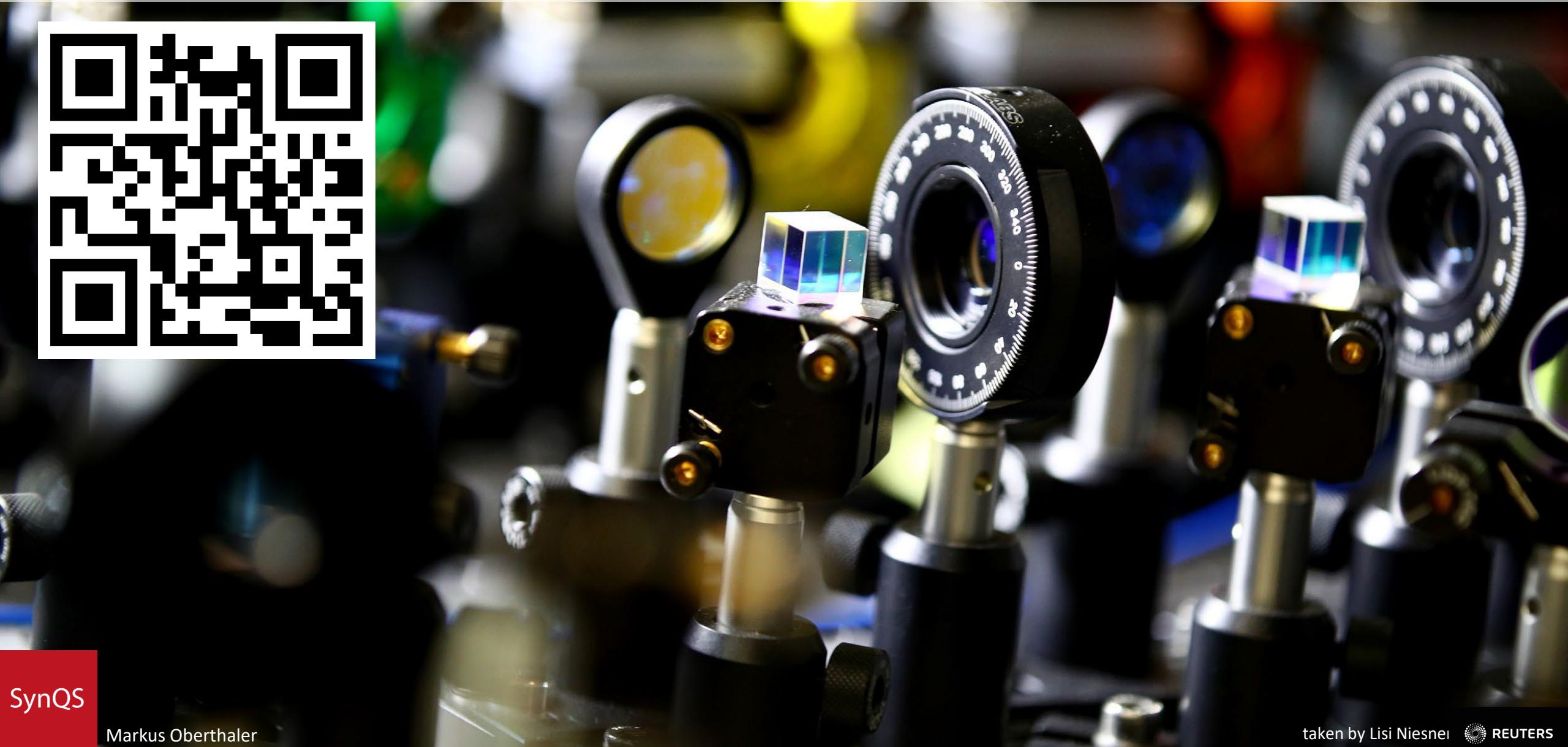
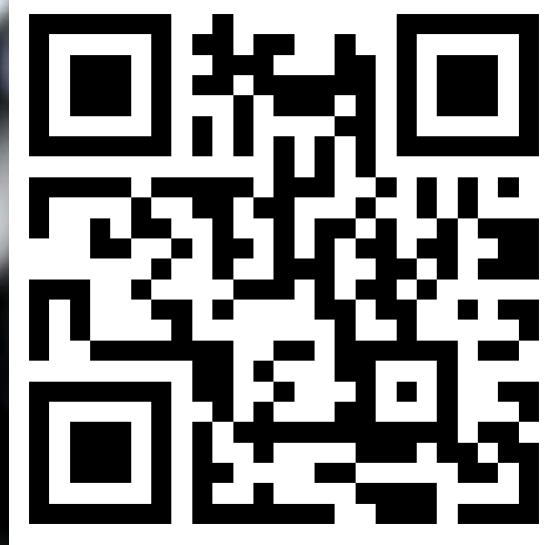


Quantum Mixtures @ Synthetic Quantum Systems/Heidelberg University

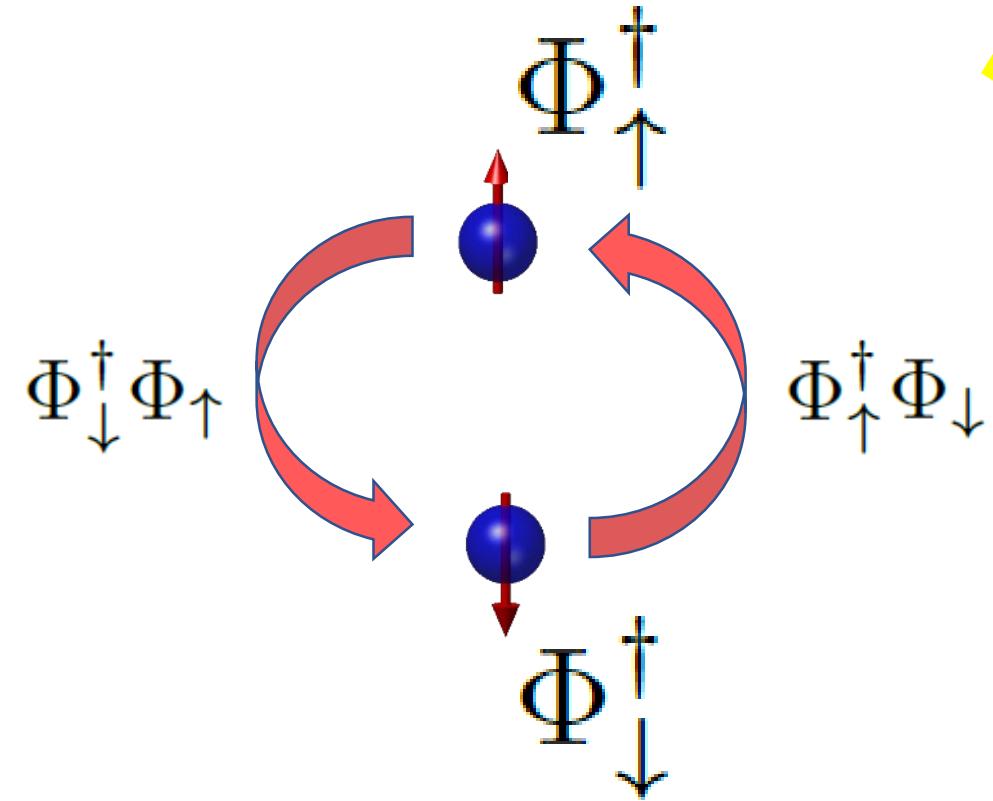


SynQS

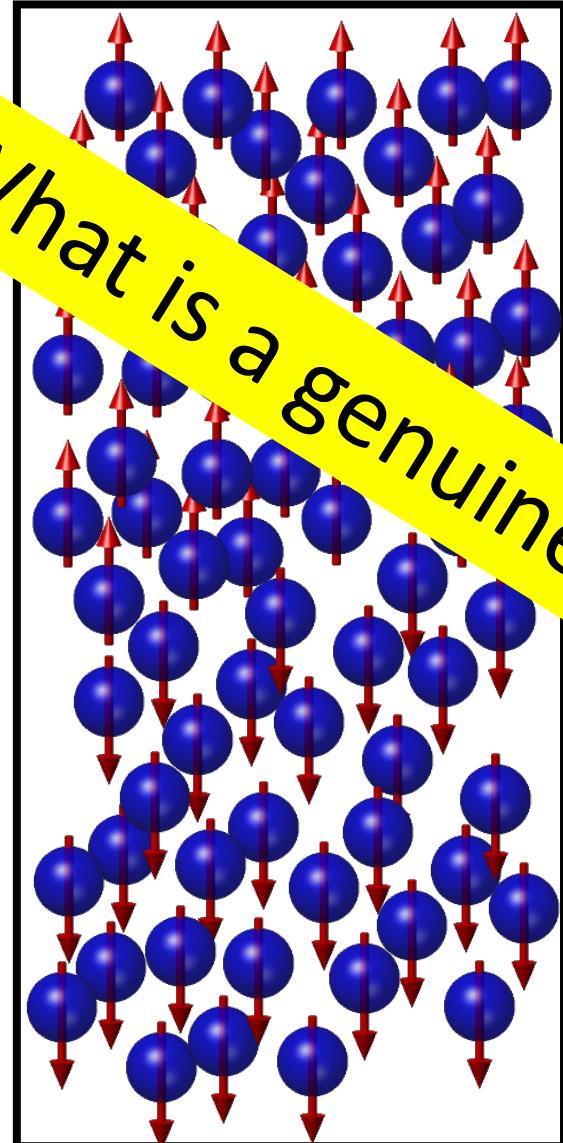
Markus Oberthaler

taken by Lisi Niesner 

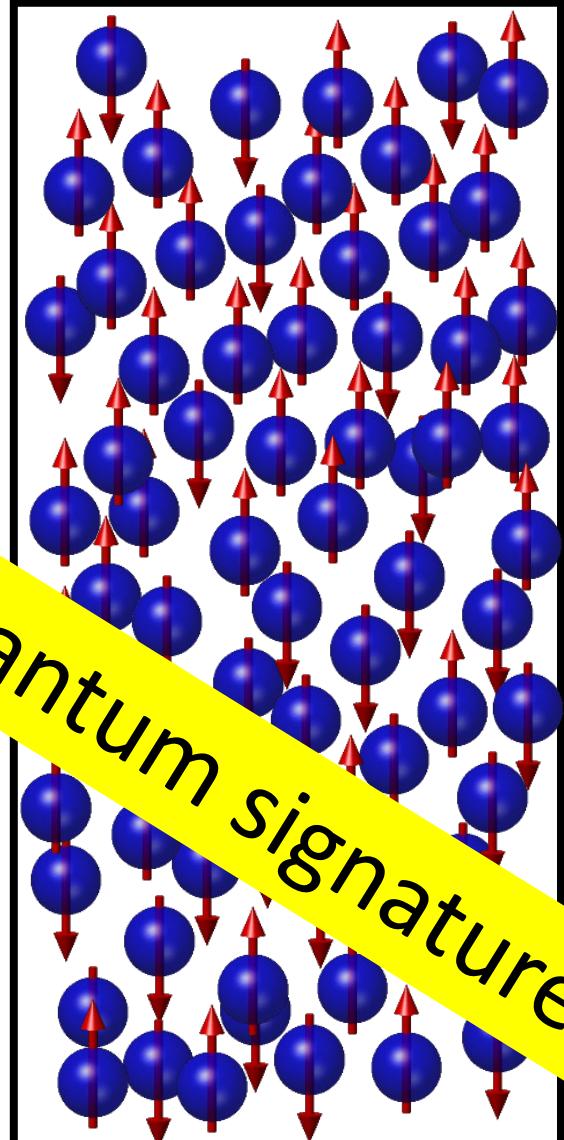
Coherent Quantum Mixtures



phase separation - immiscible

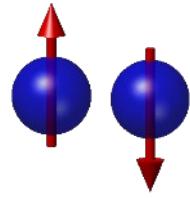


coexistence - miscible



What is a genuine quantum signature ?

Coherent mixtures with increasing complexity

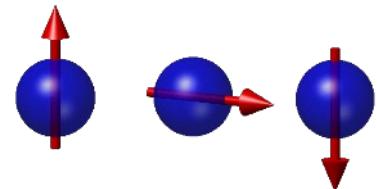


pseudo spin $\frac{1}{2}$

one dimensional

miscible to immiscible transition via linear coupling

critical scaling after a quench towards a quantum critical point



spin 1

non-local entanglement

EPR correlation from 0d to 1d

one dimensional

POVM readout

universal dynamics

topological excitations such as solitons

Question

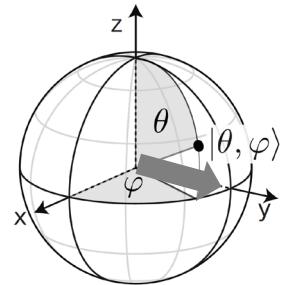
Experimental
details

Please
bear with me

Genuine quantum feature: Entanglement

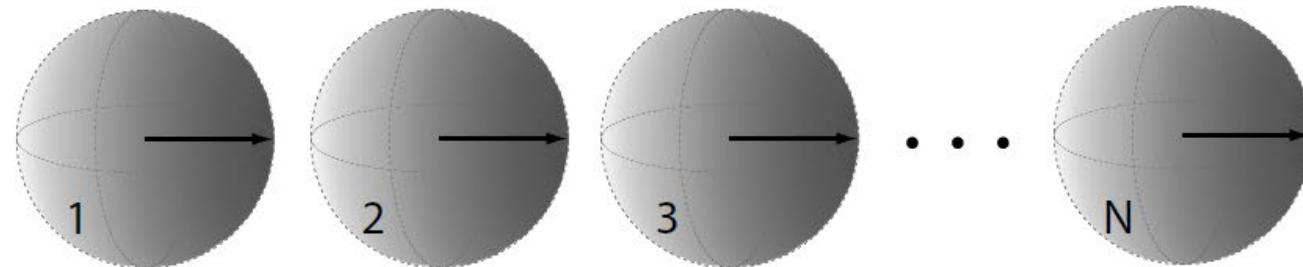
$$\rho \neq \sum_k p_k \rho_k^{(1)} \otimes \rho_k^{(2)} \otimes \cdots \otimes \rho_k^{(N)}$$

separable density matrix

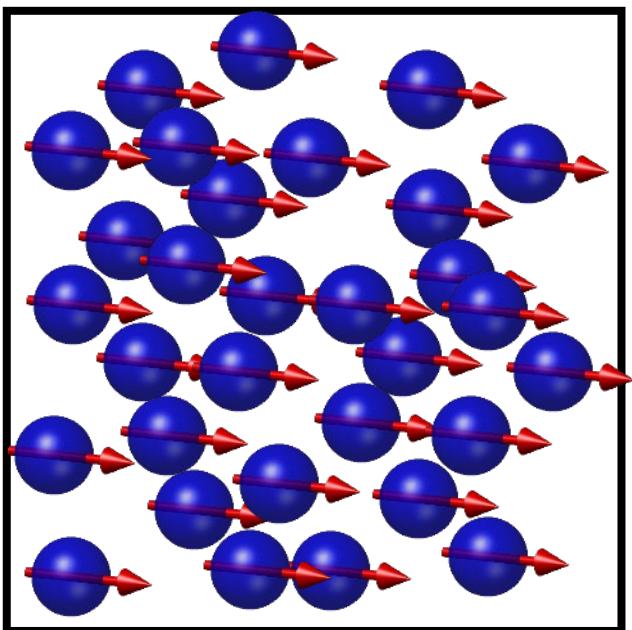


$$\sigma^{(i)} = \frac{1}{2}$$

$$\Delta\sigma_{\perp}^{(i)} = \frac{1}{2}$$



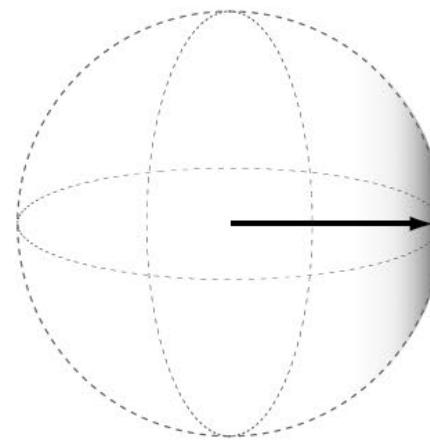
Indistinguishable uncorrelated bosons



$$J_{x,y,z} = \sum_i^N S_{x,y,z}^{(i)}$$

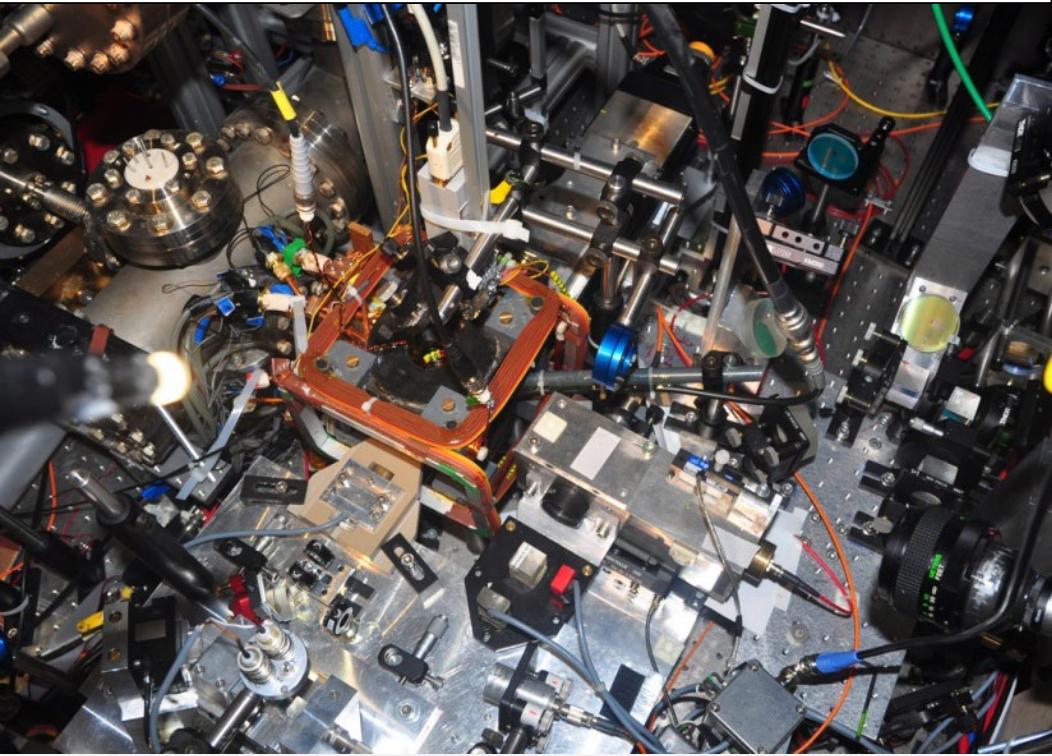
Schwinger – collective spin:

Suppression of fluctuations as witness for entanglement

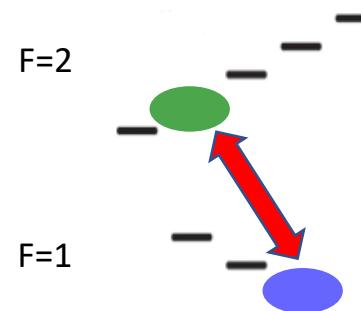


$$J = N \cdot \frac{1}{2}$$

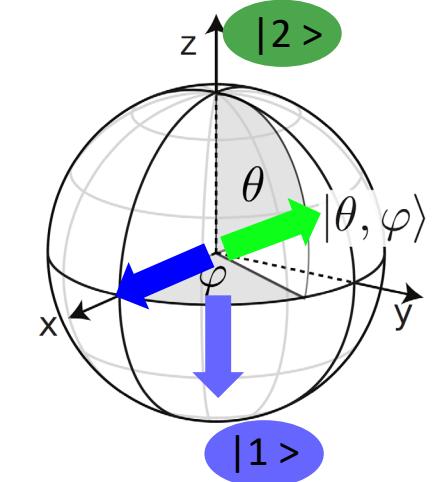
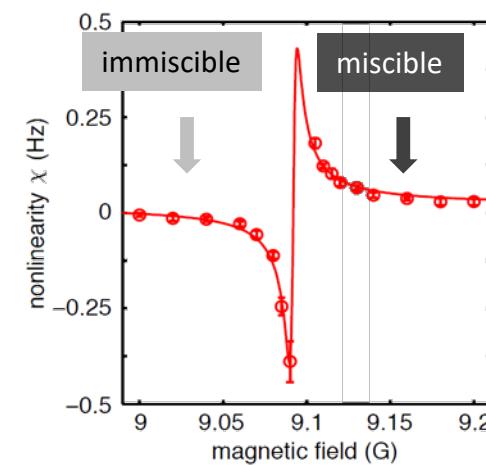
Vacuum since 2000, with one day break



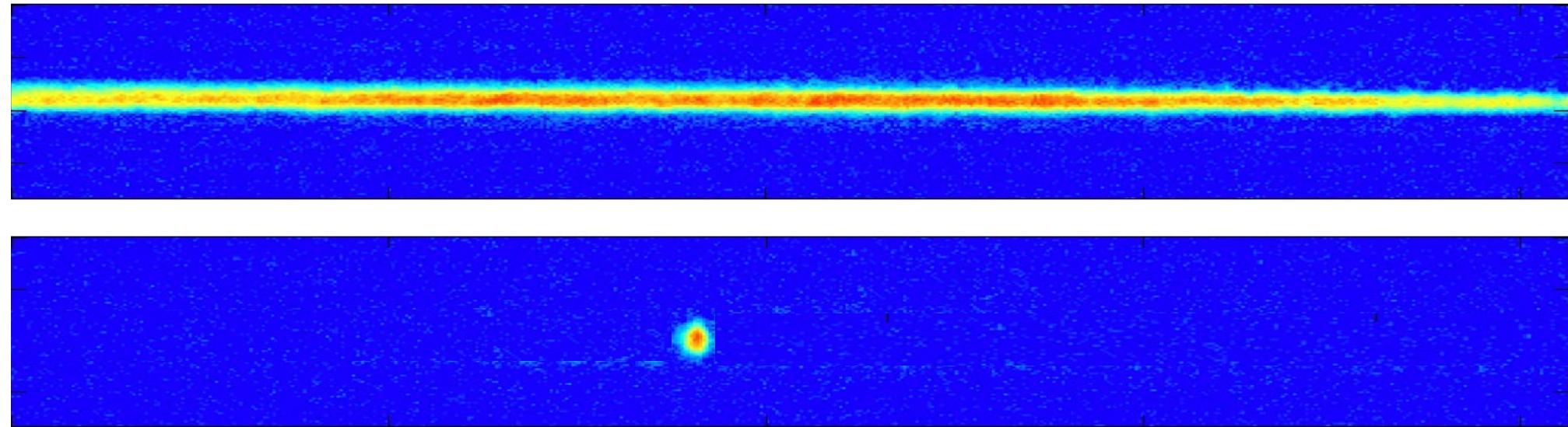
Rubidium BEC



Feshbach resonance

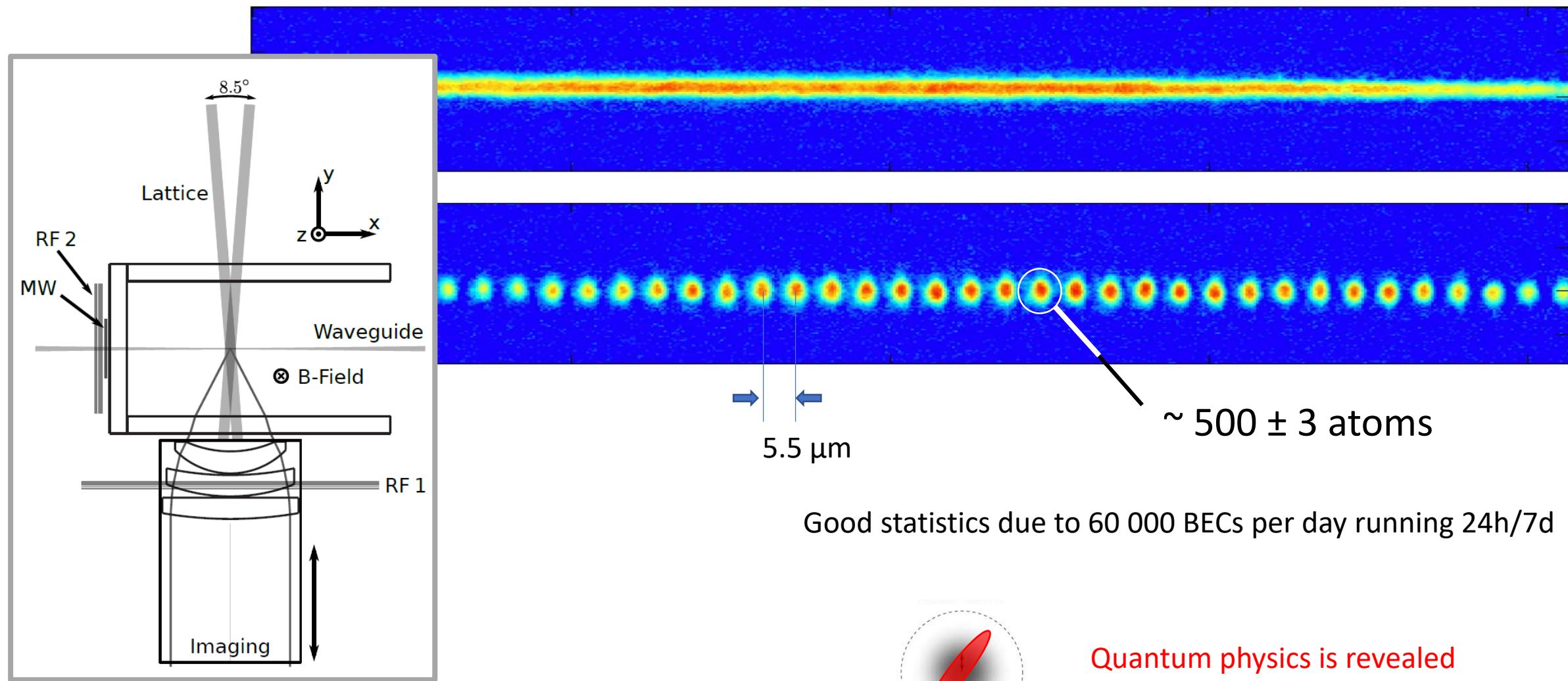


1D many mode situation → 0D single mode situation

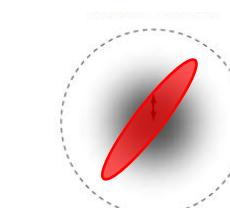


Experimental details

1D many mode situation → many 0D realizations

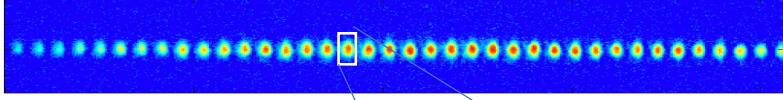


Experimental details



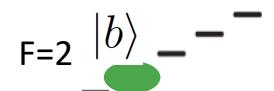
Quantum physics is revealed
by characteristic fluctuations

$$\Delta J_{\perp} = \sqrt{N/2}$$



single mode/ 0d approximation without coherent coupling

Rubidium BEC



$$\mathcal{H} = \mathcal{H}_a + \mathcal{H}_b + \mathcal{H}_{ab}$$



$$\mathcal{H}_a = \int d^3\mathbf{x} \hat{\psi}_a^\dagger(\mathbf{x}) \left(-\frac{\hbar^2}{2m} \nabla^2 + V_a(\mathbf{x}) + \frac{4\pi a_{aa}}{2m} \hat{\psi}_a^\dagger(\mathbf{x}) \hat{\psi}_a(\mathbf{x}) \right) \hat{\psi}_a(\mathbf{x})$$

$$\mathcal{H}_{ab} = \frac{4\pi a_{ab}}{m} \int d^3\mathbf{x} \hat{\psi}_a^\dagger(\mathbf{x}) \hat{\psi}_b^\dagger(\mathbf{x}) \hat{\psi}_a(\mathbf{x}) \hat{\psi}_b(\mathbf{x}).$$

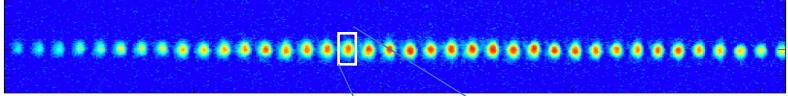
0d assumption

$$\begin{aligned}\hat{\psi}_b^\dagger(\mathbf{x}) &= \hat{a}_b^\dagger \phi_b(\mathbf{x}) \\ \hat{\psi}_a^\dagger(\mathbf{x}) &= \hat{a}_a^\dagger \phi_a(\mathbf{x})\end{aligned}$$

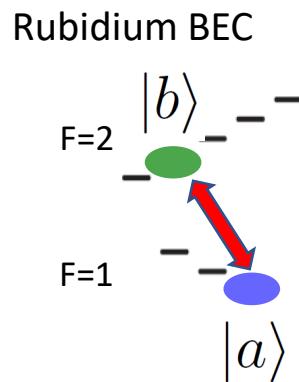
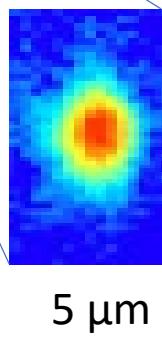
$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$$

$$\begin{aligned}\mathcal{H}_a + \mathcal{H}_b &= \frac{1}{2} \int d^3\mathbf{x} \left[\phi_a(\mathbf{x}) \omega_a \phi_a(\mathbf{x}) \hat{a}_a^\dagger \hat{a}_a + g_{aa} |\phi_a(\mathbf{x})|^4 \hat{a}_a^\dagger \hat{a}_a^\dagger \hat{a}_a \hat{a}_a \right. \\ &\quad \left. + \phi_b(\mathbf{x}) \omega_b \phi_b(\mathbf{x}) \hat{a}_b^\dagger \hat{a}_b + g_{bb} |\phi_b(\mathbf{x})|^4 \hat{a}_b^\dagger \hat{a}_b^\dagger \hat{a}_b \hat{a}_b \right],\end{aligned}$$

$$\mathcal{H}_{ab} = \int d^3\mathbf{x} g_{ab} |\phi_a(\mathbf{x})|^2 |\phi_b(\mathbf{x})|^2 \cdot \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_a \hat{a}_b$$



single mode/ 0d approximation with coherent coupling



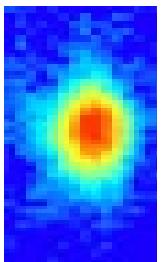
$$\tilde{\omega}_i = \int d^3\mathbf{x} \phi_i(\mathbf{x}) \omega_i \phi_i(\mathbf{x}), \quad \omega_i = -\frac{\hbar^2}{2m} \nabla^2 + V_i(\mathbf{x})$$

$$\chi_{ij} = \int d^3\mathbf{x} g_{ij} |\phi_i(\mathbf{x})|^2 |\phi_j(\mathbf{x})|^2$$

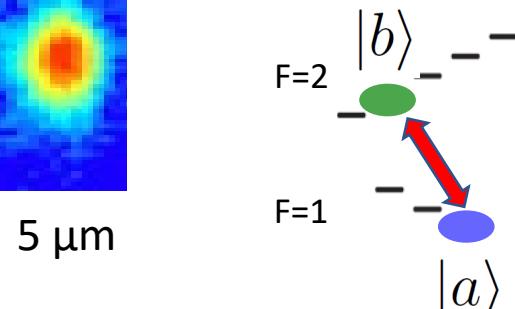
$$\mathcal{H} = \tilde{\omega}_a \hat{a}_a^\dagger \hat{a}_a + \tilde{\omega}_b \hat{a}_b^\dagger \hat{a}_b + \frac{\chi_{aa}}{2} \hat{a}_a^\dagger \hat{a}_a^\dagger \hat{a}_a \hat{a}_a + \frac{\chi_{bb}}{2} \hat{a}_b^\dagger \hat{a}_b^\dagger \hat{a}_b \hat{a}_b + \chi_{ab} \hat{a}_a^\dagger \hat{a}_b^\dagger \hat{a}_b \hat{a}_a$$

$$\mathcal{H}_{\text{cpl}} = -\frac{\hbar \Omega_r}{2} \int d^3\mathbf{x} \left(\hat{\psi}_a(\mathbf{x}) \hat{\psi}_b^\dagger(\mathbf{x}) e^{-i(\delta_c t + \varphi_0)} + \text{h.c.} \right) \Rightarrow \mathcal{H}_{\text{cpl}} = -\frac{\hbar \tilde{\Omega}}{2} \left(\hat{a}_a \hat{a}_b^\dagger e^{-i(\delta_c t + \varphi_0)} + \text{h.c.} \right)$$

$$\tilde{\Omega} = \Omega_r \int d^3\mathbf{x} \phi_a(\mathbf{x}) \phi_b(\mathbf{x})$$

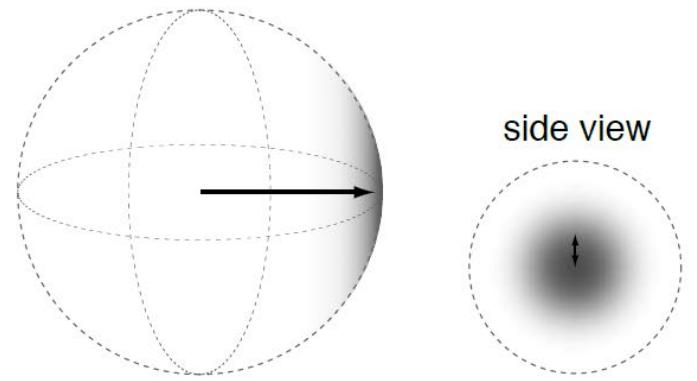


Rubidium BEC



5 μm

Schwinger collective spins simplifies H significantly



$$J = N \cdot \frac{1}{2}$$

$$\Delta J_{\perp} = \sqrt{N/2}$$

$$J_{x,y,z} = \sum_i^N S_{x,y,z}^{(i)}$$

Schwinger – collective spin:

$$\hat{J}_x = \frac{1}{2}(\hat{a}_a^\dagger \hat{a}_b + \hat{a}_b^\dagger \hat{a}_a)$$

$$\hat{J}_y = \frac{1}{2i}(\hat{a}_b^\dagger \hat{a}_a - \hat{a}_a^\dagger \hat{a}_b)$$

$$\hat{J}_z = \frac{1}{2}(\hat{a}_b^\dagger \hat{a}_b - \hat{a}_a^\dagger \hat{a}_a)$$

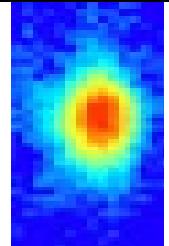
$$[\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k.$$

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$$

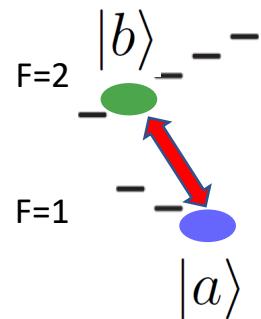
Lipkin-Meshkov-Glick Hamiltonian

Two mode Bose Hubbard

Fully connected transverse field Ising model



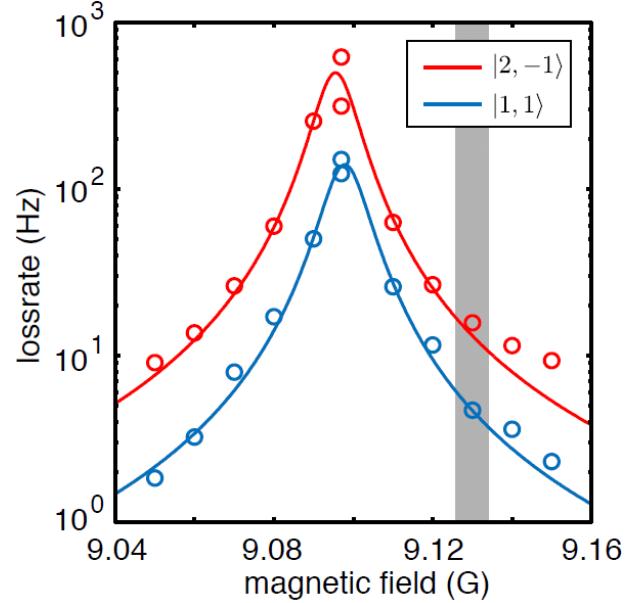
5 μm



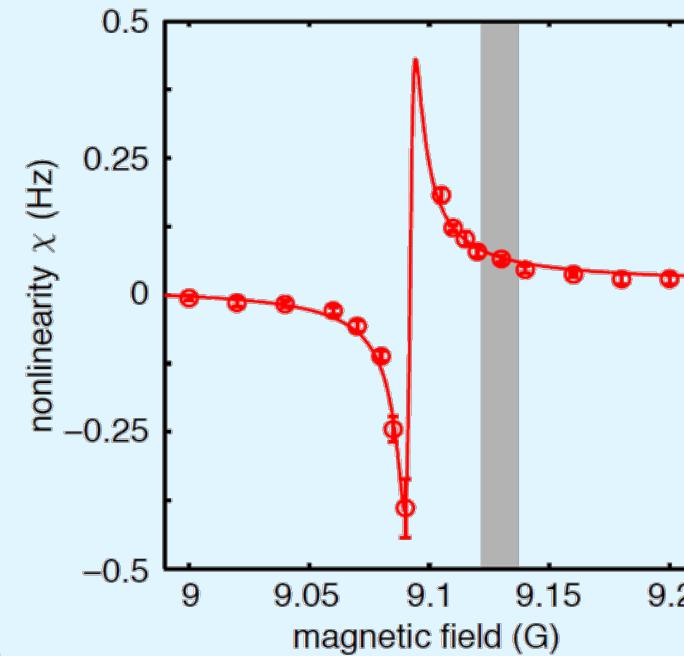
The Hamiltonian and the energy scales

$$\hat{H} = \chi \hat{J}_z^2 - \Omega \hat{J}_x + [\Delta + (g_{11} - g_{22})N] \hat{J}_z$$

Feshbach resonance -loss



Feshbach resonance



±1Hz

Magnetic field
± 100μG

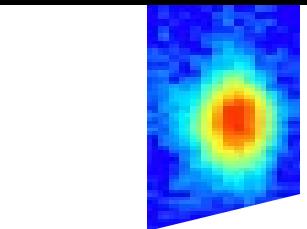
Total atom number fluctuations

N = 400 ± 40

Lipkin-Meshkov-Glick Hamiltonian

Two mode Bose Hubbard

Fully connected transverse field Ising model



$|b\rangle$
 $F=2$

1.C

*Nuclear Physics 62 (1965) 188–198; © North-Holland Publishing Co., Amsterdam
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VALIDITY OF MANY-BODY APPROXIMATION METHODS FOR A SOLVABLE MODEL

(I). Exact Solutions and Perturbation Theory

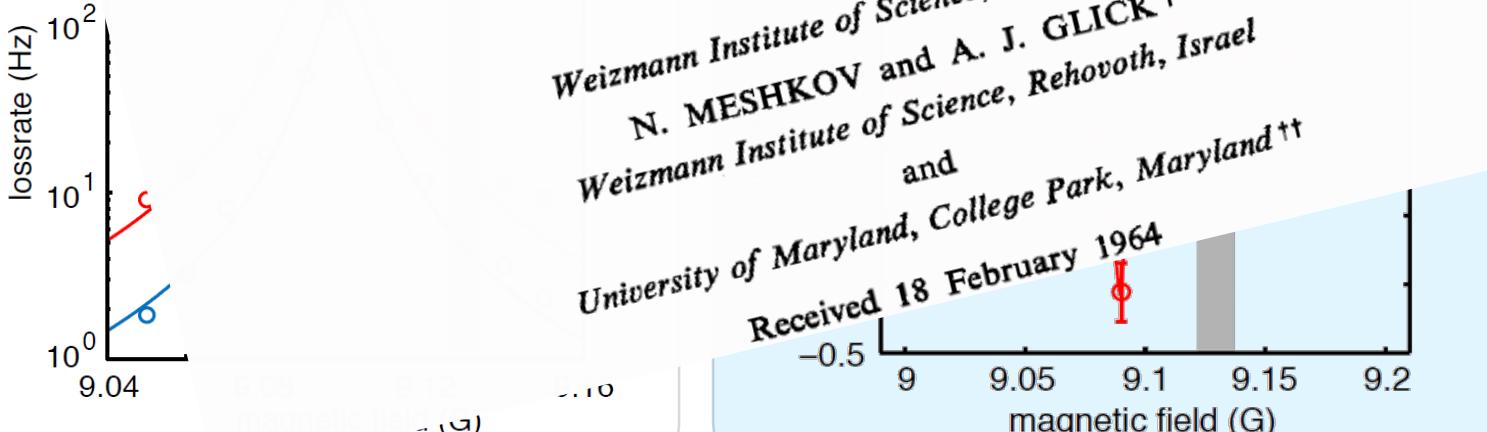
H. J. LIPKIN,

Weizmann Institute of Science, Rehovoth, Israel
N. MESHKOV and A. J. GLICK[†]

Weizmann Institute of Science, Rehovoth, Israel
and

University of Maryland, College Park, Maryland^{††}

Received 18 February 1964



The Hamiltonian and the energy scales

$$[\Delta + (g_{11} - g_{22})N] \hat{J}_z$$

$\pm 1\text{Hz}$

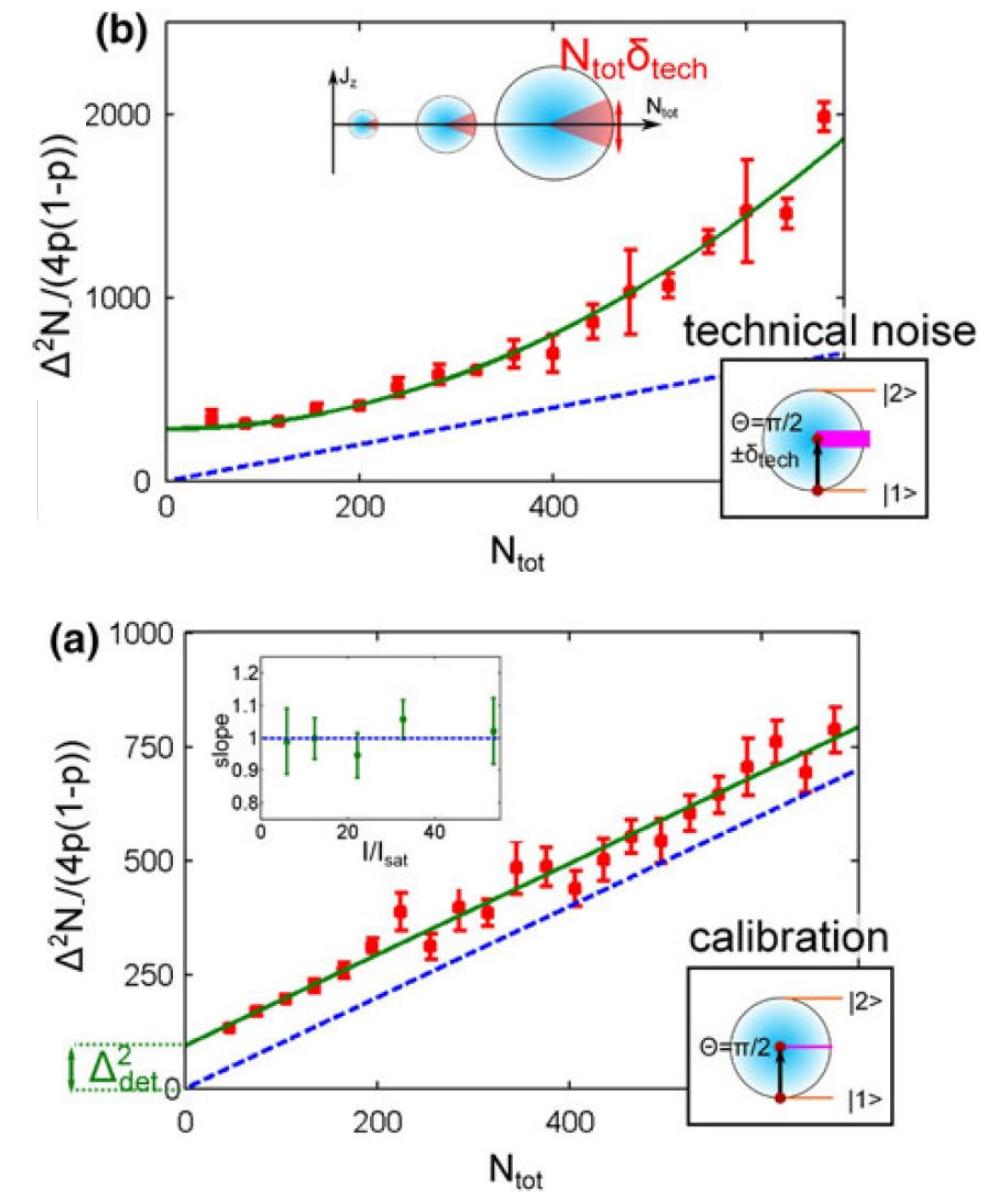
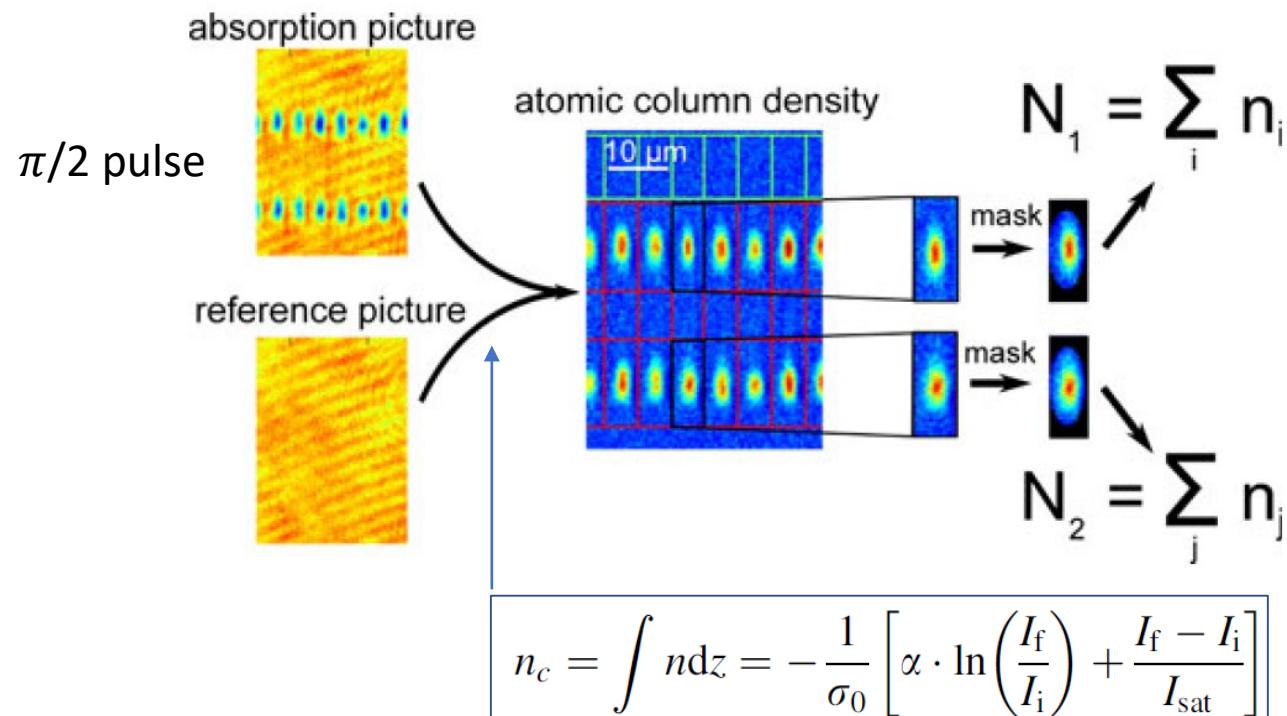
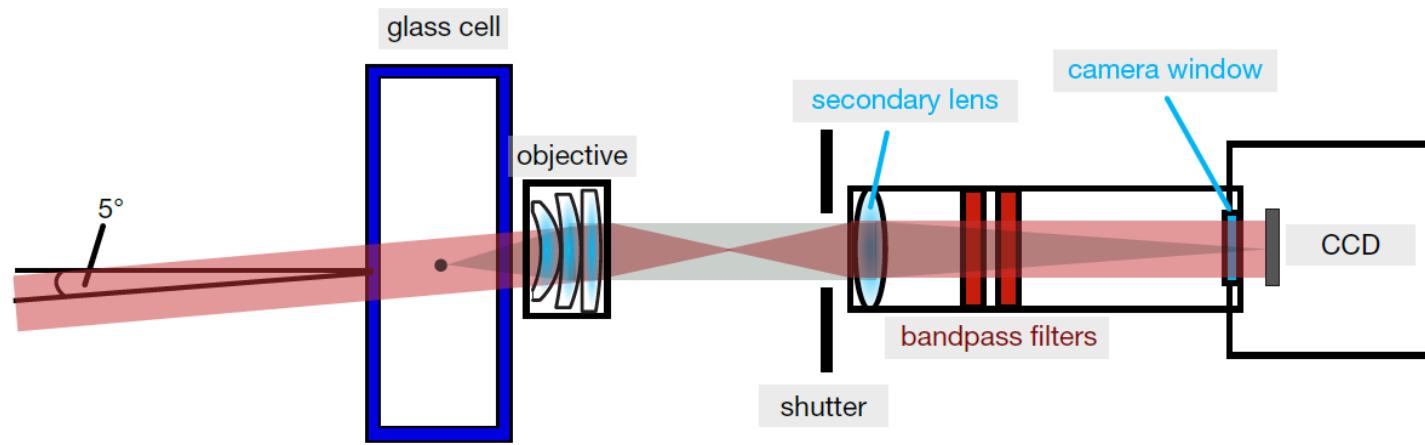
Magnetic field
 $\pm 100\mu\text{G}$

total atom number fluctuations

$N = 400 \pm 40$

Imaging @ ±3atoms

App. Phys. B 113, 69 (2013)



What has Trinitrotoluol (TNT) to do with quantum mixtures ?

PRL 113, 103004 (2014); PRA 91, 013412 (2015)

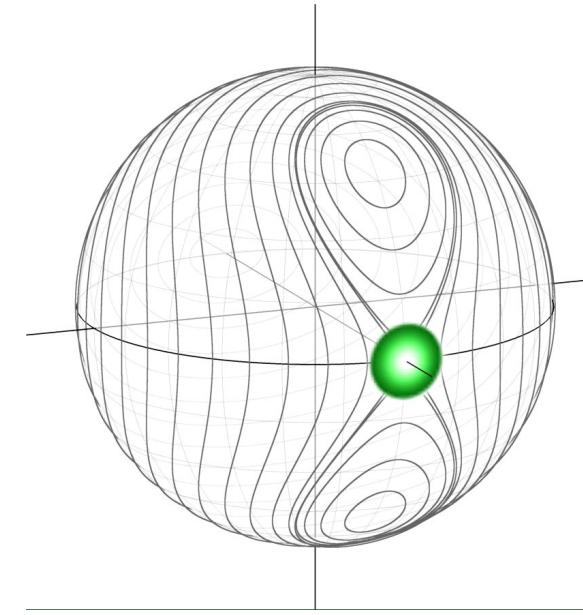
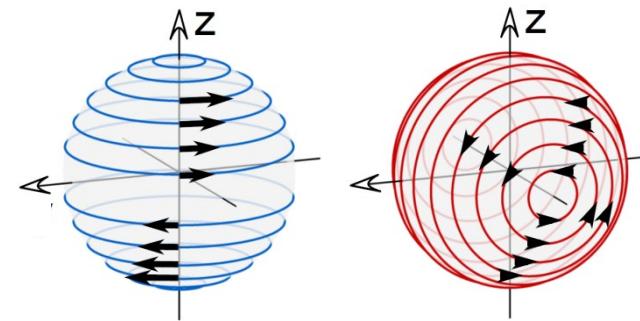
$$\hat{H} = \chi \hat{J}_z^2 - \Omega \hat{J}_x$$

rotation z axis

$$e^{\frac{-i\chi t}{\hbar} \hat{J}_z \hat{J}_z}$$

rotation x axis

$$e^{\frac{-i\Omega t}{\hbar} \hat{J}_x}$$



e.g. A. Micheli et al. Phys.Rev. A 67 , 013607 (2003)

Wolfgang
Muessel



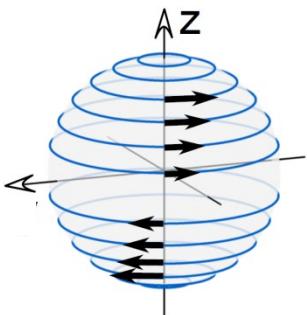
generating non-classical states – Twist aNd Turn

What about the classical limit of complex order parameter ?

$$H = \chi J_z^2 - \Omega J_x \quad \xrightarrow{\text{classical}} \quad H = \frac{\Lambda}{2} \Delta n^2 - \sqrt{1 - \Delta n^2} \cos \varphi \quad \Lambda = \frac{N\chi}{\Omega}$$

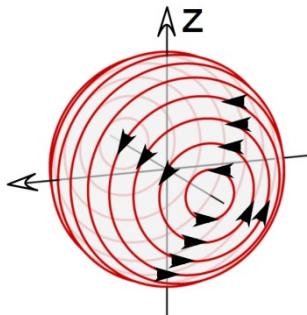
rotation z axis

$$e^{\frac{-i\chi t}{\hbar} \hat{J}_z \hat{J}_z}$$



rotation x axis

$$e^{\frac{-i\Omega t}{\hbar} \hat{J}_x}$$



$$\hat{a}^+ = \sqrt{n_a} e^{i\varphi_a}$$

$$\hat{J}_x = \frac{1}{2} (\hat{a}^+ \hat{b} + \hat{b}^+ \hat{a})$$

$$\hat{J}_y = \frac{1}{2i} (\hat{a}^+ \hat{b} - \hat{b}^+ \hat{a})$$

$$\hat{J}_z = \frac{1}{2} (\hat{a}^+ \hat{a} - \hat{b}^+ \hat{b})$$

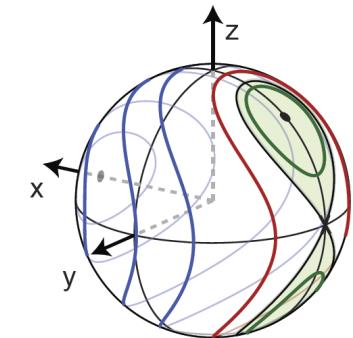
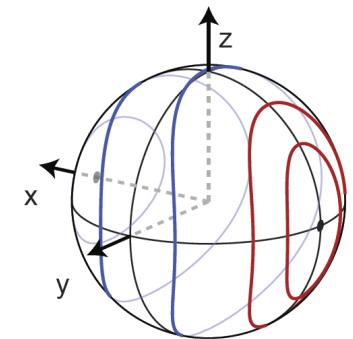
Schwinger spin

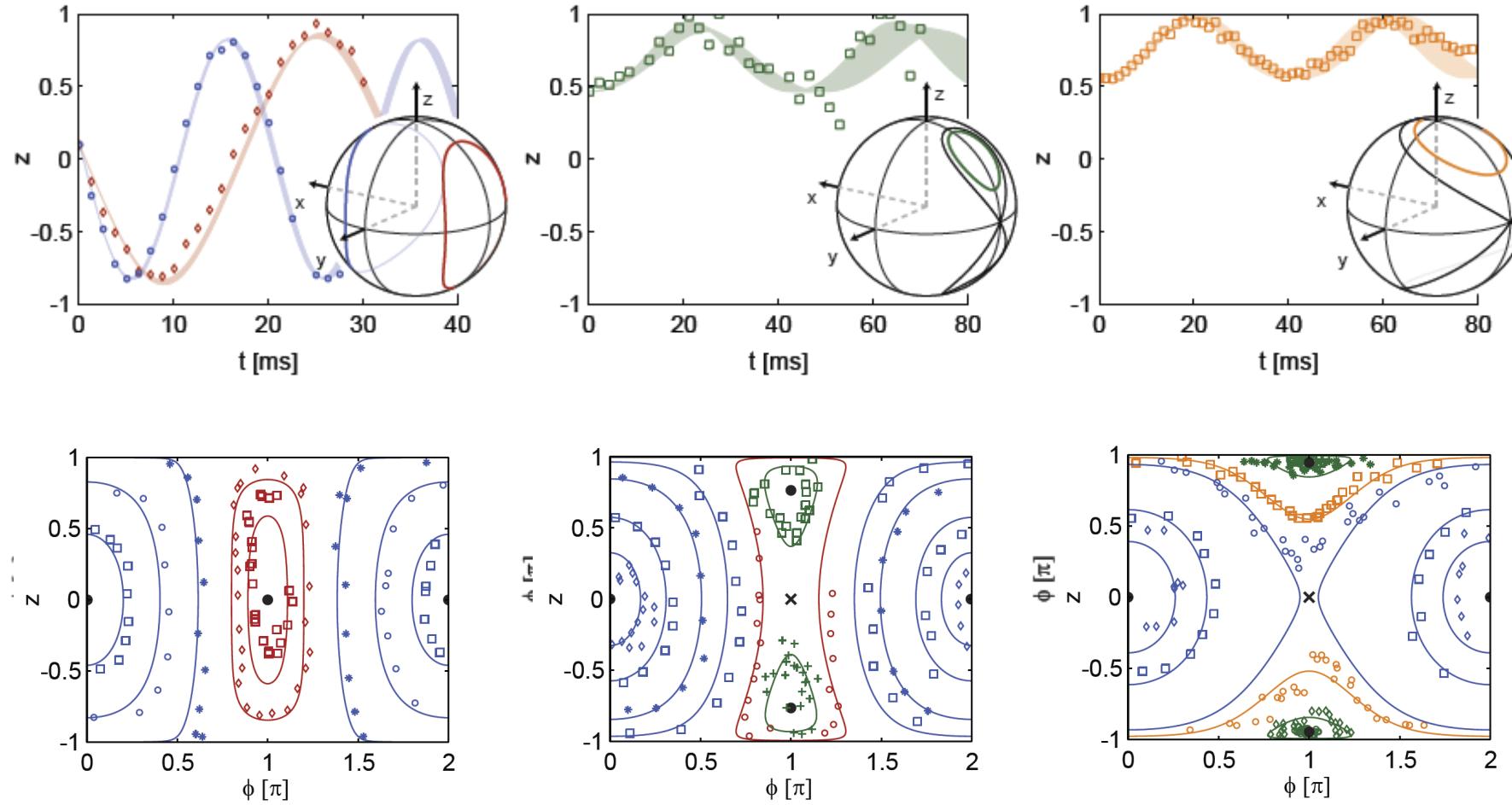
$$\hat{J}_x \cong \sqrt{n_a n_b} \cos \varphi$$

$$\hat{J}_y \cong \sqrt{n_a n_b} \sin \varphi$$

$$\hat{J}_z \cong \frac{n_a - n_b}{2}$$

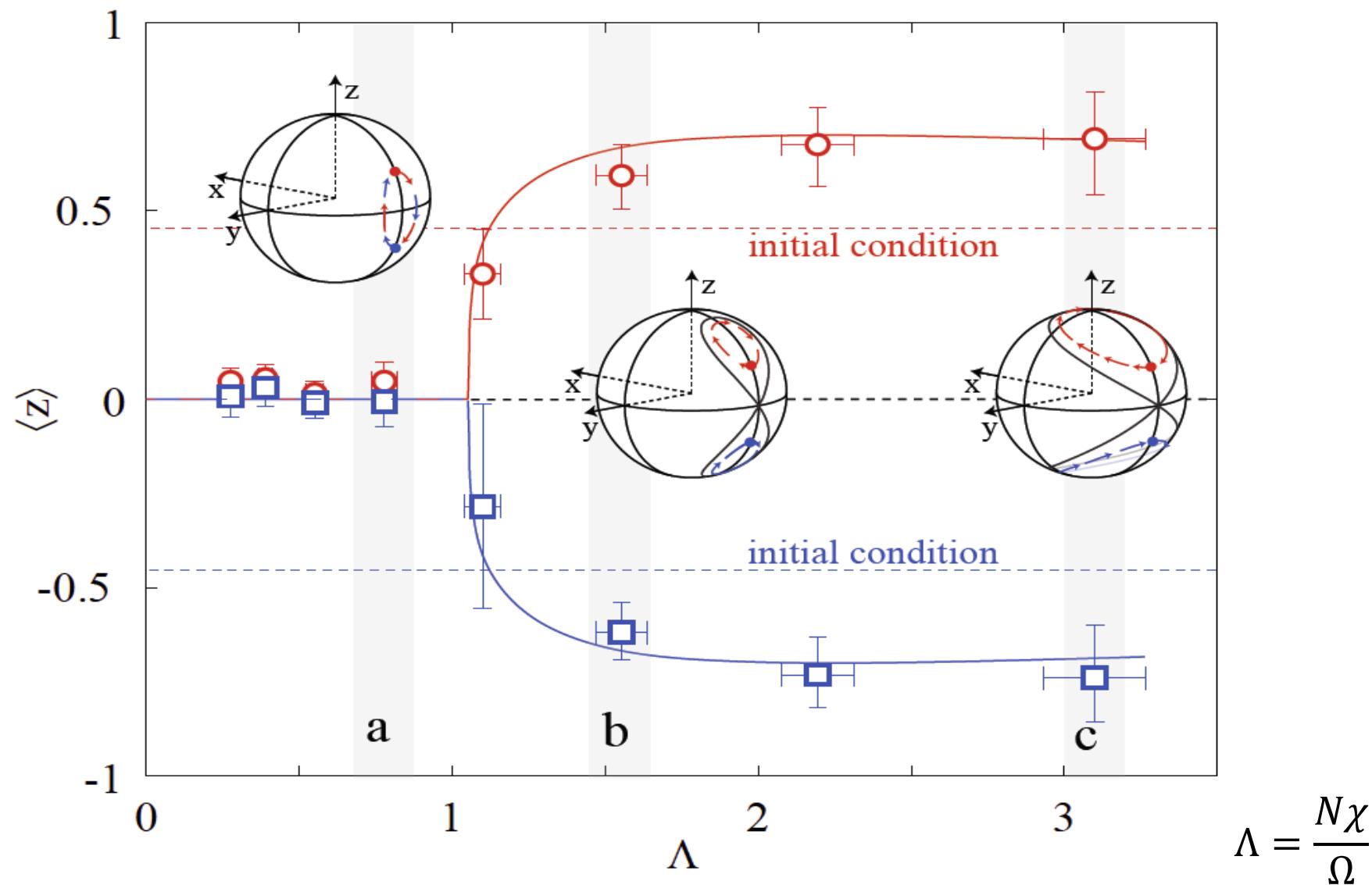
Classical description



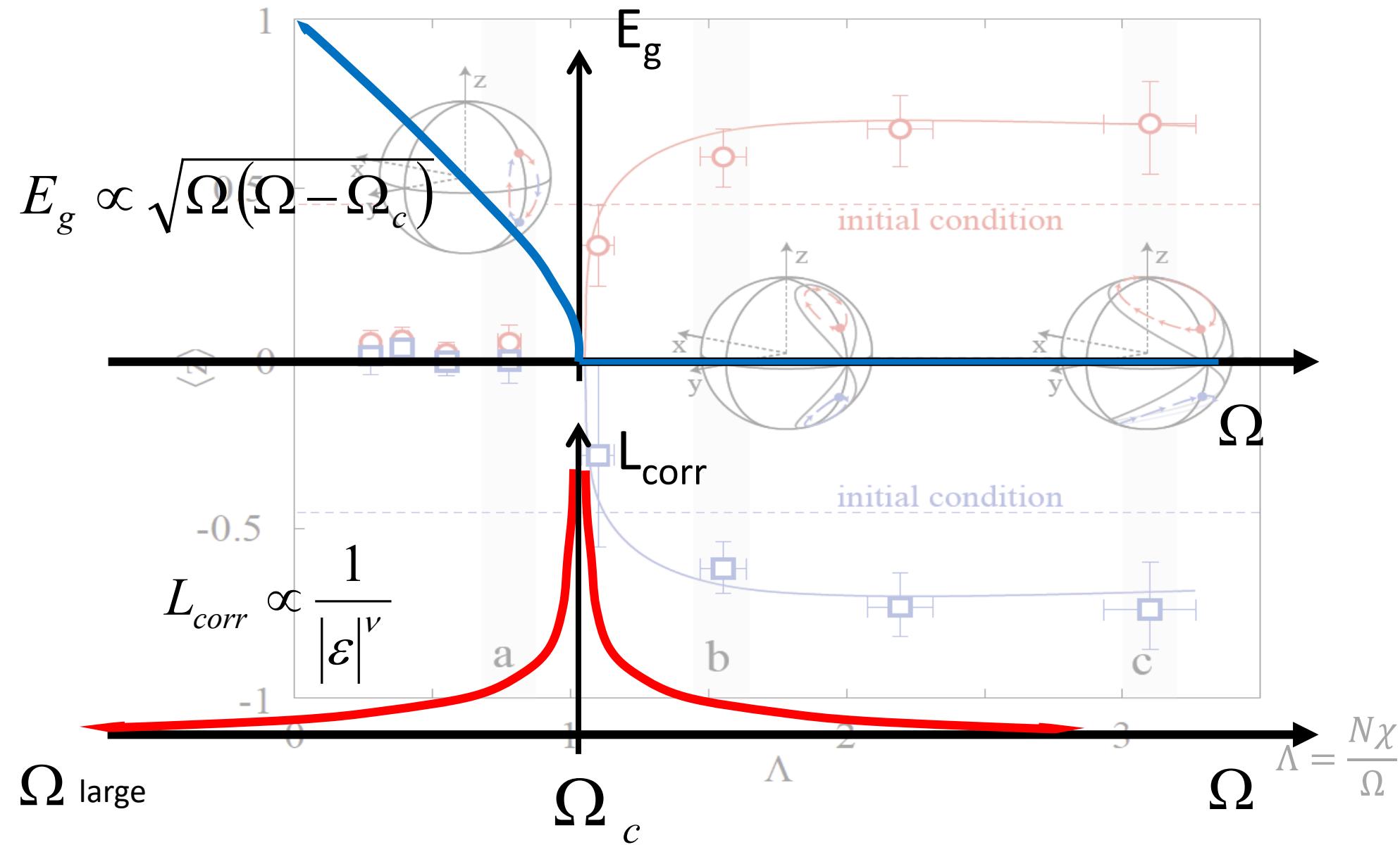


increasing interactions

Quantum bifurcation – quantum phase transition

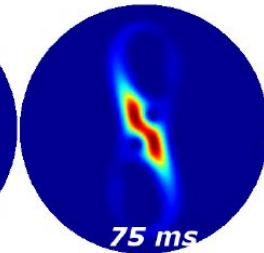
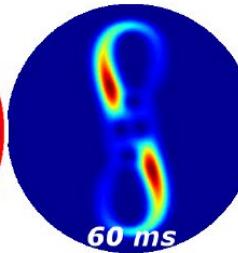
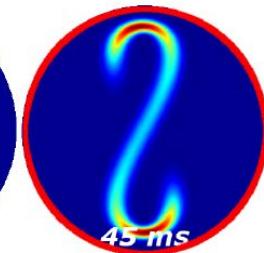
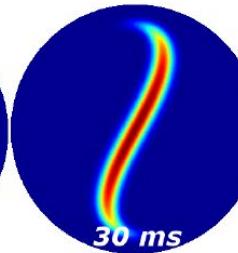
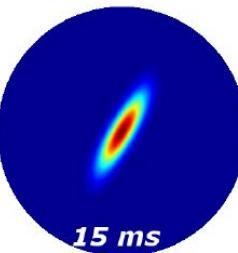
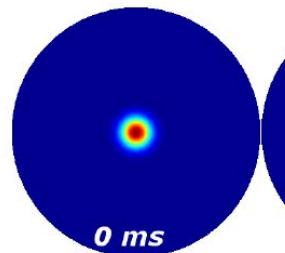
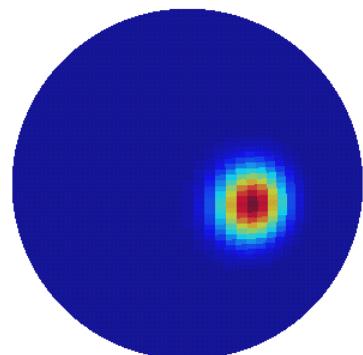
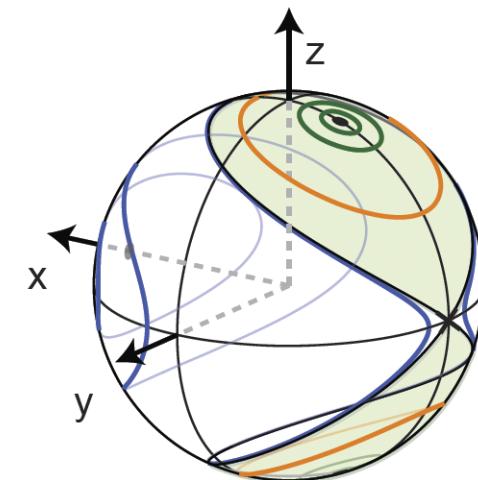
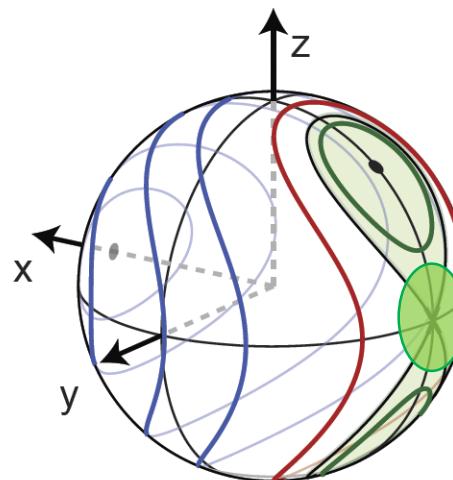
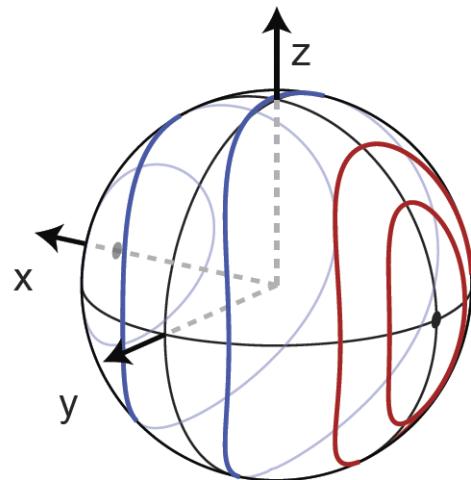


Quantum bifurcation – quantum phase transition



,interesting' many particle states

squeezing \rightarrow non Gaussian state



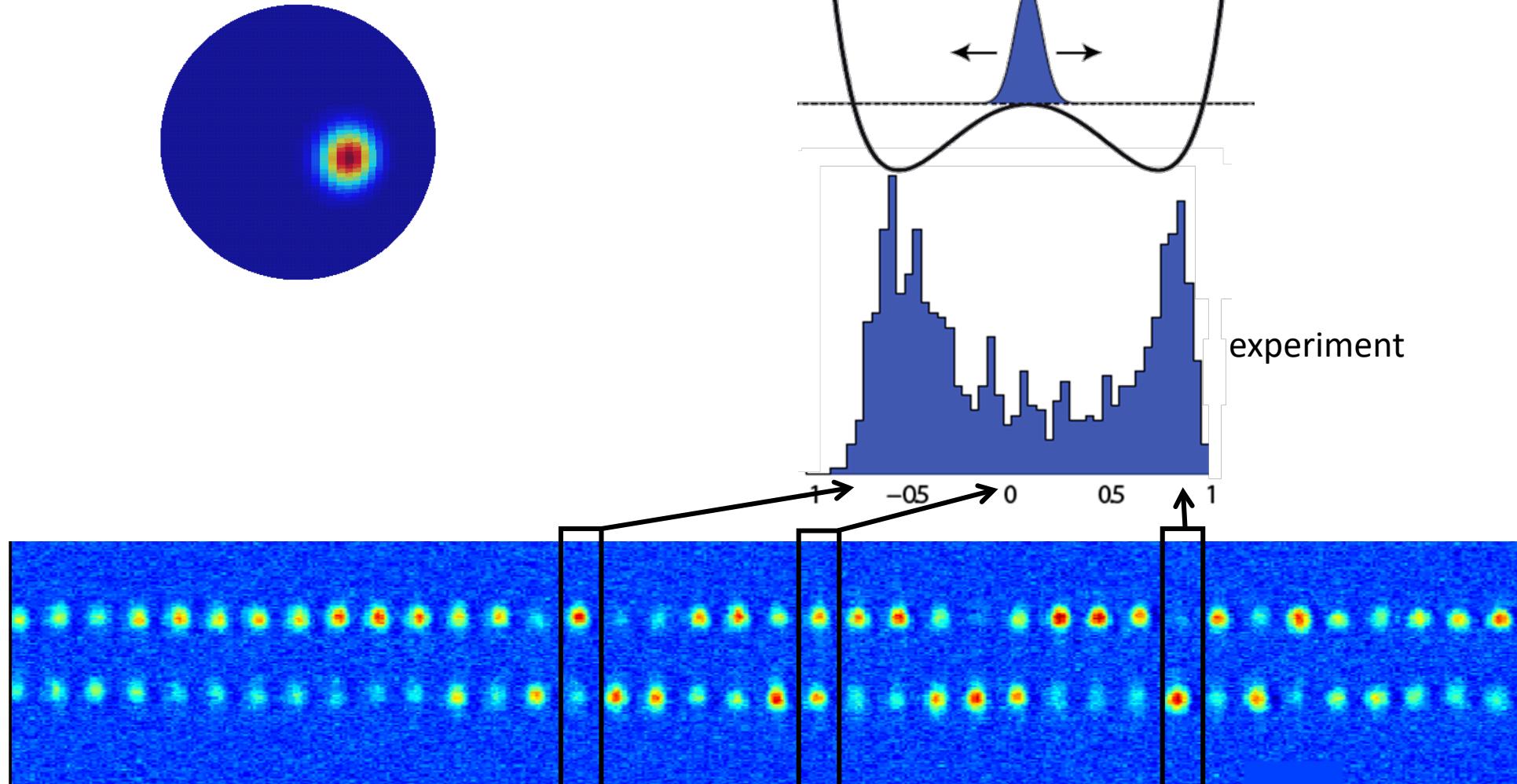
,interesting' many particle states

squeezing \rightarrow non Gaussian state

PHYSICAL REVIEW A 78, 023611 (2008)

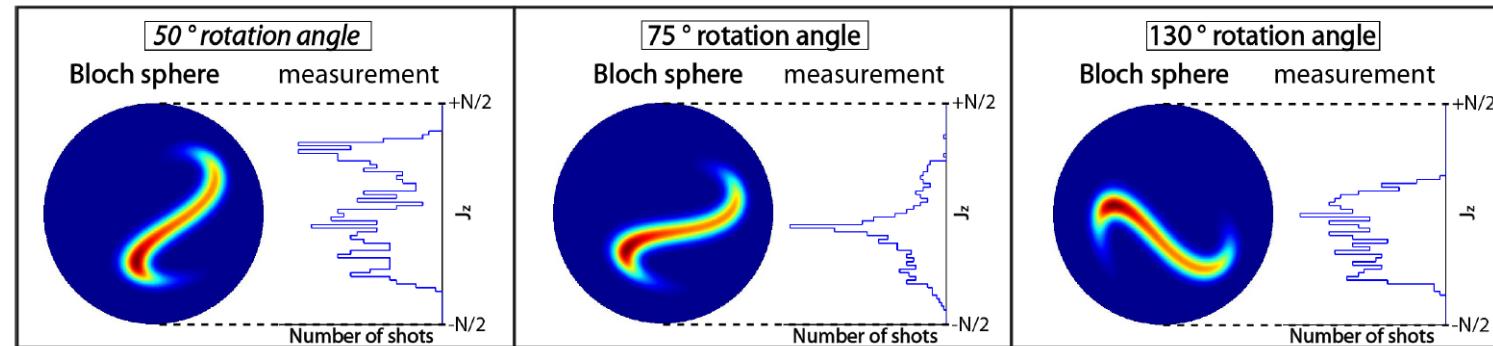
Fock-space WKB method for the boson Josephson model describing a Bose-Einstein condensate trapped in a double-well potential

V. S. Shchesnovich¹ and M. Trippenbach²

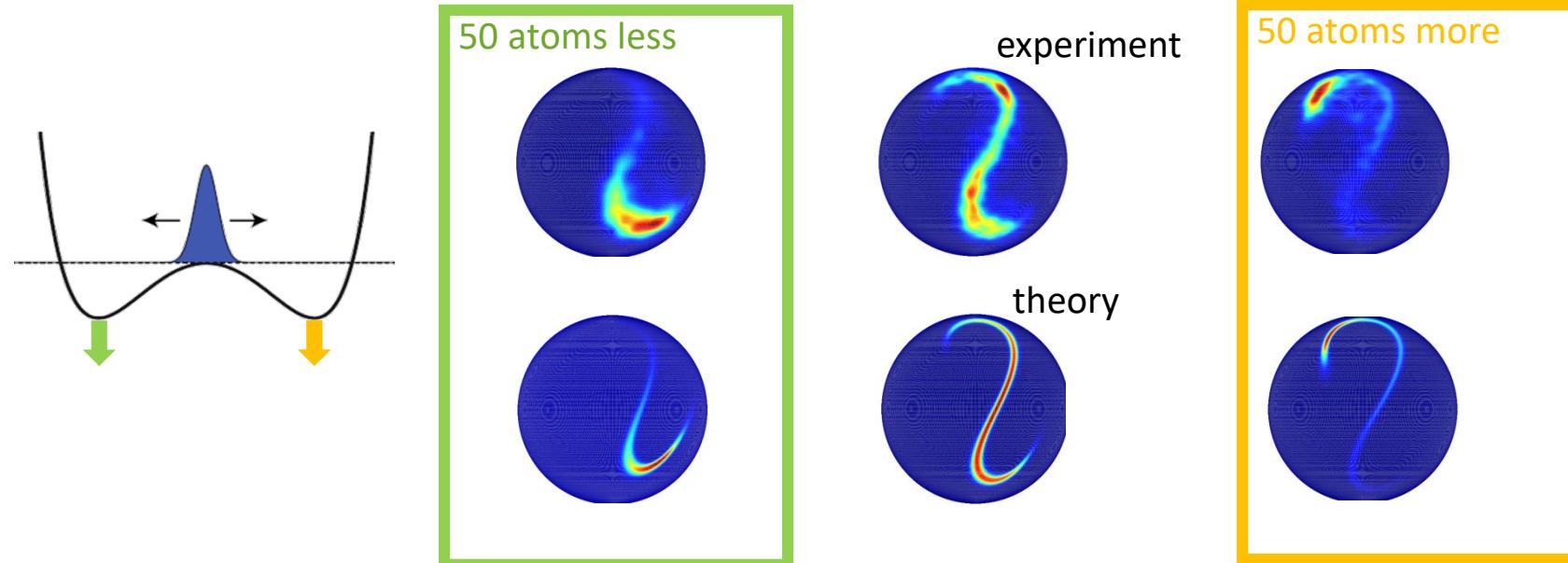


Tomography with 90000 BECs

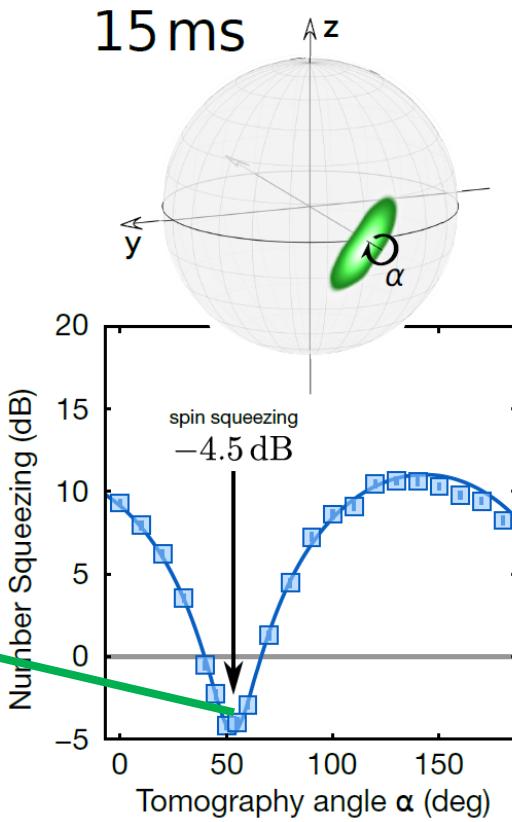
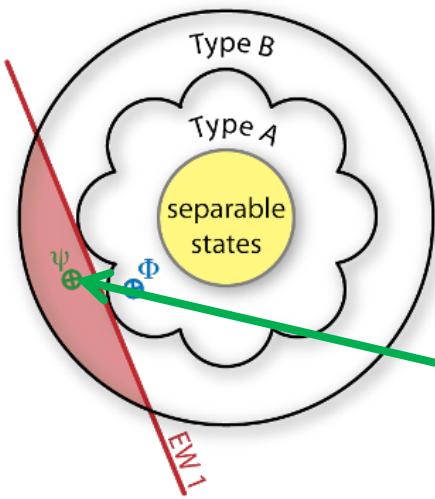
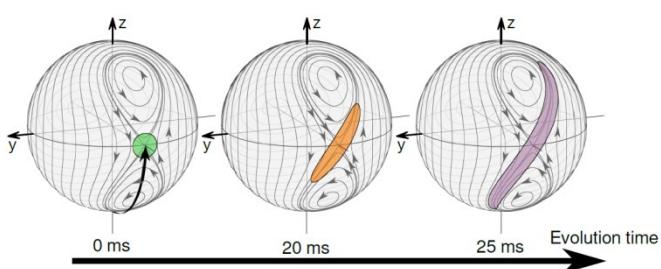
90000 BEC experiments !



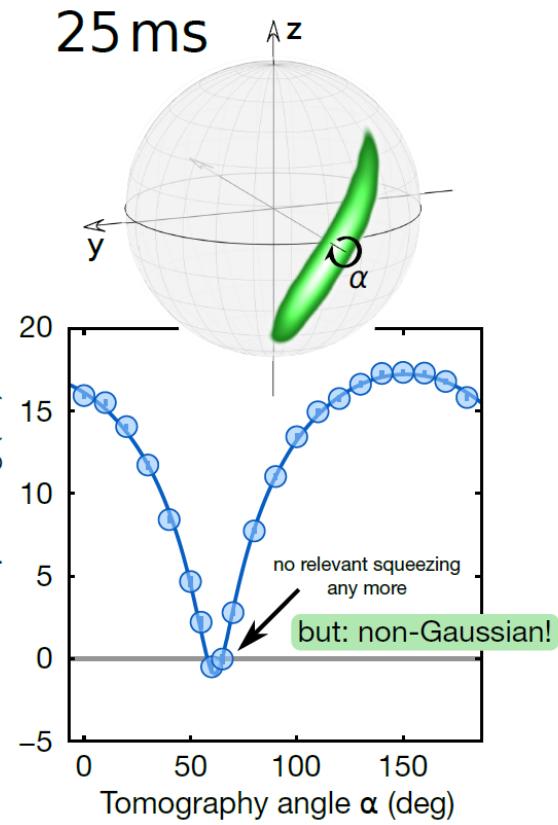
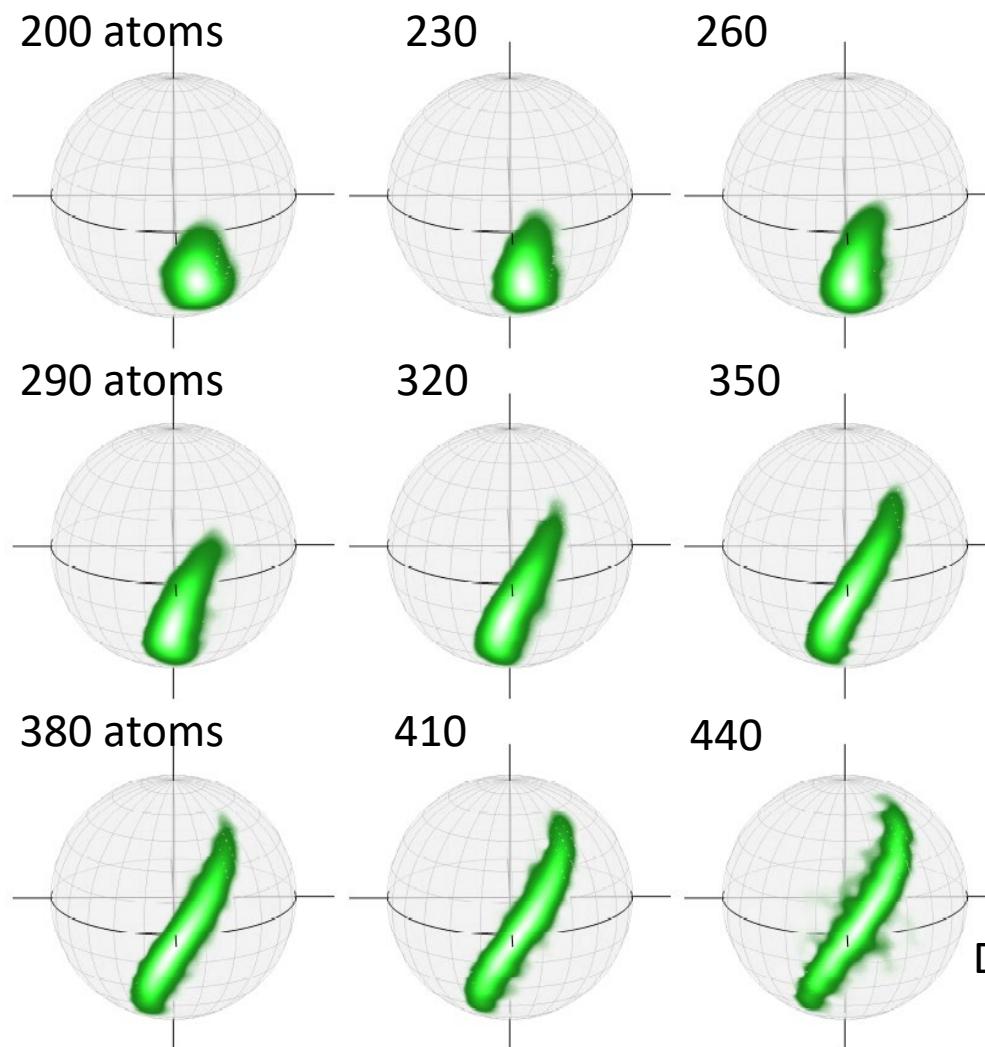
postselected: 340 ± 10 atoms



See also: Schmied, Treutlein New J. Phys. 13:065019, (2011)



experimentally reconstructed after 25ms with >100.000 BECs



Demonstrate entanglement via quantum enhanced metrology

Fisher information and entanglement of non-Gaussian spin states

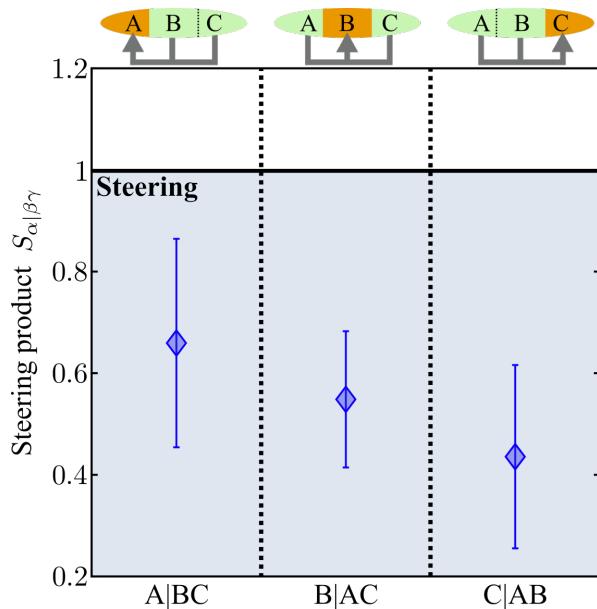
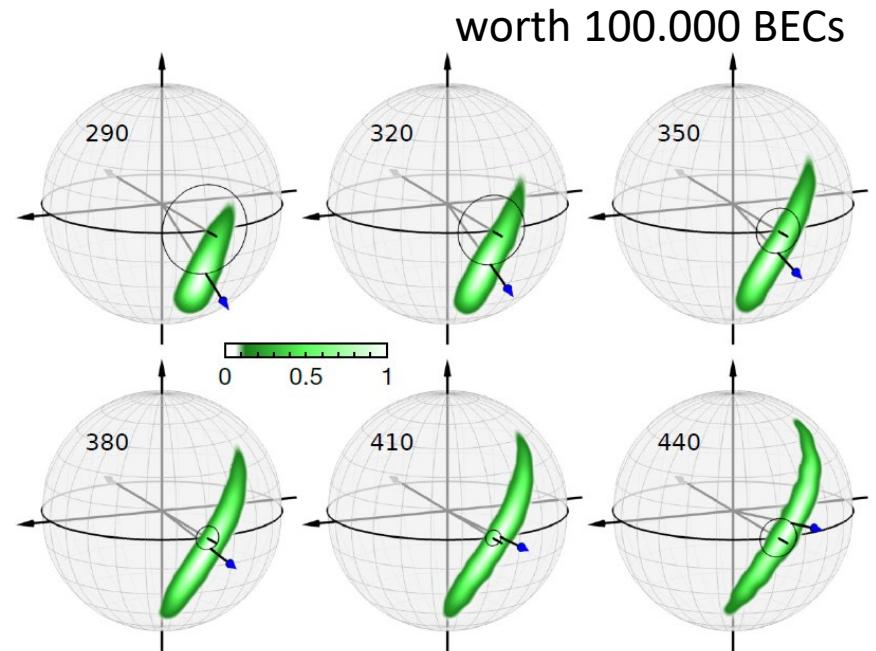
H. Strobel, W. Muessel, D. Linnemann, T. Zibold, D. B. Hume, L. Pezzè, A. Smerzi, M. K. Oberthaler

SCIENCE, 2014, Vol. 345 , 424-427

SUMMARY

Ultracold coherent mixtures reveal quantum entanglement
from squeezed to non-Gaussian states

Spinor Bose gases
an ideal platform for quantum many body physics



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Quantum metrology with nonclassical states of atomic ensembles

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