The quantum impurity problem & beyond

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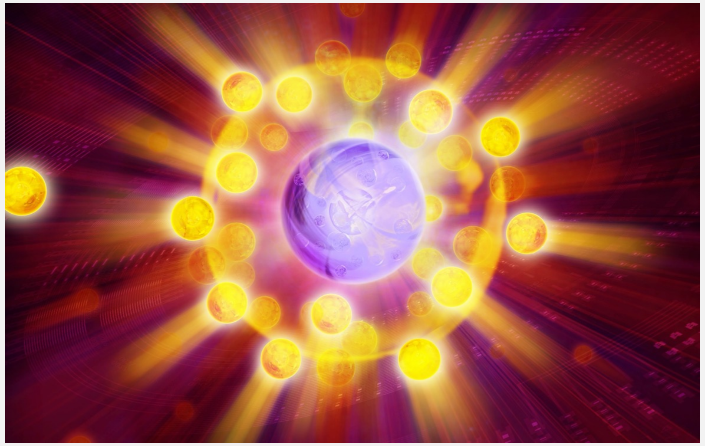
Previous Lecture

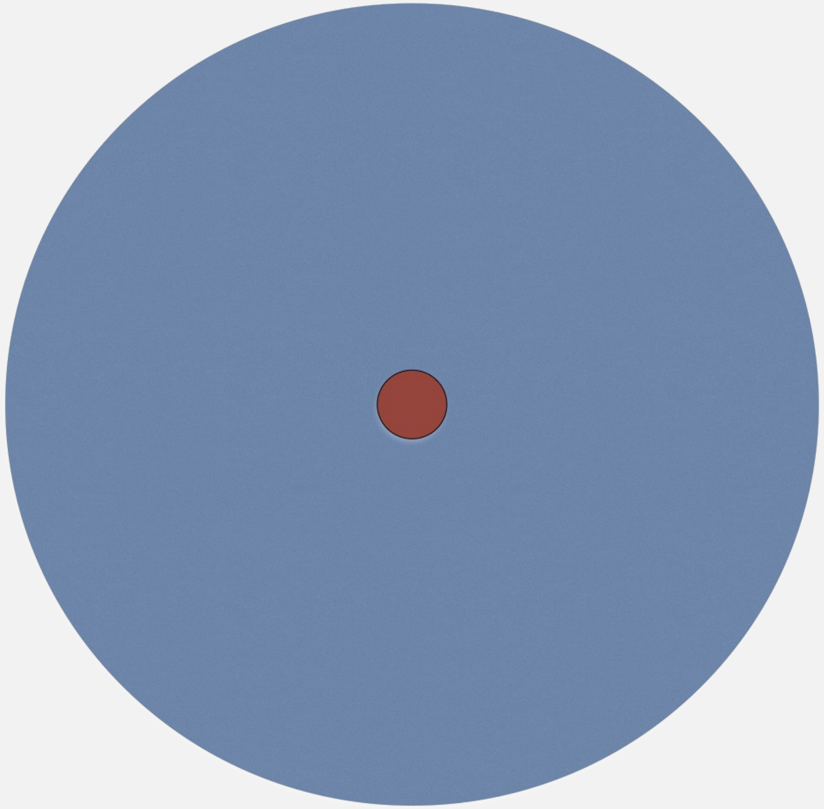
- Fermi polarons are fundamental to a range of different systems
- The concept of the polaron quasiparticle
- Polaron-molecule transition and how it relates to quantum phase transitions
- Beyond the single polaron



This Lecture:

- Energy spectrum and dynamics
- Role of temperature
- Nature of the repulsive polaron

Quantum dynamics

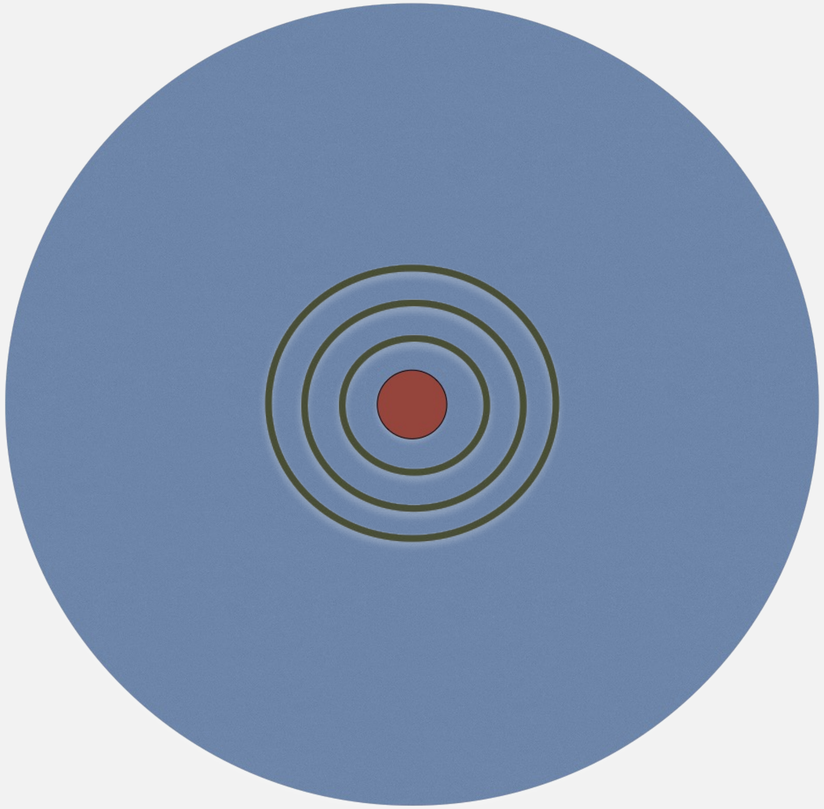


$t = 0$

Non-interacting impurity
immersed in a Fermi gas

Quantum dynamics

$$\langle \Psi(0) | \Psi(t) \rangle = \, ?$$

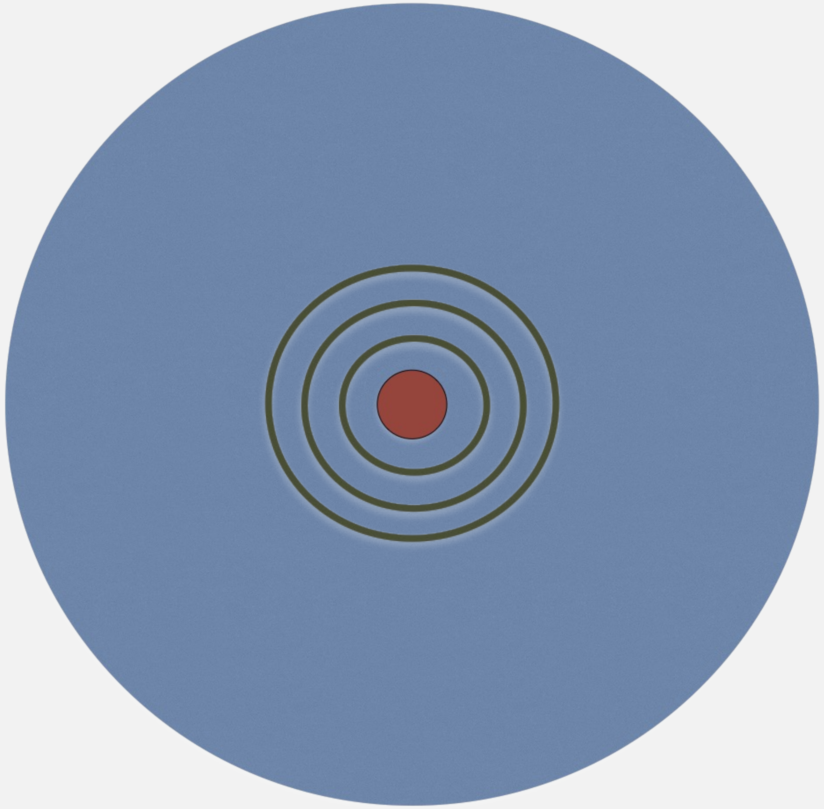


$t > 0$

Strongly interacting impurity
immersed in a Fermi gas

Quantum dynamics

$$\langle \Psi(0) | \Psi(t) \rangle = \, ?$$



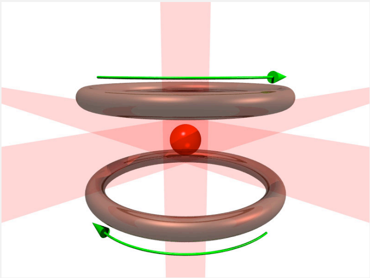
$t > 0$

Strongly interacting impurity
immersed in a Fermi gas

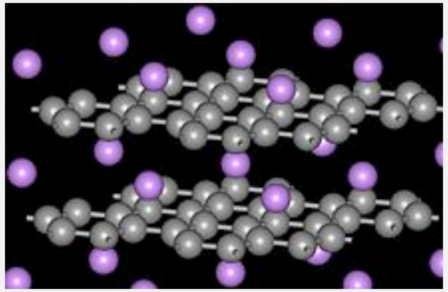
- Cold atoms can probe "ultrafast" dynamics of Fermi systems $t \lesssim \hbar/\varepsilon_F$

$\frac{\hbar^2 k_F^2}{2m}$

- Low energies compared to solid state



microseconds



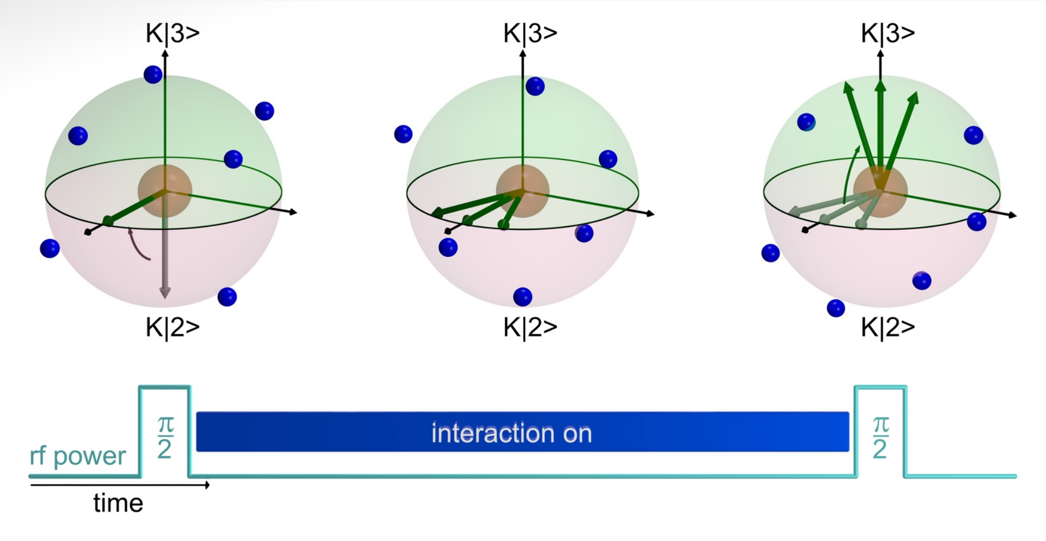
~100 attoseconds

Ramsey interferometry

- E.g., ^{40}K impurities in a ^{6}Li Fermi gas

$^{40}\mathrm{K}\,|2\rangle \rightarrow$ non-interacting

$^{40}\mathrm{K}\,|3\rangle \rightarrow$ tunable interactions

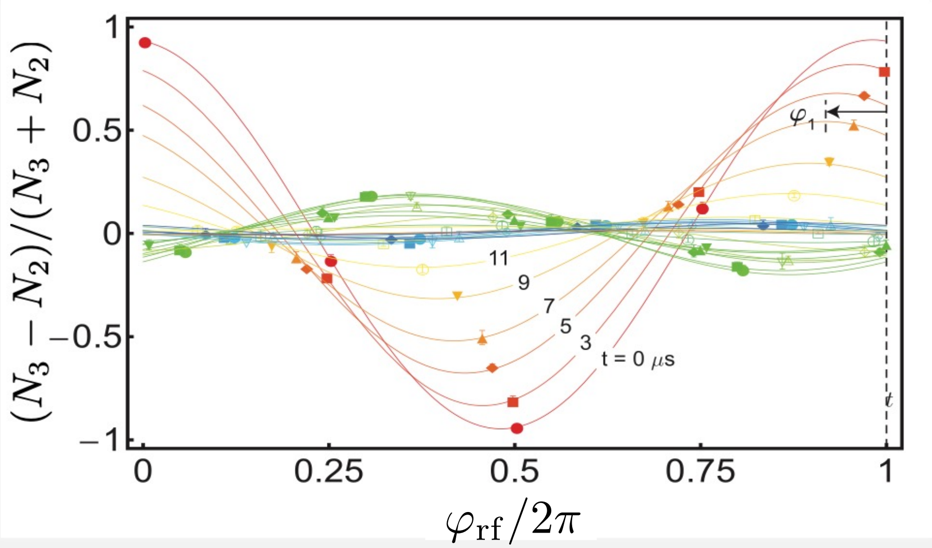


φ_{rf}

- Measurement of spin populations gives:

$$\frac{N_3 - N_2}{N_3 + N_2} = -\,\mathrm{Re}\left[e^{i\varphi_{\mathrm{rf}}} S(t)\right]$$

where $S(t) = \langle \Psi_0(t) | \Psi_{\mathrm{int}}(t) \rangle$



M. Cetina et al., Science 354, 96 (2016)

Dynamics and energy spectrum

- Ramsey response (overlap):

$$S(t) = \langle \Psi_0(t) | \Psi_{\text{int}}(t) \rangle = \langle \psi_0 | e^{-i\hat{H}t/\hbar} | \psi_0 \rangle$$

with non-interacting state $|\psi_0\rangle = \hat{c}_0^\dagger |\mathrm{FS}\rangle$

- Expansion in eigenstates:

$$S(t) = \langle \psi_0 | \, e^{-i\hat{H}t/\hbar} \, | \psi_0 \rangle = \sum_j |\langle \psi_0 | \phi_j \rangle|^2 \, e^{-iE_j t/\hbar}$$

$|\phi_j\rangle$, E_j

$\sum_j |\phi_j\rangle\langle\phi_j|$

Dynamics and energy spectrum

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- General form of eigenstates:

$$|\psi\rangle = \left[\alpha_0 \hat{c}_{\mathbf{0}}^\dagger + \sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{kq}} \hat{c}_{\mathbf{q}-\mathbf{k}}^\dagger \hat{f}_{\mathbf{k}}^\dagger \hat{f}_{\mathbf{q}} + \cdots \right] |\text{FS}\rangle$$

$$S(t) = \sum_j \underbrace{|\alpha_{0,j}|^2}_{z_j} e^{-iE_j t/\hbar}$$

- Fourier transform gives spectral function:

$$A(\omega) = \text{Re} \int_0^\infty \frac{dt}{\pi} e^{i\omega t} S(t)$$

$$= \sum_j |\alpha_{0,j}|^2 \delta(\omega - E_j)$$

$S(t) = S^*(-t)$

Dynamics and energy spectrum

- Ramsey response (overlap):

$$S(t) = \langle \Psi_0(t) | \Psi_{\text{int}}(t) \rangle = \langle \psi_0 | e^{-i\hat{H}t/\hbar} | \psi_0 \rangle$$

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Approximate

- ~~General form of~~ eigenstates:

$$|\psi\rangle = \left[\alpha_0 \hat{c}_{\mathbf{0}}^\dagger + \sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{kq}} \hat{c}_{\mathbf{q}-\mathbf{k}}^\dagger \hat{f}_{\mathbf{k}}^\dagger \hat{f}_{\mathbf{q}} \cancel{+ \cdots} \right] |\text{FS}\rangle$$

$|\phi_j\rangle$ $\alpha_{0,j}$

truncate basis

$$S(t) = \sum_j |\alpha_{0,j}|^2 e^{-iE_j t/\hbar}$$

- Fourier transform gives spectral function:

$$A(\omega) = \text{Re} \int_0^\infty \frac{dt}{\pi} e^{i\omega t} S(t)$$

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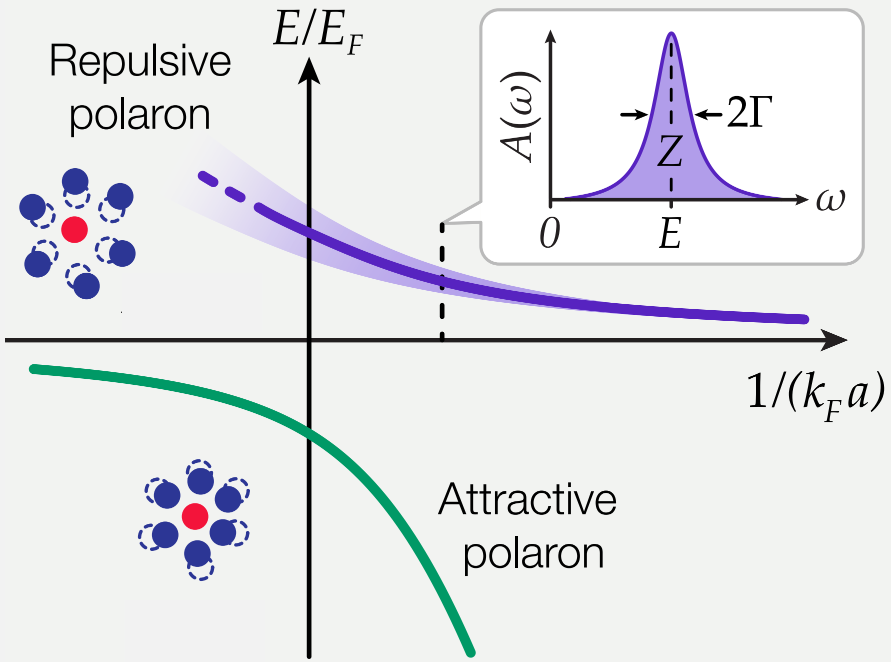
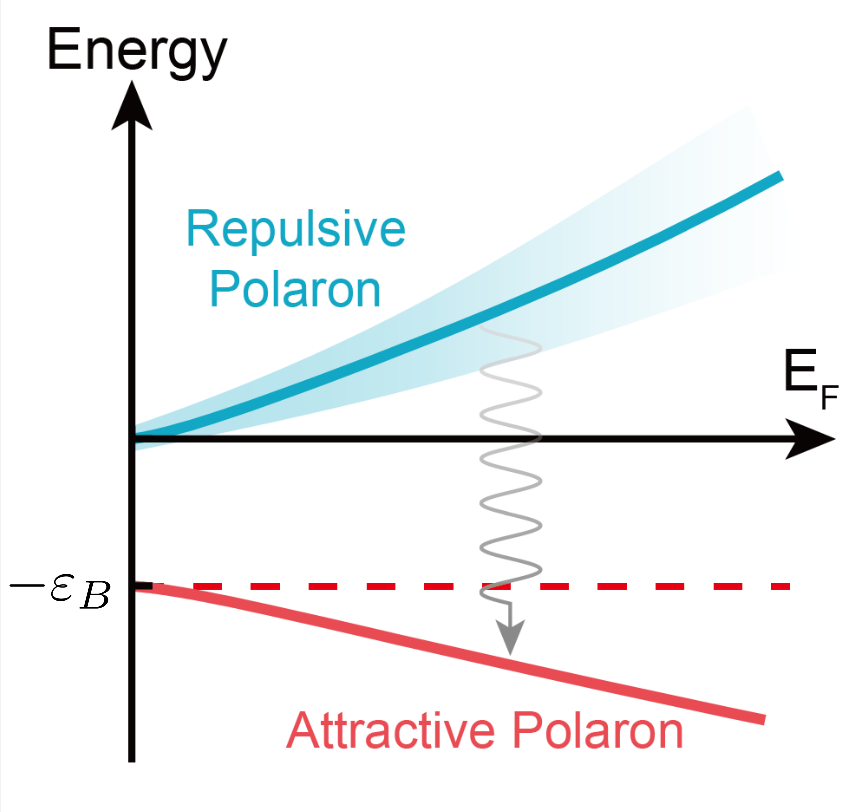
Energy spectrum



- Two quasiparticle branches
  - Attractive polaron $a \to 0^-$
  - Repulsive polaron $a \to 0^+$

- Alternative plot (*a* > 0 or 2D):

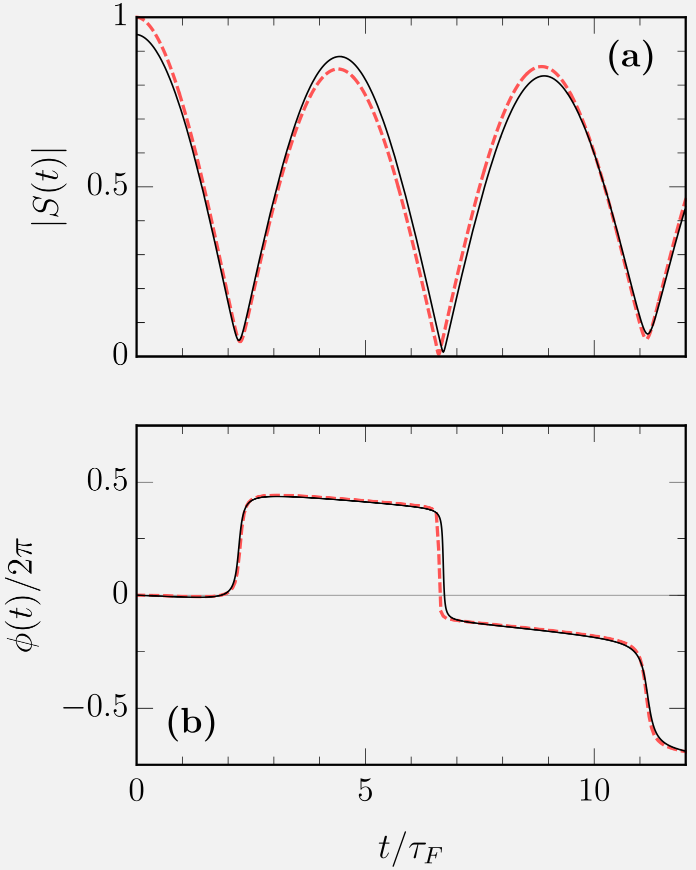


Coherent dynamics

$$S(t) = |S(t)| e^{-i\phi(t)}$$

- Polaron interference when there are two well-defined branches

$$S(t) \simeq Z_{\text{att}}\, e^{-i E_{\text{att}} t} + Z_{\text{rep}}\, e^{-i E_{\text{rep}} t}$$



$Z_{\text{att}} \approx Z_{\text{rep}}$

- Short-time dynamics

$$S(t) = \langle \psi_0 | \, e^{-i\hat{H}t/\hbar} \, |\psi_0\rangle \simeq 1 - \frac{it}{\hbar} \langle \psi_0 | \, \hat{H} \, |\psi_0\rangle$$

$$- \frac{t^2}{\hbar^2} \langle \psi_0 | \, \hat{H}^2 \, |\psi_0\rangle + \ldots$$

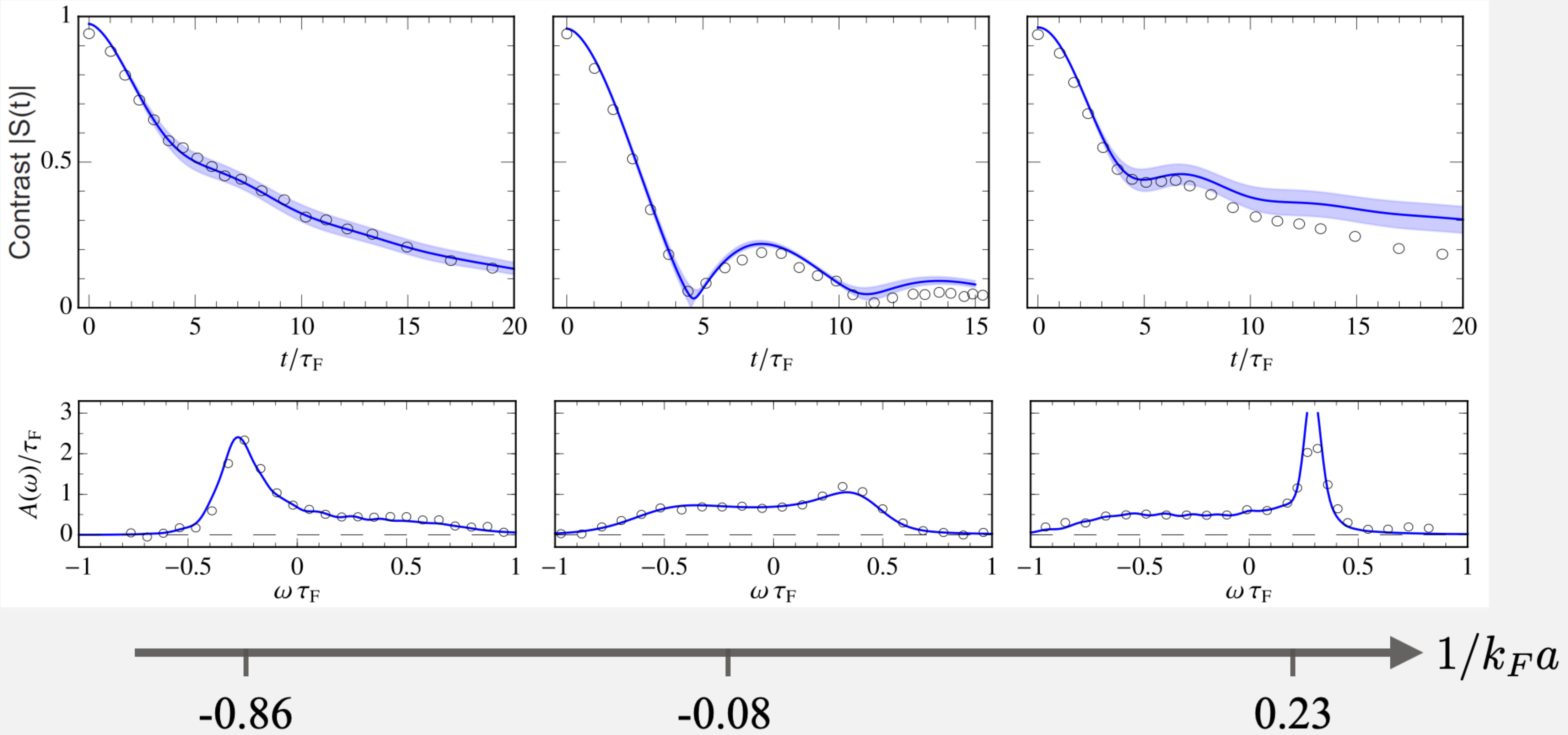
- *Non-analytic* behaviour due to short-range interactions

$$S(t) \simeq 1 - \frac{8e^{-i\pi/4}(m/m_r)^{3/2}}{9\pi^{3/2}} \left(\frac{t}{\tau_F}\right)^{\frac{3}{2}}$$

Parish & Levinsen, PRB 94, 184303 (2016)

Ramsey response (Li-K)

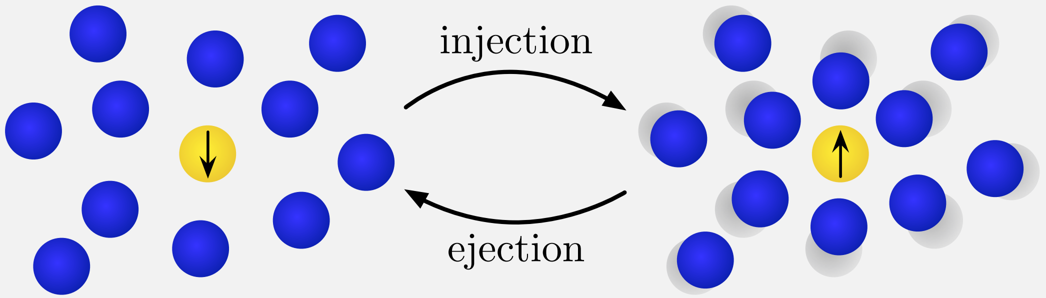
→ Experiment captured by single excitation + temperature



Cetina et al., Science 354, 96 (2016)
Liu, Levinsen & Parish, PRL 122, 205301 (2019)

Probing the spectrum

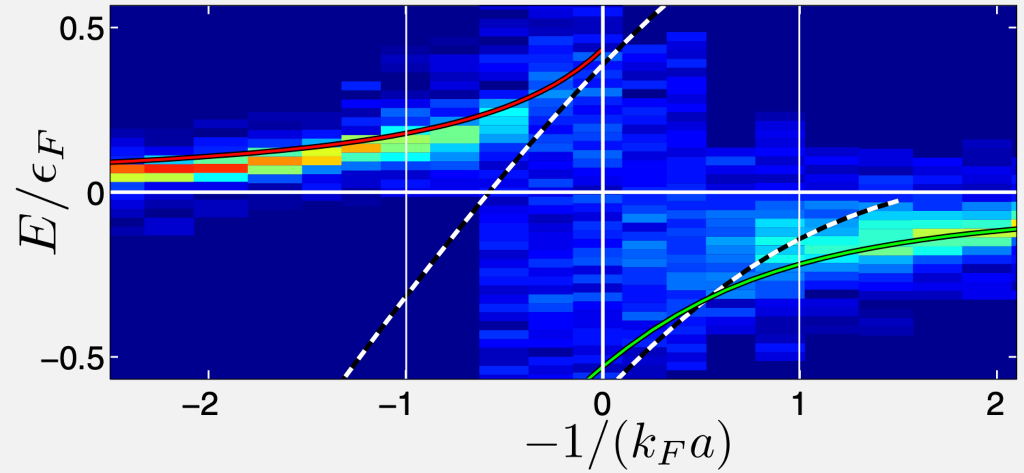
- Rf spectroscopy



- rf pulse drives transition between non-interacting & interacting spin states
- Transfer rate gives impurity spectral function (momentum-averaged)

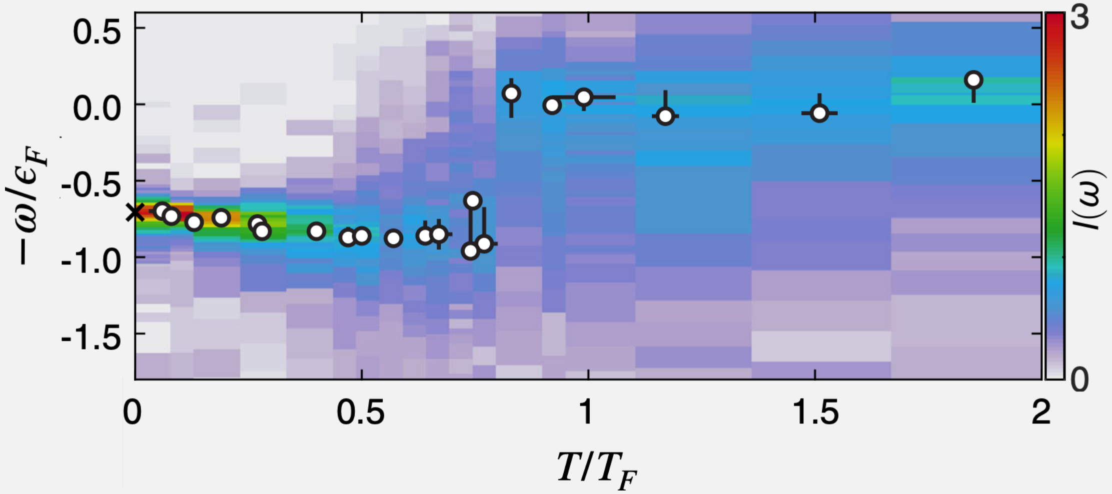
Injection spectrum

Kohstall et al., Nature 2012



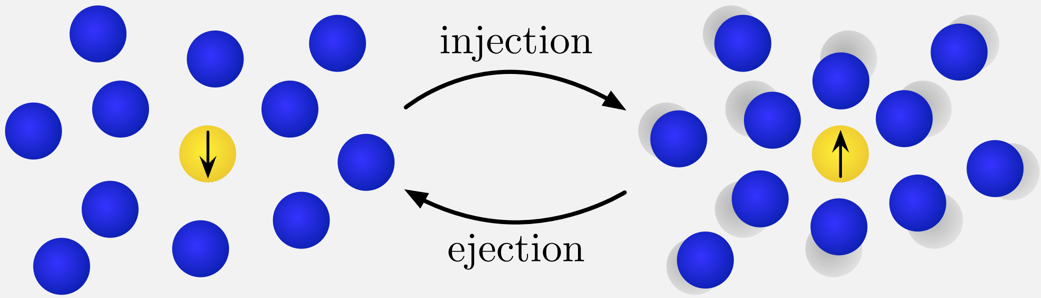
Ejection spectrum @ unitarity

Yan et al., PRL 2019



Probing the spectrum

- Rf spectroscopy



- rf pulse drives transition between non-interacting & interacting spin states

$$I_{\mathrm{ej}}(\omega) = e^{\beta\omega} e^{\beta \Delta F} I_{\mathrm{inj}}(-\omega)$$

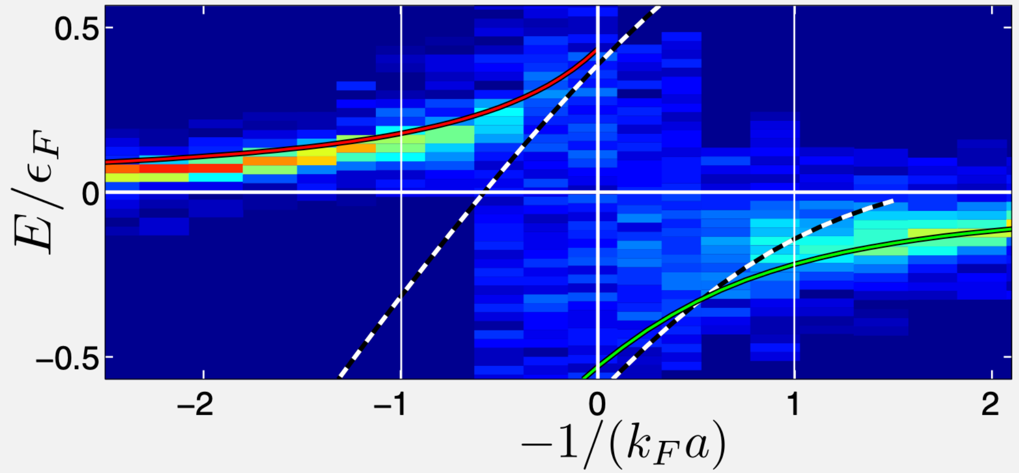
$\beta = 1/k_B T$

$\Delta F = F - F_0$

Liu, Shi, Levinsen, & Parish, PRL 2020;
Liu, Shi, Parish, & Levinsen, PRA 2020

Injection spectrum

Kohstall et al., Nature 2012



Ejection spectrum @ unitarity

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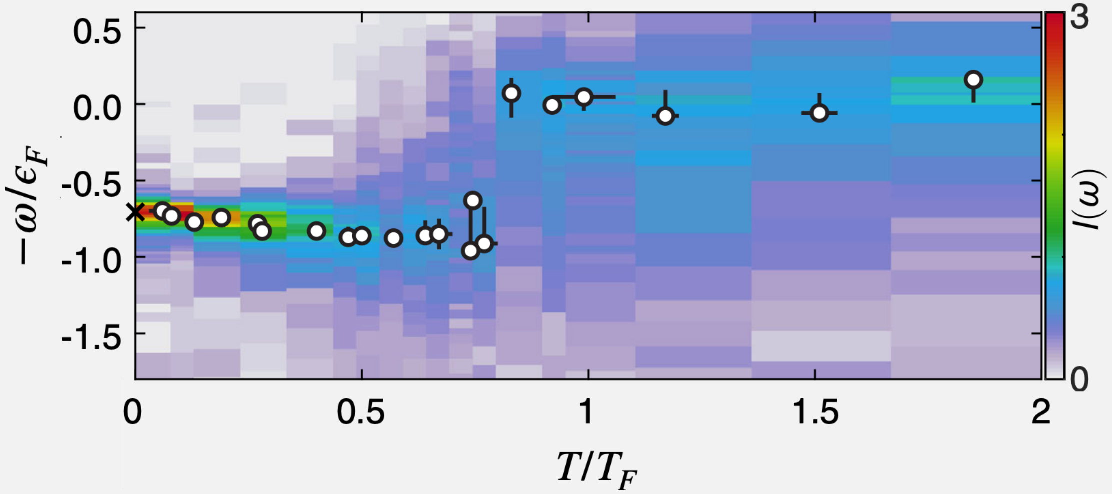
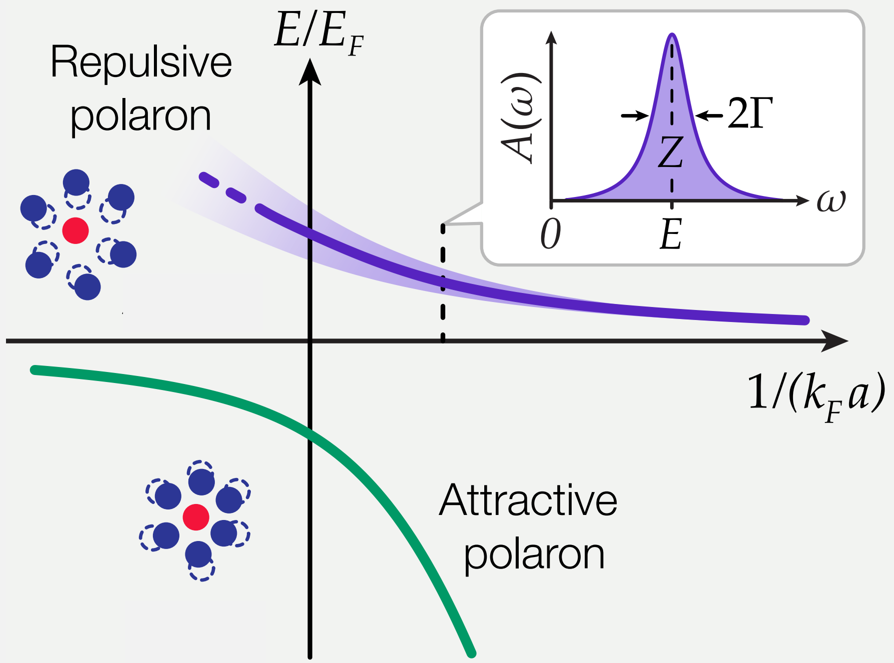


Repulsive polaron branch

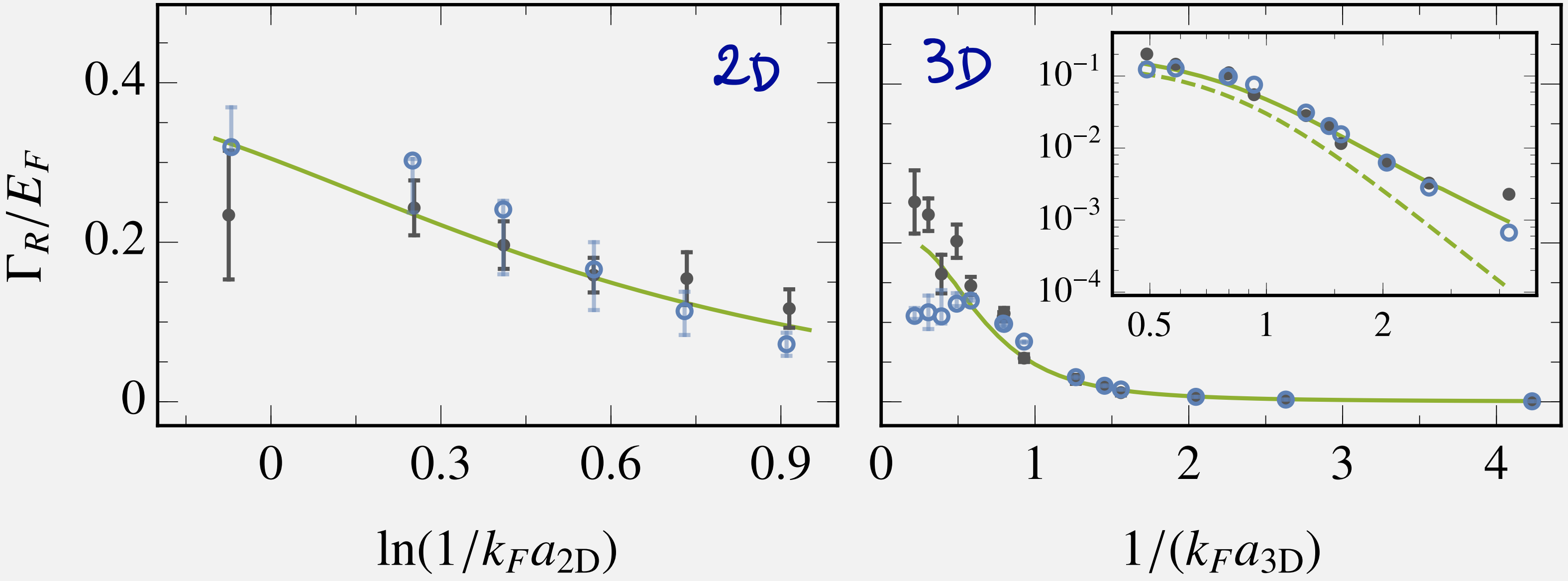
- Metastable excited quasiparticle



Does the decay into lower energy states determine the quasiparticle lifetime?

Repulsive polaron branch

- Extracted width of polaron peak

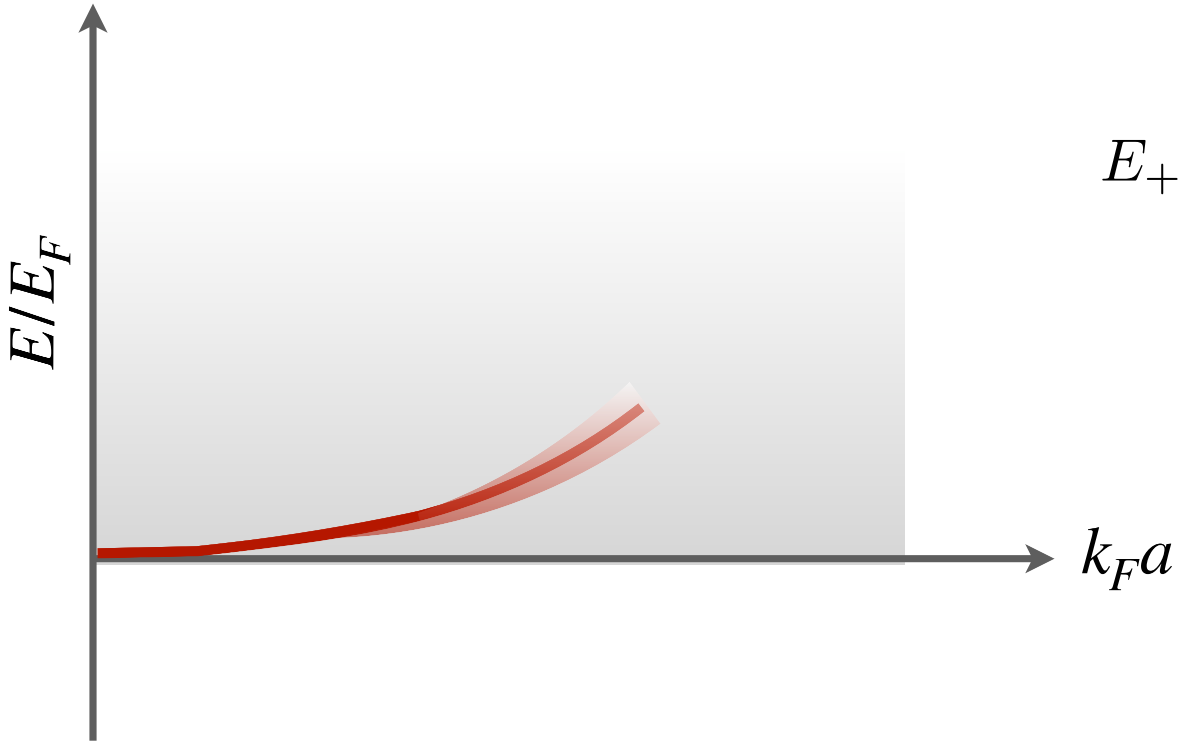


NO decay into lower branch…

Adlong, Liu, Scazza, Zaccanti, Oppong, Fölling, Parish, & Levinsen, PRL 2020

Repulsive polaron lifetime

- Dominated by “many-body dephasing”



$$E_+ = \frac{4k_F a_{3\mathrm{D}}}{3\pi} E_F + O(a_{3\mathrm{D}}^2)$$

$$\Gamma \simeq \frac{8(k_F a_{3\mathrm{D}})^4}{9\pi^3} E_F$$

Adlong, Liu, Scazza, Zaccanti, Oppong, Fölling, Parish, & Levinsen, PRL 2020

Repulsive polaron lifetime

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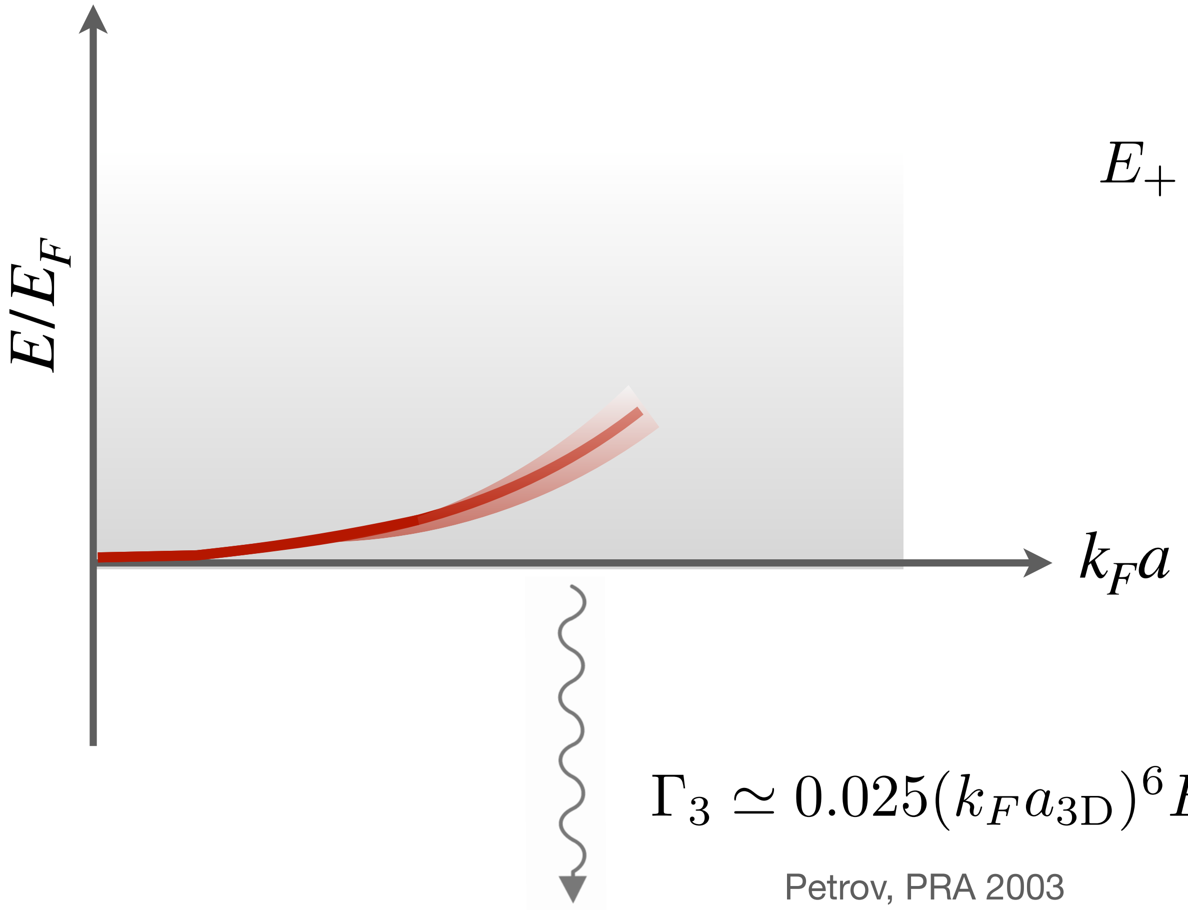

$$E_+ = \frac{4k_F a_{3\mathrm{D}}}{3\pi} E_F + O(a_{3\mathrm{D}}^2)$$

$$\Gamma \simeq \frac{8(k_F a_{3\mathrm{D}})^4}{9\pi^3} E_F$$

$$\Gamma_3 \simeq 0.025(k_F a_{3\mathrm{D}})^6 E_F \ll \Gamma$$

Petrov, PRA 2003

Adlong, Liu, Scazza, Zaccanti, Oppong, Fölling, Parish, & Levinsen, PRL 2020

Lecture summary

- Cold-atom experiments can probe ultrafast coherent dynamics of fermionic systems
  - Universal short-time dynamics
  - Interference between branches
- Relationship between injection and ejection rf spectra at non-zero T
- Repulsive polaron lifetime dominated by many-body dephasing

Acknowledgements

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- Nelson Darkwah Oppong (LMU)
- Simon Fölling (LMU)