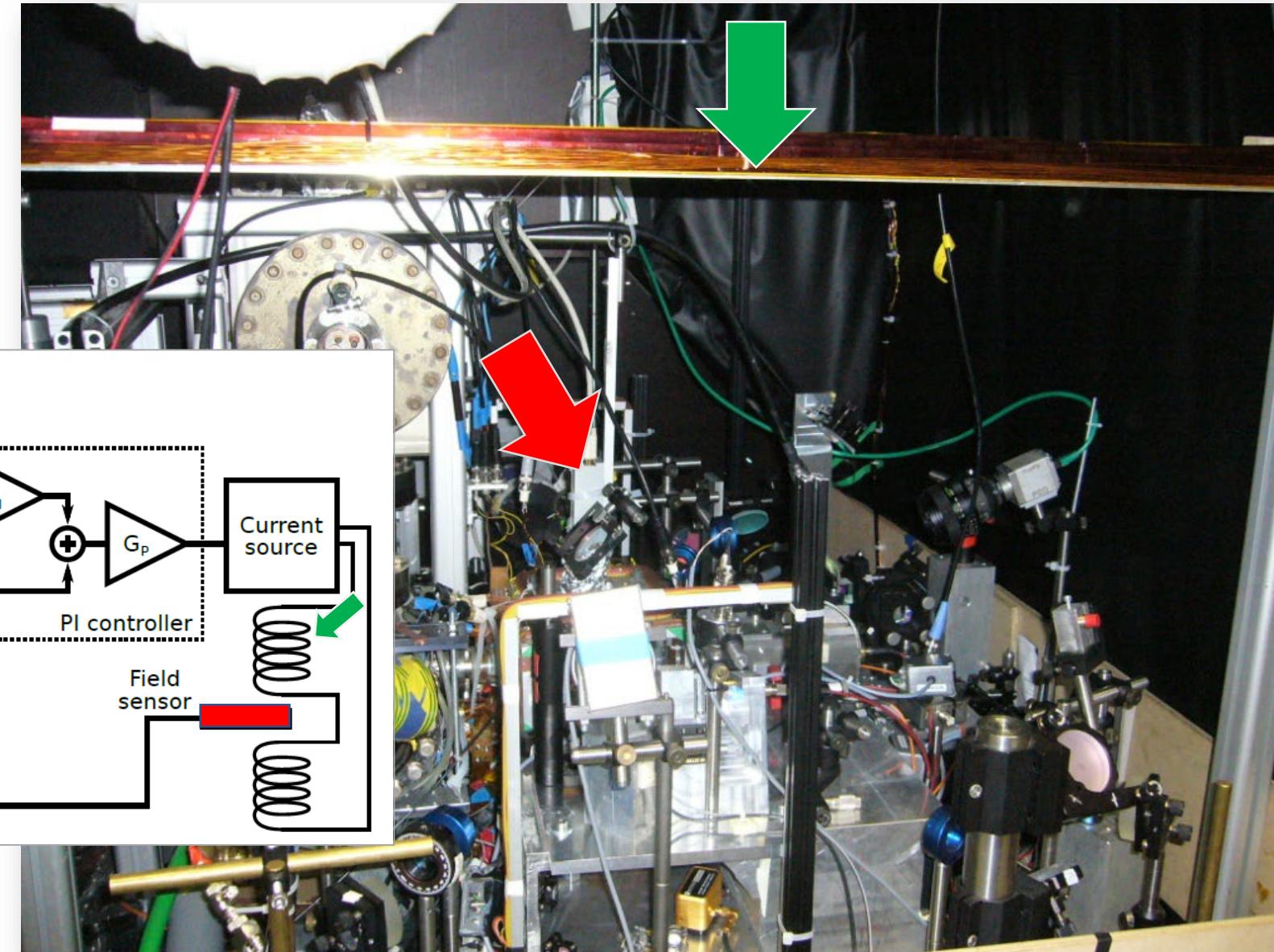
$97.7 a_B$

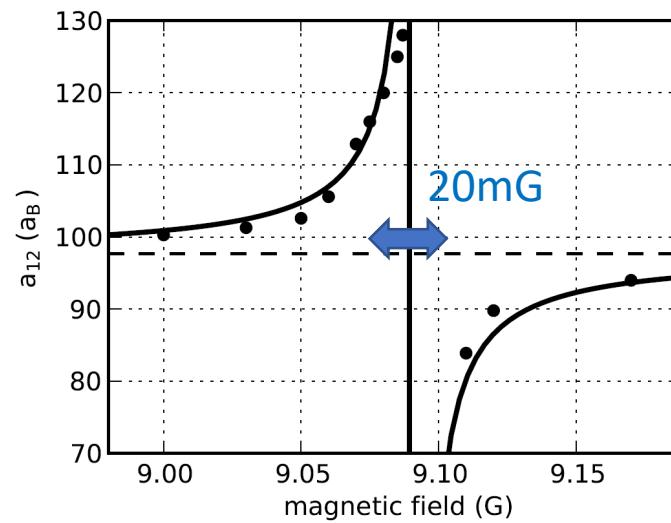


Experimental details



Is a magnetic field stability @ 9.1G an issue
for a standard setup?



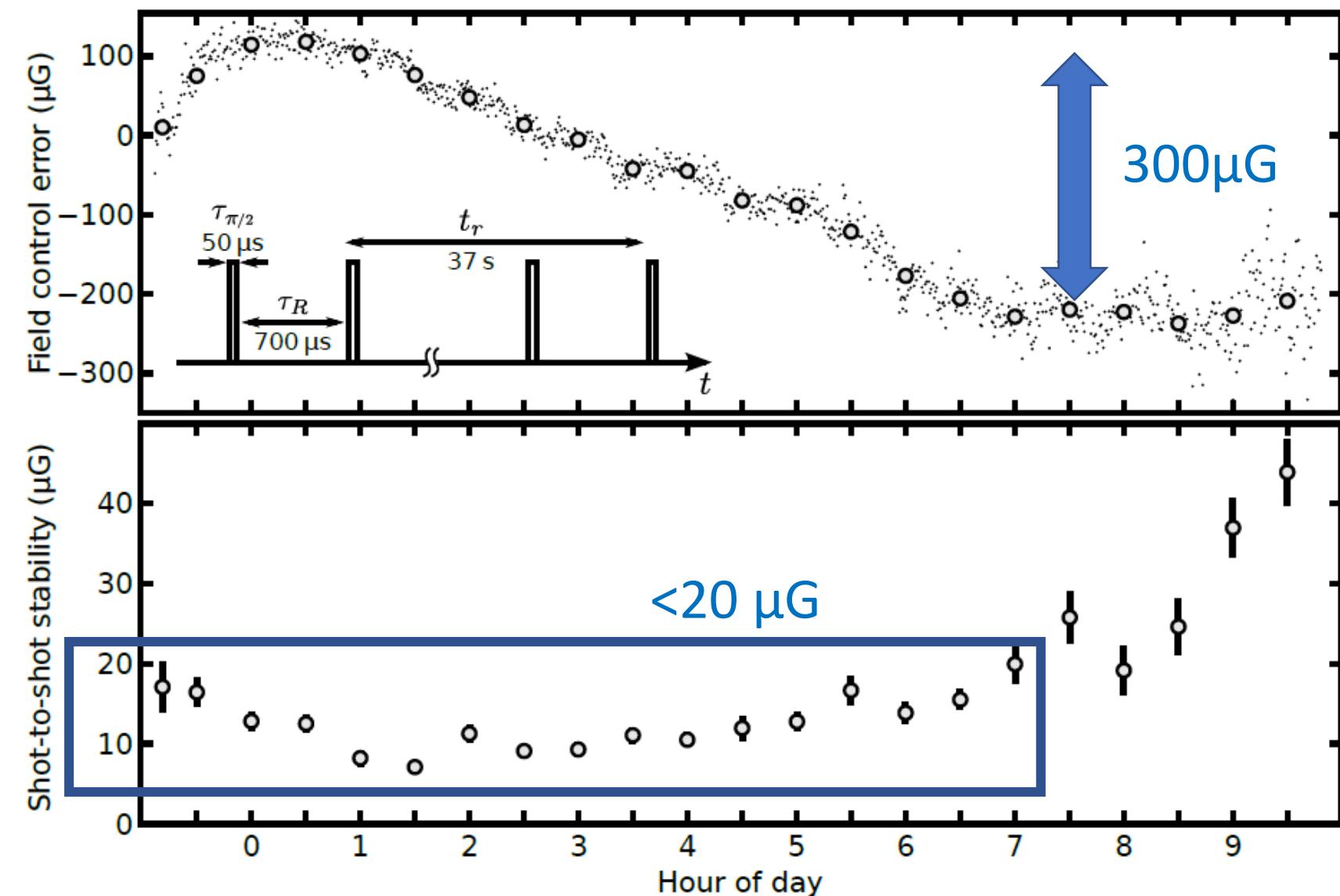


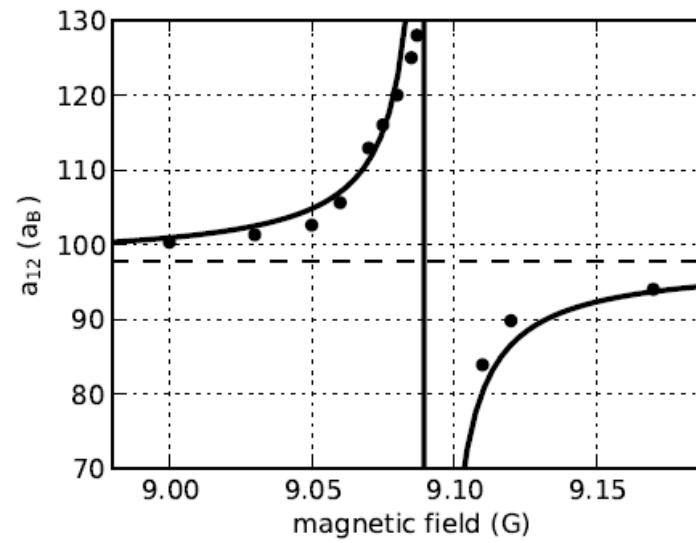
Is a magnetic field stability @ 9.1G an issue for a standard setup?

NO

Experimental details

Limited by temperature dependence of flux-gate sensor:
 $300\mu\text{G} = 75\text{mK}$ change





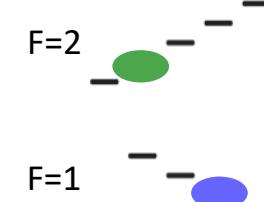
Question

$|1\rangle$

$|2\rangle$

What happens if the superposition @ t=0 is generated?

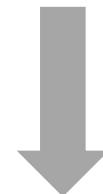
Rubidium BEC



miscible



$$\sqrt{g_{11}g_{22}} < g_{12}$$



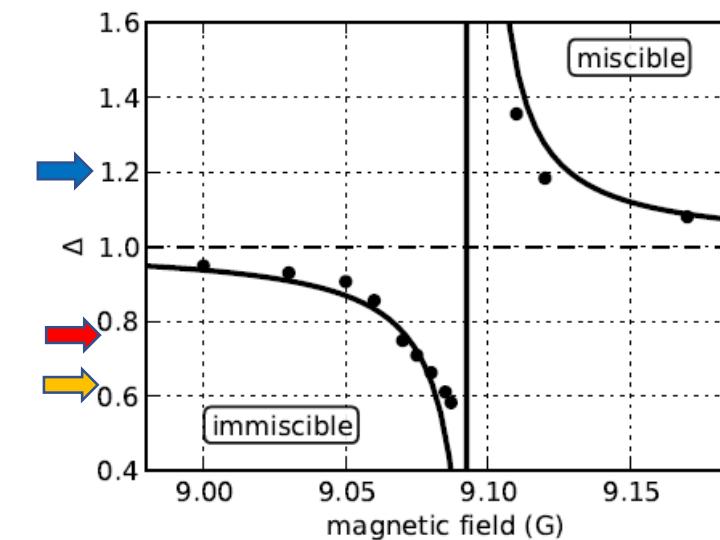
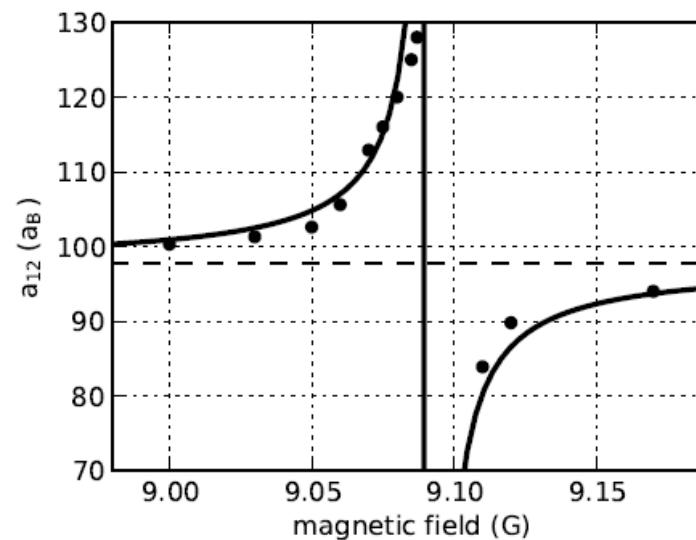
$$\Delta = \frac{g_{11}g_{22}}{g_{12}^2} < 1$$

immiscible



$$E_{\text{unif}} = \frac{g_{11}}{2} \frac{N_1^2}{L} + \frac{g_{22}}{2} \frac{N_2^2}{L} + g_{12} \frac{N_1 N_2}{L}$$

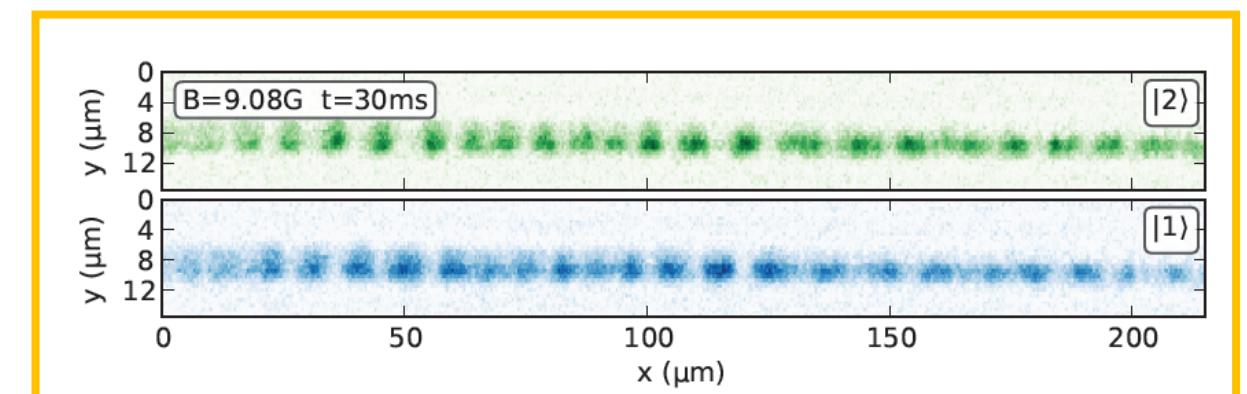
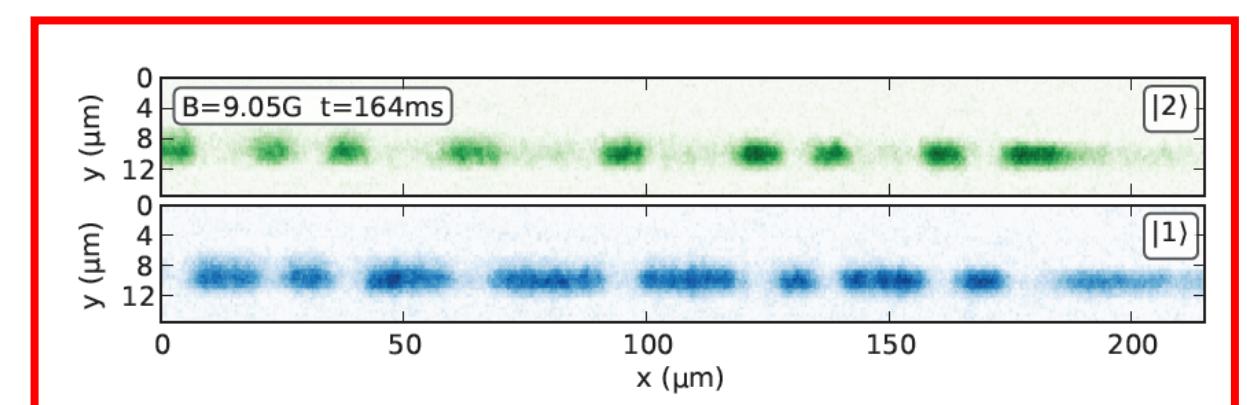
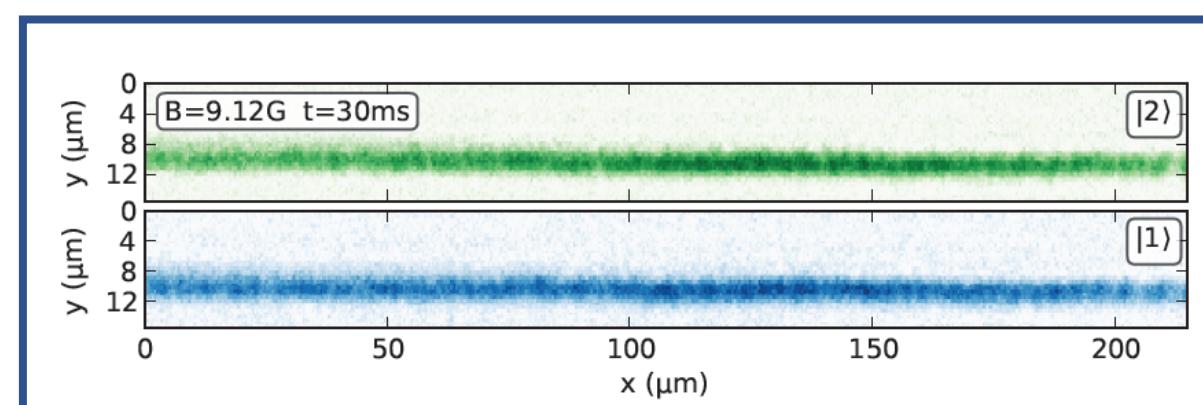
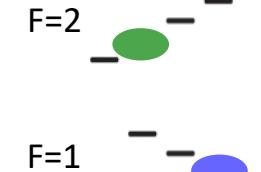
$$E_{\text{sep}} = \frac{g_{11}}{2} \frac{N_1^2}{L} + \frac{g_{22}}{2} \frac{N_2^2}{L} + \sqrt{g_{11}g_{22}} \frac{N_1 N_2}{L}$$

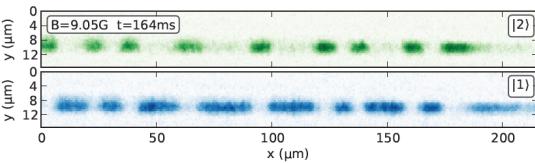
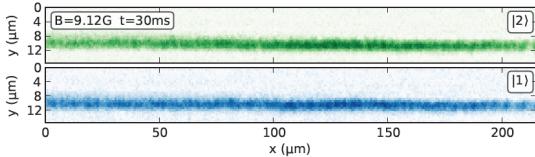


Rubidium BEC

$|1\rangle$ $|2\rangle$

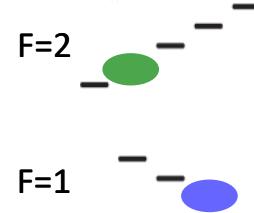
superposition @ $t=0$ is generated?





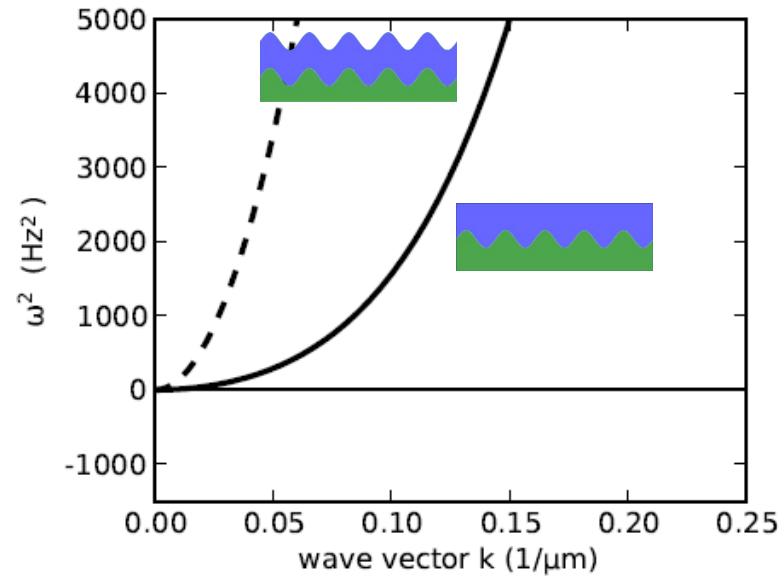
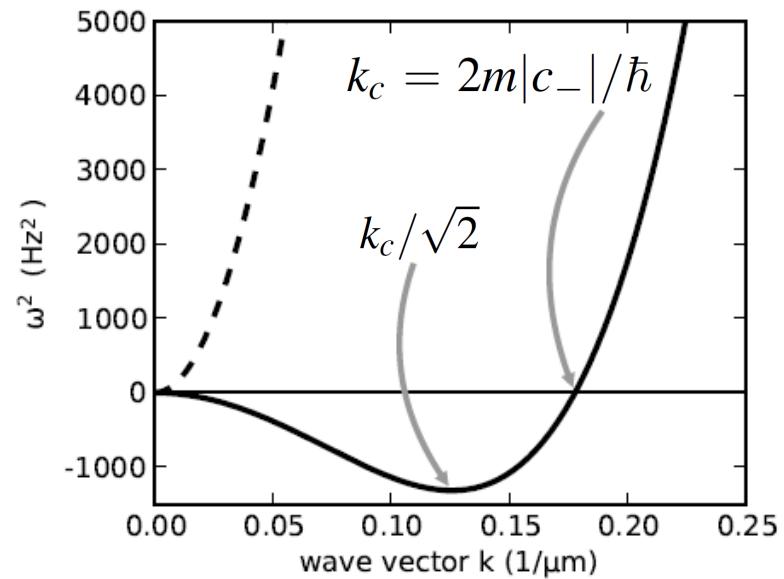
Question

Where does the length scale come from ?



Bogoliubov treatment: $\hbar^2 \omega_{\pm}^2(k) = c_{\pm}^2 \hbar^2 k^2 + \left(\frac{\hbar^2 k^2}{2m} \right)^2$

$$c_{\pm}^2 = \frac{1}{2} \left[(\tilde{c}_1^2 + \tilde{c}_2^2) \pm \sqrt{(\tilde{c}_1^2 - \tilde{c}_2^2)^2 + 4(g_{12}^2/g_{11}g_{22})\tilde{c}_1^2\tilde{c}_2^2} \right]$$



immiscible

<1

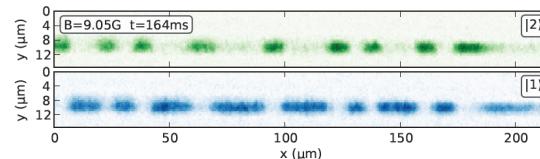
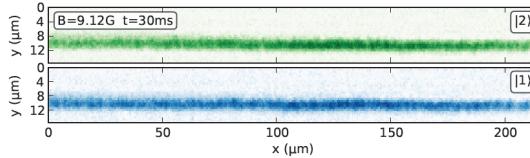
>1

miscible

$$\Delta = \frac{g_{11}g_{22}}{g_{12}^2}$$

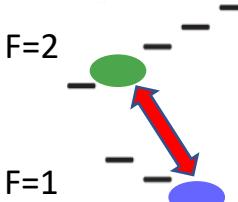
1

1

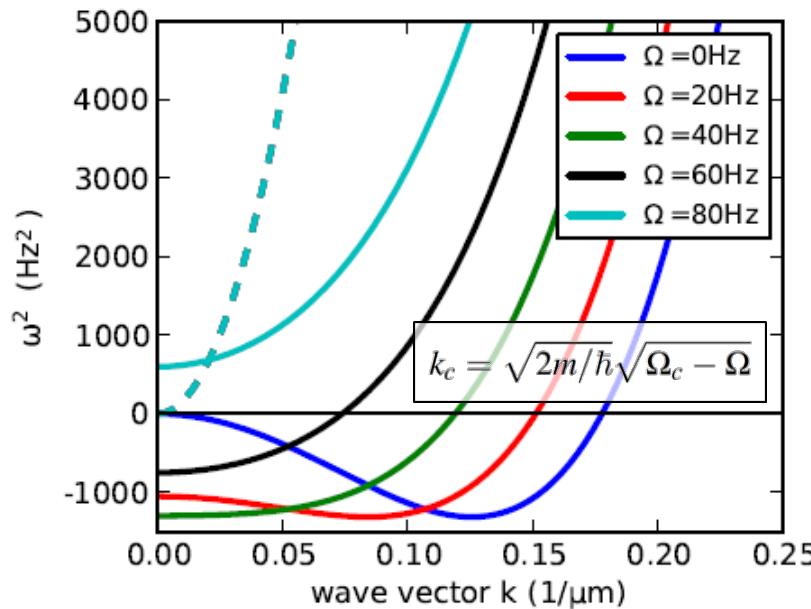


Question

What happens if linear coupling is added ?



$$\hat{H}_{\text{cpl}} = -\frac{1}{2} \int dx \left[\hbar \tilde{\Omega} \hat{\Psi}_1^\dagger \hat{\Psi}_2 + \hbar \tilde{\Omega}^* \hat{\Psi}_2^\dagger \hat{\Psi}_1 \right] + \frac{1}{2} \hbar \delta \int dx \left[\hat{\Psi}_2^\dagger \hat{\Psi}_2 - \hat{\Psi}_1^\dagger \hat{\Psi}_1 \right]$$

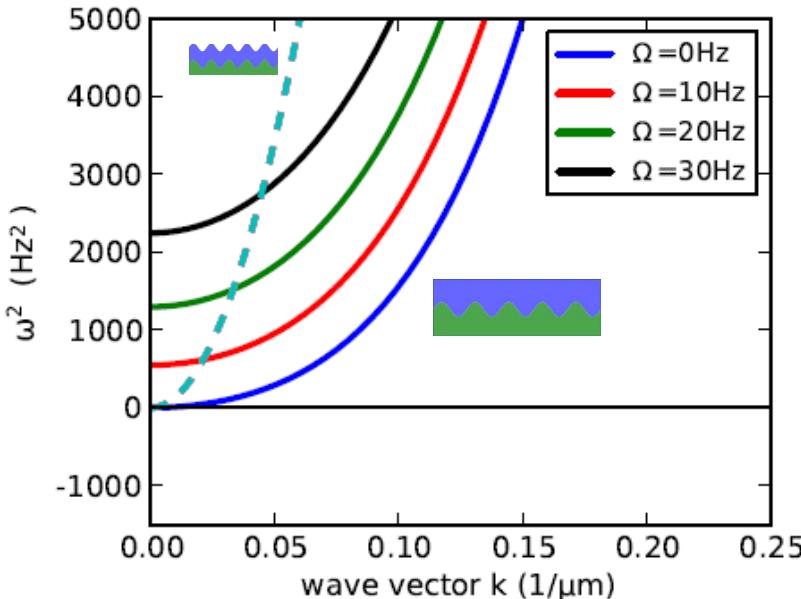


$\Omega / n g_{11}$

miscible

immiscible

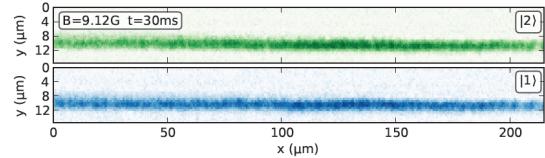
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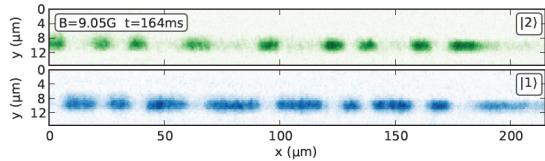
>1

miscible

$$\Delta = \frac{g_{11}g_{22}}{g_{12}^2}$$



miscible



immiscible

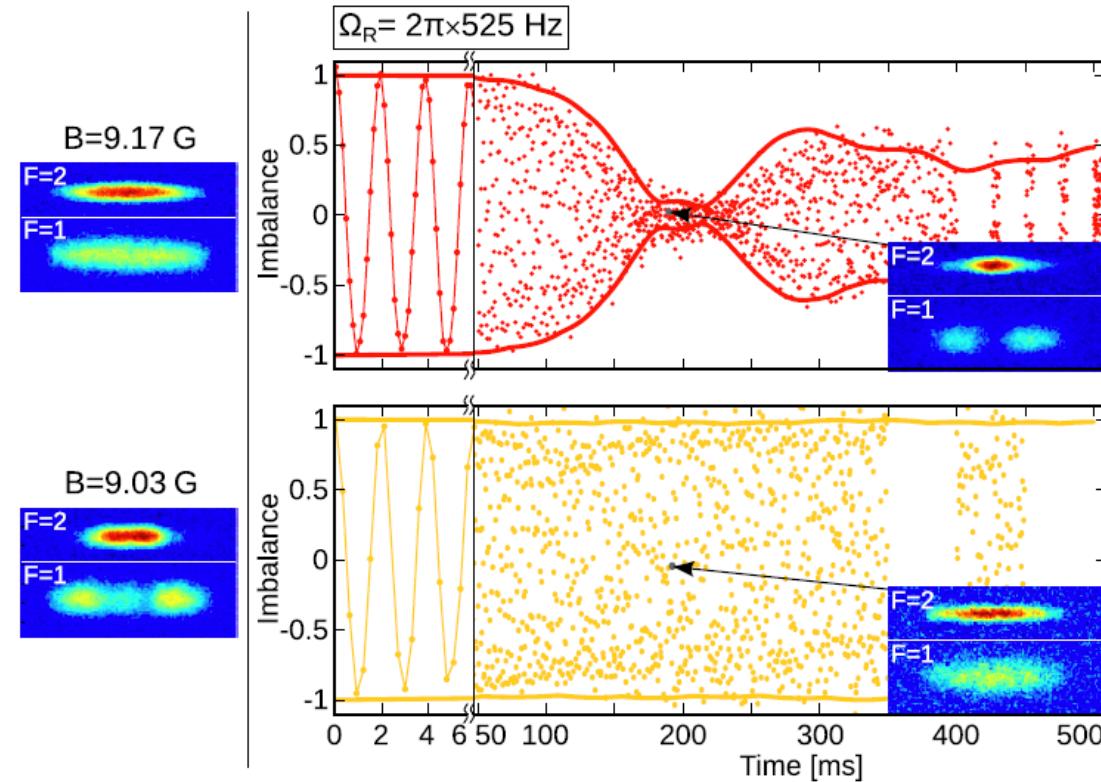
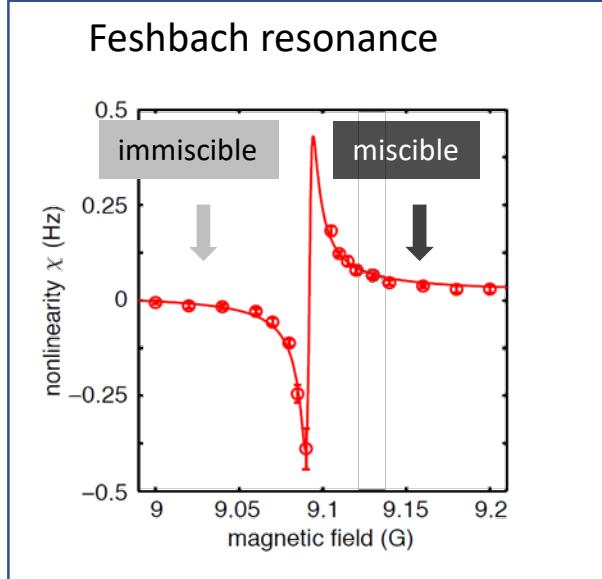
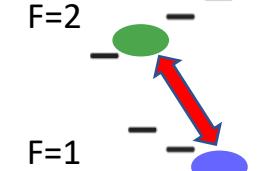
Question

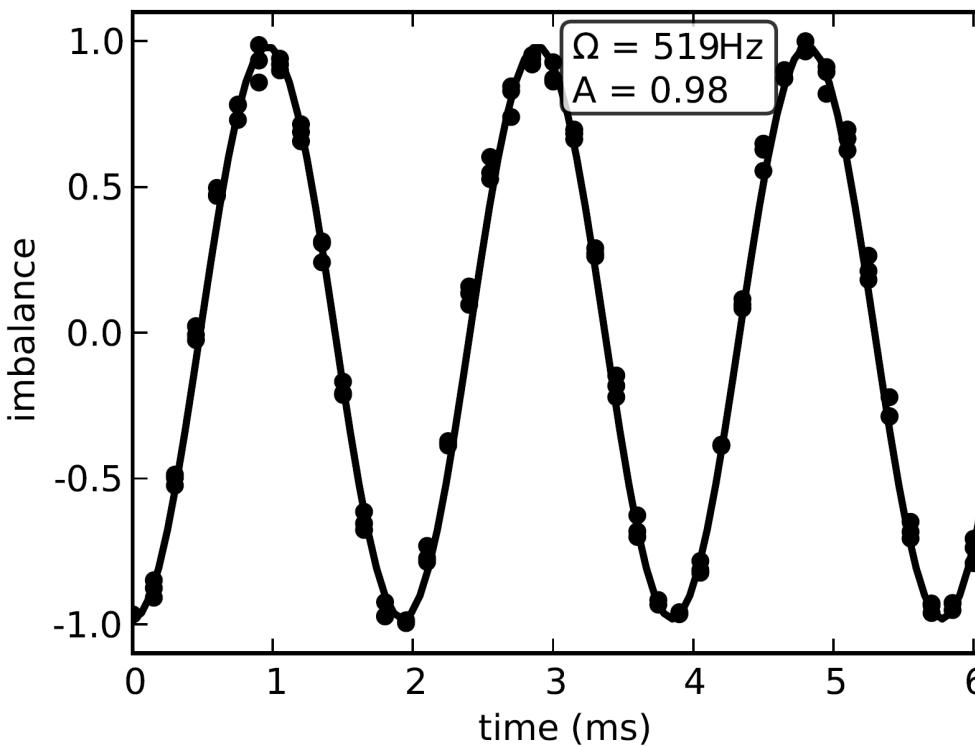
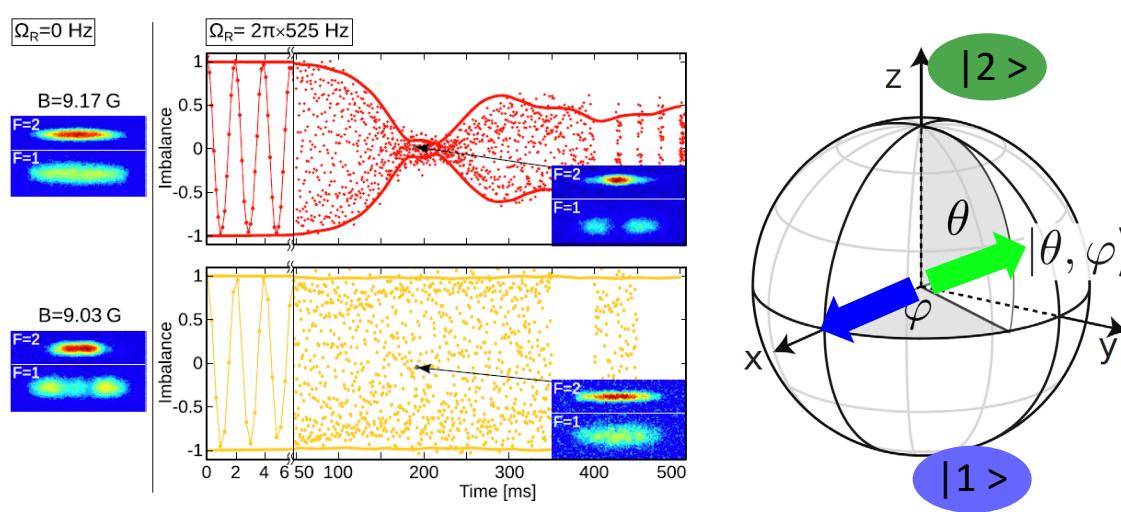
|1>

|2>

Rabi oscillations in the miscible and immiscible regime ?

Rubidium BEC





$$\hat{H}_{\text{cpl}} = -\frac{1}{2}\hbar \begin{pmatrix} \delta & \tilde{\Omega} \\ \tilde{\Omega}^* & -\delta \end{pmatrix} = -\frac{1}{2}\hbar \begin{pmatrix} \delta & \Omega e^{i\varphi} \\ \Omega e^{-i\varphi} & -\delta \end{pmatrix}$$

dressed states

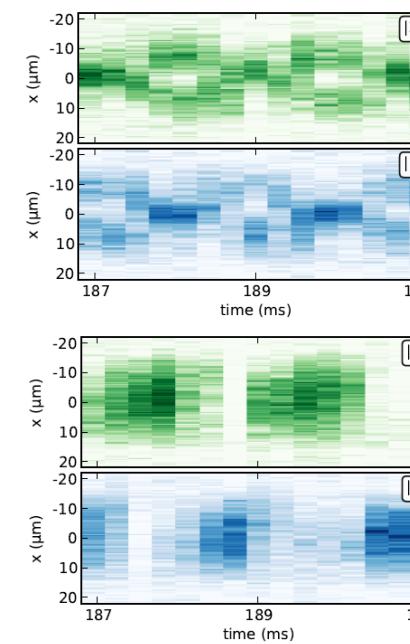
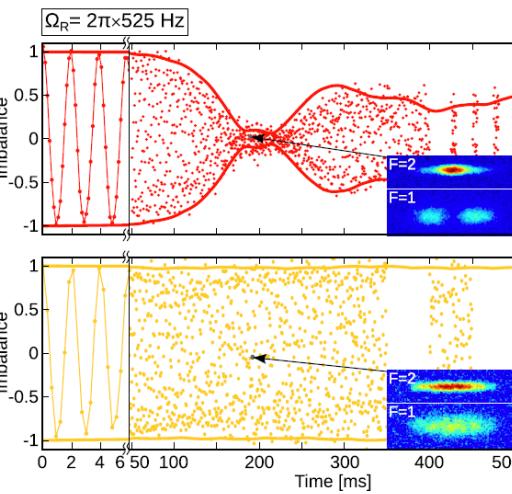
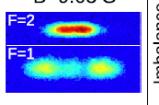
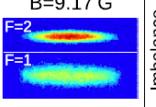
$$|+\rangle = e^{i\varphi/2} \sin(\theta/2)|1\rangle + e^{-i\varphi/2} \cos(\theta/2)|2\rangle$$

$$|-\rangle = e^{i\varphi/2} \cos(\theta/2)|1\rangle - e^{-i\varphi/2} \sin(\theta/2)|2\rangle$$

$$E_{\pm} = \frac{\hbar}{2}(\mp\Omega_{\text{eff}} - \delta) \quad \Omega_{\text{eff}} = \sqrt{\Omega^2 + \delta^2}$$

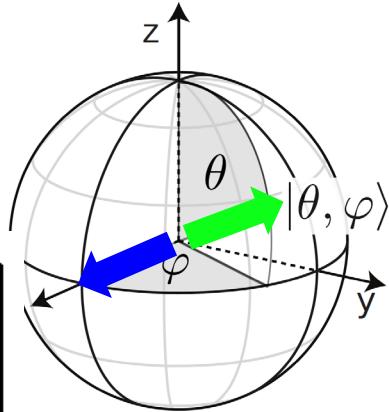
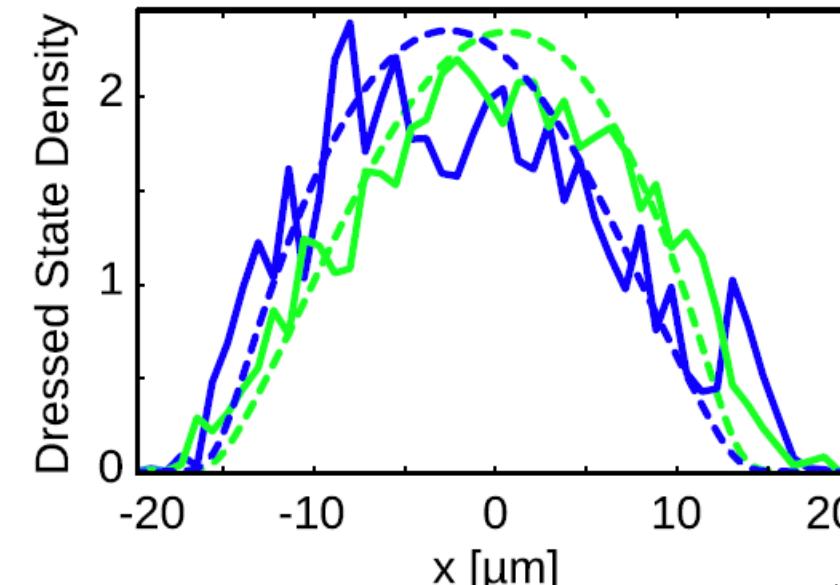
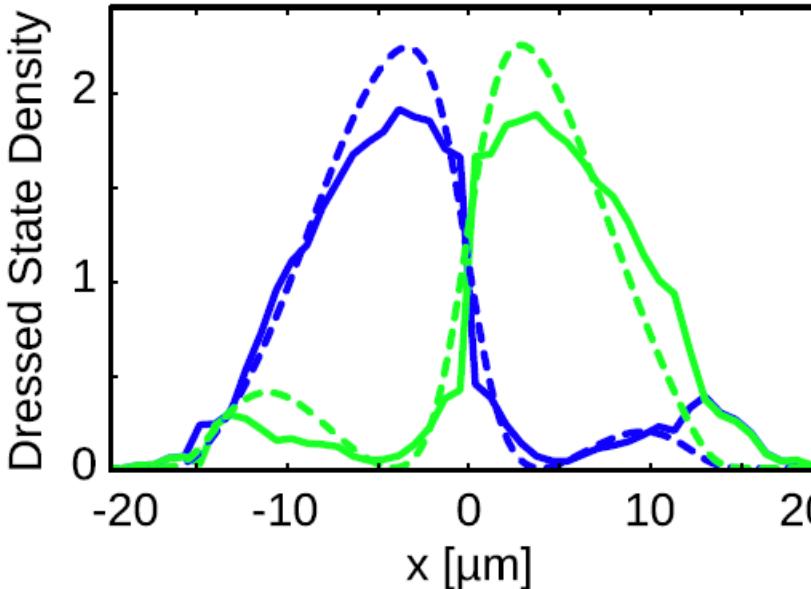
$$|\psi(t)\rangle = \cos(\Omega/2 t)|1\rangle + \sin(\Omega/2 t)|2\rangle$$

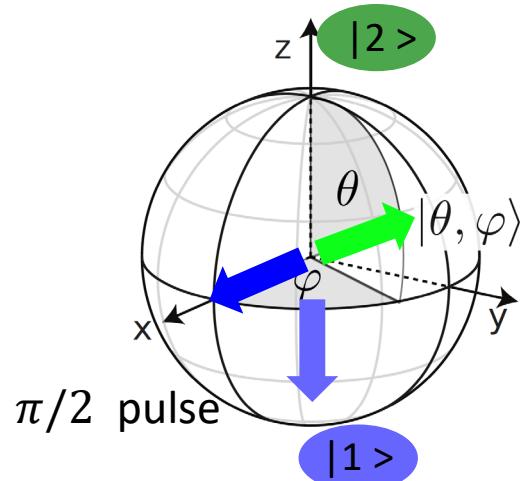
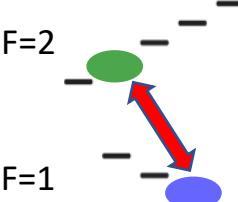
$$= \frac{1}{\sqrt{2}} \left(e^{-i\Omega/2 t}|+\rangle + e^{+i\Omega/2 t}|-\rangle \right)$$

$\Omega_R=0 \text{ Hz}$ 

$$\begin{aligned}
 |\psi(t)\rangle &= \cos \alpha \cdot e^{-i\Omega t/2} e^{-i\varphi/2} |+\rangle + \sin \alpha \cdot e^{+i\Omega t/2} e^{+i\varphi/2} |-\rangle \\
 &= \frac{1}{\sqrt{2}} (\cos \alpha \cdot e^{-i(\Omega t+\varphi)/2} + \sin \alpha \cdot e^{+i(\Omega t+\varphi)/2}) |1\rangle \\
 &\quad + \frac{1}{\sqrt{2}} (\cos \alpha \cdot e^{-i(\Omega t+\varphi)/2} - \sin \alpha \cdot e^{+i(\Omega t+\varphi)/2}) |2\rangle
 \end{aligned}$$

Dressed states reconstructed: miscibility \leftrightarrow immiscibility

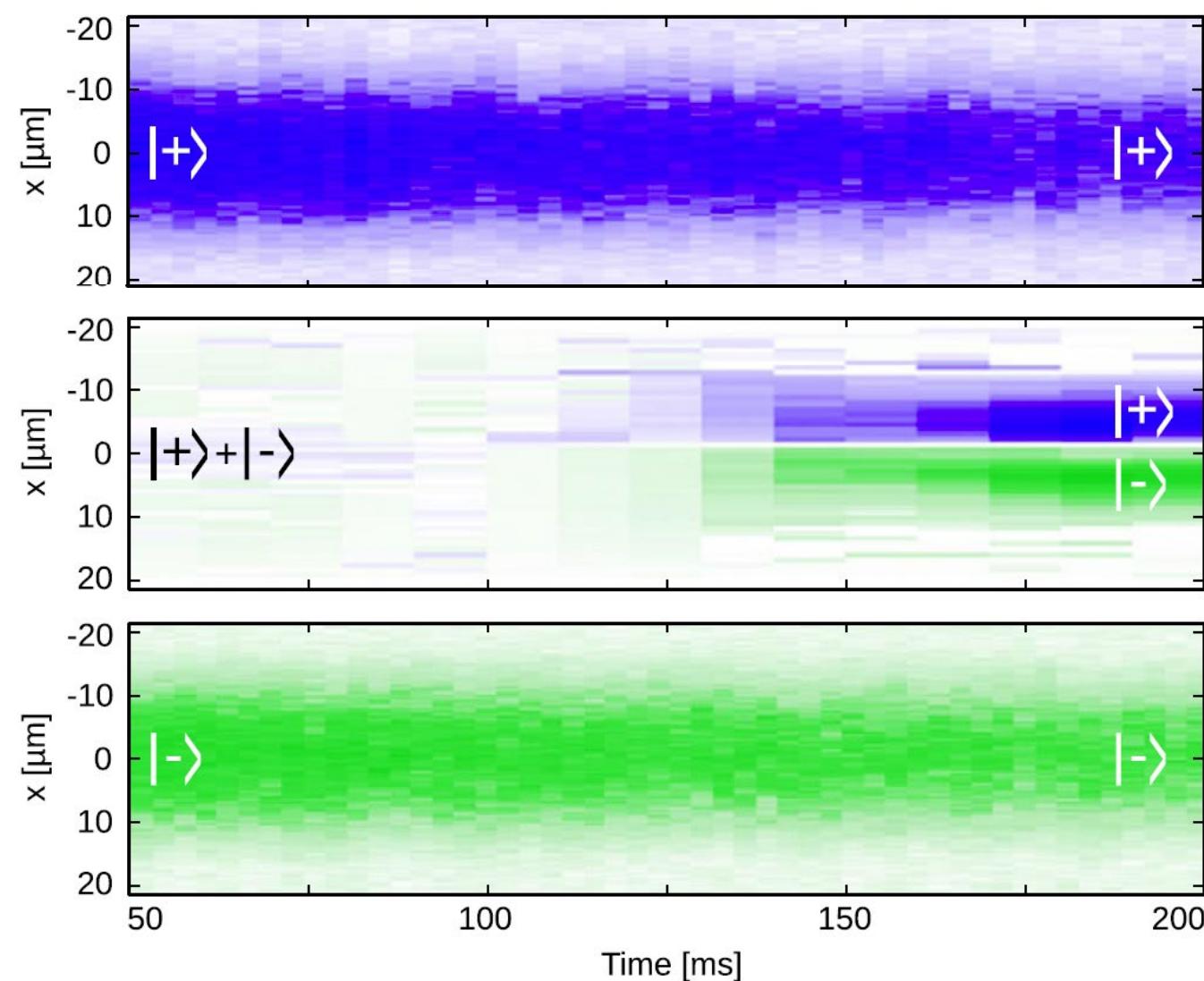




Question

Rabi oscillations in the miscible regime with initial condition

$|1\rangle + |2\rangle$



Dynamic stability of dressed condensate mixtures

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(Received 10 July 2003; published 17 November 2003)

See also: C.P. Search, P.R. Berman PRA 63, 043612 (2001)

$$a_{++} = a_{--} = \frac{1}{4}(a_{11} + a_{22} + 2a_{12})$$

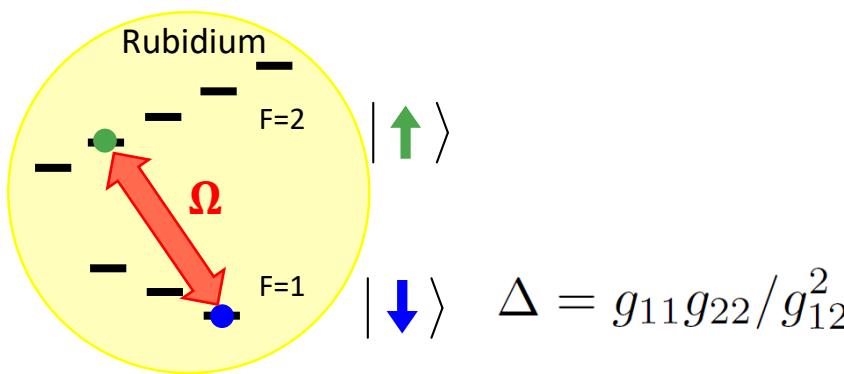
$$a_{+-} = \frac{1}{2}(a_{11} + a_{22})$$

Miscibility of dressed states:

$$a_{+-}^2 < a_{++}a_{--}$$

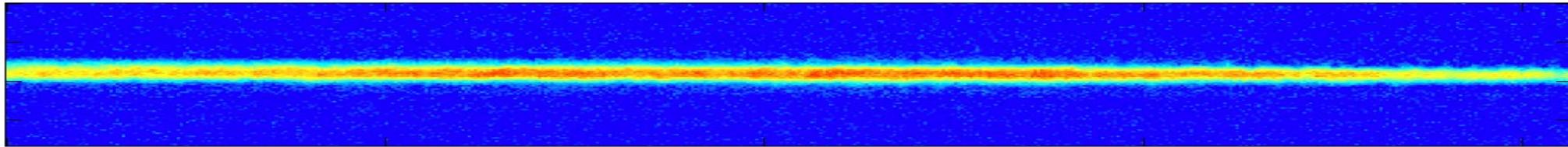
implies immiscibility of bare states $a_{11} \sim a_{22}$:

$$a_{12}^2 > a_{11} a_{22}$$



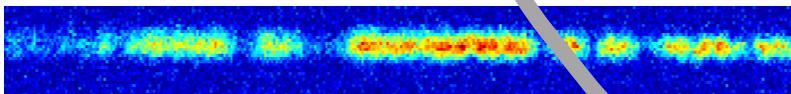
Standard miscible-immiscible transition

density

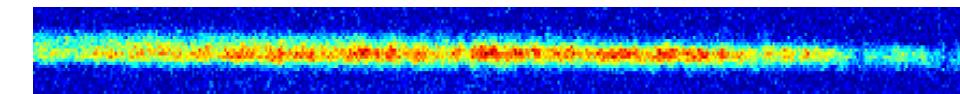


Ω/ng_{11}

immiscible



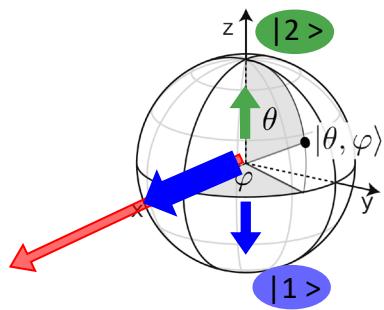
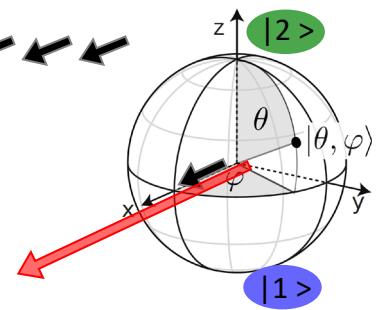
miscible

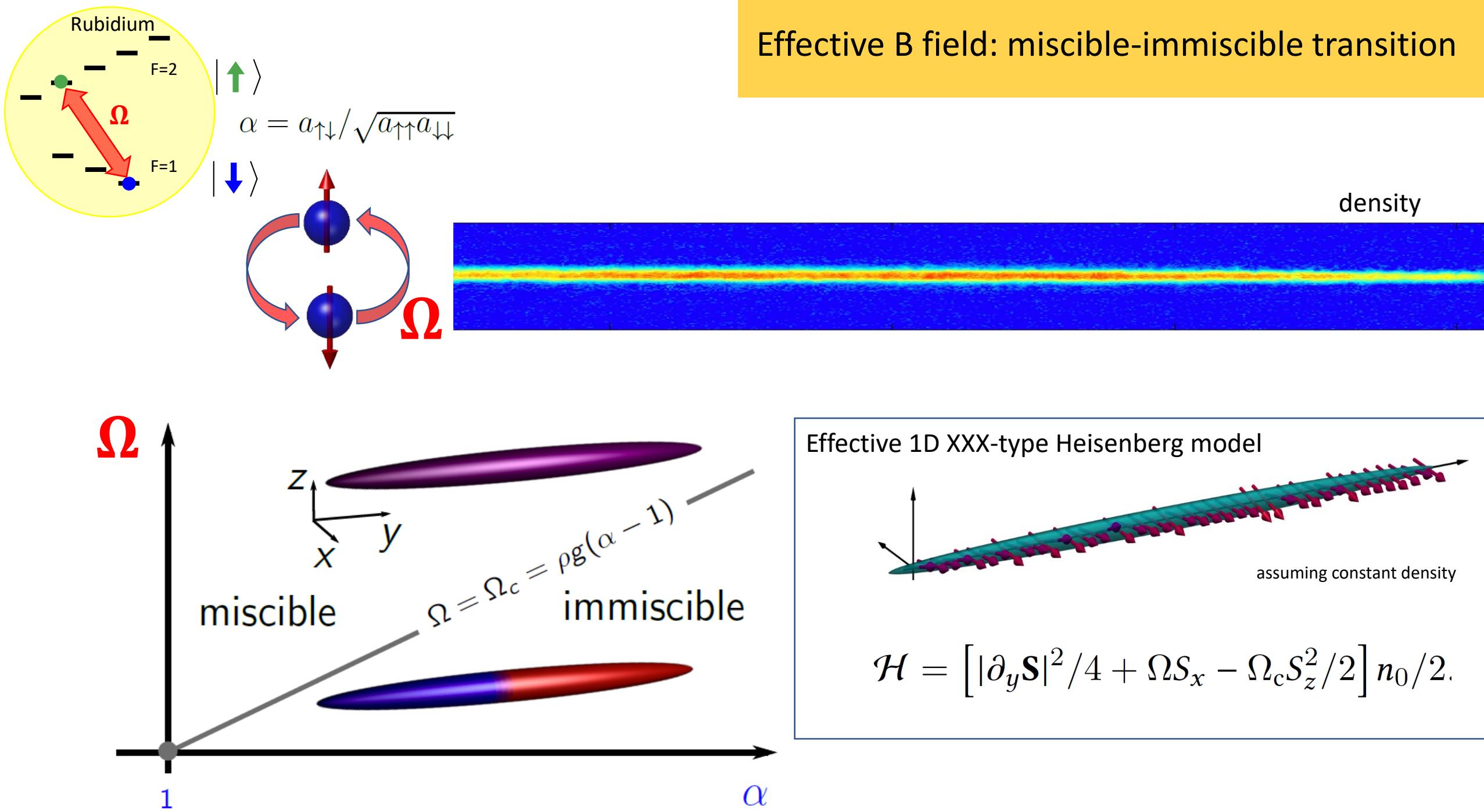


immiscible

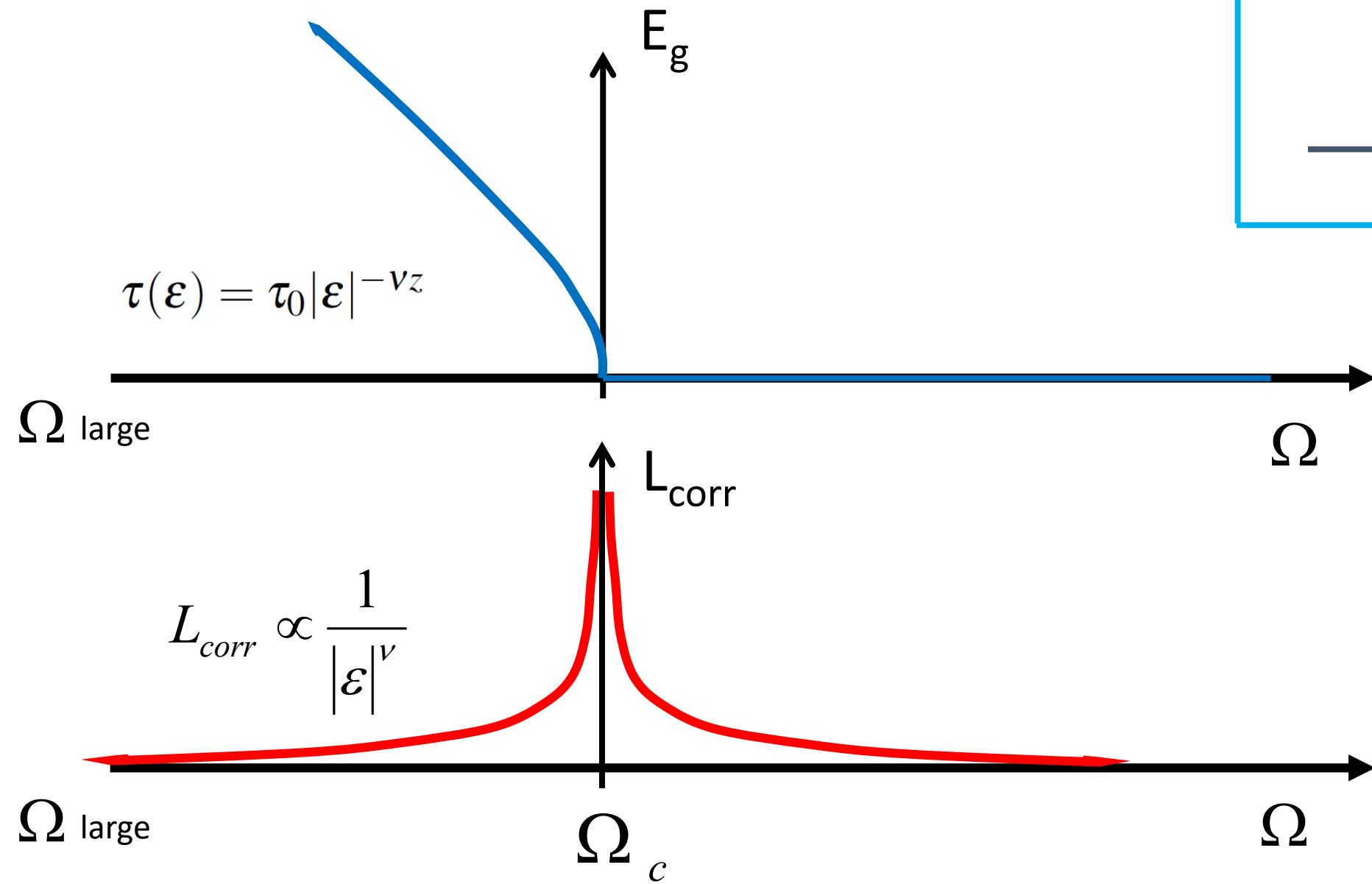
miscible

Δ

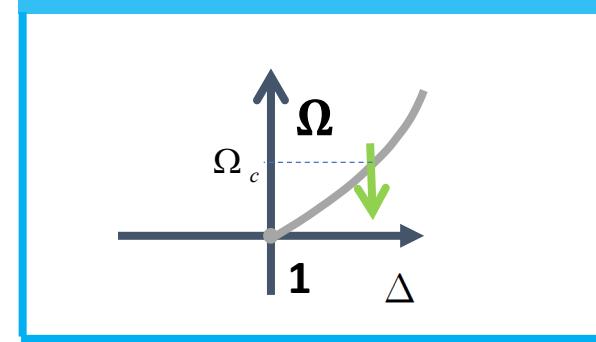


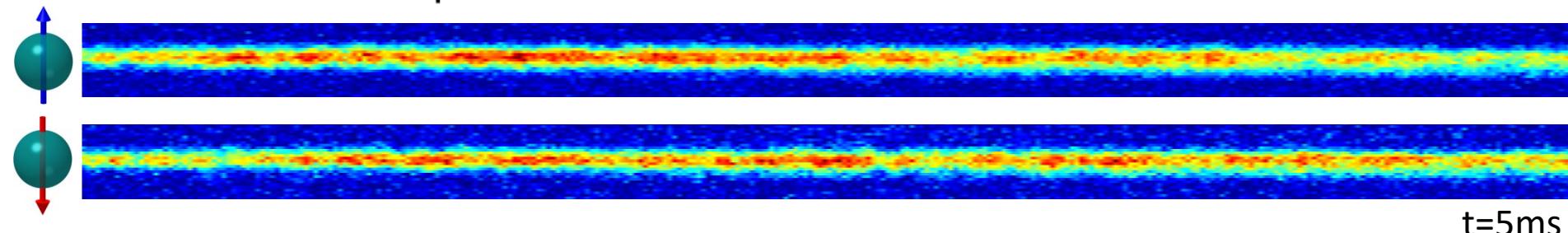
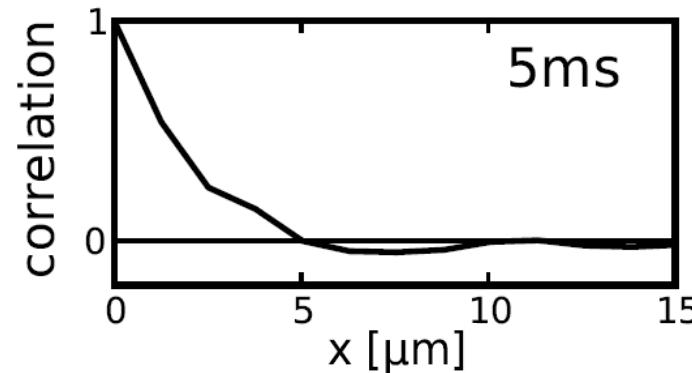
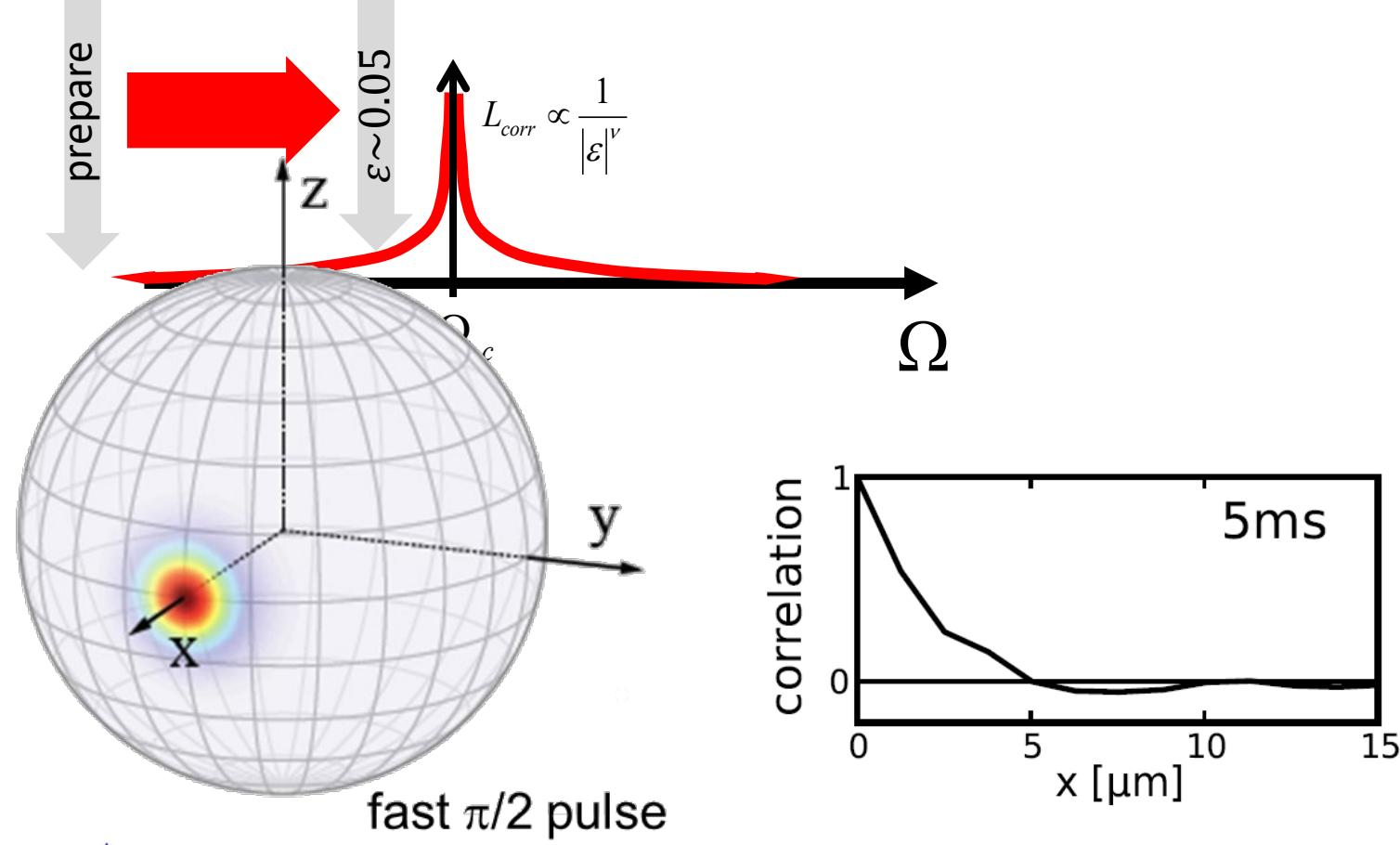


Characteristic time & length



Relevant parameters



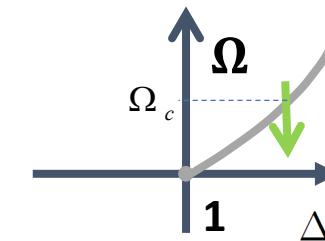


$$\langle J_z(x)J_z(x') \rangle = \langle \Delta n(x)\Delta n(x') \rangle$$

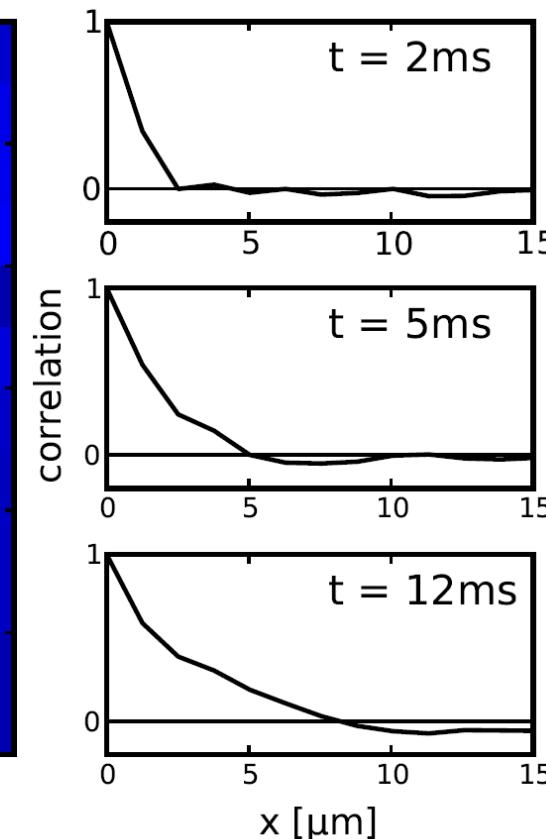
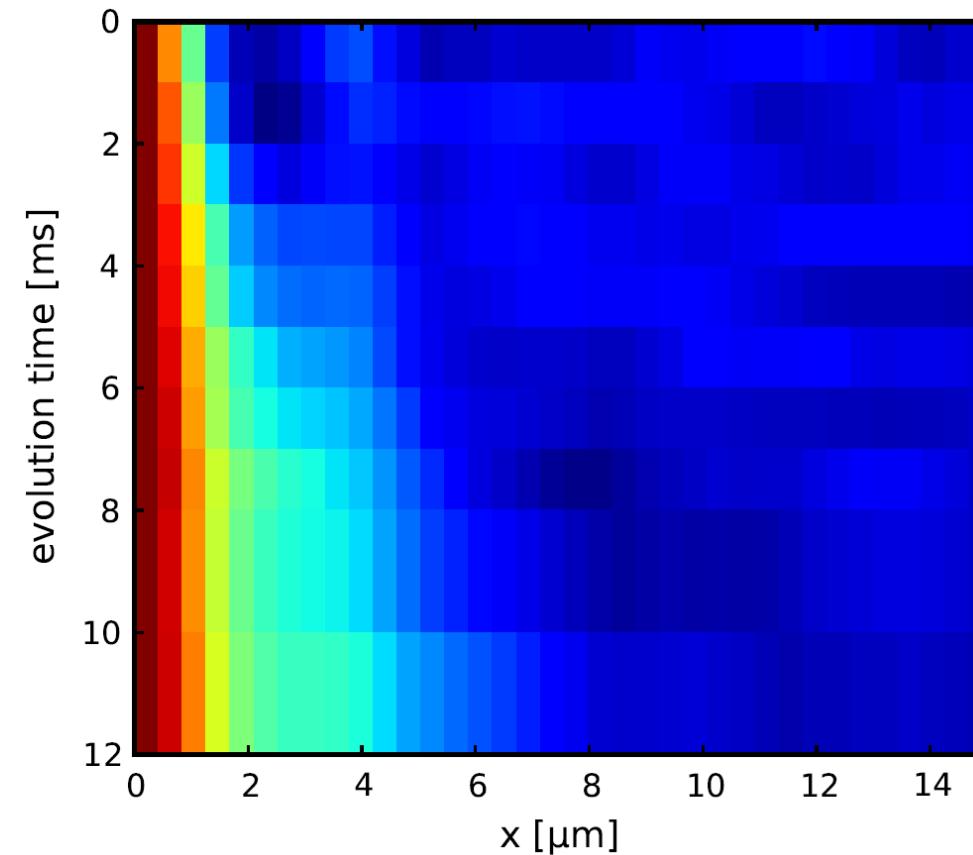
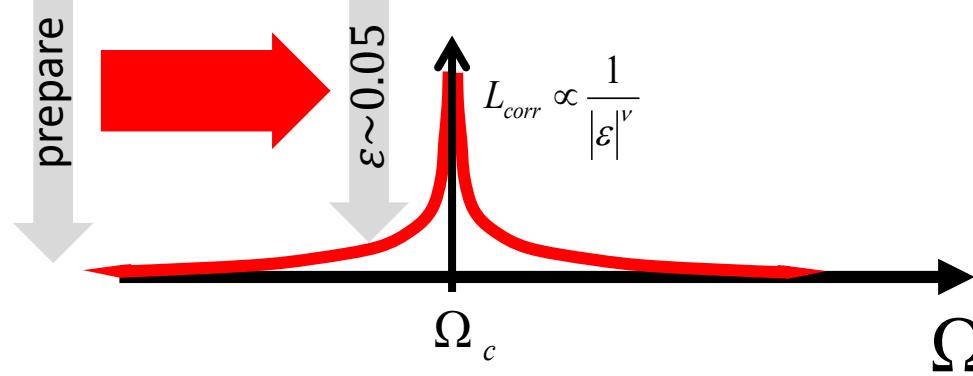
with 1 μm spatial resolution !!!
 $\sim 5\mu\text{m}$ spin healing length for $\Omega=0$

Access correlation length

PRL 115, 245301 (2015)

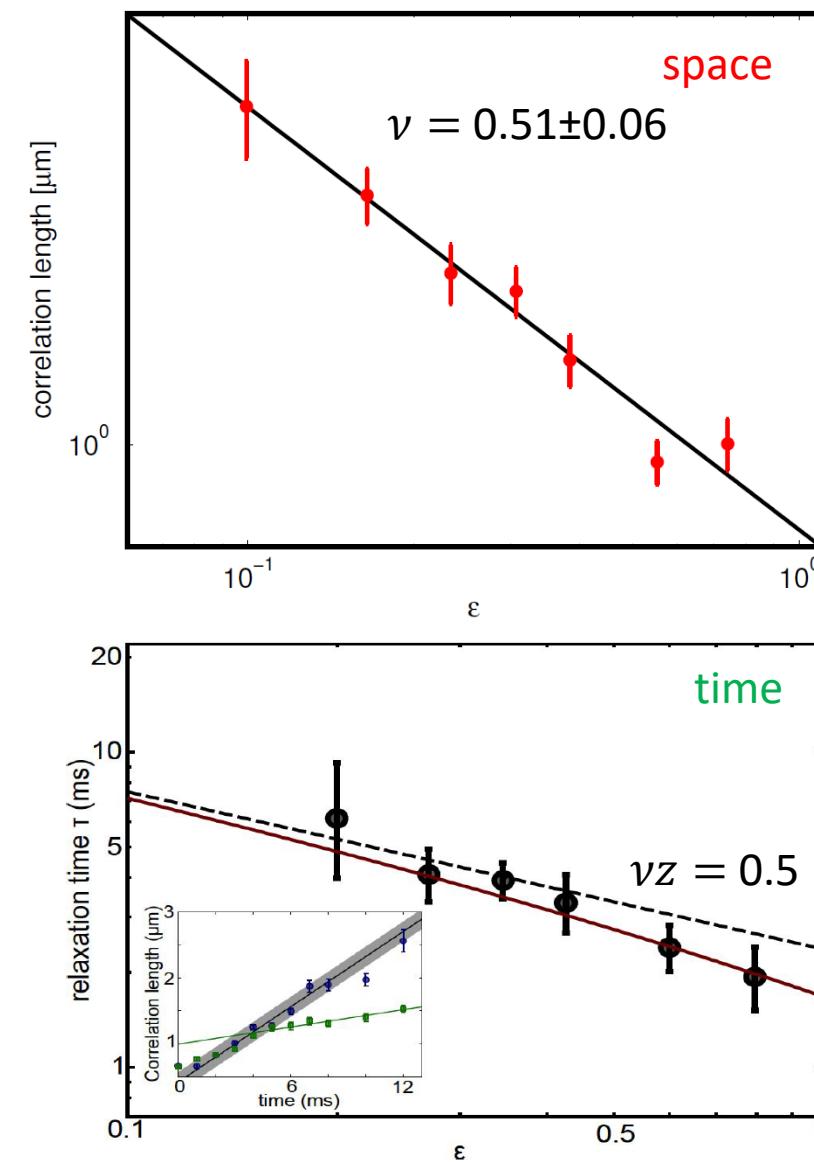
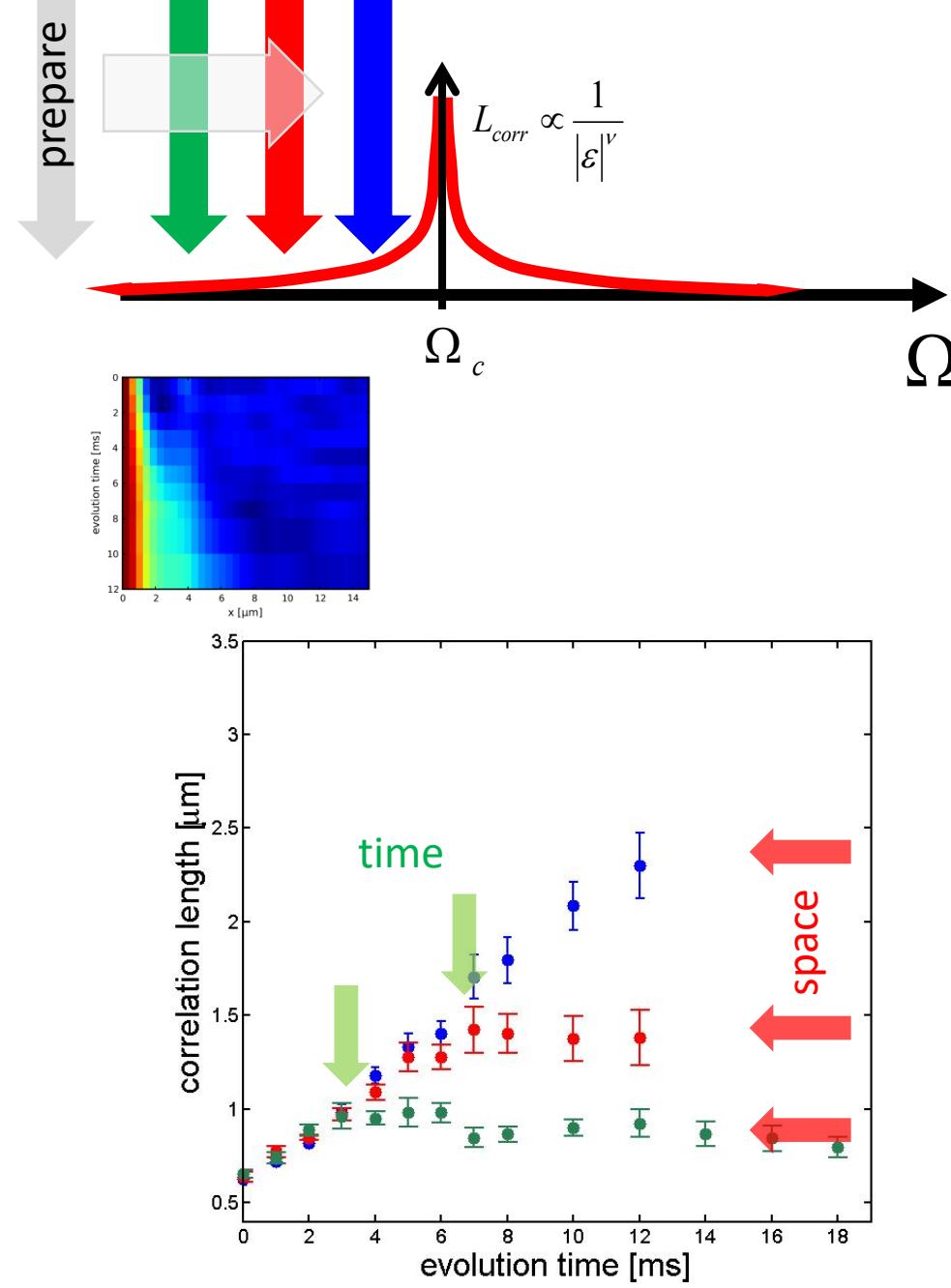


Temporal evolution of correlation functions



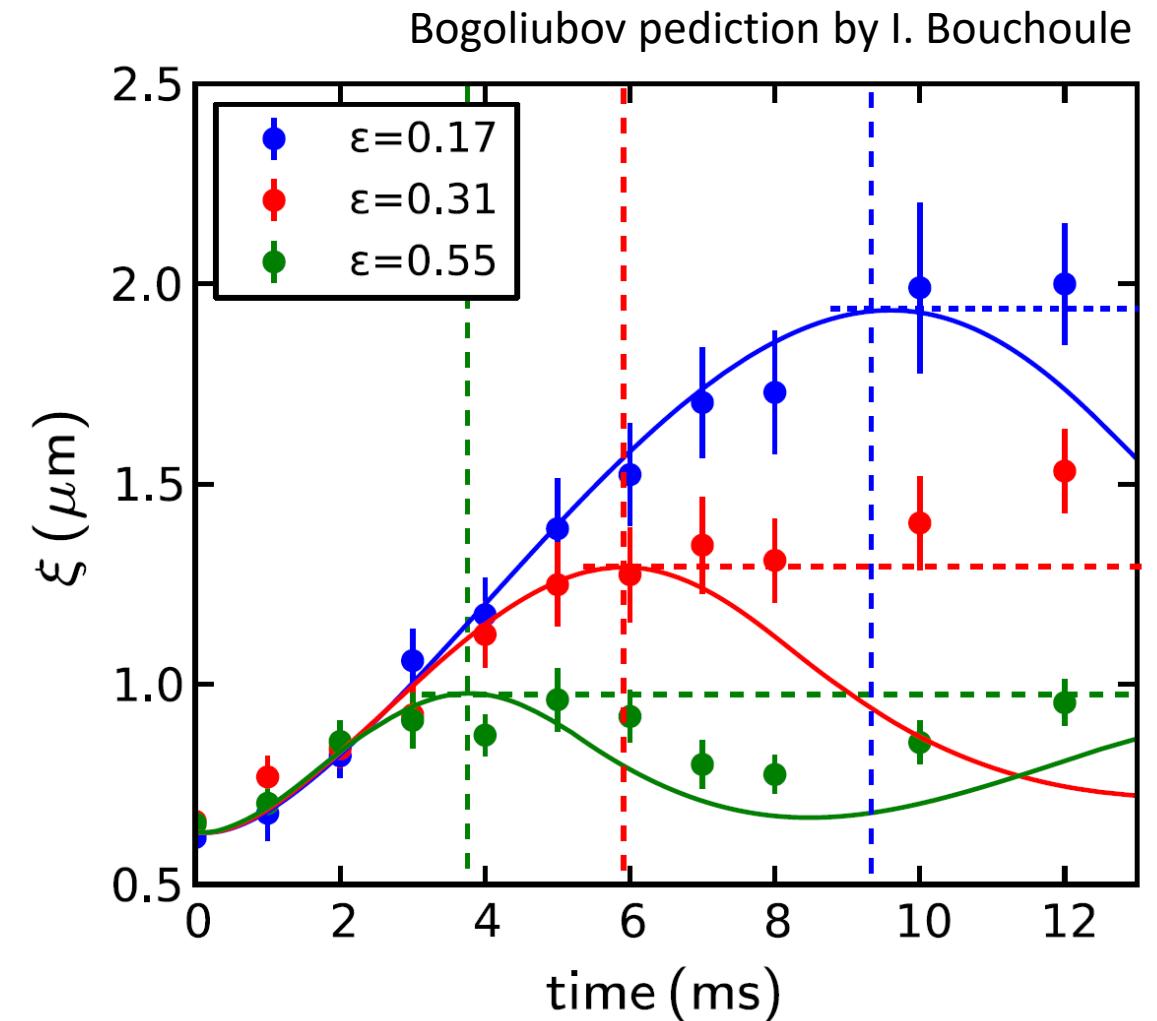
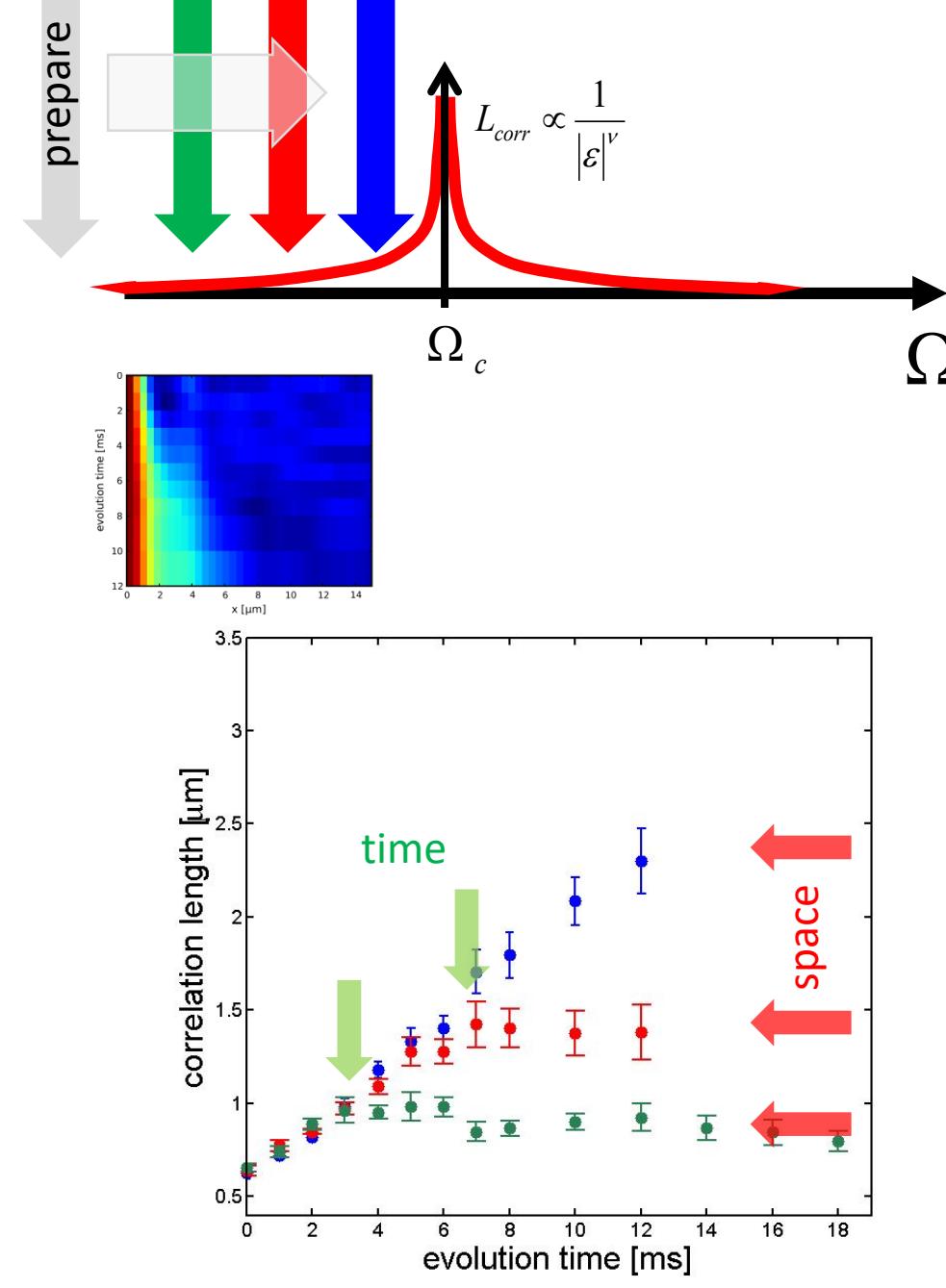
Scaling of correlation functions

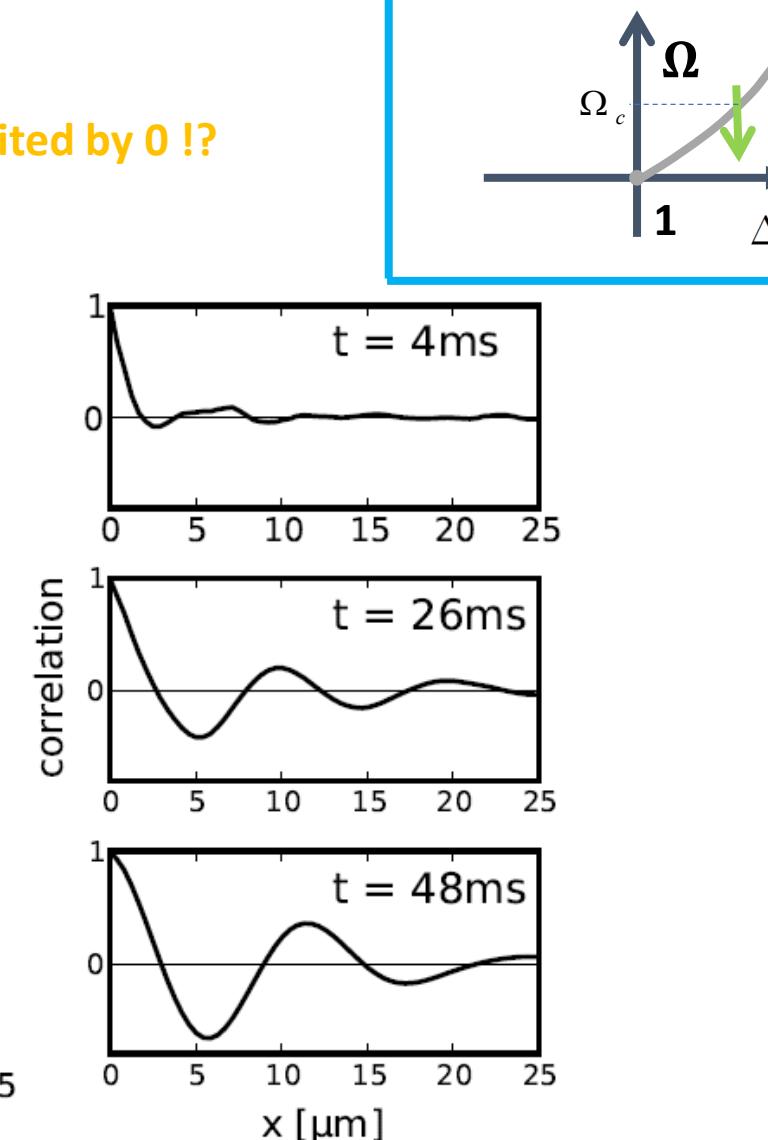
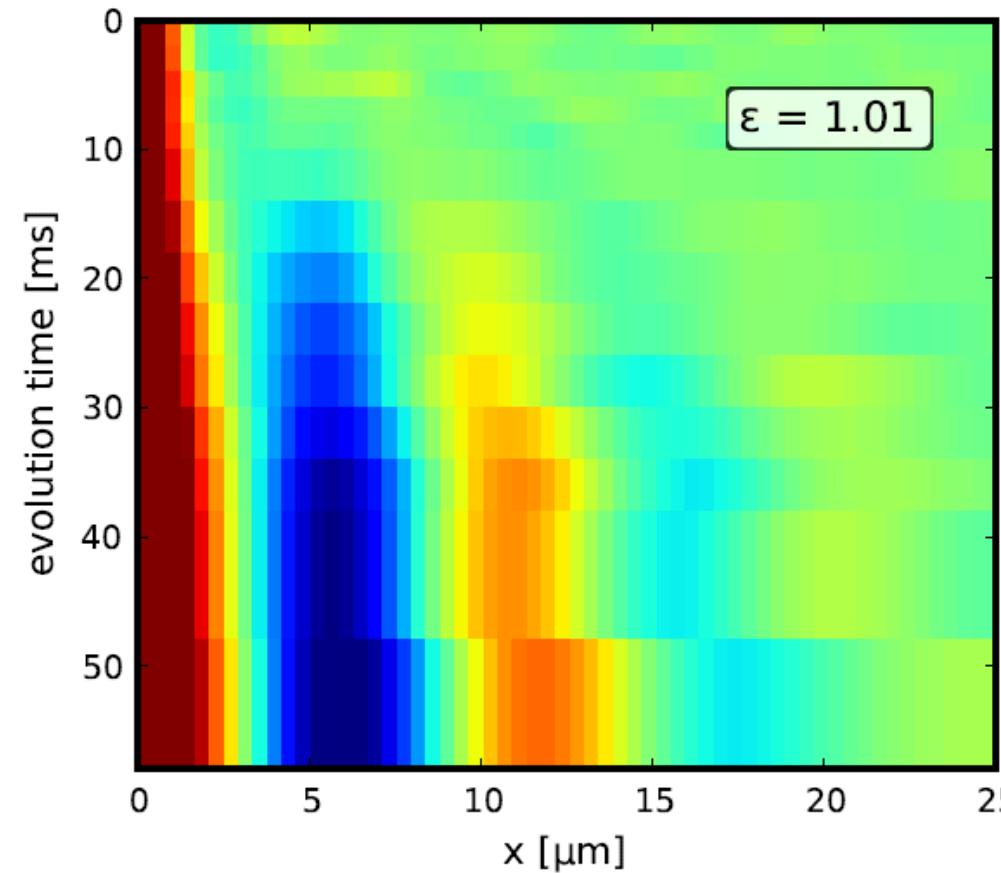
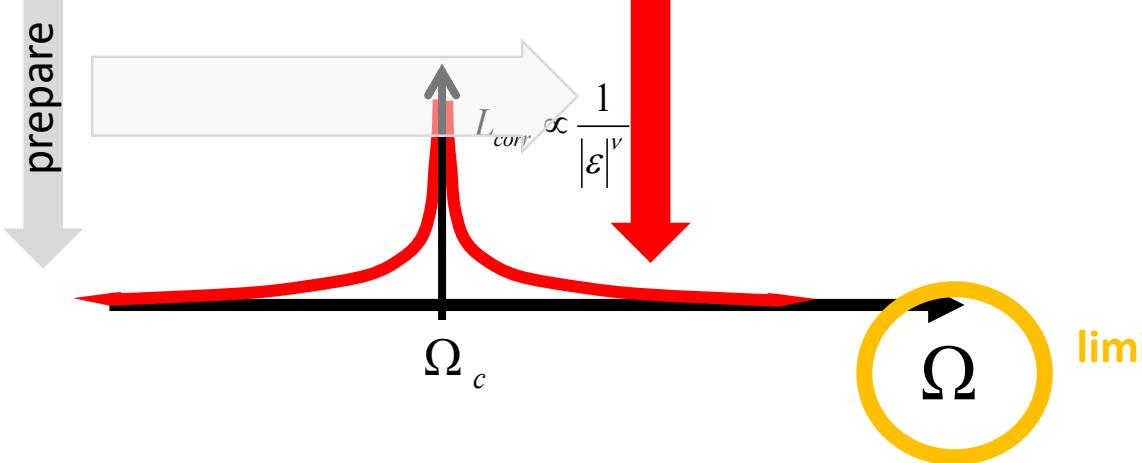
PRL 115, 245301 (2015)



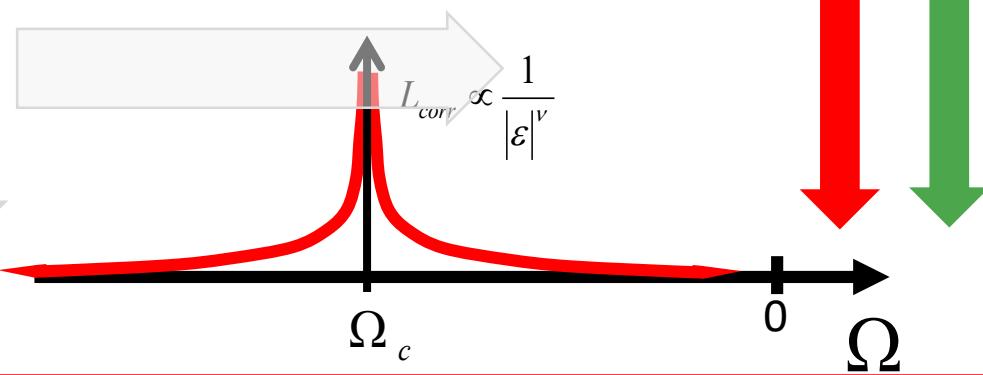
Scaling of correlation functions

PRL 115, 245301 (2015)

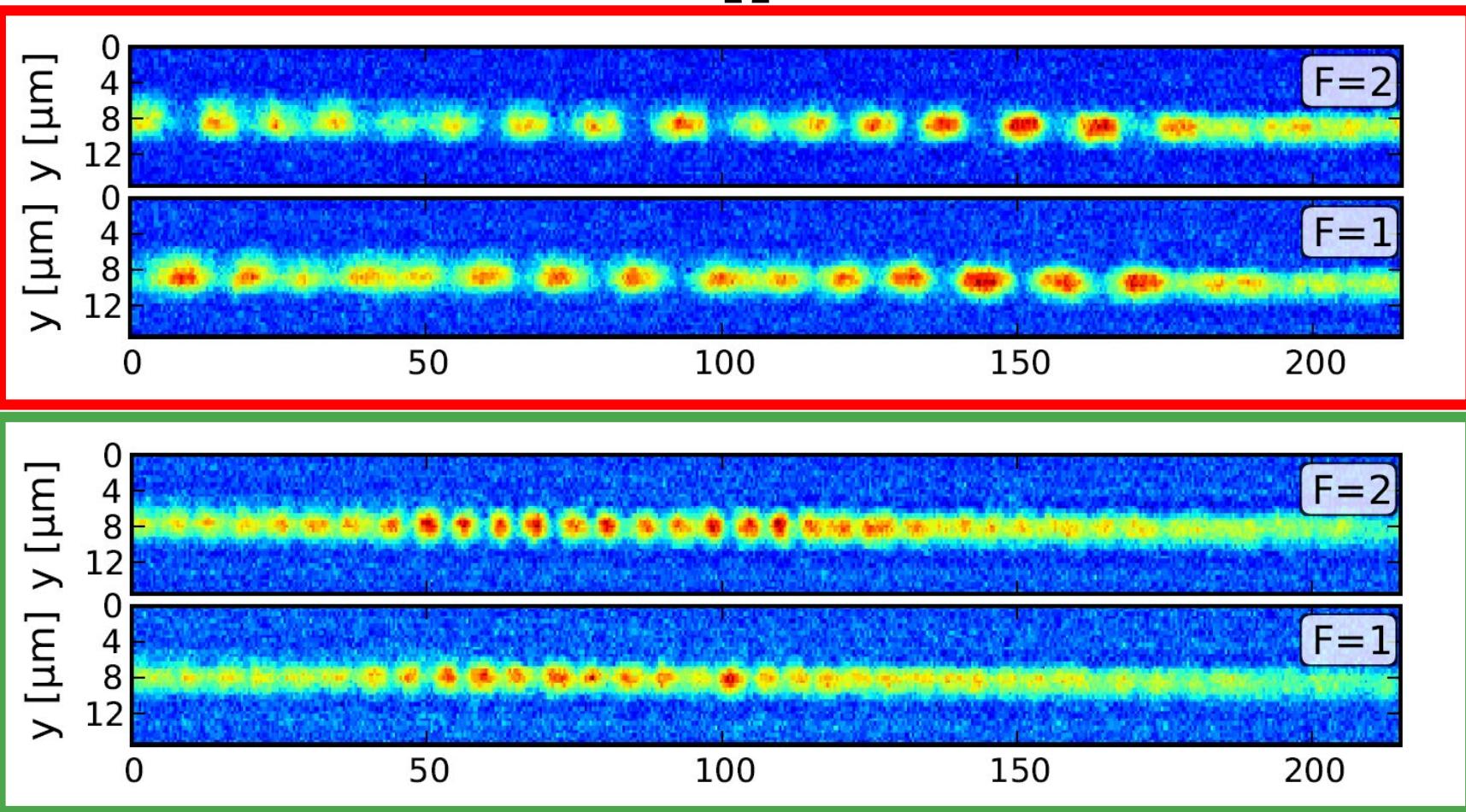




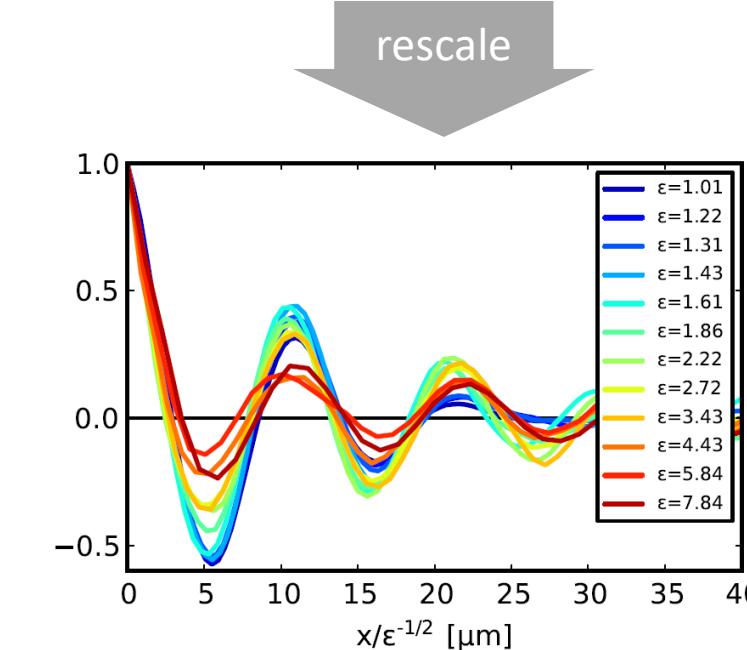
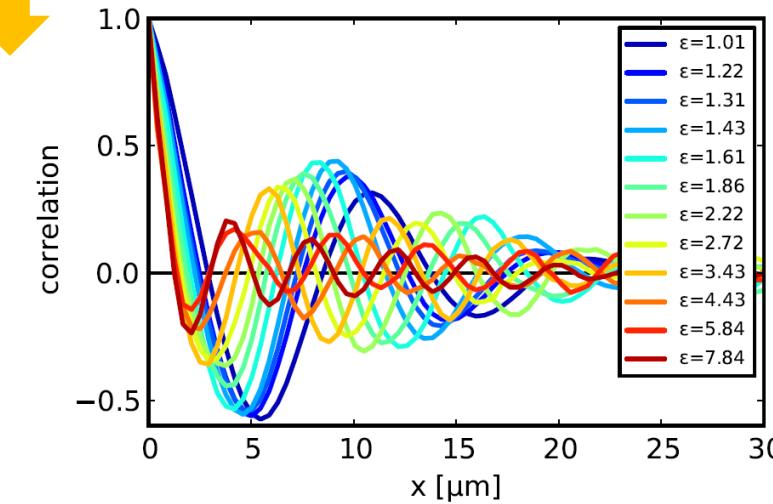
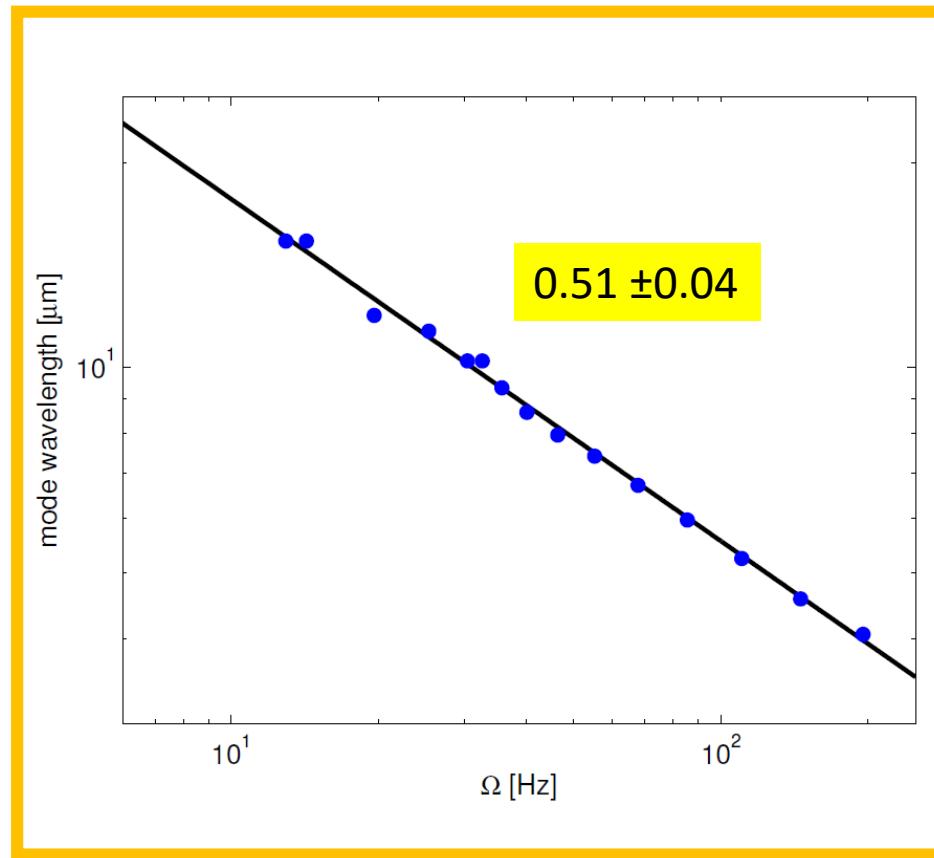
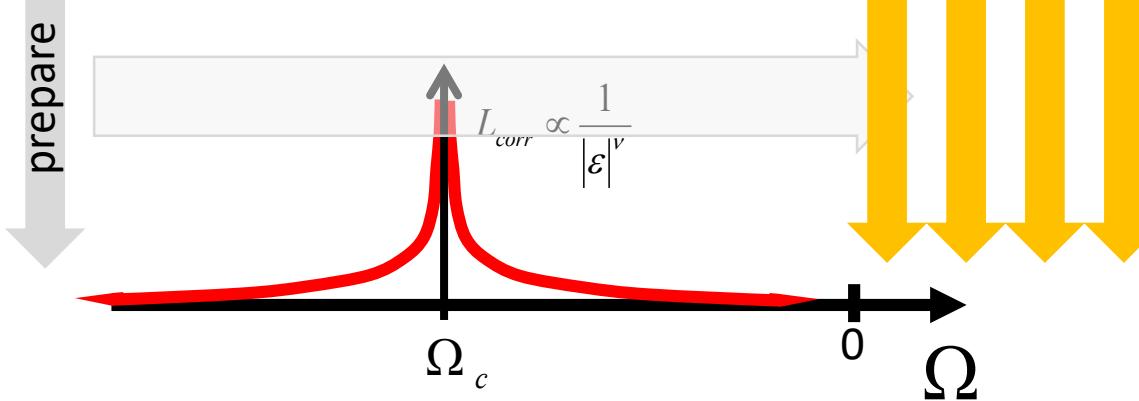
prepare



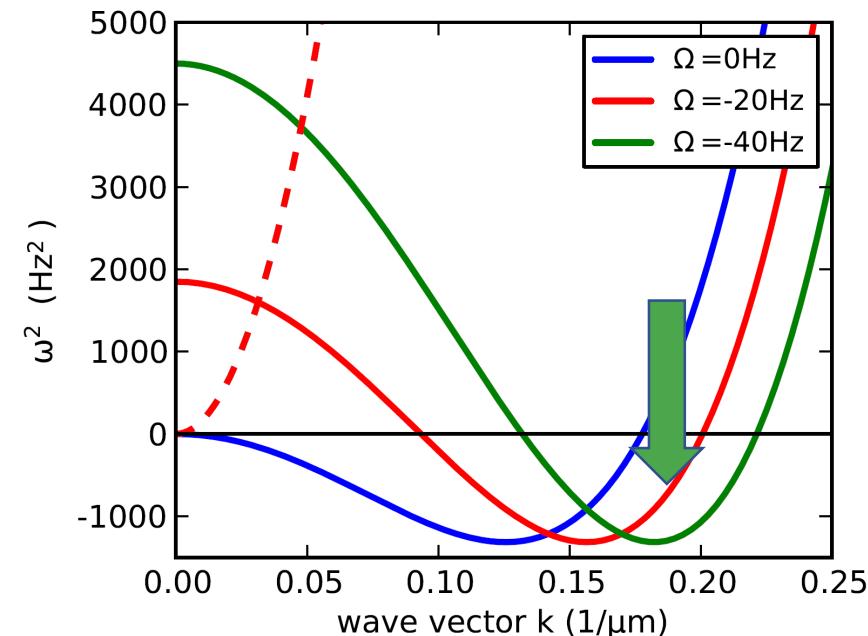
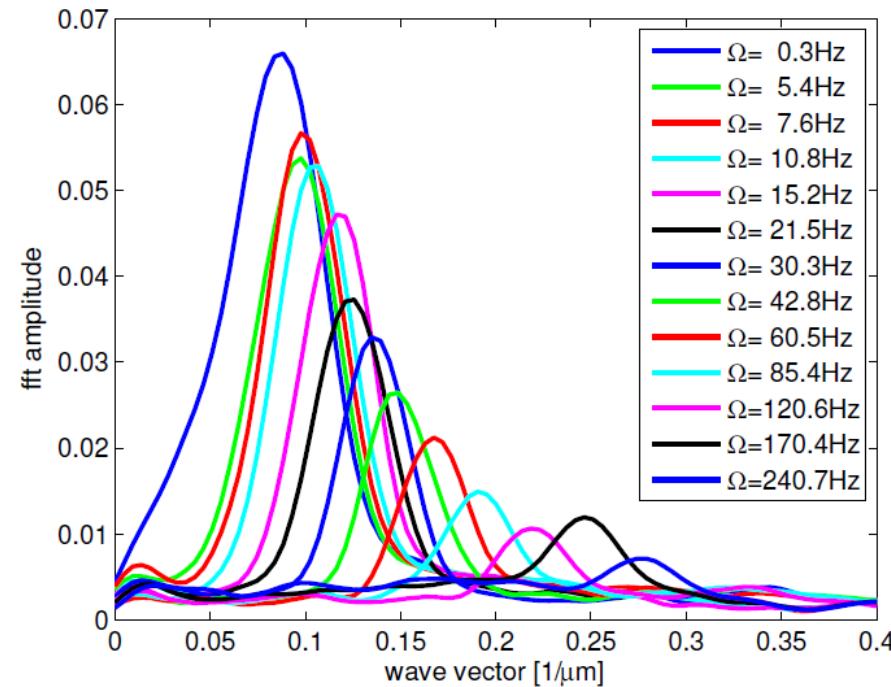
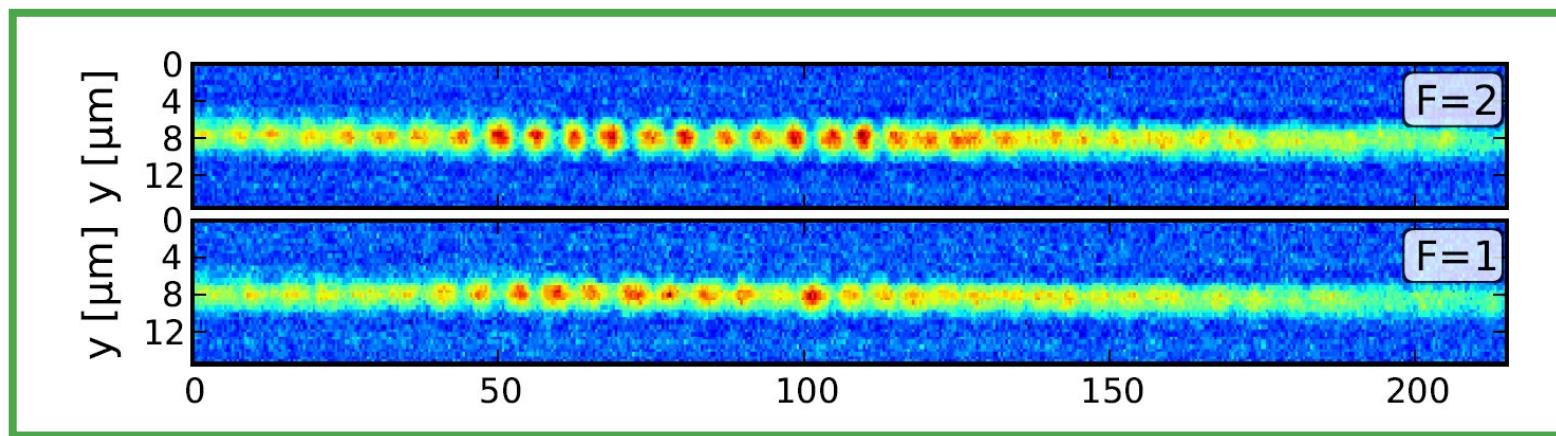
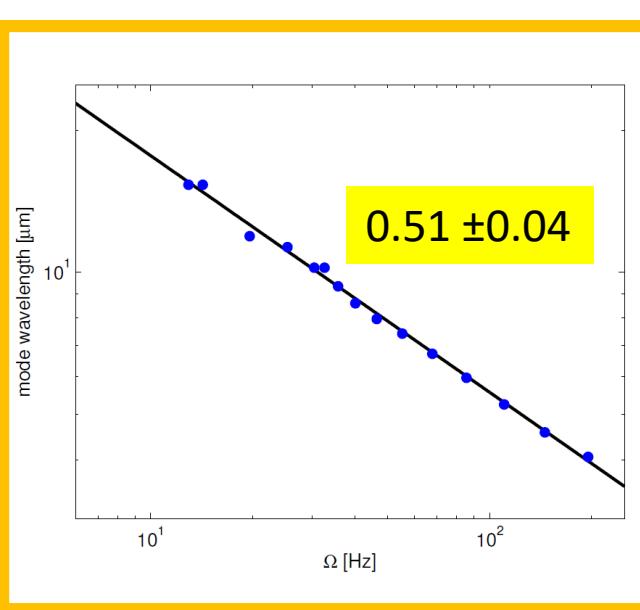
extension to positive Ω

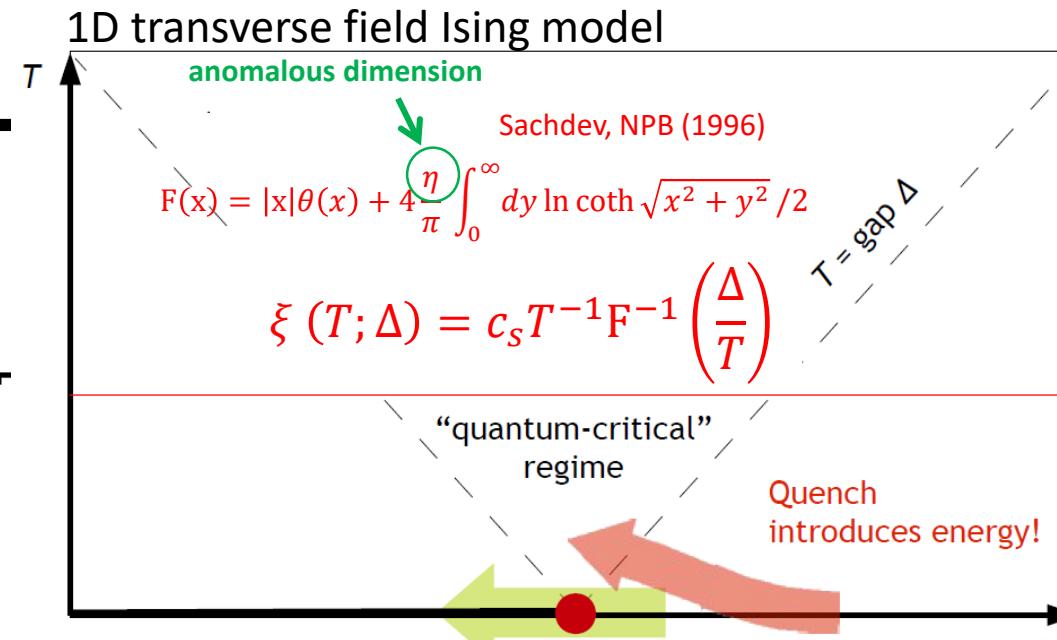
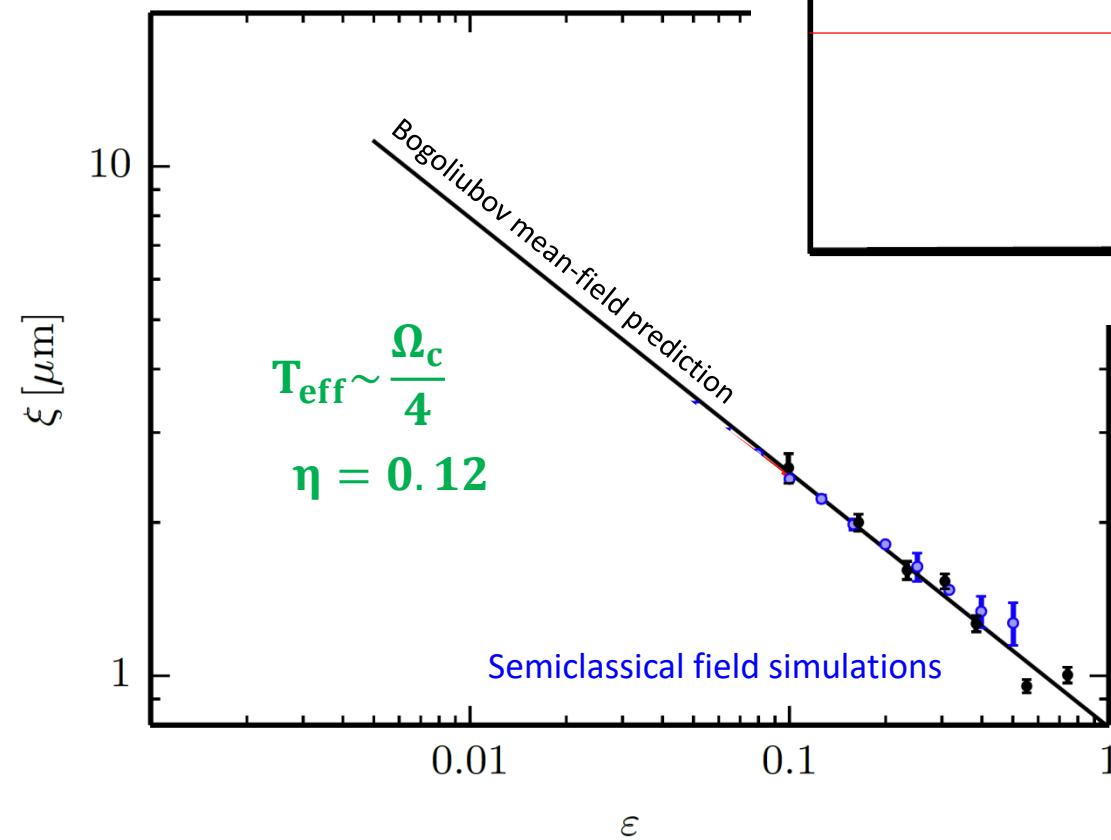
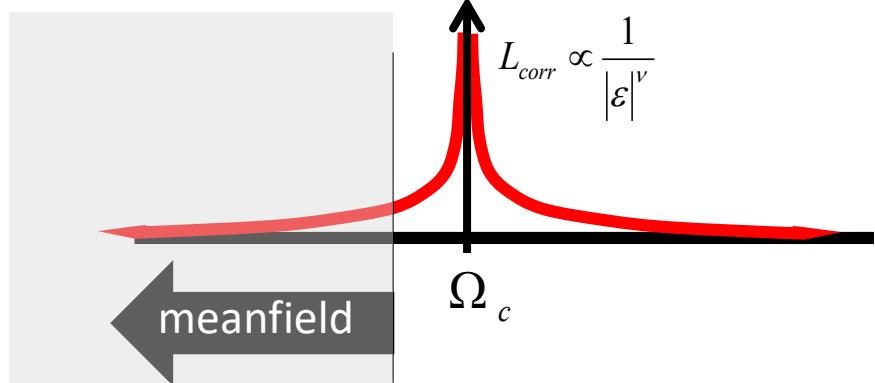


Scaling in the immiscible regime



Bogoliubov for positive and negative Ω

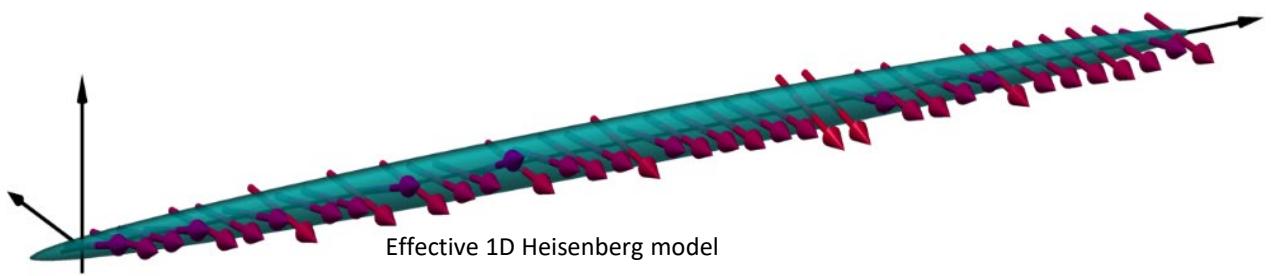




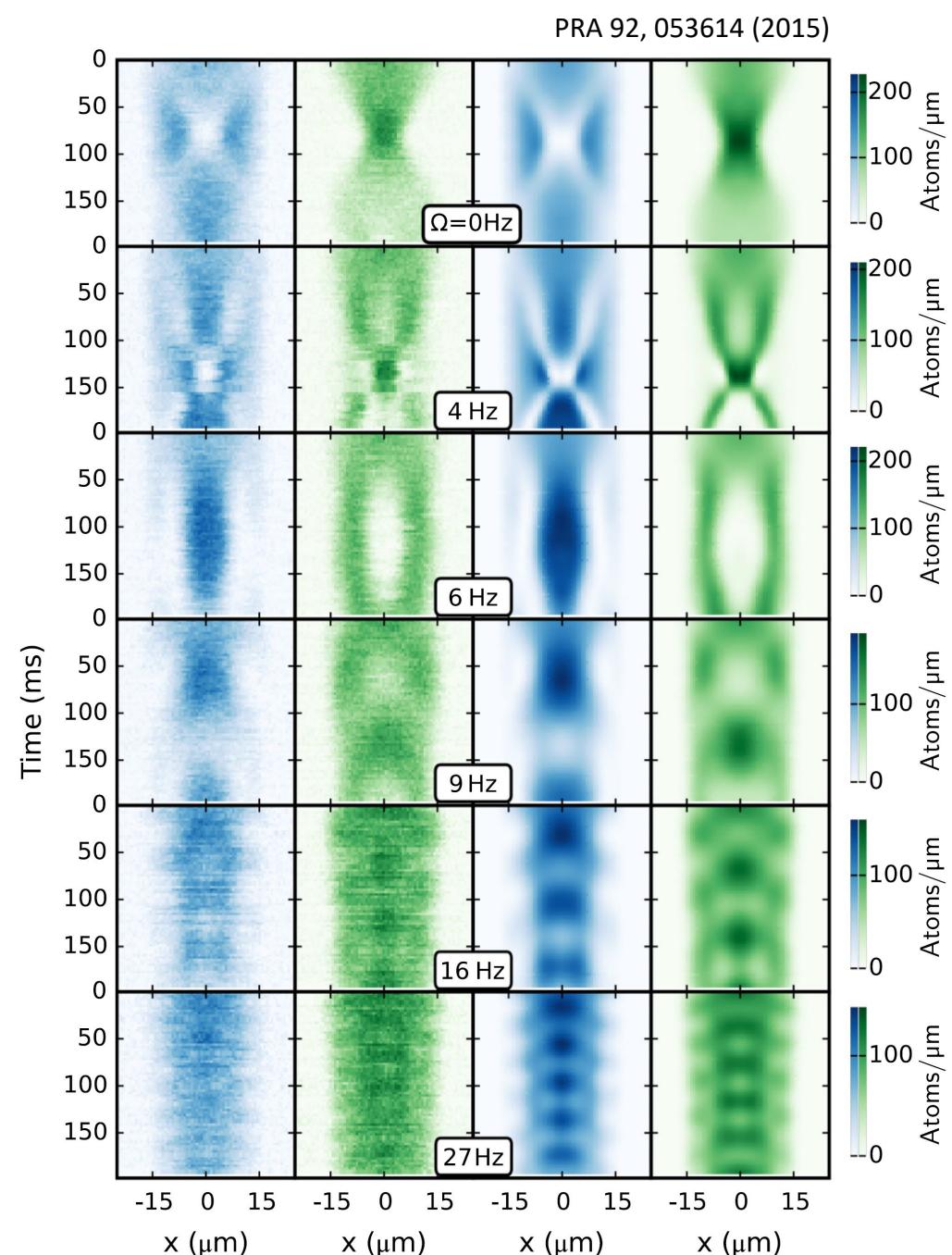
SUMMARY

Ultracold gas systems simulate classical
non-linear coupled equations with high precision

Two component interacting gases + linear coupling
additional experimentally accessible control

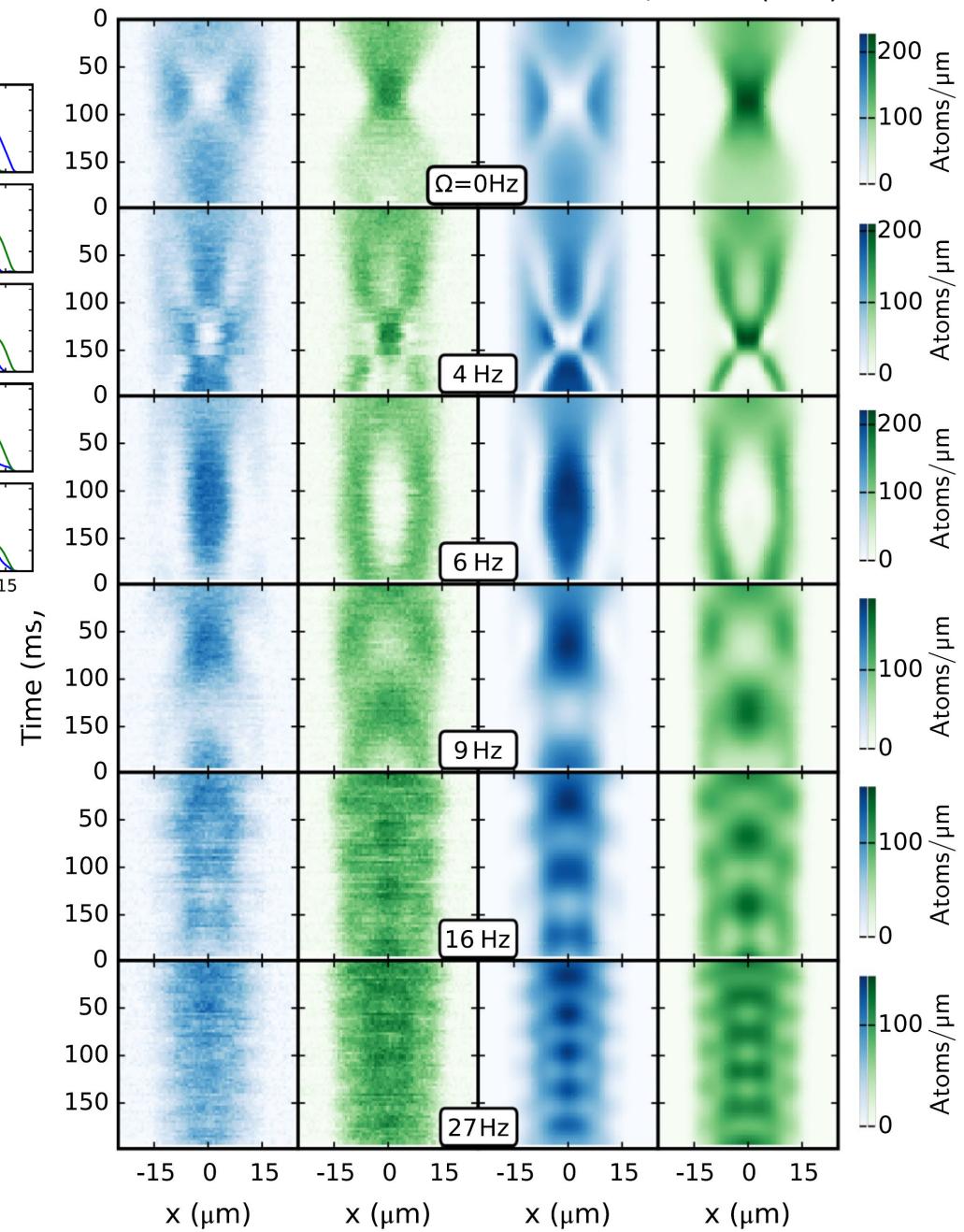
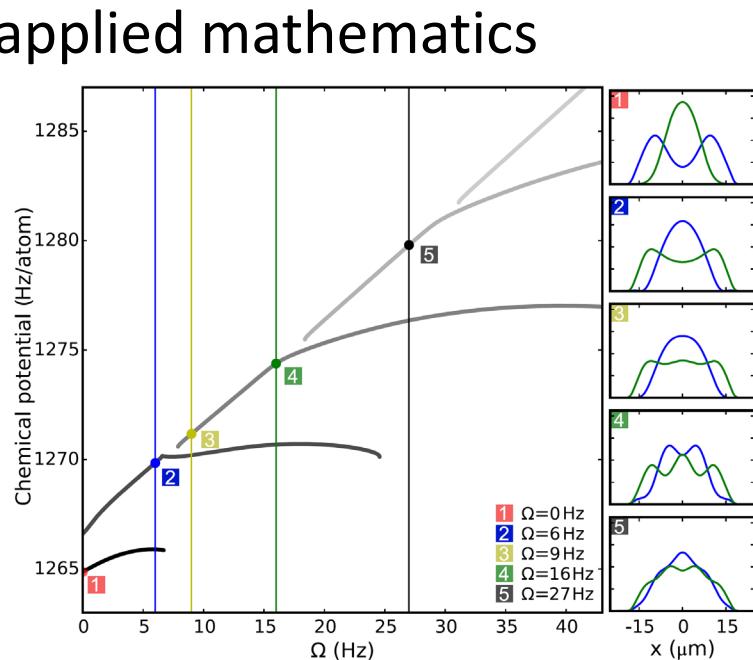
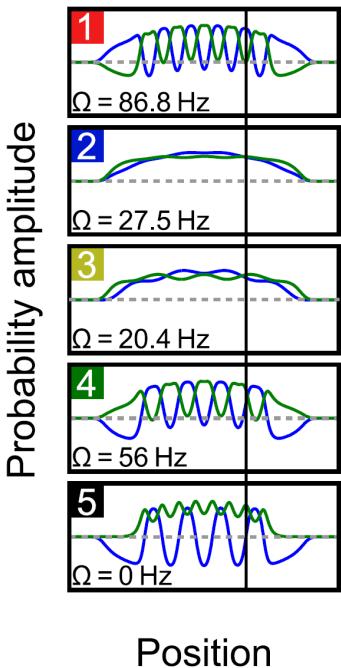
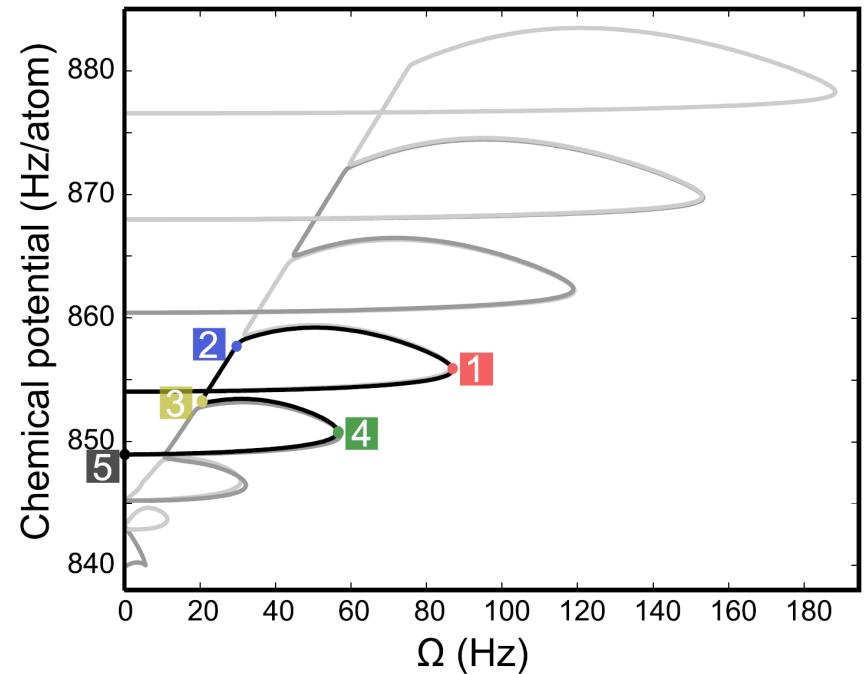


Many new developments:
Trento Group – Lectures by Giacomo Lamporesi
Bourdel Group – Poster 17, tomorrow – Alfred Hammond
Tarruell Group – Poster 9, yesterday – Craig Chisholm



Nonlinear dressed states – applied mathematics

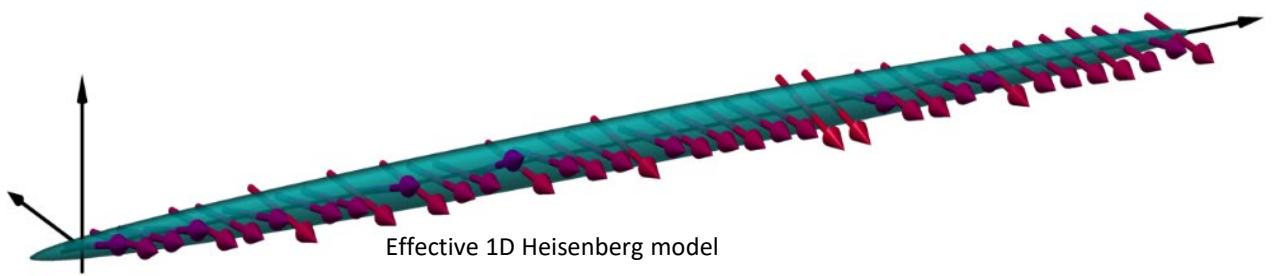
Panos Kevrekidis



SUMMARY

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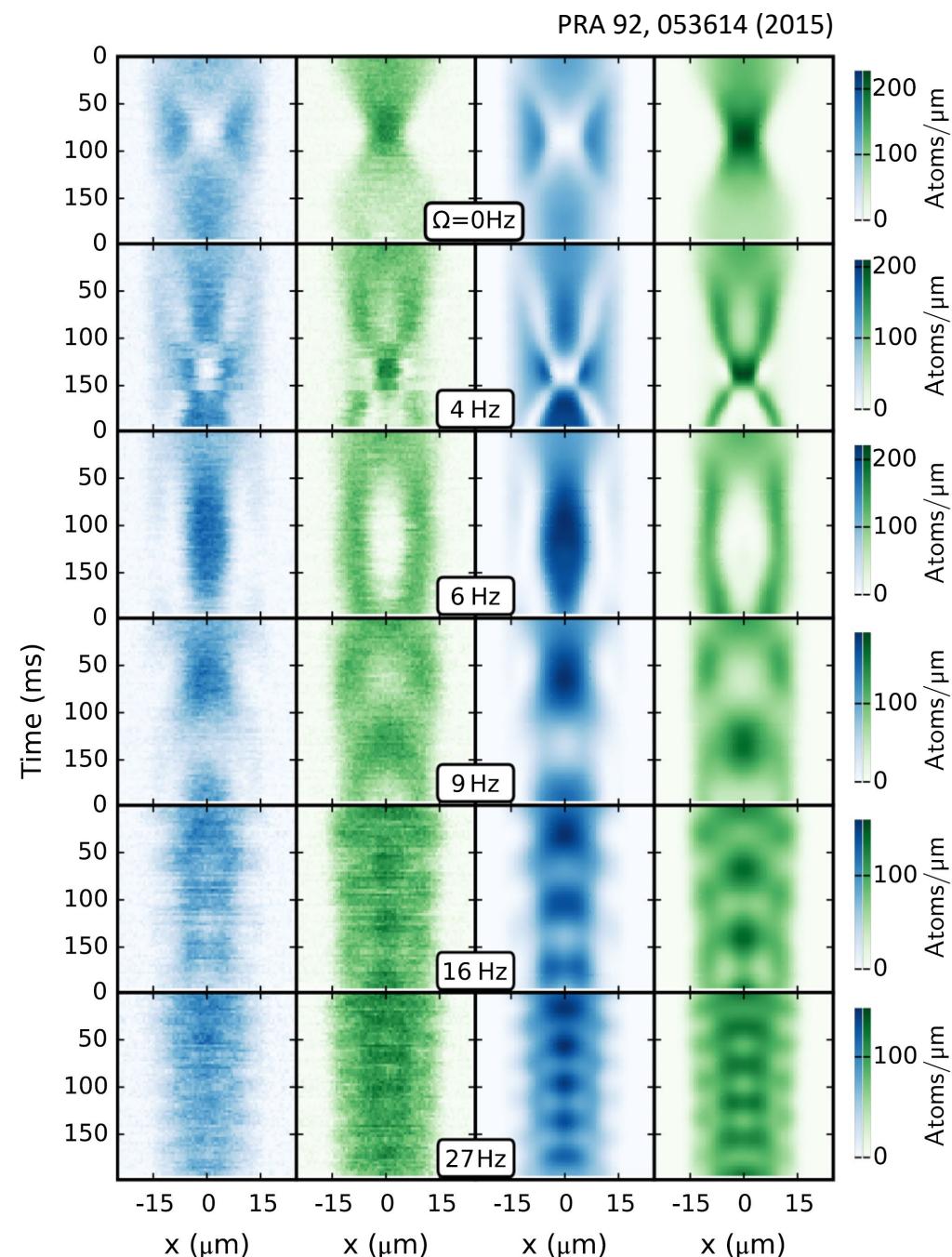


Many new developments:

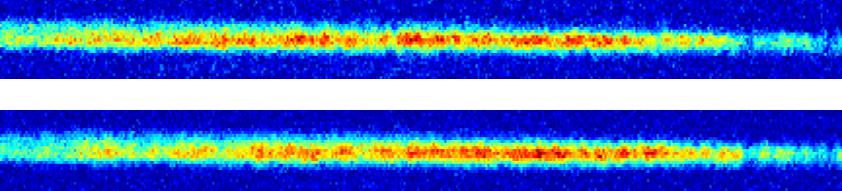
Trento Group – Lectures by Giacomo Lamporesi

Bourdel Group – Poster 17, tomorrow – Alfred Hammond

Tarruell Group – Poster 9, yesterday – Craig Chisholm



Ginzburg criterium



$$\begin{aligned} J_x &= 2\rho^{-1} \sqrt{\rho_\downarrow \rho_\uparrow} \cos \varphi \\ J_y &= 2\rho^{-1} \sqrt{\rho_\downarrow \rho_\uparrow} \sin \varphi \\ J_z &= \rho^{-1} (\rho_\downarrow - \rho_\uparrow) \end{aligned}$$

$$\begin{aligned} \varphi &= \varphi_\downarrow - \varphi_\uparrow \\ \phi_j &= \sqrt{\rho_j} \exp(i\varphi_j) \end{aligned}$$

$$H_0 = \sum_{i=\downarrow,\uparrow} \int dy \Phi_j^\dagger \left[-\frac{\hbar^2}{2m} \partial_y^2 + V(y) \right] \Phi_j,$$

$$H_{\text{cpl}} = \frac{\hbar}{2} \int dy \left[\Omega \left(\Phi_\downarrow^\dagger \Phi_\uparrow + \text{h.c.} \right) + \delta \left(\Phi_\downarrow^\dagger \Phi_\downarrow - \Phi_\uparrow^\dagger \Phi_\uparrow \right) \right],$$

$$H_{\text{int}} = \frac{1}{2} \sum_{i,j=\downarrow,\uparrow} g_{ij} \int dy \Phi_i^\dagger \Phi_i \Phi_j^\dagger \Phi_j.$$

Rewritten in spin operators

$$\begin{aligned} H = & \frac{1}{2} \int dy \left\{ m^{-1} (\partial_y \sqrt{\rho})^2 + \frac{\rho}{4m} |\partial_y \mathbf{J}|^2 + \rho (V + mv_{\text{eff}}^2) \right. \\ & \left. + g\rho^2 + \frac{g\rho^2}{2} (\alpha - 1) [1 - (J_z)^2] + \Omega\rho J_x + \delta\rho J_z \right\}. \end{aligned}$$

Assuming constant density

$$\mathcal{H} = \rho \left[|\partial_y \mathbf{J}|^2 / 4 + v_{\text{eff}}^2 + \Omega J_x - \Omega_c J_z^2 / 2 \right] / 2$$

Ginzburg criterium

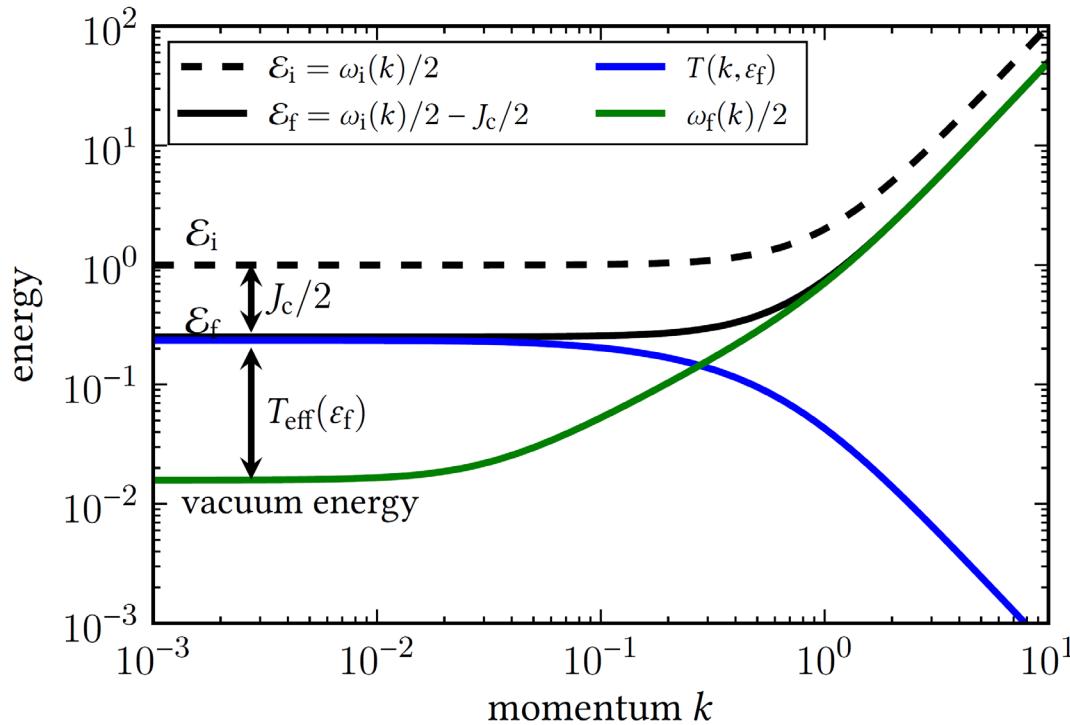
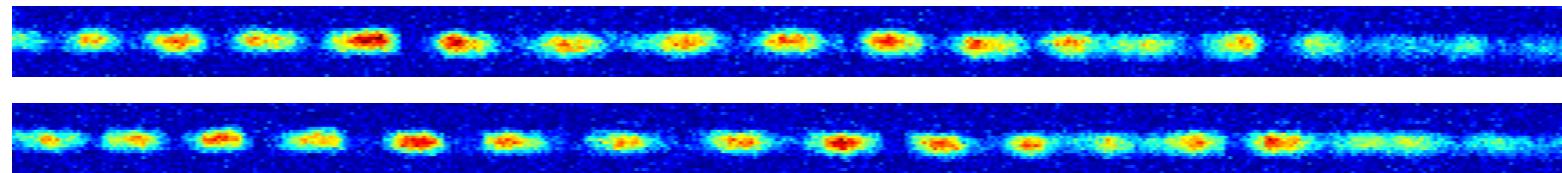
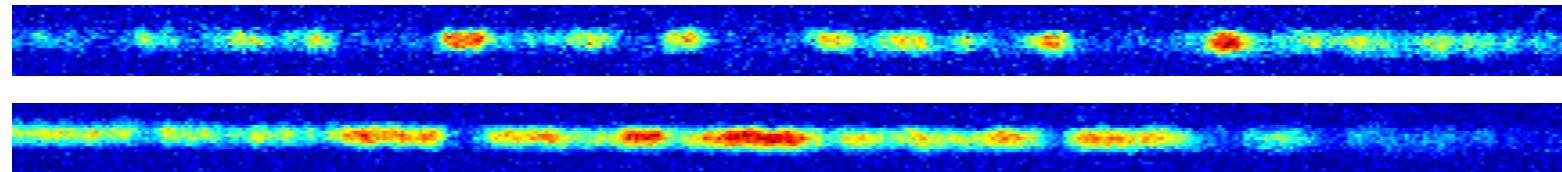
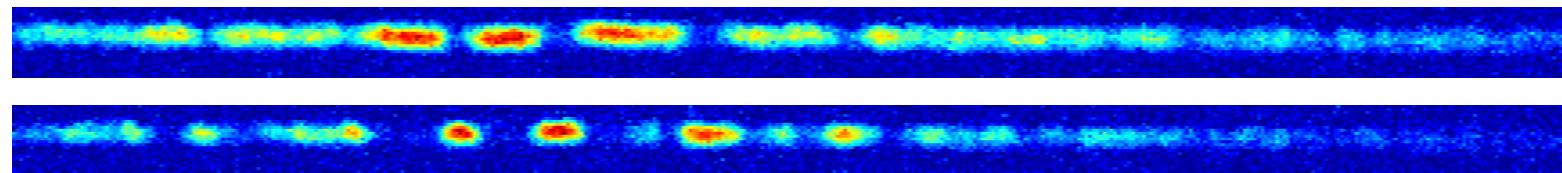


Figure III.6: The figure illustrates the energetic configuration before and after the quench at the example of a quench to $\epsilon = 0.001$, which leads to a thermal post-quench quasi-particle occupation. The infrared parts of the pre-quench energy spectrum (dashed black line), \mathcal{E}_i , which is identical to the pre-quench vacuum energy spectrum, the post-quench energy spectrum (solid black line), \mathcal{E}_i , the post-quench vacuum energy spectrum (solid green line), and, finally, the resulting mode temperatures (solid blue line). Details are discussed in the main text.

$$\xi(\hat{t}) = \xi_0 (\tau_Q / \tau_0)^{\nu/(1+\nu z)}$$

 $\tau_Q = 1ms$  $\tau_Q = 5ms$  $\tau_Q = 10ms$  $\tau_Q = 30ms$ 