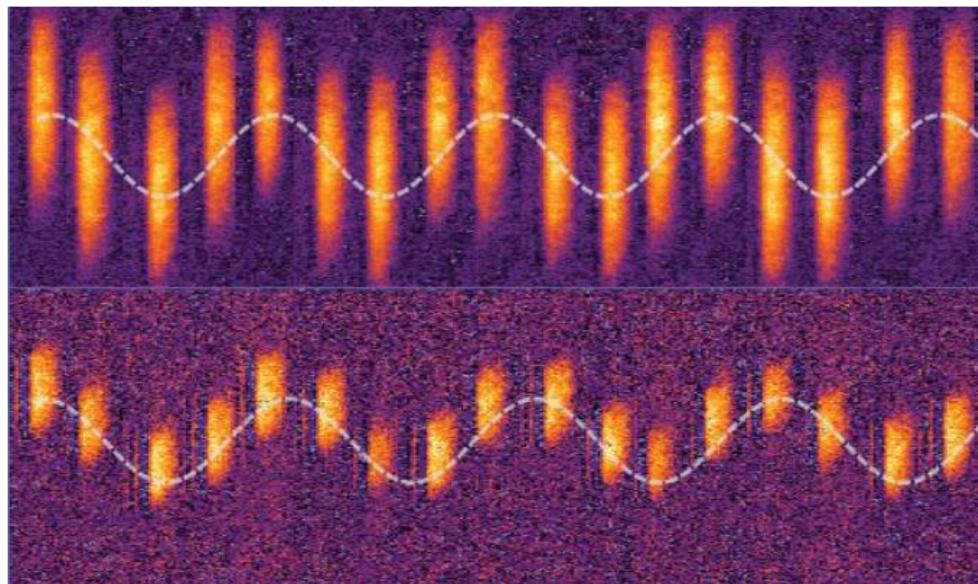


Dual Bose-Fermi Superfluids

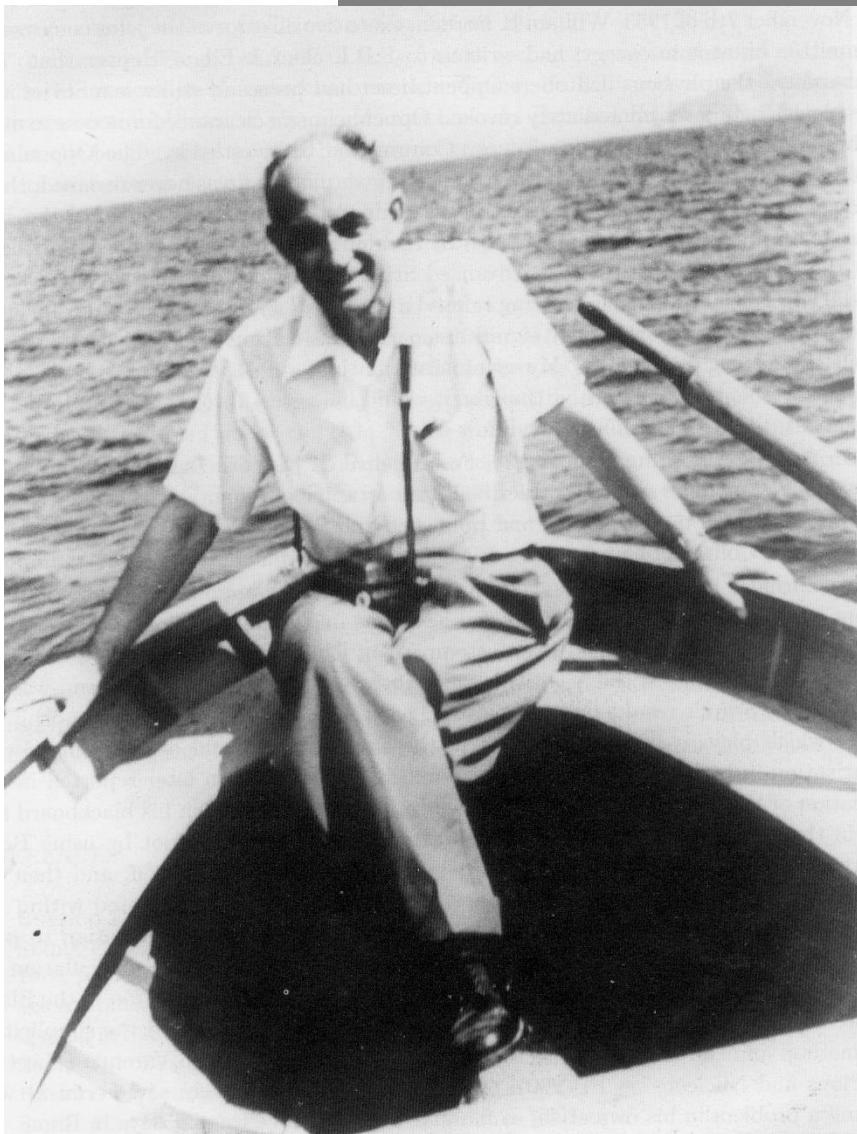


C. Salomon



Enrico Fermi School on Mixtures of Quantum Gases
Varenna, July 18, 2022

Bose and Fermi Statistics



Enrico Fermi on lake Como

ITALIAN PHYSICAL SOCIETY

PROCEEDINGS

OF THE

INTERNATIONAL SCHOOL OF PHYSICS

"ENRICO FERMI"

COURSE CLXIV

edited by M. INGUSCIO, W. KETTERLE and C. SALOMON

Directors of the Course

VARENNA ON LAKE COMO

AL LIVIANA DI FISICA

VILLA MONASTERO

Lake Como, Italy

20 – 30 June 2006

Ultra-cold Fermi Gases

2007

Istituto Nazionale di Fisica Nucleare (INFN)

European Science Foundation
through QUDEDIS and IOP program

IOS
Press

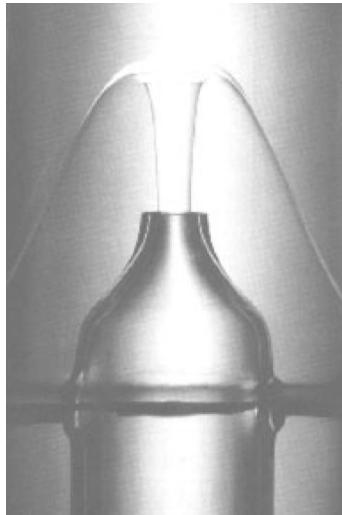
Varenna 2006

Outline of the two lectures

- Dual Bose-Fermi superfluid with ${}^6\text{Li}$ - ${}^7\text{Li}$ isotopes
 - Center of mass modes and link with Equation of State
 - Measurement of critical velocity for superfluid counterflow
 - Lifetime of the Bose Fermi mixture: a simple formula !
 - Link with Tan's contact
- 1) I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, and C. Salomon, *Science*, **345**, 1035, 2014
- 2) M. Delehaye, S. Laurent, I. Ferrier-Barbut, S. Jin, F. Chevy, C. Salomon, *PRL*, **115**, 265303, 2015
- 3) Y. Castin, I. Ferrier-Barbut and C. Salomon
Comptes-Rendus Acad. Sciences, Paris, **16**, 241, 2015
- 4) S. Laurent, M. Pierce, M. Delehaye, T. Yefsah, F. Chevy, C. Salomon
Phys. Rev. Lett., **118**, 103403, 2017

111 years of quantum fluids

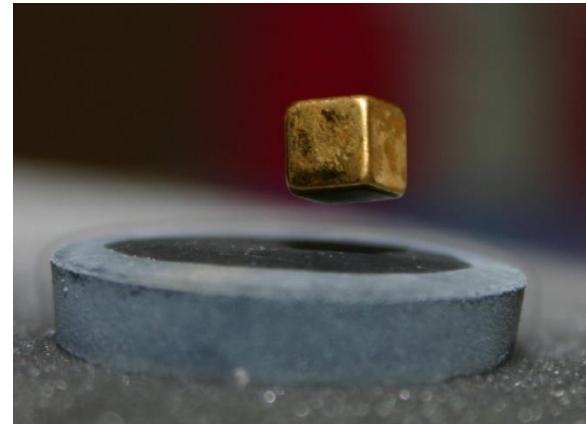
Bose Einstein condensate



${}^4\text{He}$

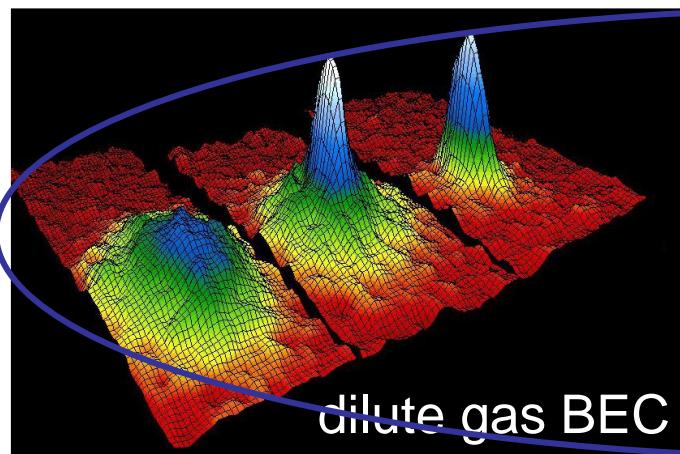
$T \sim 2.2 \text{ K}$

Superconductivity



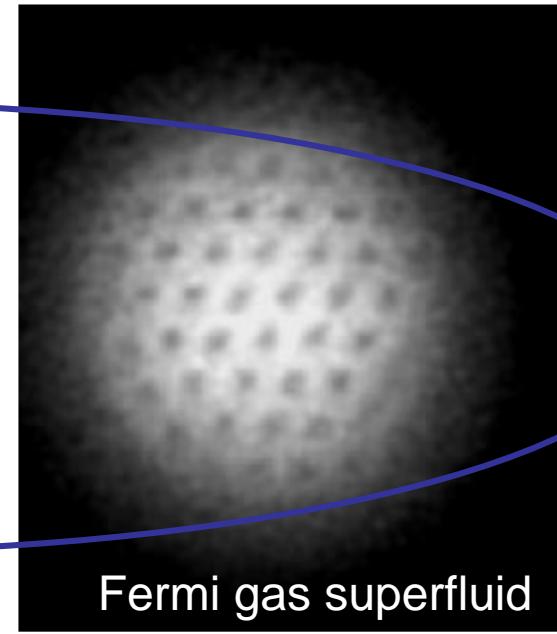
High T_c
77 K

${}^3\text{He}$
2.5 mK



100 nK

+ BEC of light and polaritons



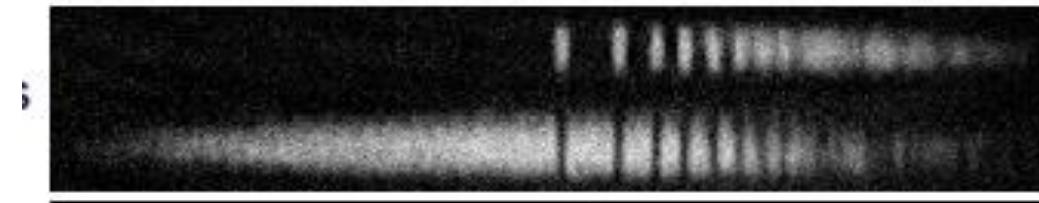
Mixtures of Superfluids

Bose-Bose superfluid mixtures first observed long ago !

Two hyperfine states in Rb at JILA (Myatt et al. '97) and vortex production
Mixtures of BEC's at LENS, Hamburg,
Spinor condensates at MIT, Hamburg, Berkeley, ENS,

Dark-bright soliton production in two Rb BEC, Engels group, PRL 2011

Rb



$|2, 2\rangle$

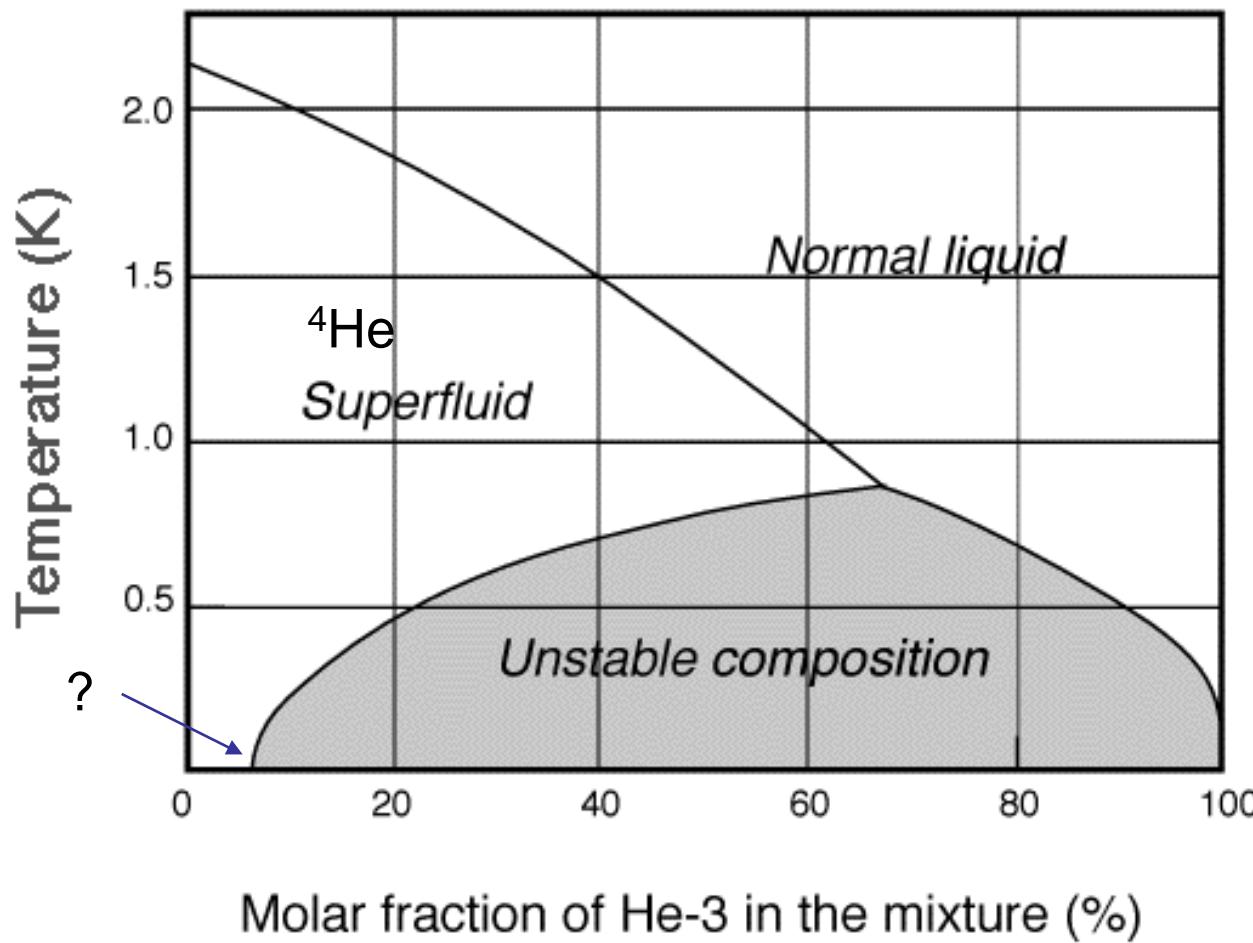
$|1, 1\rangle$

Many groups are studying Bose-Fermi mixtures but
until 2014 none had achieved double Bose-Fermi superfluidity

Today at least 3 examples, ^6Li - ^7Li at ENS (2014)
 ^6Li - ^{41}K at USTC, ^6Li - ^{174}Yb at Washington Univ., (2016)

....

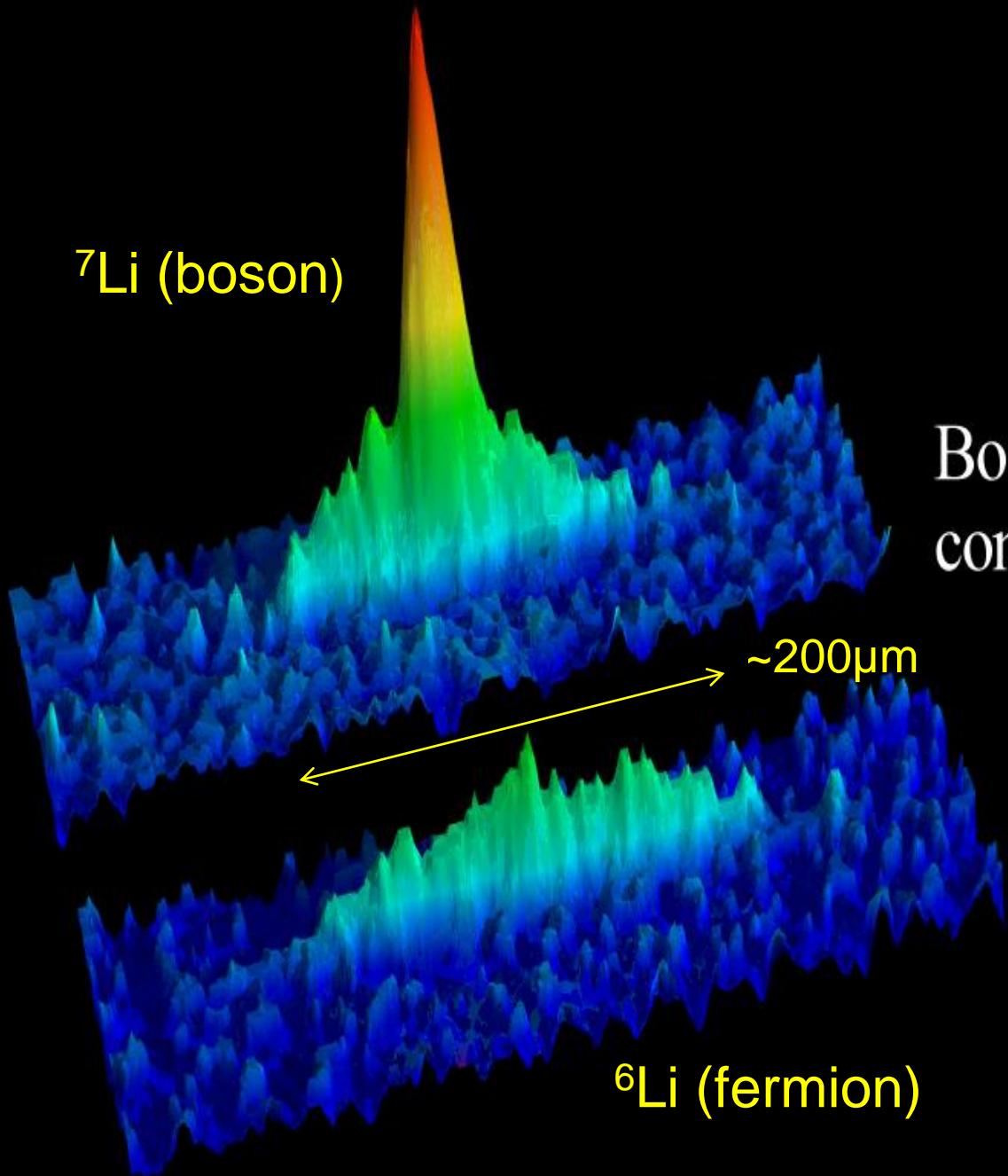
Searching for superfluid Bose-Fermi systems: ^4He - ^3He mixture



Volovik, Mineev, Khalatnikov, JETP, 42, 342 (1975): Fermi liquid theory of mixture

Expected $T_c \sim 1$ to $20 \mu\text{K}$?

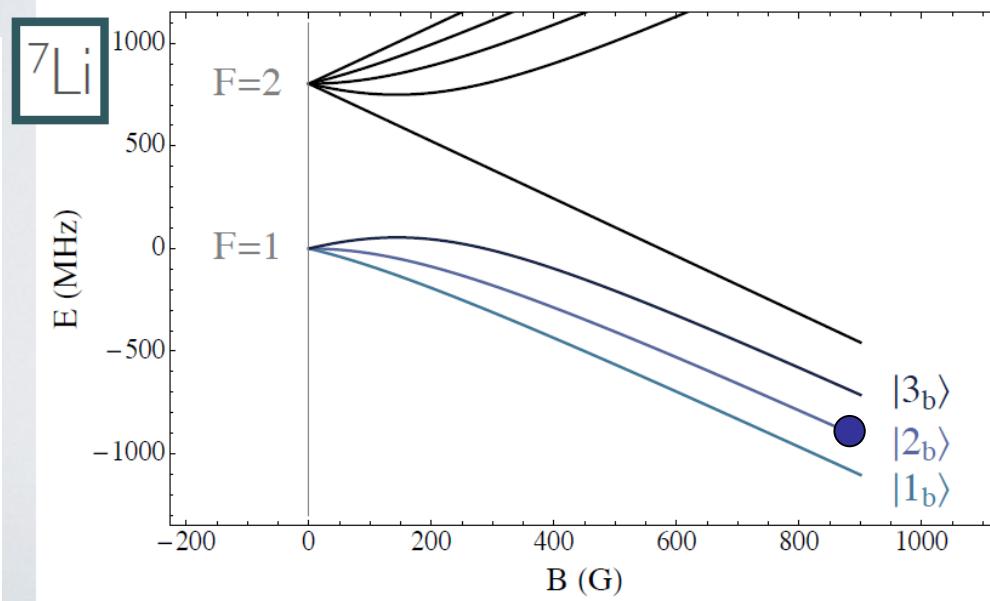
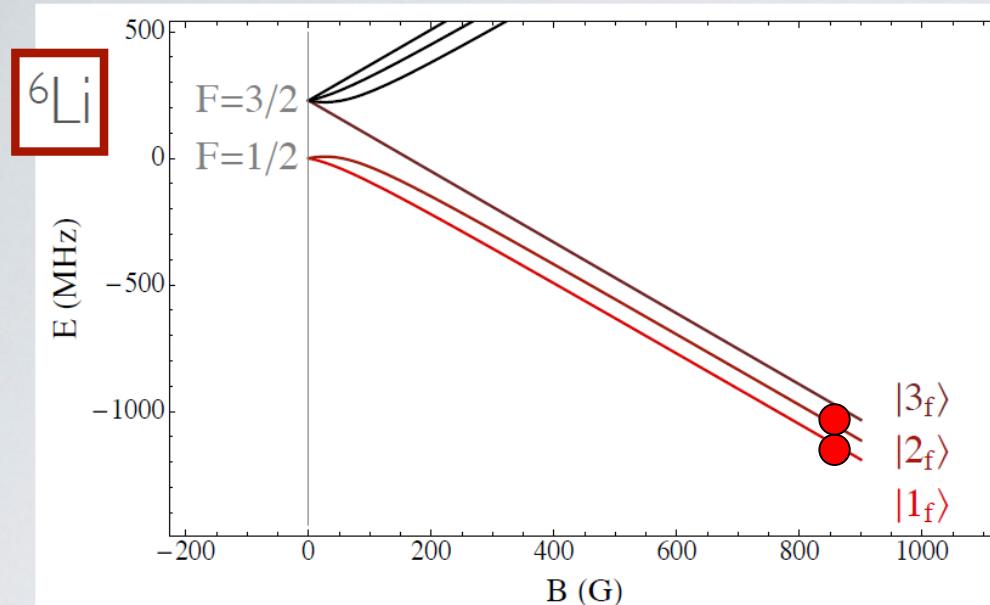
ENS 2001



Bose-Einstein
condensate



^7Li and ^6Li isotopes

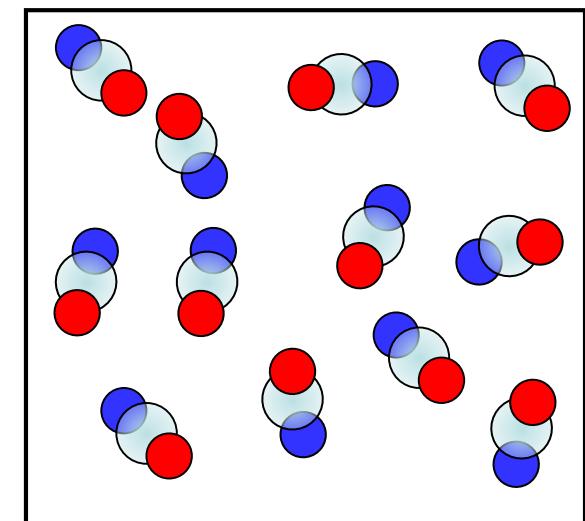
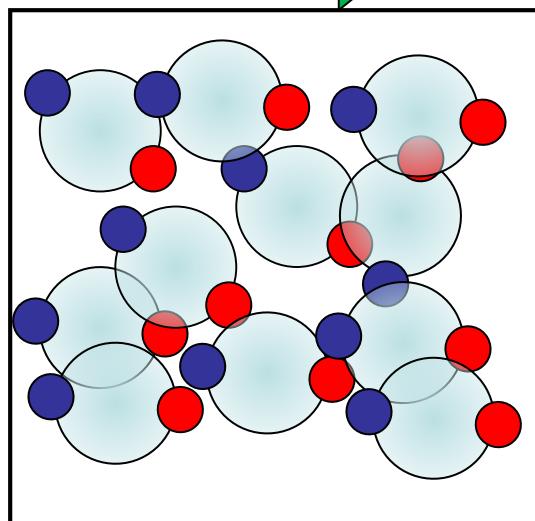
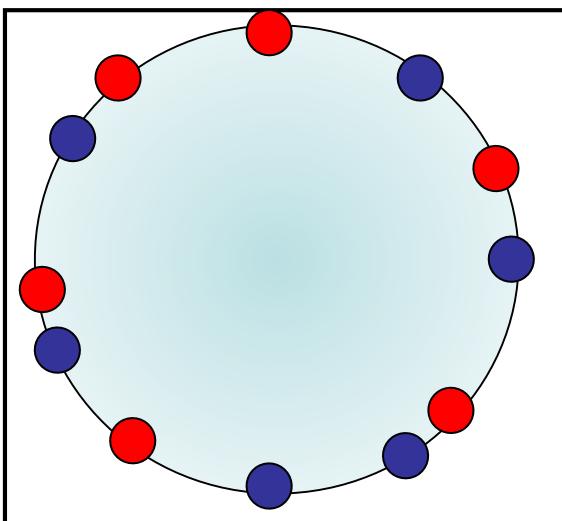


Fermions with two spin states and attractive interaction the BCS-BEC Crossover

Leggett, Nozières, schmidt-Rink,.. '80

Increasing attraction strength

$$T_c \approx T_F e^{-\pi/2k_F|a|}$$



BCS regime:

$$k_F|a| \ll 1$$

Cooper pairs $k, -k$

Well localized in

Momentum: $k \sim k_F$

Delocalized in position

On resonance

$$na^3 \gg 1$$

$$k_F a \geq 1$$

Pairs stabilized by

Fermi sea

Size of pairs

$$\hbar v_F / \Delta \sim k_F^{-1}$$

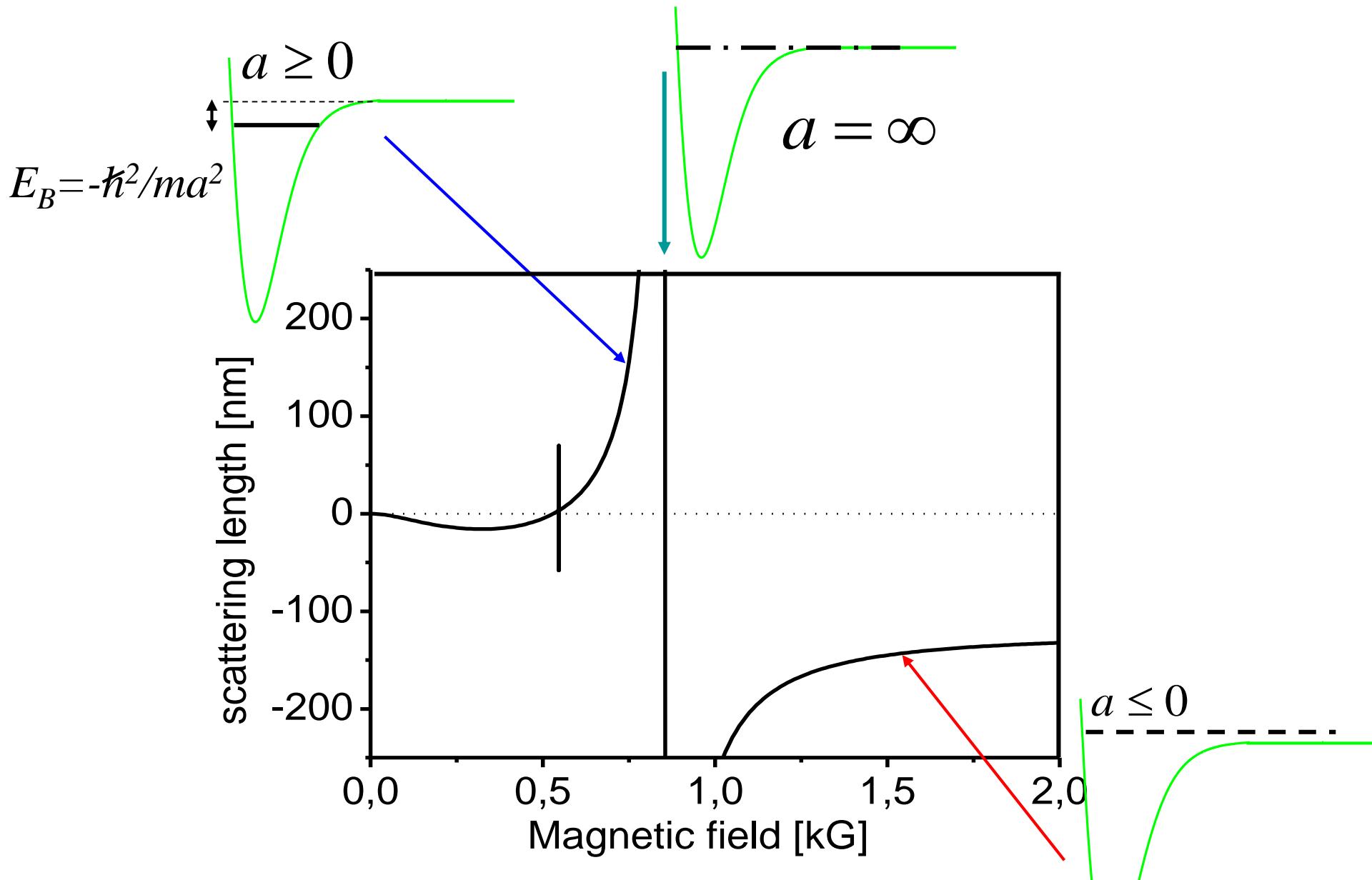
a Molecular Condensate

A two-body bound state

strongly bound molecules

Size: $a \ll n^{-1/3}$

$n^{-1/3}$: average distance
between particles



Tuning interactions:
Fano-Feshbach resonance. Ex: Lithium 6

Fermi Superfluid in the BEC-BCS Crossover

Fermions with two spin states $\uparrow \downarrow$ and tunable attractive interaction

M. Greiner, C. Regal & D. Jin, Nature 2003 with potassium 40

Emergence of a molecular Bose–Einstein condensate from a Fermi gas



*S. Jochim et al., Science 2003, with Lithium 6
Bose-Einstein condensation of molecules*

Followed by large number of studies of BCS-BEC crossover, both theory and experiment, see book edited by W. Zwerger

Equation of State of Quantum Gases

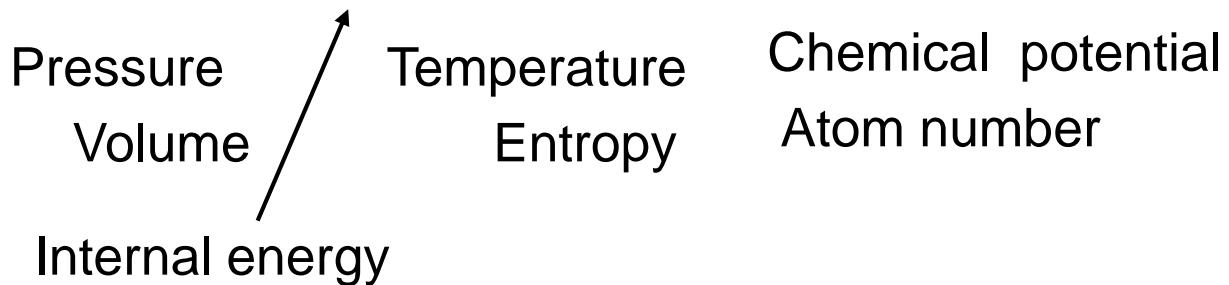
$$PV = Nk_B T$$

is a useful but incomplete equation of state !

Complete information is given by **thermodynamic potentials**:

Grand potential

$$\Omega = -PV = E - TS - \mu N$$



$P(\mu, T)$ is an equation of state of the gas

S. Nascimbène et al., Nature, 463, 1057, (2010) temperature dependence and spin imbalance.

See also:

M. Horikoshi et al., Science, 327, 442 (2010), M. Zwierlein et al., MIT (2012)

The Equation of State of a Cold Gas

Q. Zhou, T.L. Ho,
Nature Physics, 09

C. Cheng, S.Yip,
PRB (2007)

The pressure is obtained from *in situ* images

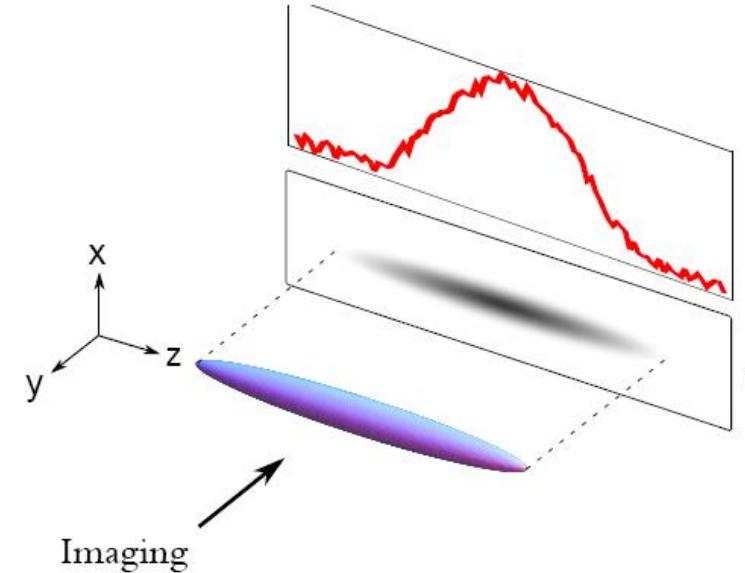
$$P(\mu_z, T) = \frac{m\omega_r^2}{2\pi} \bar{n}(z)$$

$$\bar{n}(z) = \int dx dy n(x, y, z)$$

Doubly-integrated density profile

Local density approx.

$$\mu(r) = \mu_0 - V(r)$$



$P(\mu_z, T)$ is an Equation of State of the locally homogeneous gas

Universal Equation of State at unitarity

$$1/k_F a = 0$$

Thermodynamics is universal

T. L.Ho, E. Mueller, '04

The system has continuous scale invariance

Pressure depends only on $\mu/k_B T$

Critical temperature $T_c = 0.16 T_F$ is universal

At very low Temp.

$$\mu = \xi E_F$$

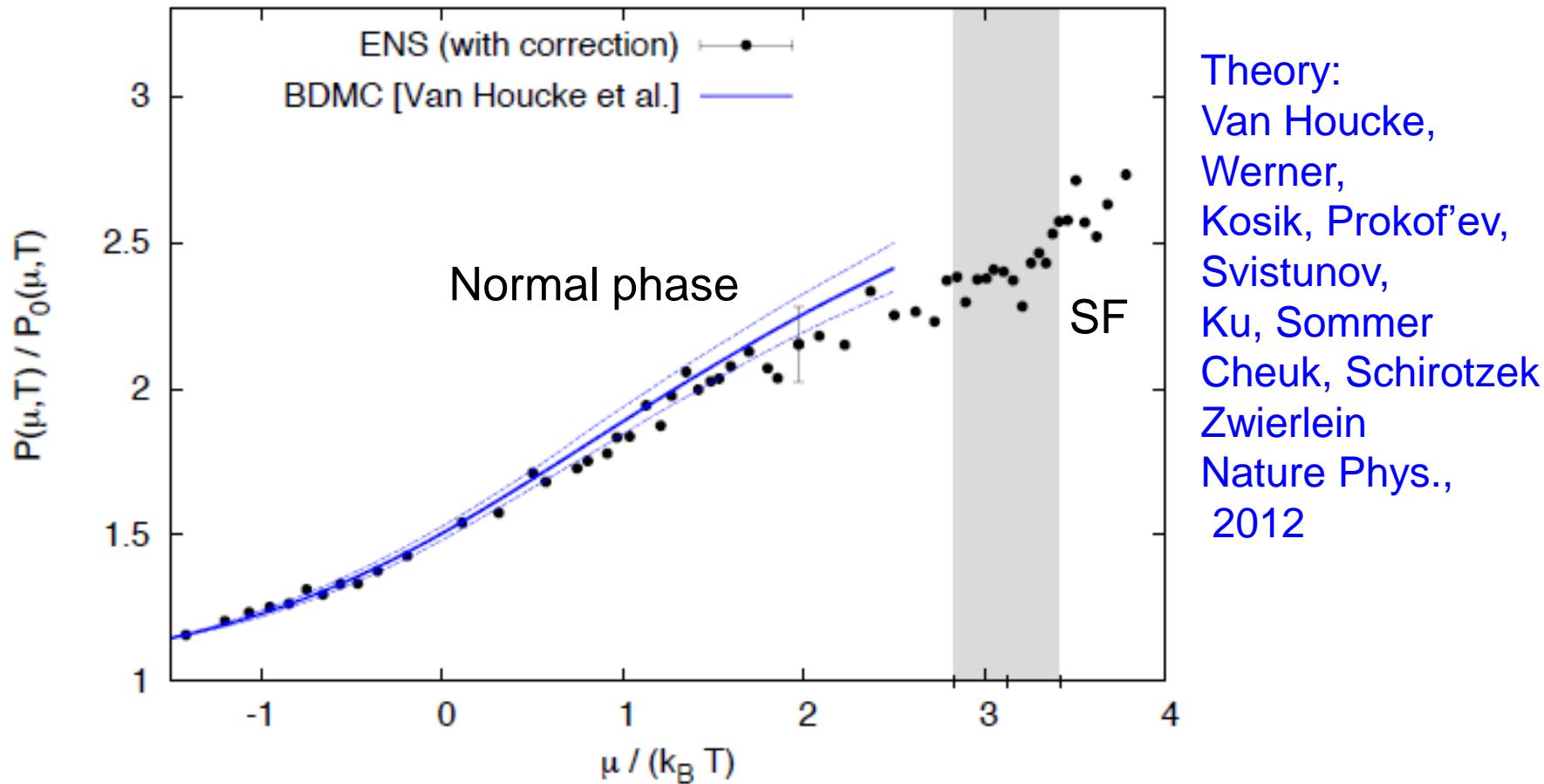
$$\xi = 0.37 E_F$$

*Carlson, Hausmann,
MIT, Ku et al.,
Science 2012*

Universal Equation of State at Unitarity

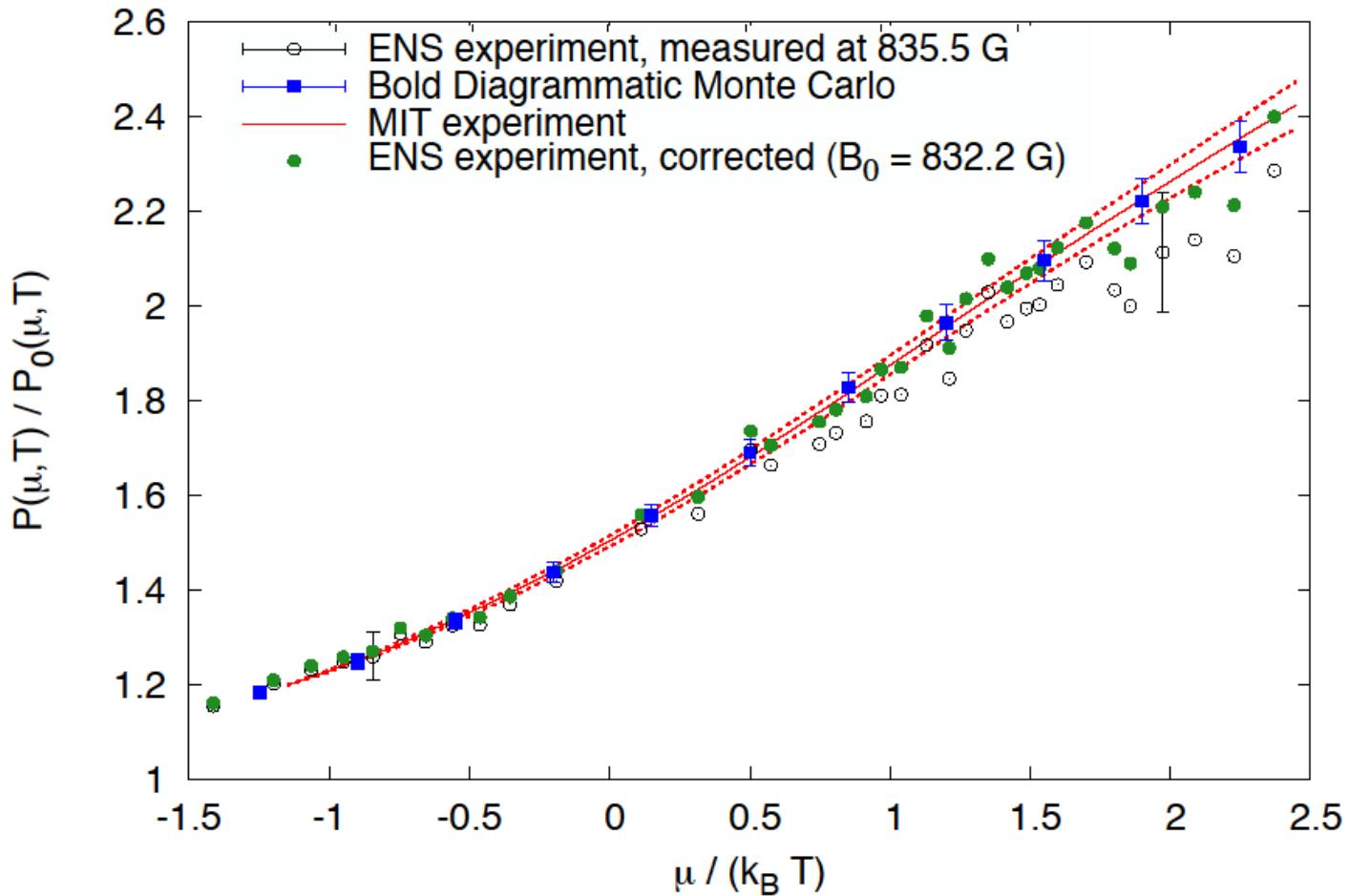
Comparison with Bold Diagrammatic Monte-Carlo

S. Nascimbène, N. Navon, K. Jiang, F. Chevy, C. Salomon, Nature 2010



5% agreement with a Many-Body theory in strongly interacting regime

Comparison with Bold Diagrammatic Monte-Carlo and MIT (2012)

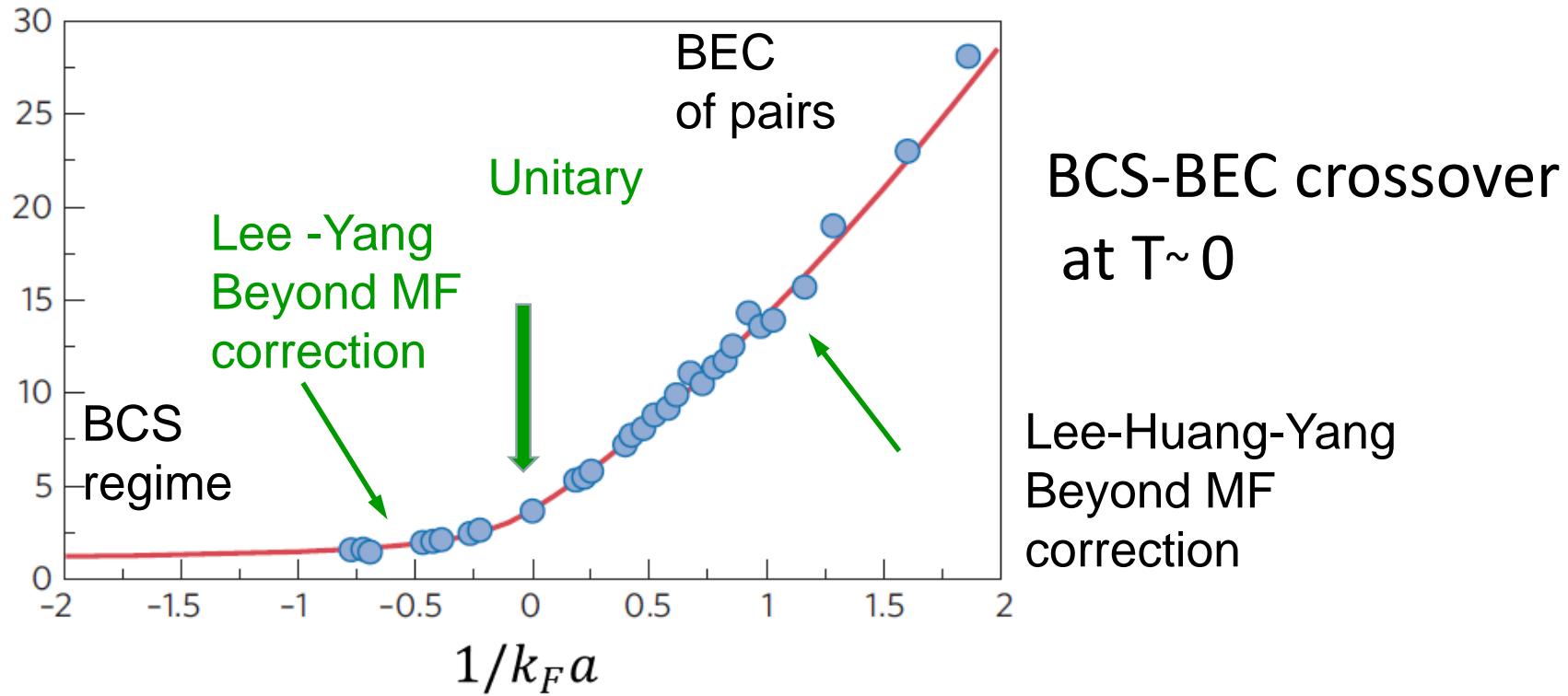


Theory:
Van Houcke,
Werner,
Kosik, Prokof'ev,
Svistunov,
Ku, Sommer
Cheuk, Schirotzek
Zwierlein
Nature Phys.,
2012

5% agreement with a Many-Body theory in strongly interacting regime

Equation of State of Fermi gas in the BEC-BCS crossover

Pressure equation of state $P/P_0 = f(1/k_F a)$

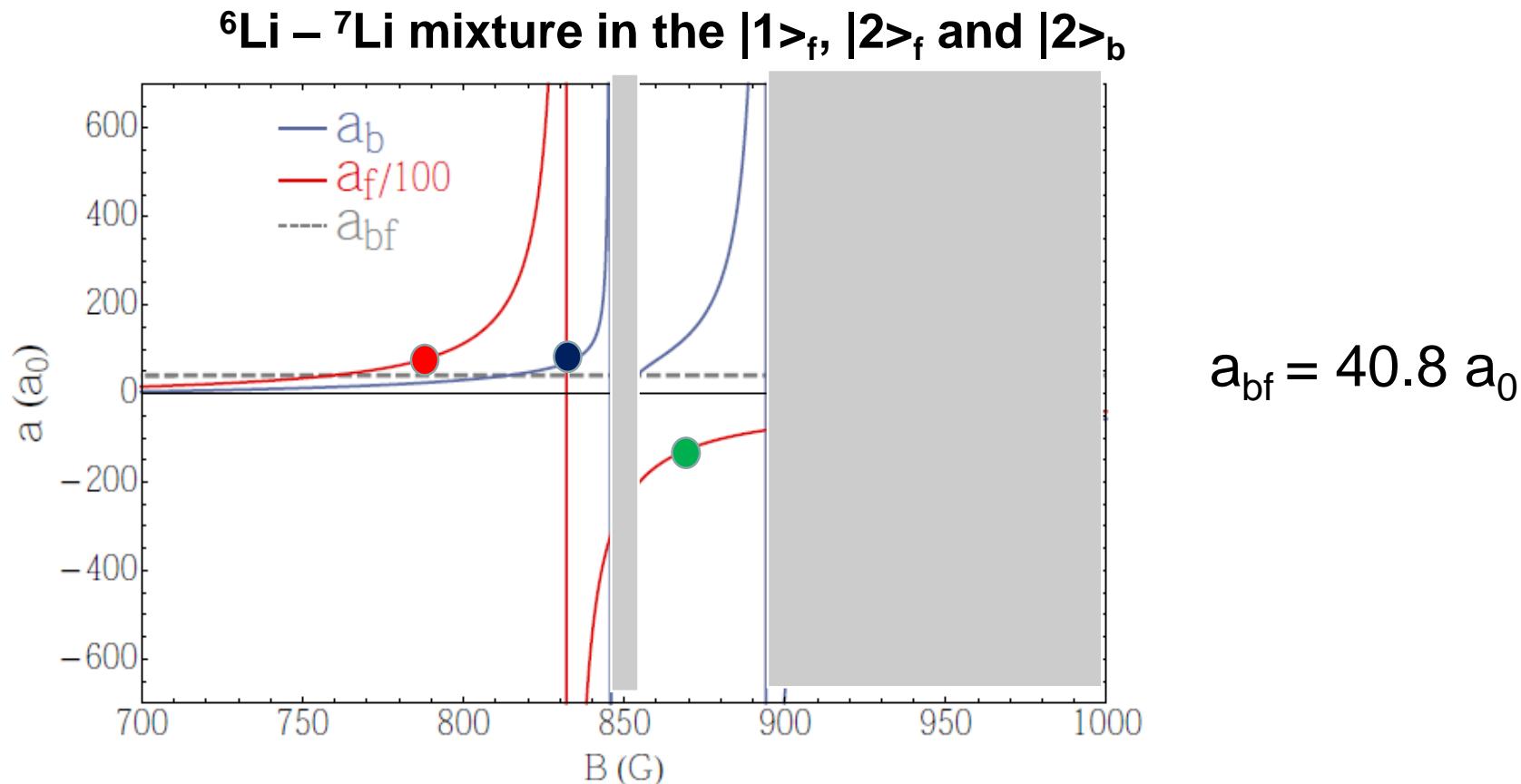


An example of quantum simulation in the strongly correlated regime

Bose-Fermi superfluidity recipe

Requirements:

- Low a_{bf} (no interspecies demixing)
- High $|a_f|$ (high fermionic superfluid T_c)
- Positive a_{bb} (stable BEC)



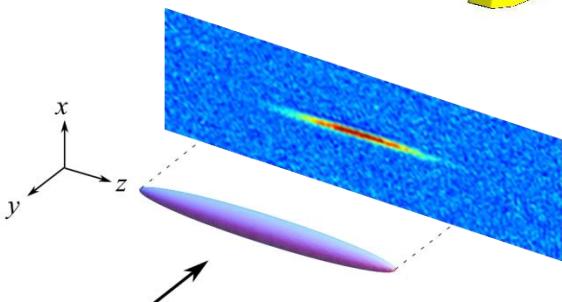
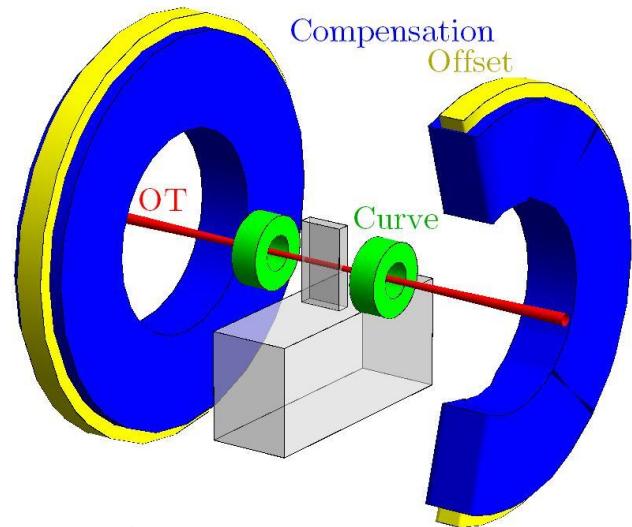
Experimental Setup

Magneto-optical trap of bosonic ^7Li and fermionic ^6Li

After evaporation in a magnetic trap we load the atoms in a single beam optical trap (OT) with magnetic axial confinement. $T \sim 40 \mu\text{K}$

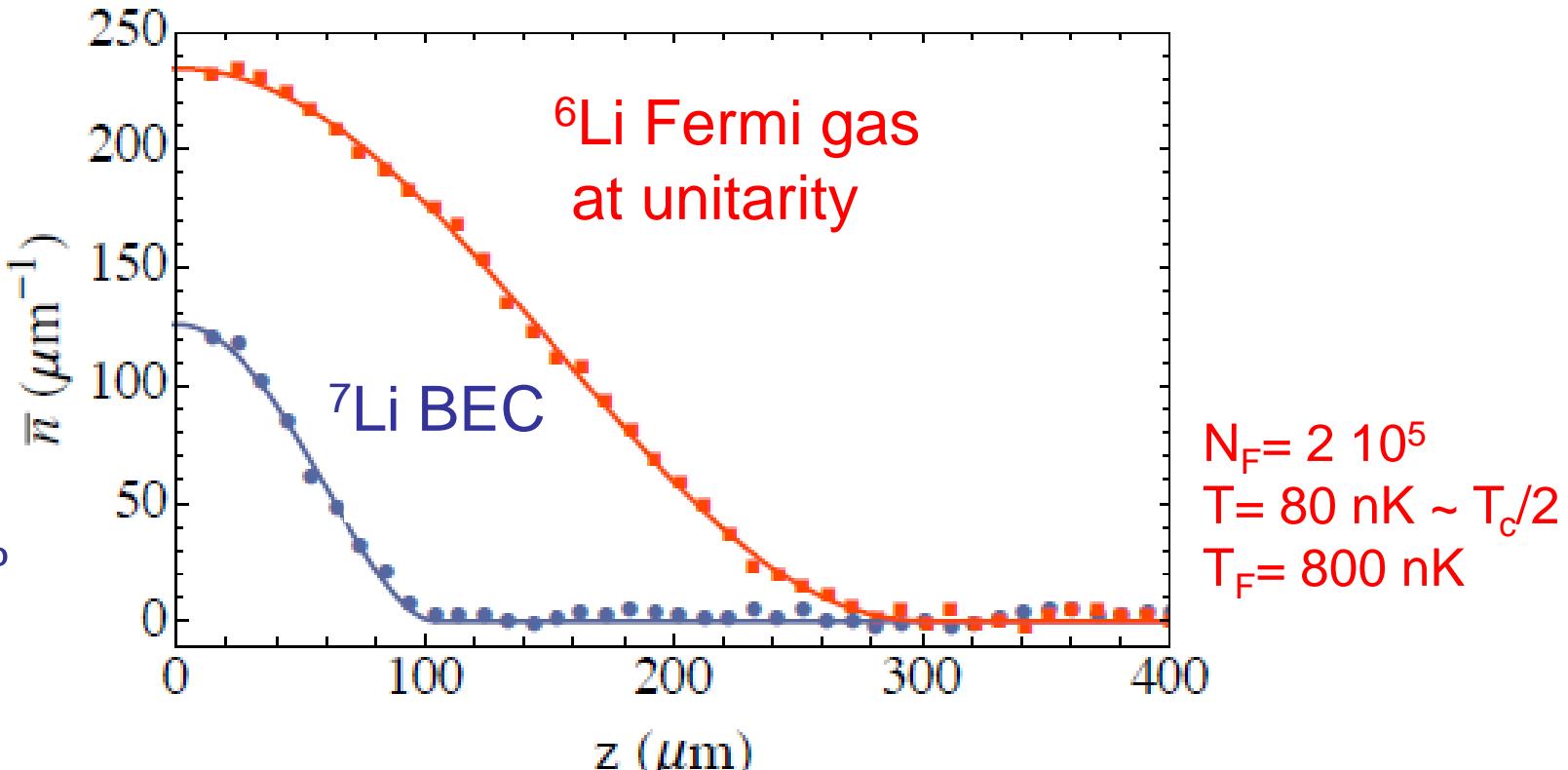
Evaporative cooling of mixture in OT
 ~ 4 second ramp, $T \sim 50\text{-}80 \text{ nK}$

Absorption imaging of the *in-situ* density distributions or Time of Flight



In situ density profiles

$N_B = 2 \cdot 10^4$
 $T = 80 \text{ nK}$
 $N_0/N_B > 80\%$
 $T < T_c/2$

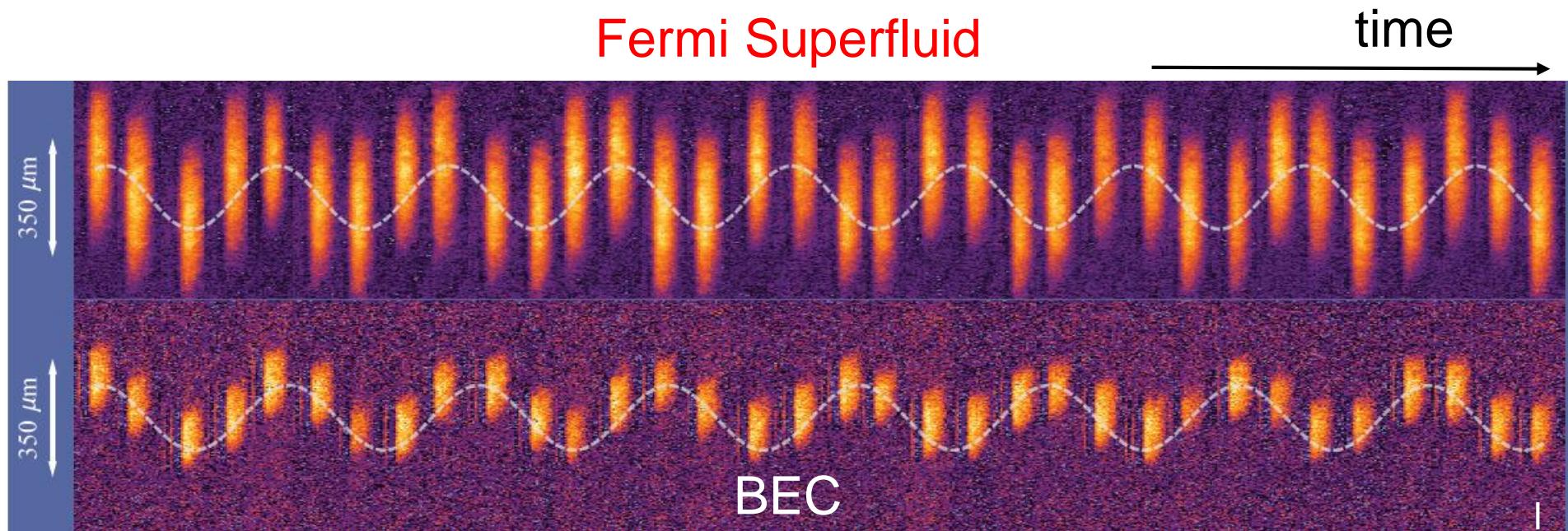


Trap frequencies: $v_z = 15.6 \text{ Hz}$
for bosons, $v_{\text{rad}} = 440 \text{ Hz}$

Lifetime of mixture: 7s in shallowest trap

Unitary ${}^6\text{Li}$ Fermi gas can cool any species fulfilling the requirements to BEC
See also ${}^6\text{Li}-{}^{41}\text{K}$, USTC, China, PRL '16, and ${}^6\text{Li}-{}^{173}\text{Yb}$, UWash, PRL'17

Long-lived Oscillations of both Superfluids



$$\tilde{\omega}_6 = 2\pi \times 17.06(1) \text{Hz}$$

$$\tilde{\omega}_7 = 2\pi \times 15.40(1) \text{Hz}$$

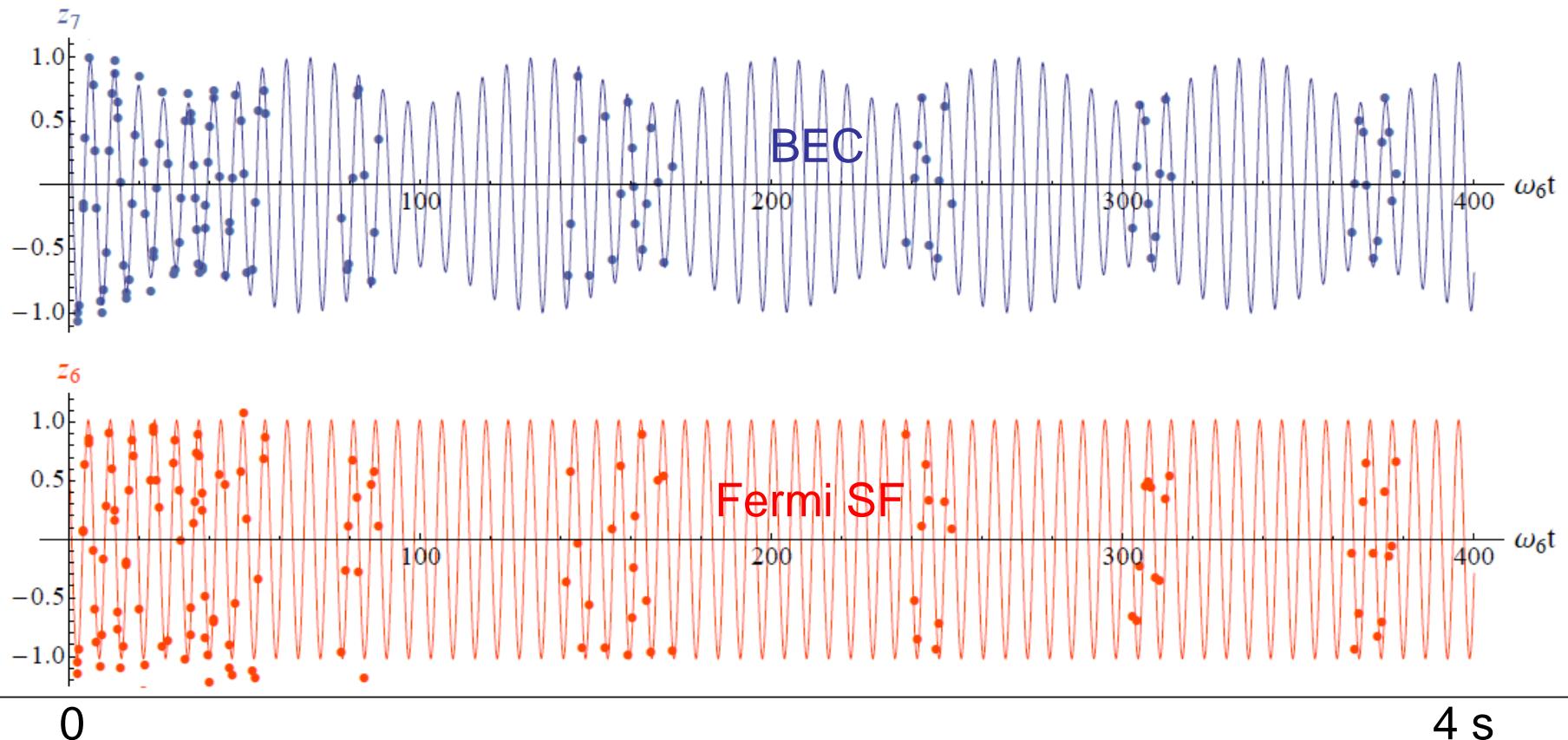
Coupled Superfluids

$$\omega_6 = 2\pi \times 17.14(3) \text{Hz}^{400 \text{ ms}}$$

$$\omega_7 = 2\pi \times 15.63(1) \text{Hz}$$

Single Superfluid
Ratio = $(7/6)^{1/2} = (m_7/m_6)^{1/2}$

Oscillations of both superfluids



Very small damping !

Modulation of the ${}^7\text{Li}$ BEC amplitude by $\sim 30\%$ at

$$(\tilde{\omega}_6 - \tilde{\omega}_7)/2\pi$$

Coherent energy exchange between the two oscillators

Mean field model

1.5% down shift in ${}^7\text{Li}$ BEC frequency

BEC osc. amplitude beat at frequency $(\tilde{\omega}_6 - \tilde{\omega}_7)/2\pi$

Weak interaction regime: $k_F a_{bf} \ll 1$ and $N_7 \ll N_6$

Boson effective potential $V_{eff} = V(r) + g_{bf} n_6(r)$ with $g_{bf} = \frac{2\pi\hbar^2 a_{bf}}{m_{67}}$
 $m_{67} = m_6 m_7 / (m_6 + m_7)$

LDA $n_6(r) = n_6^0 (\mu_6^0 - V(r))$

Where $n_6(\mu)$ is the Eq. of State of the stationary Fermi gas.

For the small BEC: $V(r) \ll \mu_6^0$

Expand $n_6(r) \approx n_6^0 (\mu_6^0) - V(r) \frac{dn_6^0}{d\mu_6} + \dots$

Boson effective potential and link with Equation of State

Thomas Fermi radius of BEC << TF radius of Fermi Superfluid:

$$V_{eff} = g_{bf} n_6(0) + V(r) \left[1 - g_{bf} \left(\frac{dn_6^{(0)}}{d\mu_6} \right)_0 \right]$$

The potential remains harmonic with rescaled frequency

$$\tilde{\omega}_7 = \omega_7 \sqrt{1 - g_{bf} \left(\frac{dn^{(0)}}{d\mu_6} \right)_0}$$

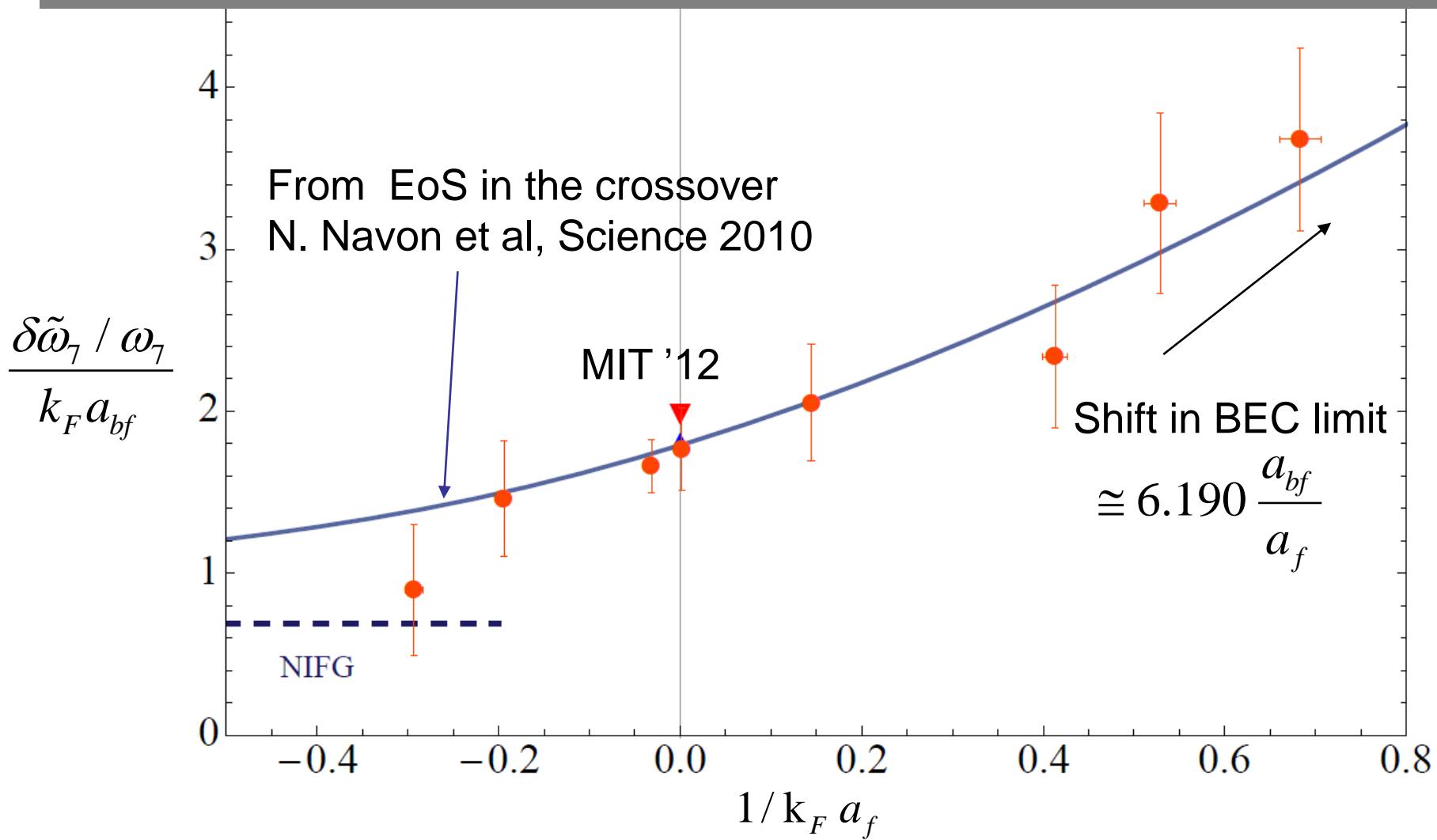
A new means to access properties of the EoS !

The equation of state $n(\mu)$ at low T is known in the BEC-BCS crossover
N. Navon et al., Science, 2010, M. Ku et al., Science 2012

Example: at unitarity, $1/a=0$

From Thomas Fermi radius of ${}^6\text{Li}$ superfluid, we find: $\tilde{\omega}_7 = 2\pi \times 15.43 \text{ Hz}$
very close to the measured value: $\tilde{\omega}_7 = 2\pi \times 15.40(1) \text{ Hz}$

Equation of State and Bose-Fermi Coupling in BEC-BCS crossover



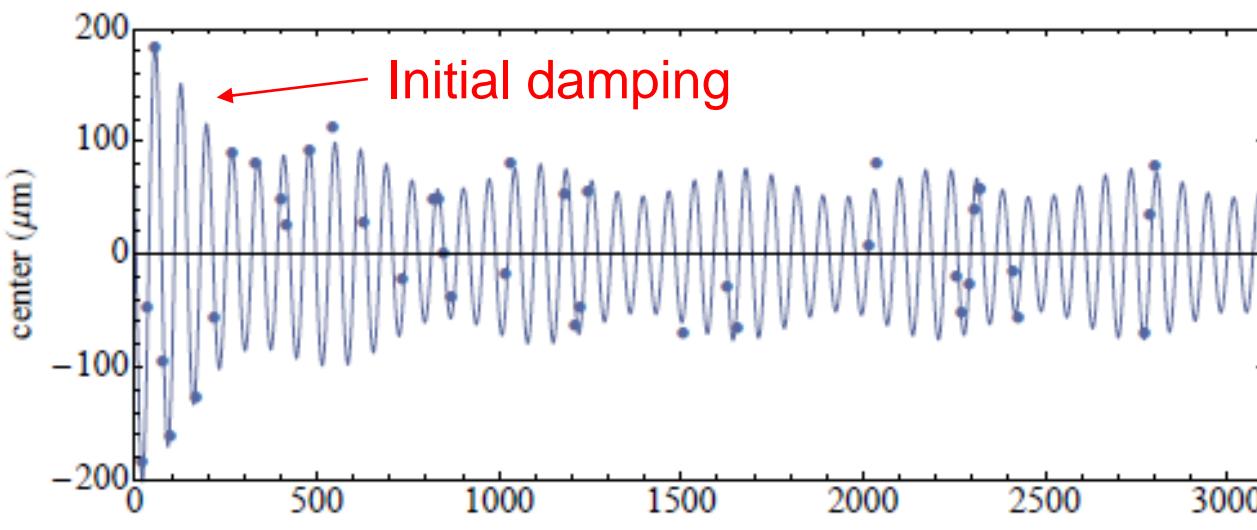
What is the critical velocity for superfluid counterflow ?

Increase initial displacement



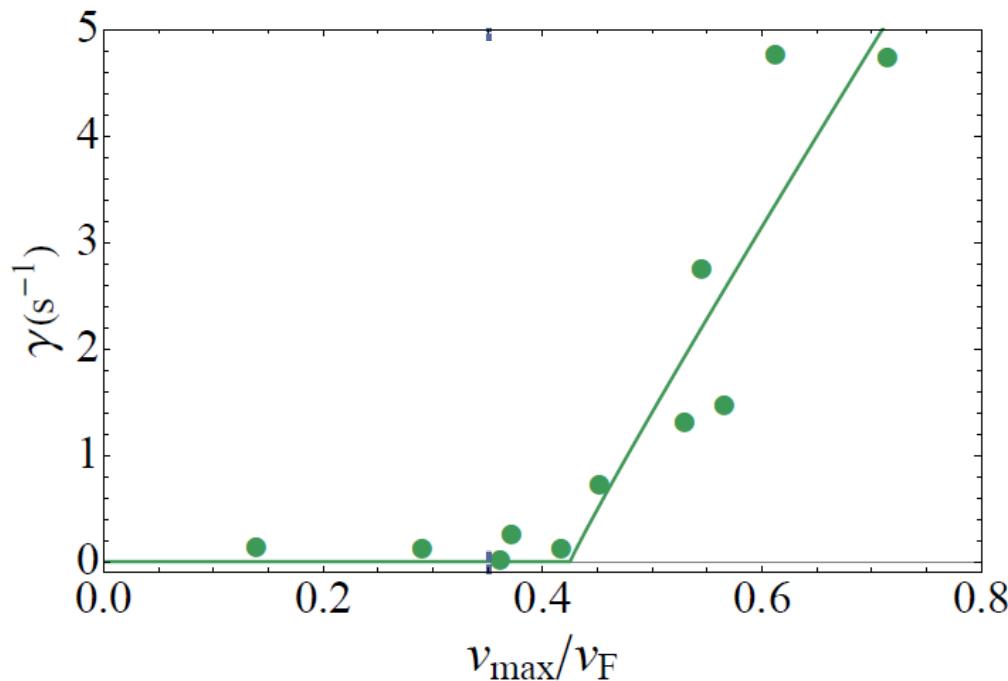
Increase relative velocity

Critical velocity for superfluid counterflow



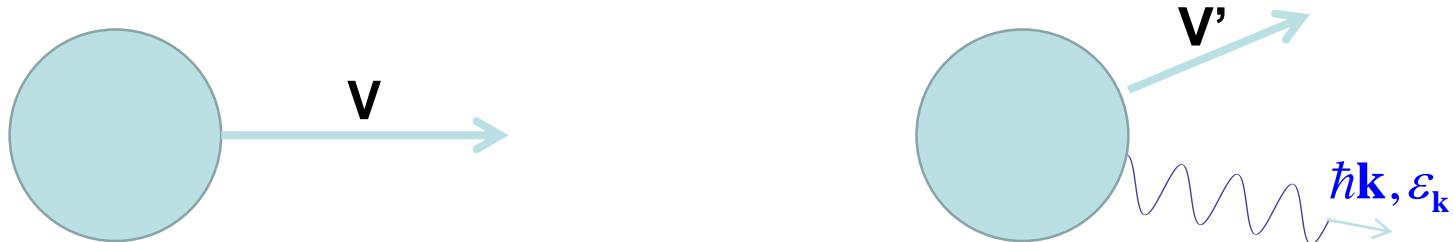
$$d = d_0 \exp(-\gamma t) + d'$$
$$\gamma = 3.1 \text{ s}^{-1}$$

Time(ms)



$V_c = 2 \text{ cm/s}$
is quite high !

Landau criterion



Momentum Conservation : $M\mathbf{V} = M\mathbf{V}' + \hbar\mathbf{k}$

Energy Conservation : $MV^2 / 2 = MV'^2 / 2 + \varepsilon_{\mathbf{k}}$

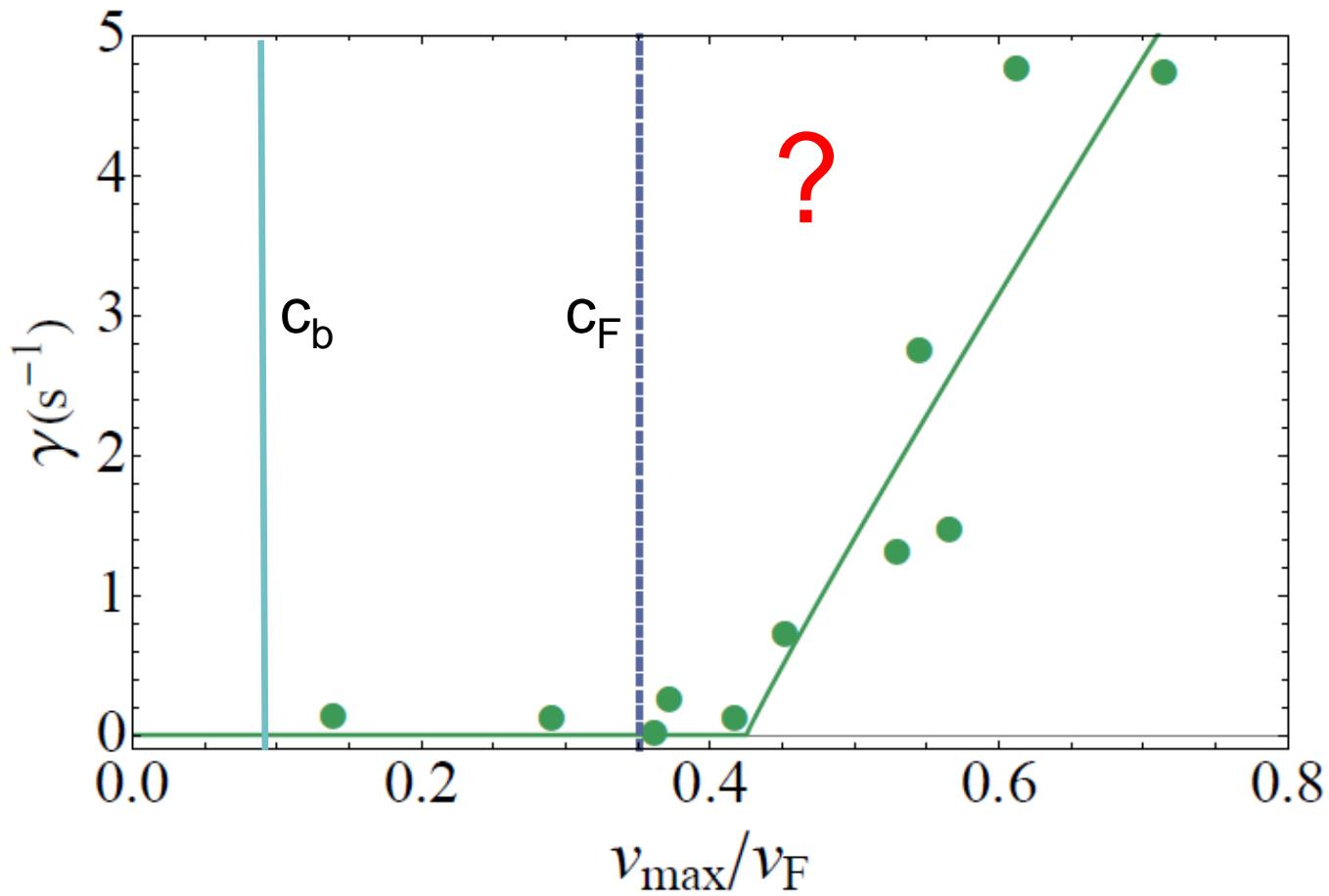
$$\hbar kV \geq \hbar\mathbf{k} \cdot \mathbf{V} = \varepsilon_k + \hbar^2 k^2 / 2M \geq \varepsilon_k$$

Motion of impurity is damped by the creation of elementary excitations if:

$$V \geq V_c = \min_k \left(\frac{\varepsilon_k}{\hbar k} \right)$$

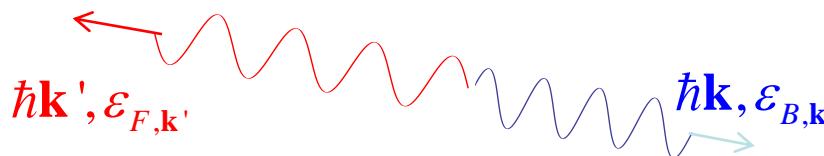
For a linear excitation spectrum $\varepsilon_k \sim kc$, $V_c = c$, the sound velocity

Critical velocities



Revisiting Landau criterion for a Bose-Fermi mixture @ T=0

Y. Castin, I. Ferrier-Barbut and C. Salomon
Comptes-Rendus Acad. Sciences, Paris, **16**, 241 (2015)



1 Excitation in the bosonic superfluid $E_{B,\mathbf{k}} = \varepsilon_{B,\mathbf{k}} + \hbar\mathbf{k} \cdot \mathbf{V}_B$

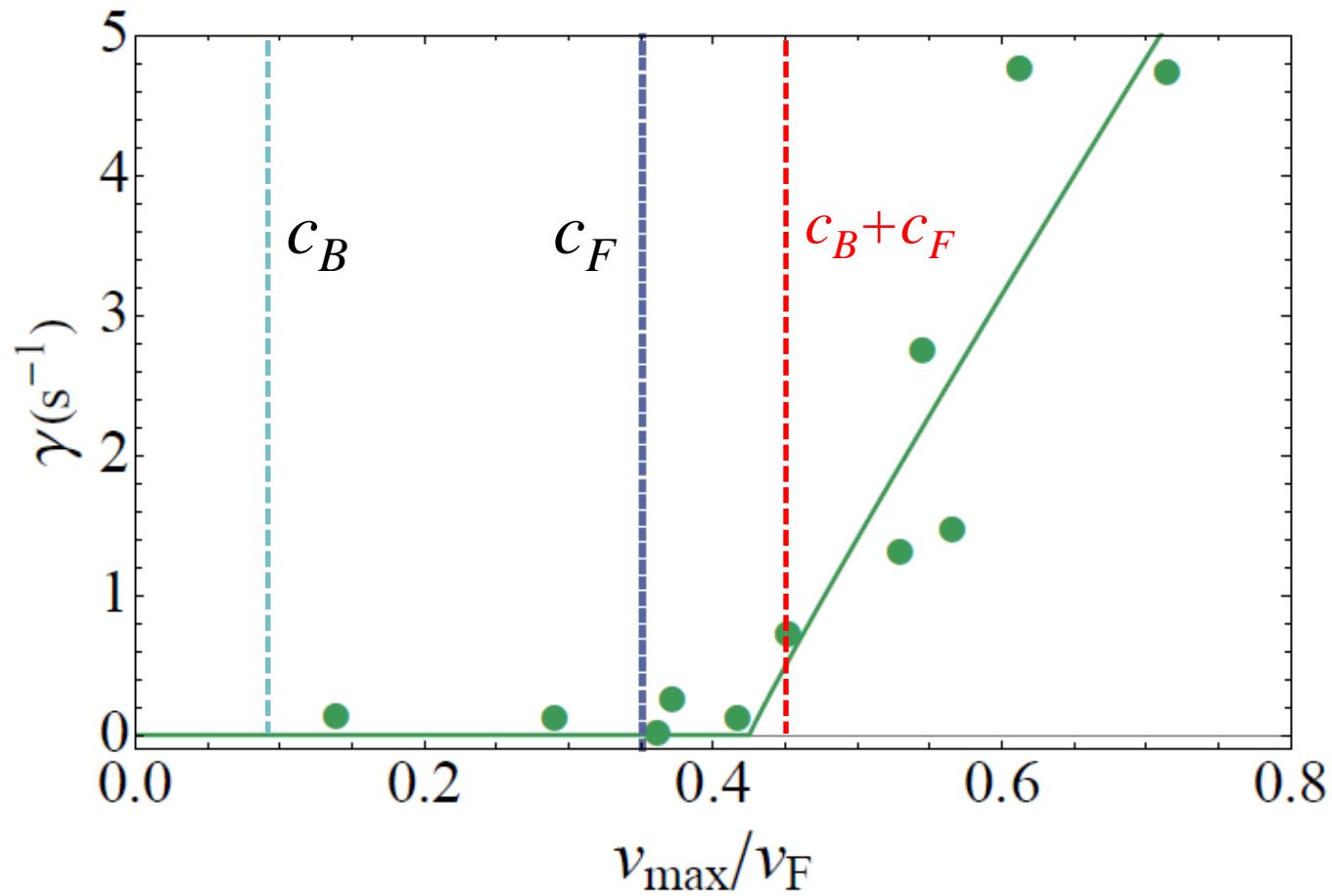
1 Excitation in the fermionic superfluid $E_{F,\mathbf{k}'} = \varepsilon_{F,\mathbf{k}'} + \hbar\mathbf{k}' \cdot \mathbf{V}_F$

Energy-momentum conservation: $E_{B,\mathbf{k}} + E_{F,\mathbf{k}'} = 0 \quad \mathbf{k} + \mathbf{k}' = 0$

$$|\mathbf{V}_B - \mathbf{V}_F| \geq \min_k \left(\frac{\varepsilon_{B,k} + \varepsilon_{F,-k}}{\hbar k} \right) \quad \text{Sound Modes: } V_c = c_B + c_F$$

See also Abbad et al. EPJD 69, 126 (2015), F. Chevy, PRA **91**, 063606 (2015),
W. Zheng and H. Zhai, Phys. Rev. Lett. 113, 265304 (2014)

Counter-flow critical velocity



Next lecture

What is the lifetime of the
Bose-Fermi mixture ?

Three-body recombination as a probe of
quantum correlations in a strongly interacting system