


# The quantum impurity problem & beyond

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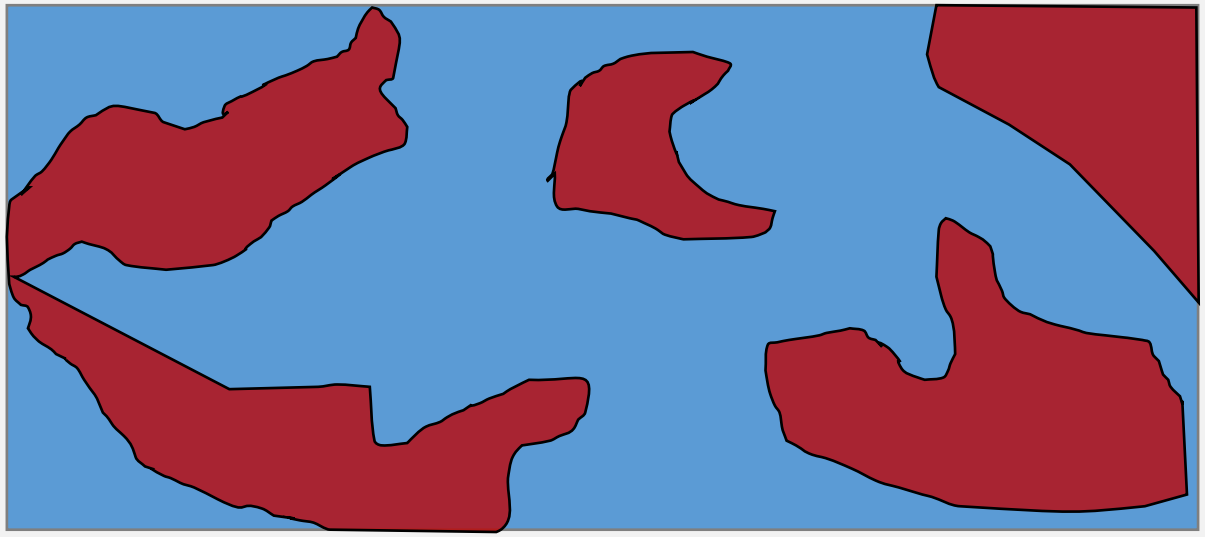
# Background

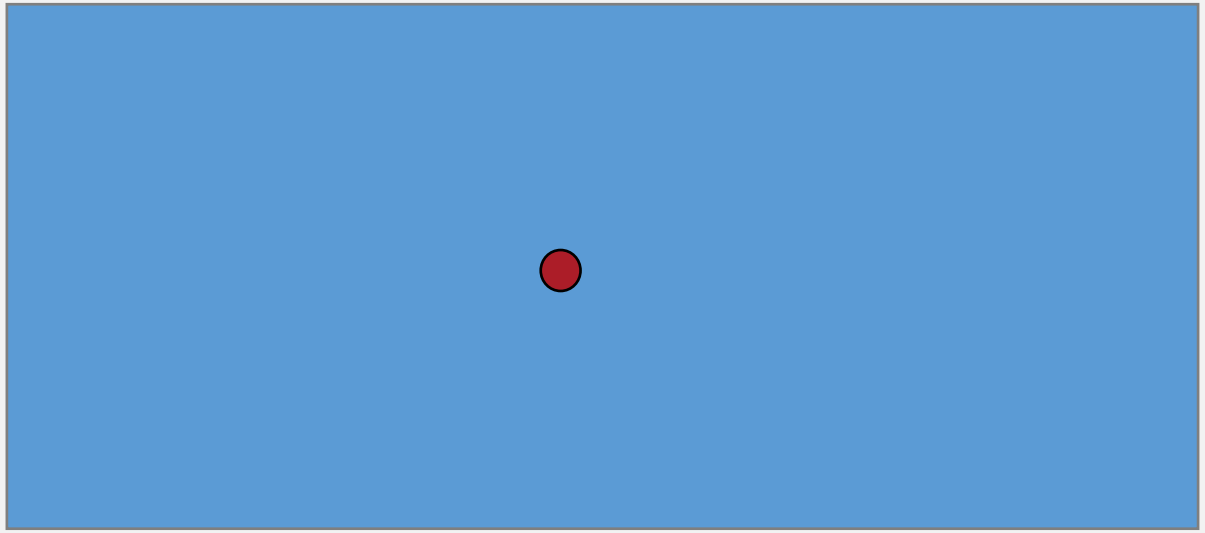
- Consider an interacting quantum mixture with, e.g., 2 components:



- Complicated many-body problem!
- How to tackle?

# Background

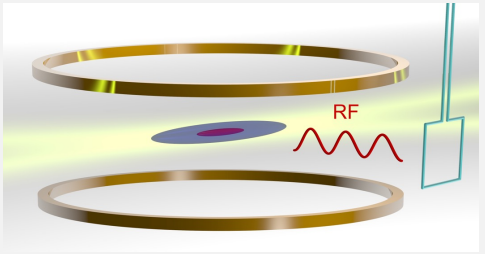
- Consider limit of a single quantum impurity particle in a background medium:



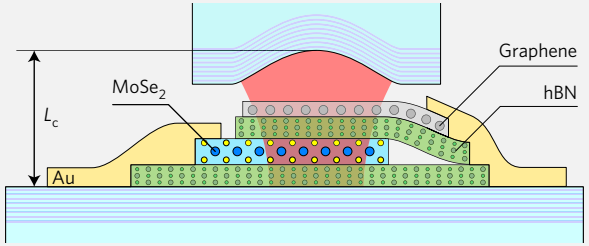
- Simpler problem and **fundamental** to a range of different systems

## Impurity scenarios:

- Ultracold atomic gases



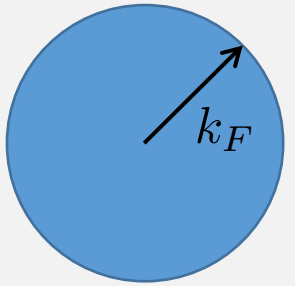

- Excitons in 2D semiconductors




*Sidler et al, Nature Physics 2016*

- Protons in neutron stars

# Focus of these lectures



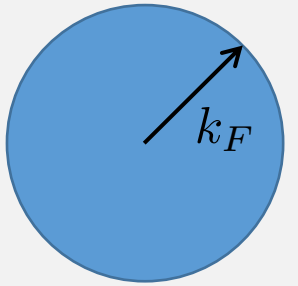

- Medium is an ideal Fermi gas
  - The so-called “Fermi polaron” problem
- Non-trivial problem despite the medium being simple and well understood

  For instance:

  - Possibility of *transitions* where impurity abruptly changes its character
  - Orthogonality catastrophe when the impurity is infinitely heavy

$$\langle \psi_{int} | \psi_0 \rangle = 0$$

# Focus of these lectures

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## Outline

- Introduction to Fermi polarons
  - Theoretical description
- Ground-state behaviour
  - Single-impurity transitions
- Energy spectrum and dynamics

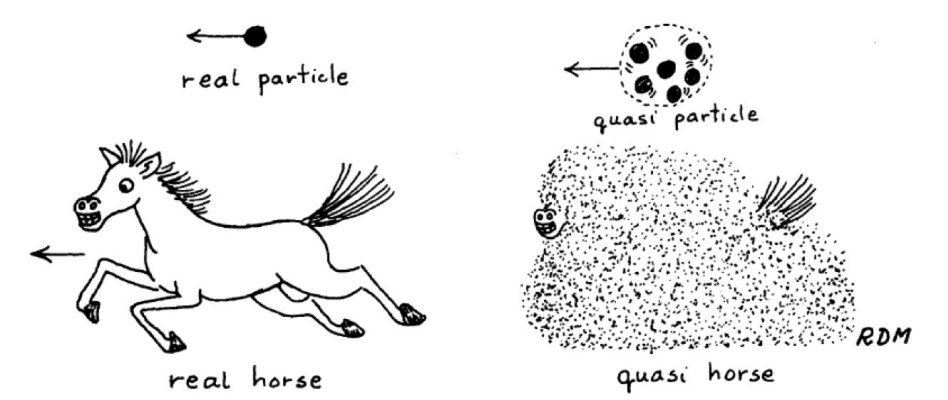
## Reviews:


- Chevy & Mora, *Rep. Prog. Phys*. **73** 112401 (2010)
- Massignan, Zaccanti & Bruun, *Rep. Prog. Phys.* **77**, 034401 (2014)
- MMP & Levinsen, *Annu. Rev. Cold Atoms Mol.* **3**, 1 (2015)
- Schmidt et al., *Rep. Prog. Phys.* **81**, 024401 (2018)
- Scazza, Zaccanti, Massignan, MMP & Levinsen, arXiv:2204.06984

# Quantum impurity in a Fermi gas

- Impurity becomes “dressed” by particle-hole excitations of the Fermi sea

  → Fermi polaron *quasiparticle*




- Polarons resemble original impurity, but with modified properties, e.g., mass

- Theoretical description

$$\hat{H} = \sum_{\mathbf{k}} \left( \epsilon_{\mathbf{k},I} \hat{c}^{\dagger}_{\mathbf{k}} \hat{c}_{\mathbf{k}} + \epsilon_{\mathbf{k}} \hat{f}^{\dagger}_{\mathbf{k}} \hat{f}_{\mathbf{k}} \right)$$

$$+ g \sum_{\mathbf{k}\mathbf{k}'\mathbf{q}} \hat{f}^{\dagger}_{\mathbf{k}} \hat{f}_{\mathbf{k}+\mathbf{q}} \hat{c}^{\dagger}_{\mathbf{k}'} \hat{c}_{\mathbf{k}'-\mathbf{q}}$$

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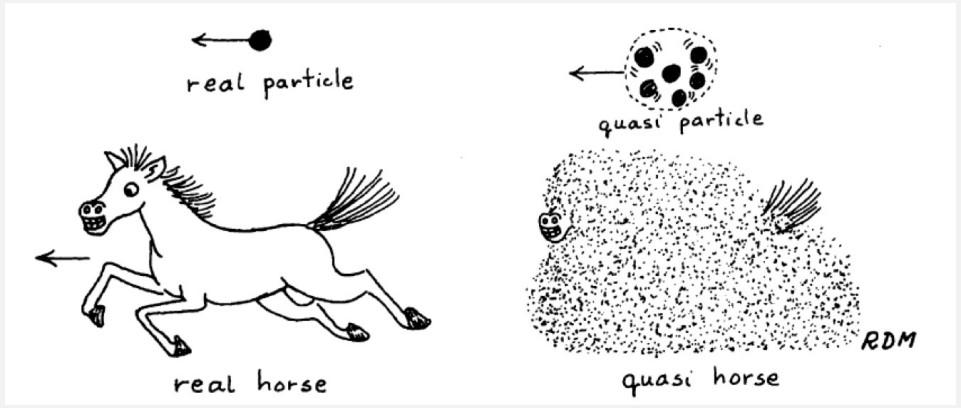


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Short-range attractive interactions
(atom-atom, electron-exciton, etc. …)

Lippmann-Schwinger equation:

$$\frac{1}{g} = \frac{m_r}{2\pi a} - \sum_{\mathbf{k}}^{\Lambda} \frac{1}{\epsilon_{\mathbf{k},I} + \epsilon_{\mathbf{k}}} \qquad \text{(3D)}$$

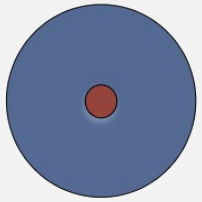
reduced mass

scattering length

# Polaron ground state

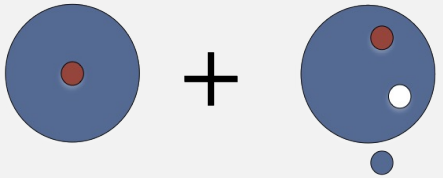
- Start with the case of vanishing impurity-fermion interactions

$$\hat{c}_0^\dagger |\mathrm{FS}\rangle$$



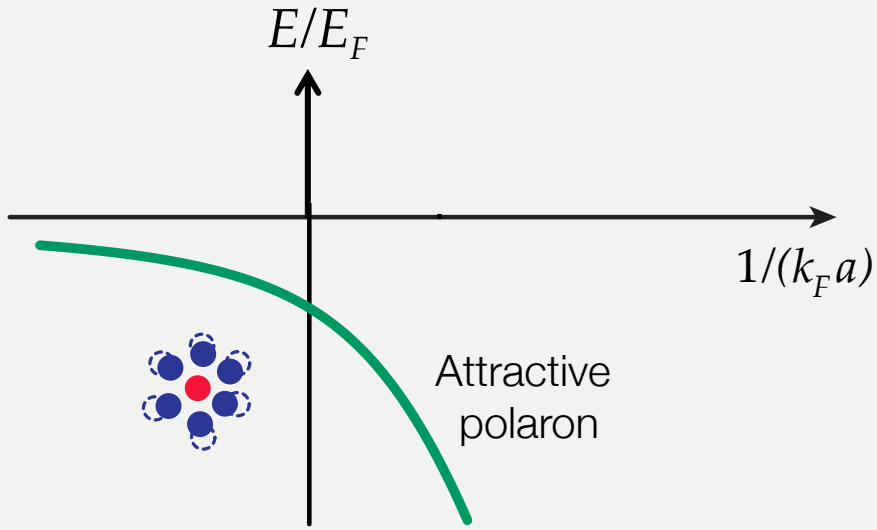
- Turning on interactions creates particle-hole excitations
- Variational state ("Chevy ansatz"):

$$|\psi\rangle = \left[\alpha_0 \hat{c}_{\mathbf{0}}^\dagger + \sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{kq}} \hat{c}_{\mathbf{q}-\mathbf{k}}^\dagger \hat{f}_{\mathbf{k}}^\dagger \hat{f}_{\mathbf{q}}\right] |\mathrm{FS}\rangle$$

$$\simeq$$ 

Chevy, PRA 74, 063628 (2006)

- Minimize $\langle\psi|\hat{H}|\psi\rangle$ w.r.t **α** parameters to obtain polaron properties:
  - energy *E*, residue $|\alpha_0|^2$, mass…



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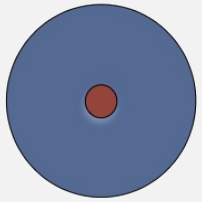

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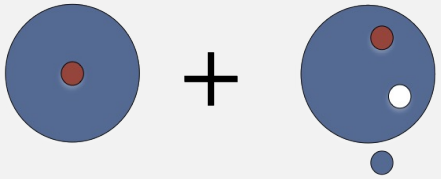
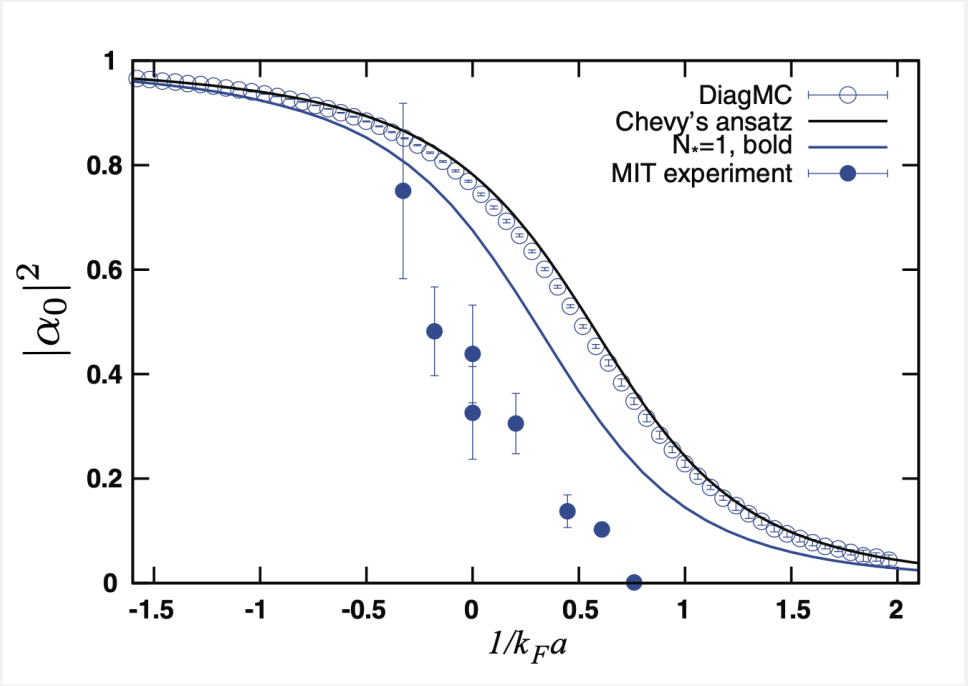
$$\simeq$$ 

Chevy, PRA 74, 063628 (2006)

- Ansatz is accurate* for all interactions!

**Comparison with QMC (equal masses)**

Vlietinck et al., PRB 87, 115133 (2013)




Why so accurate?

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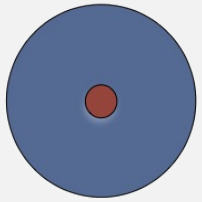

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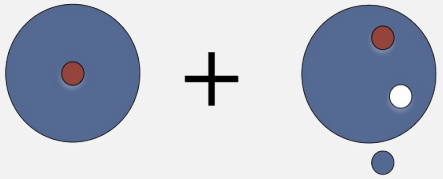
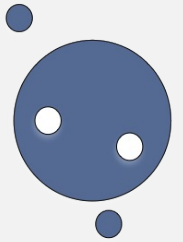
$$\simeq$$ 

Chevy, PRA 74, 063628 (2006)

- Near cancellation of processes with 2+ excitations due to Fermi statistics

Combescot & Giraud, PRL 101, 050404 (2008)



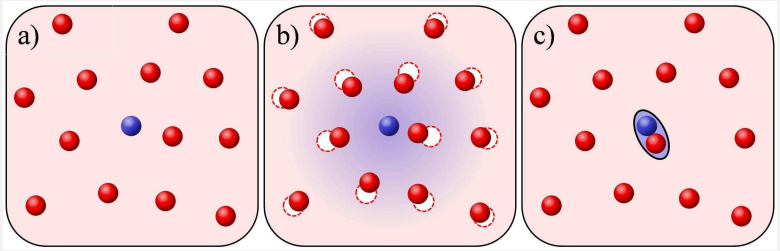
- Describes two-body (molecule) bound state in limit $1/k_F a \to \infty$

$$\sum_{\mathbf{k}} \varphi_{\mathbf{k}} \hat{c}_{-\mathbf{k}}^\dagger \hat{f}_{\mathbf{k}}^\dagger |0\rangle$$

See also: Suris 2003

# Ground-state transitions

- Impurity abruptly binds fermion to form (dressed) molecule




Schirotzek et al., PRL 2009

$$|\psi\rangle = \left[\alpha_0 \hat{c}^\dagger_{\mathbf{0}} + \sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{kq}} \hat{c}^\dagger_{\mathbf{q}-\mathbf{k}} \hat{f}^\dagger_{\mathbf{k}} \hat{f}_{\mathbf{q}} + \cdots\right] |\mathrm{FS}\rangle$$

$|\alpha_0|^2 \to 0$

$$|\psi\rangle = \left[\sum_{\mathbf{k}} \alpha_{\mathbf{k}} \hat{c}^\dagger_{-\mathbf{k}} \hat{f}^\dagger_{\mathbf{k}} + \cdots\right] |\mathrm{FS}'\rangle$$

Prokof'ev & Svistunov, PRB 77, 020408(R) (2008);
Punk et al., PRA 2009; Bruun & Massignan, PRL 2010 …

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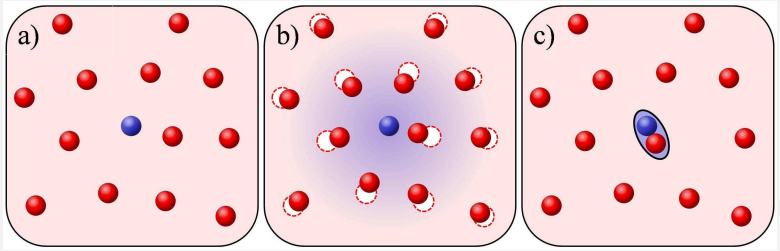


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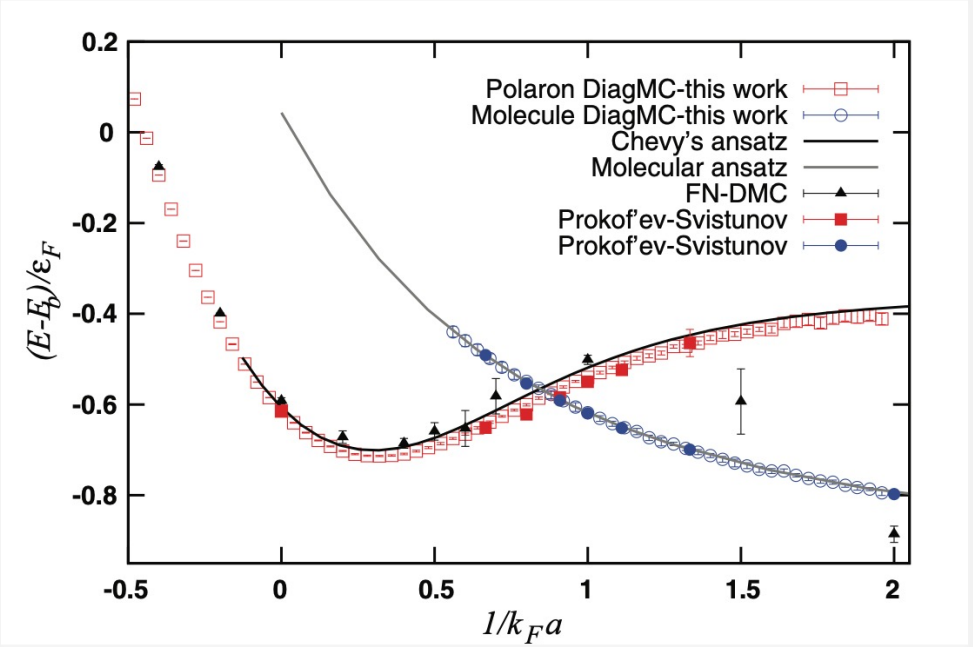
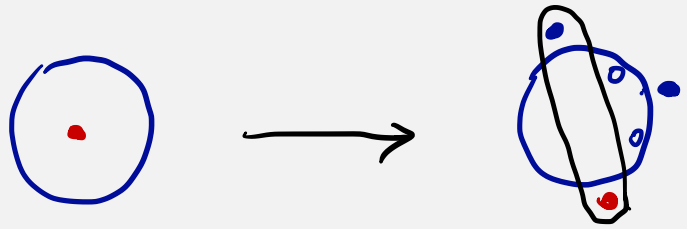
Prokof'ev & Svistunov, PRB 77, 020408(R) (2008);
Punk et al., PRA 2009; Bruun & Massignan, PRL 2010 …

- Sharp transition according to theory:




Vlietinck et al., PRB 87, 115133 (2013)

Involves multiple excitations

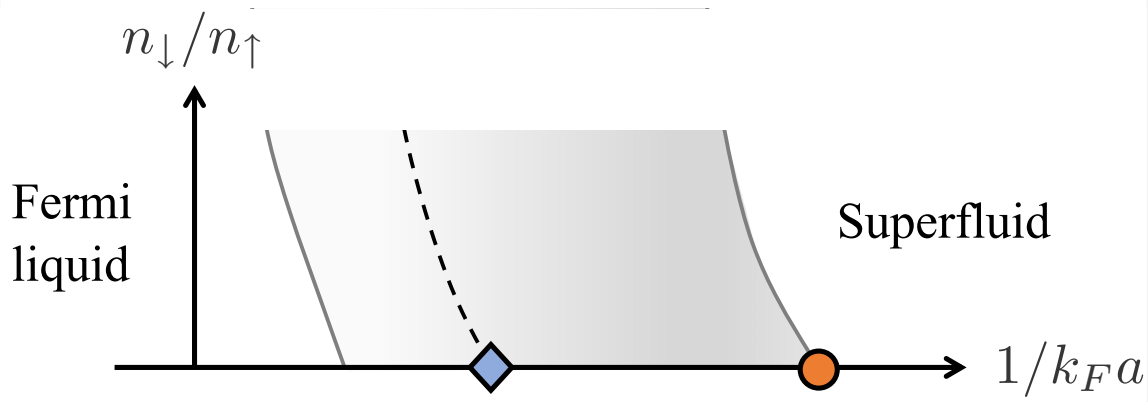


# Many impurities?

- Impurity transitions have implications for the phase diagram of quantum mixtures

  → Fermi liquid versus BEC

- E.g., spin-imbalanced Fermi gases, equal masses:

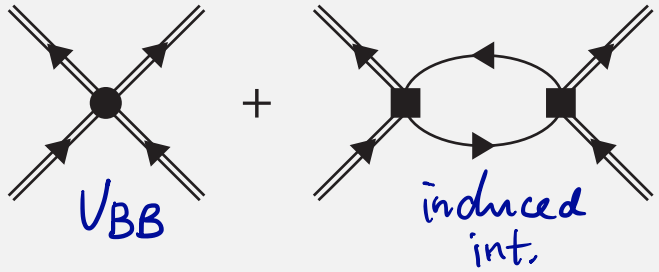



Experiments: MIT, JILA, ENS, Innsbruck, LENS, LMU, Rice …

- Single-impurity picture fails at transition
  - Preempted by phase separation (consistent with experiment)

- Due to polaron-polaron interactions

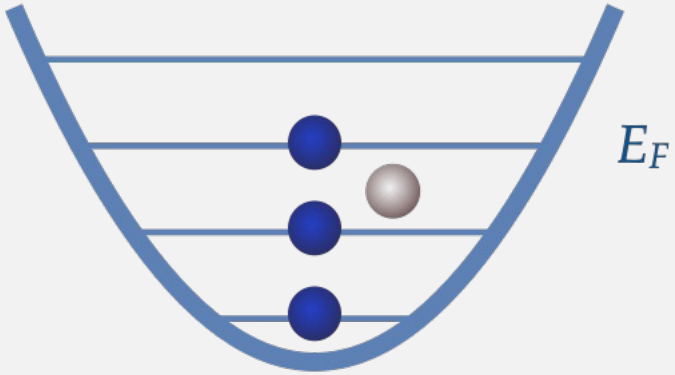
  BEC of dressed molecules is unstable if interactions are *attractive*



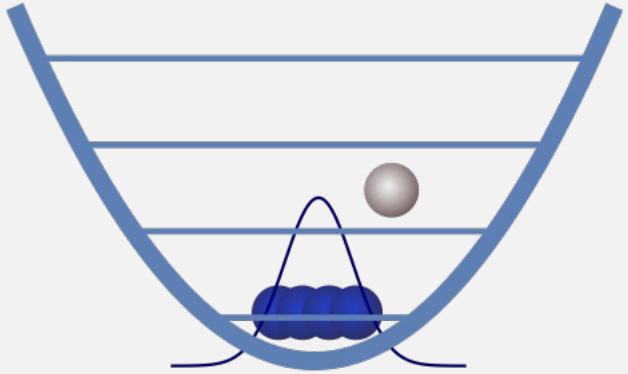

$$u = U_{BB}\left[1 - \frac{k_F a}{(k_F a)_{\text{tcp}}}\right]$$

Parish, Adlong, Liu Levinsen PRA 2021

# Fermi vs Bose polarons




Fermi gas + impurity



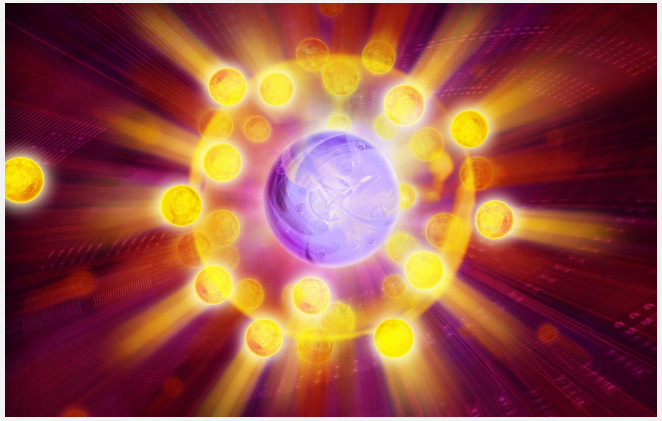
BEC + impurity

- Non-interacting medium
- Abrupt transitions for mobile impurity
- Orthogonality catastrophe for $m_I \to \infty$

- Weakly interacting medium
  - Boson-boson scattering length
- *Smooth* crossover at single-impurity level
- Analogue of OC when $a_{BB} \to 0$
- Effect of Efimov trimers

# Lecture summary

- Fermi polarons are fundamental to a range of different systems
- The concept of the polaron quasiparticle
- Polaron-molecule transition and how it relates to quantum phase transitions
- Beyond the single polaron



Next:

- Energy spectrum and dynamics
- Role of temperature
- Nature of the repulsive polaron