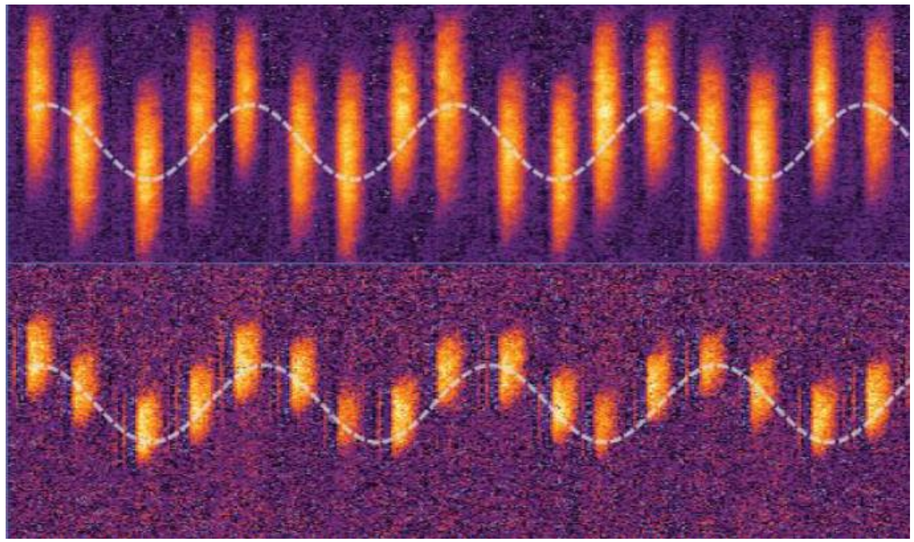
Lecture 2: an impurity in a strongly interacting Fermi gas

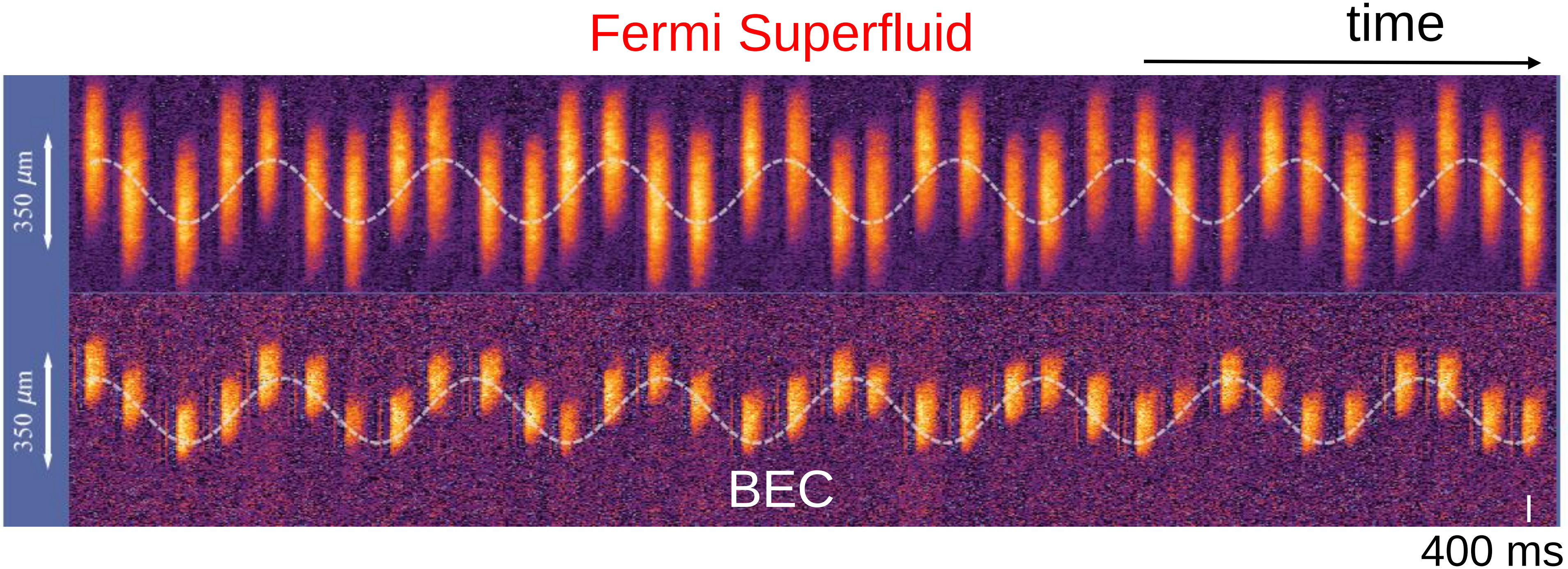


C. Salomon

Laboratoire Kastler Brossel
Physique quantique et applications

Enrico Fermi School
Mixtures of quantum gases,
Varenna, July 20, 2022

Lifetime of dual Bose-Fermi superfluids with ^6Li-^7Li isotopes

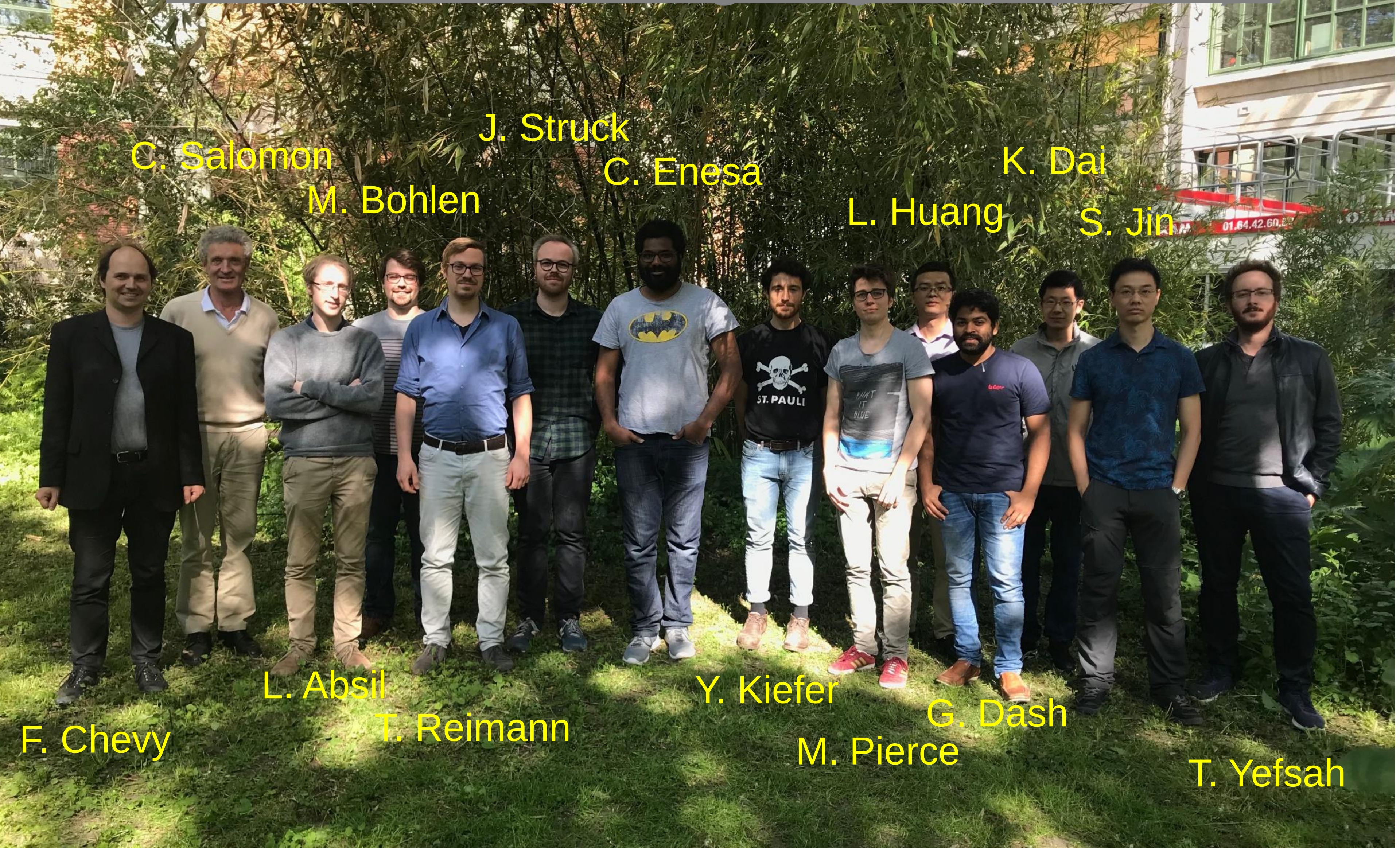


At unitarity, the lifetime is 7 seconds in shallow optical trap

Question: what is the lifetime of the Bose-Fermi mixture ?

How does it vary with $1/k_f a_f$, with a_{bf} , and with density ?

The ENS Fermi gas group (2018)



Theory Y. Castin, F. Werner, X. Leyronas (ENS), S. Stringari (Trento), A. Recati, T. Ozawa, O. Goulko (Amherst), C. Lobo, J. Lau (Southampton), I. Danaila (Rouen)

Outline

- Tan's two-body contact
- Three-body recombination in quantum gases
- Lifetime of Bose-Fermi mixture
  Theory: a simple formula !
- Experiments with ^{6}Li-^{7}Li isotopes

S. Laurent, M. Pierce, M. Delehaye, T. Yefsah, F. Chevy, C. Salomon
Phys. Rev. Lett., **118**, 103403, 2017

I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce,
B. S. Rem, F. Chevy, and C. Salomon, Science, **345**, 1035, 2014

M. Delehaye, S. Laurent, I. Ferrier-Barbut, S. Jin,
F. Chevy, C. Salomon, PRL, 115, 265303, 2015

Tan's two-body Contact: spin ½ Fermi gas

n_1: density of spin up particles = N_1/V
n_2: density of spin down particles = N_2/V

Contact density : density-density correlator when $r_1 - r_2 \rightarrow 0$

$$\langle n_1(\mathbf{R}+\mathbf{r}_1)\, n_2(\mathbf{R}+\mathbf{r}_2)\rangle \longrightarrow \frac{1}{16\pi^2|\mathbf{r}_1-\mathbf{r}_2|^2}\mathcal{C}(\mathbf{R}).$$

Take a small sphere with radius $s \ll a$ and volume $4/3\,\pi s^3$
integrate over r_1 and r_2 and make $s \longrightarrow 0$

$$N_{\rm pair}(\mathbf{R}, s) \longrightarrow \frac{s^4}{4}\mathcal{C}(\mathbf{R}).$$

$C(R)$ has dimension $(\text{length})^{-4}$

and not N_1N_2/V^2 that scales as s^6

The local pair density is the small volume limit of $N_1N_2/V^{4/3}$

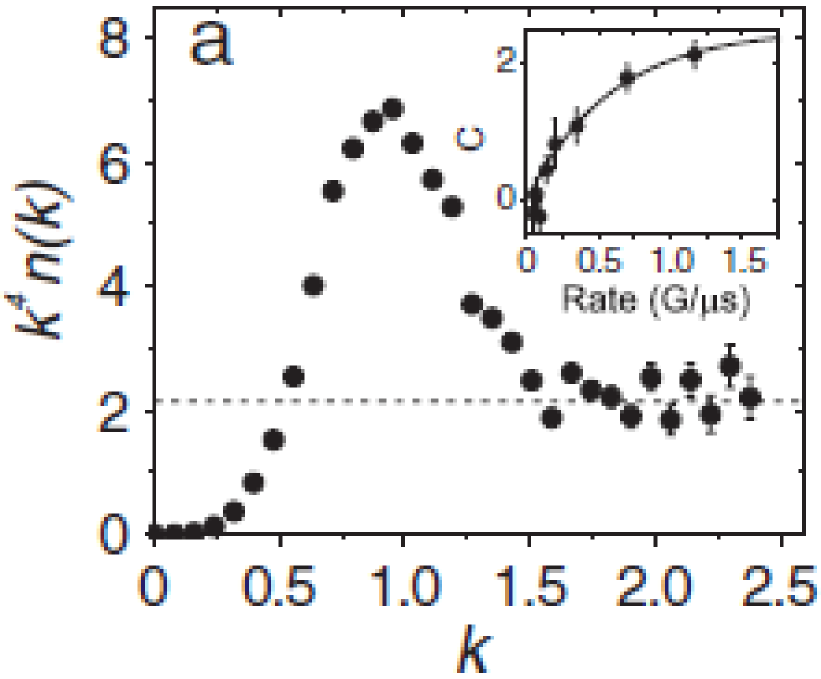
Because of the strong correlations there are
many more pairs in a small volume

Shina Tan, Annals of Physics **323** (2008) 2952-2990;

Tan's Contact and Debbie Jin

Tail of the momentum distribution
at large k

$$k^4 n_\sigma(k) \to C_2 \quad when\ k \to \infty$$

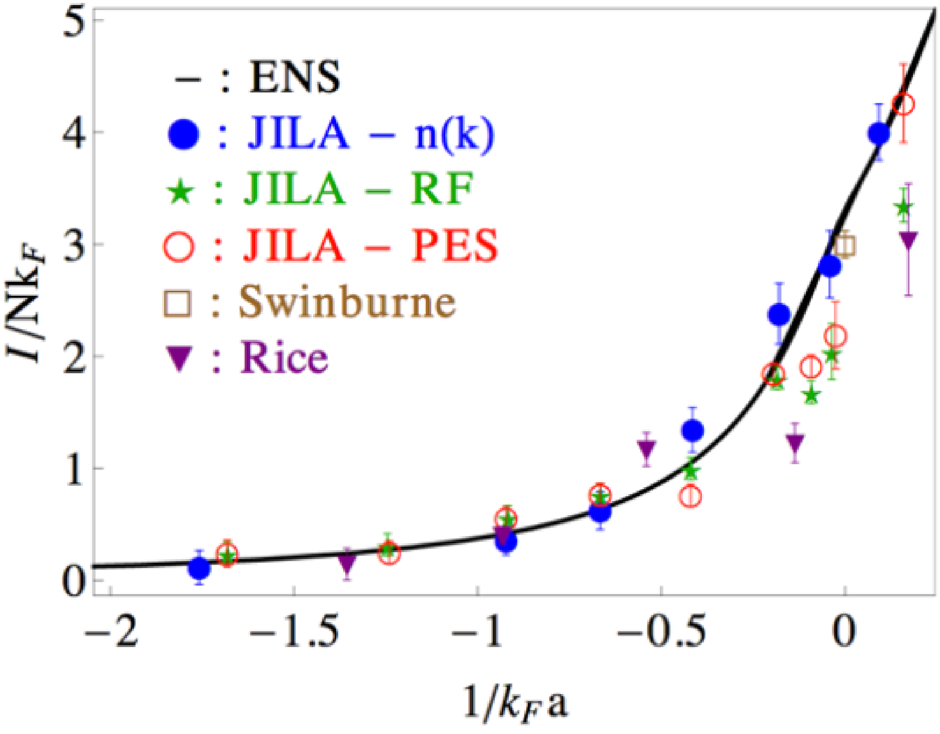


JILA: Stewart et al., Jin's group, PRL, 2010

Tan's Contact

Adiabatic energy relation

$$C_2 = -\frac{4\pi m_f}{\hbar^2}\frac{\partial E}{\partial(1/a)}$$ at constant entropy



@ T=0

Black curve: from equation of state of homogeneous gas integrated over the trap

ENS, Navon et al., Science, 2010

Tan's Contact

Molecule photo-association

Rice University: G. Partridge et al., PRL 2005
ENS interpretation: Werner et al.,E PJ B 2009

Radio-frequency spectroscopy

JILA: Stewart et al., PRL 2010
MIT: B. Mukherjee et al., PRL 2019, temperature dependence in box potential

Equation of state and adiabatic energy relation

JILA: Stewart et al., PRL 2010
ENS: Nascimbène et al., Nature 2010

Bragg spectroscopy

Swinburne: S. Hoinka et al., PRL 2013
C. Carcy et al., PRL 2019

Loss rate of impurity gas

ENS: Laurent et al., PRL 2017

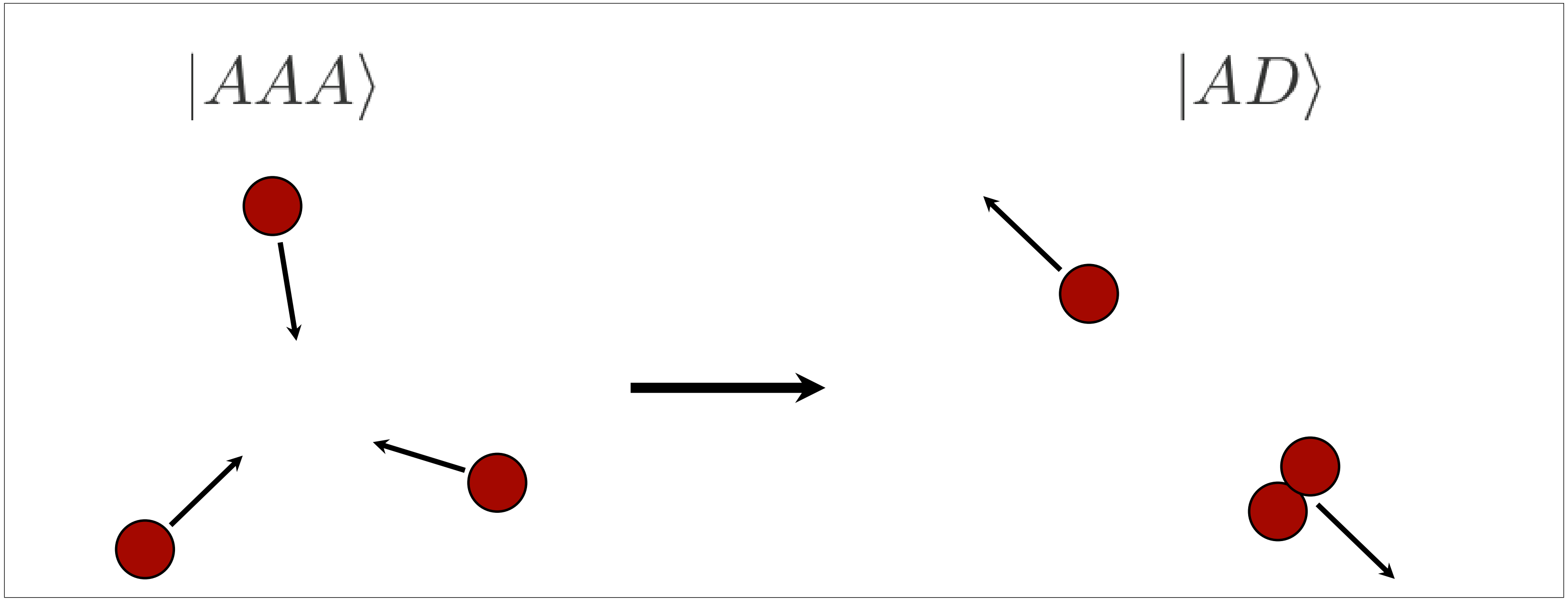
2-body and 3-body Contact for bosons

Cambridge, Fletcher et al., 2018

Theory: *Tan, Hausmann, Punk, Zwerger, Strinati, Perali, Rossi, Kosik, Werner,Castin Svistunov, Prokofiev, Hu, Liu, Drummond,….*

Three-body recombination
(1) Bose gas

- Three-body loss for Bose gases with scattering length a



$$\dot{n} = -L_3 n^3$$

Rate constant : $\gamma_3 = -\frac{\dot{n}}{n} = n^2 L_3(a, T, E_t)$

Two-body rate constant : $\gamma_2 = n\sigma v$

Bosons: three-body loss rate as a probe of few–body physics

For na^3 <<1 and at low enough temperature

L_3 scales as a^4 modulated by Efimov features

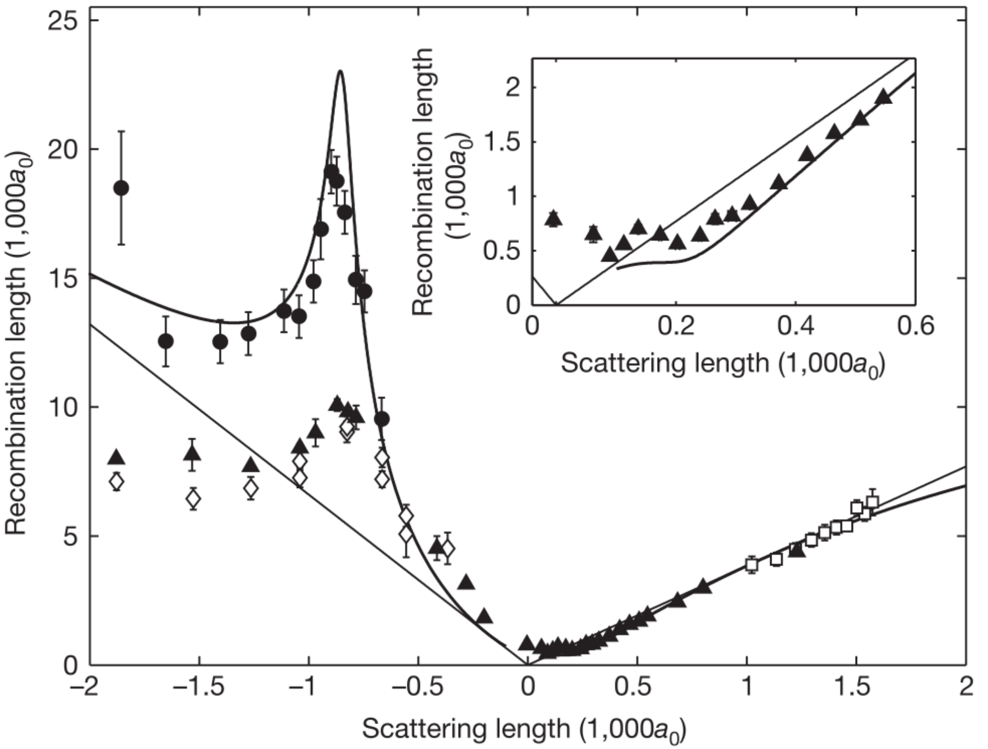
Fedichev et al., PRL 77, (1996)
D'Incao et. al., PRL **93**, 123201 (2004),
Braaten and Hammer, PRA (2008)

Universal Efimov trimers:

infinite set of weakly bound states

Kraemer *et al.*, *Nature*, **440**, (2006)

^{7}Li: Pollack *et al.*, *Science*, **326**, (2009)
^{7}Li: N. Gross et al., PRL **103**, (2009)
^{7}Li: B. Rem et al. PRL **110**, (2013)
^{39}K: Zaccanti *et al.*, *Nat. Phys.* **5**, (2009)
^{85}Rb: Wild *et al.*, *PRL*, **108**, (2012)



Universality of Efimov trimers: discrete scale invariance
Innsbruck: B. Huang et al., PRL 112, 190401 (2014)
Chicago: S. Tung et al., PRL 113, 240402 (2014)

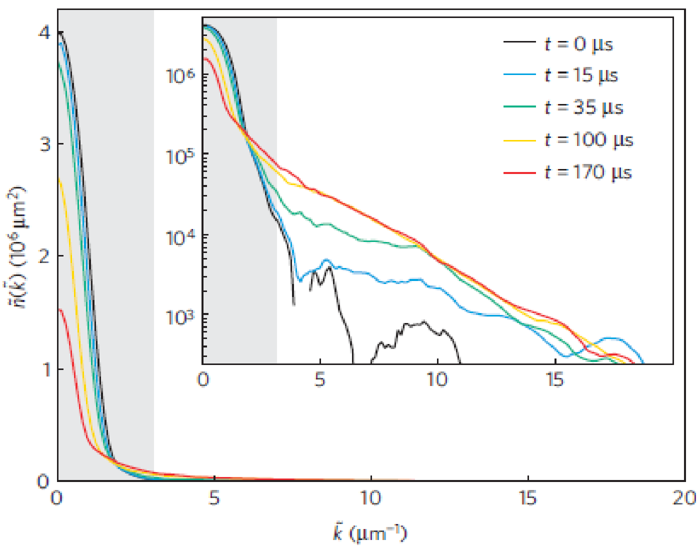
3-body loss as a probe of many-body physics

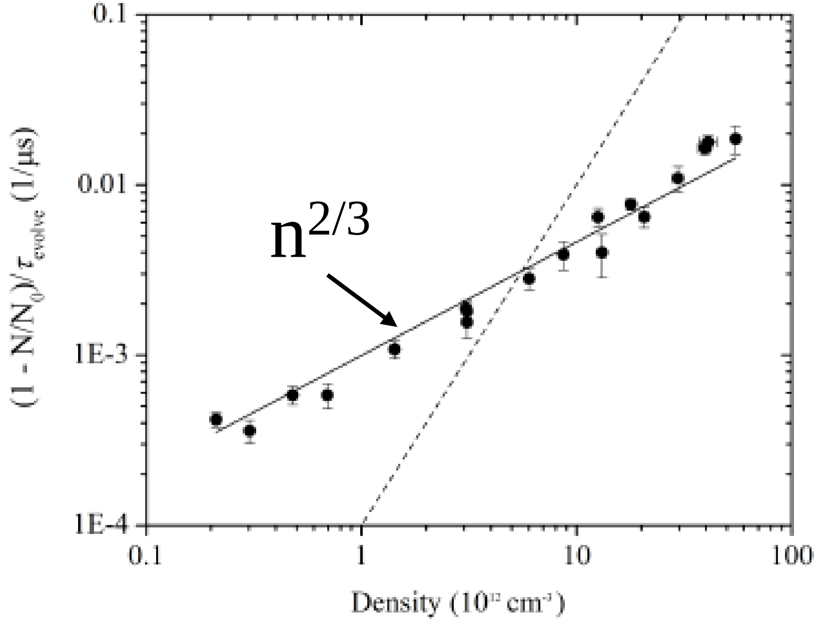
JILA: E. A. Burt, R. W. Ghrist, C. J. Myatt, M. J. Holland, E. A. Cornell, and C. E. Wieman, PRL 1997

Third-order coherence of BEC matter wave field:

Density-density-density correlations reduced by 3! in a BEC compared to thermal gas at same density (Kagan, Svistunov, Shlyapnikov)

BEC quenched to unitarity, ^{85}Rb JILA, ^{39}K Cambridge; universal features





P. Makotyn, et al., Nat. Physics 2014

C. E. Klauss et al.PRL 2017

Universal prethermal dynamics of Bose gases quenched to unitarity

C. Eigen, …., Z. Hadzibabic, *Nature* **563**, 221 (2018).

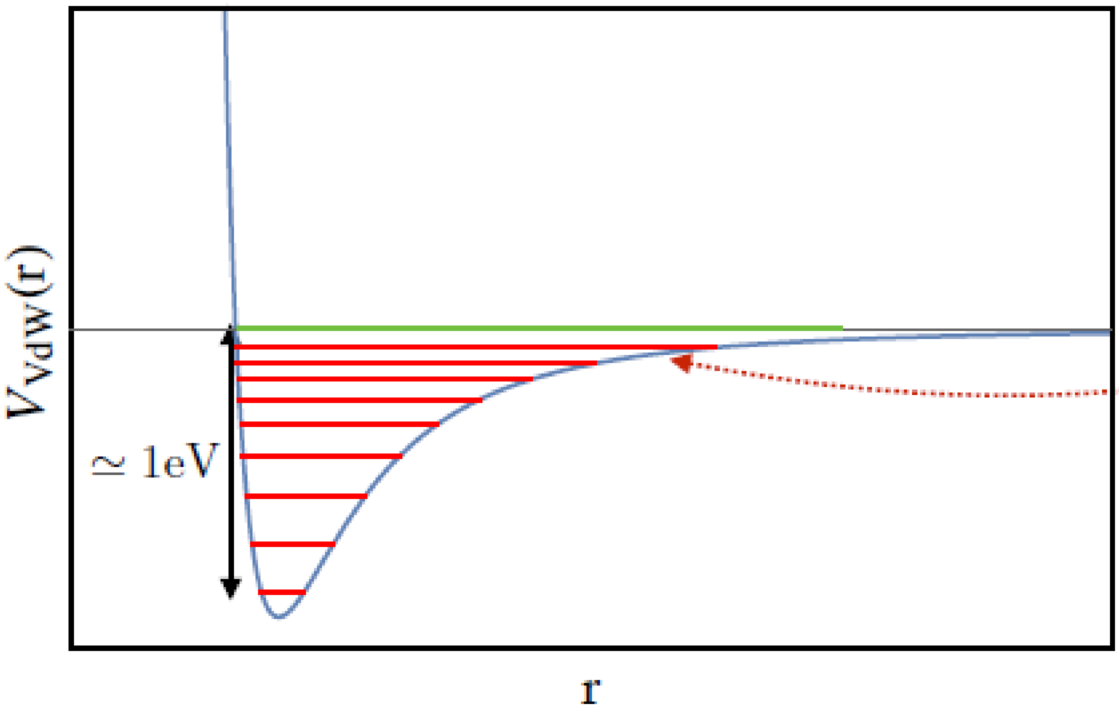
Two- and three-body contacts in the unitary Bose gas

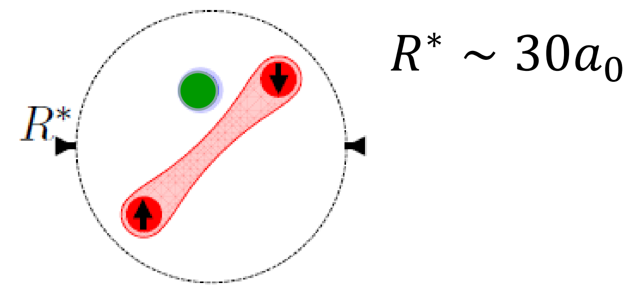
R. J. Fletcher, .. Z. Hadzibabic, *Science* **355**, 377 (2017).

Three-body recombination in Bose-Fermi mixture

As a_{bf} is small, bosons act as a weakly coupled impurity i immersed in a Fermi gas with large a_f

Three-body recombination: i, ↓ , ↑





$$\psi \sim \frac{1}{r\sqrt{2\pi a_f}} e^{-r/a_f}$$

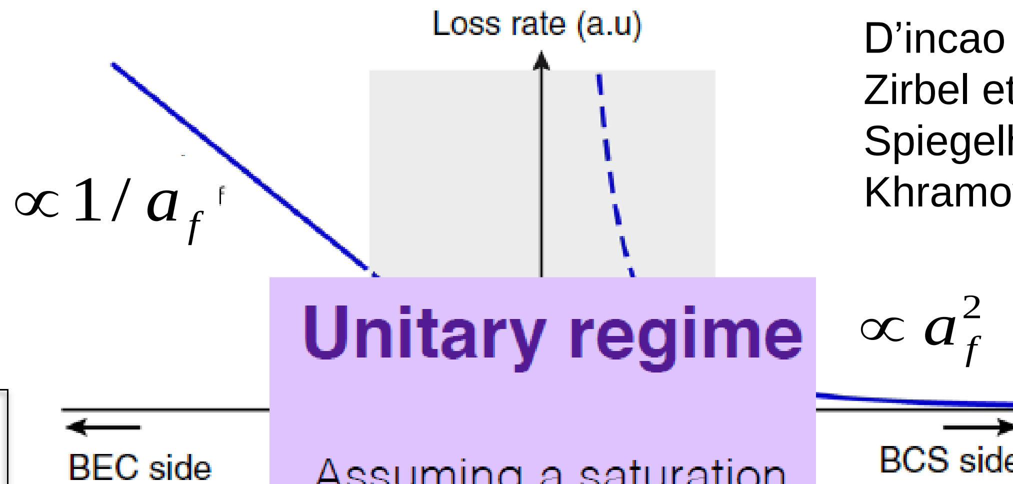
Decay to a deeply bound molecular state

Binding energy transferred to kinetic energy of collision partners

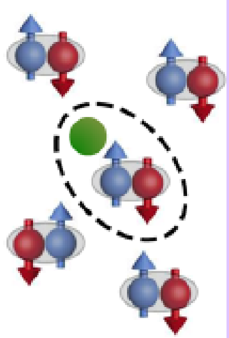
Atom and molecule leave the trap

A weakly coupled impurity in a resonant Fermi gas

D'incao and Esry, PRL 2008
Zirbel et al., PRL 2008
Spiegelhalder et al., PRL 2009
Khramov et al., PRA 2012



BEC Side
« Two » body Dimer-impurity losses
$\dot{n}_i = -L_{di} n_d n_i$
$L_{di} \propto 1/a_f$



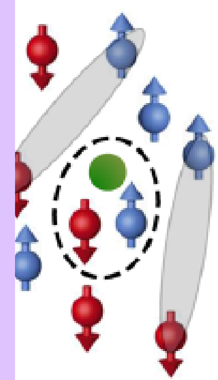
Unitary regime

Assuming a saturation

$$a \simeq n_f^{-1/3}$$

We expect :

$$\Gamma_{if} \propto n_f^{4/3}$$



BCS Side
Three body losses
$\dot{n}_i = -L_{ffi} n_i n_f^2$
$L_{ffi} \propto a_f^2$

JILA: Zirbel et al.,
PRL 100, 143201 (2008)

Unitarity: ?

A weakly coupled impurity in a resonant Fermi gas

Kagan, Svistunov, Shlyapnikov, JETP, 1985

$$P(R<R^*) = \int_{R<R^*} d^3\boldsymbol{r}_1 d^3\boldsymbol{r}_2 d^3\boldsymbol{r}_3 \left\langle \hat{\Psi}_1^\dagger(\boldsymbol{r}_1)\hat{\Psi}_2^\dagger(\boldsymbol{r}_2)\hat{\Psi}_i^\dagger(\boldsymbol{r}_3)\hat{\Psi}_i(\boldsymbol{r}_3)\hat{\Psi}_2(\boldsymbol{r}_2)\hat{\Psi}_1(\boldsymbol{r}_1)\right\rangle$$

Weak coupling between the impurity and the resonant fermions

$$\Downarrow$$

$$P(R<R^*) = \int_{R<R^*} d^3\boldsymbol{r}_1 d^3\boldsymbol{r}_2 d^3\boldsymbol{r}_3 \underbrace{\left\langle \hat{\Psi}_1^\dagger(\boldsymbol{r}_1)\hat{\Psi}_2^\dagger(\boldsymbol{r}_2)\hat{\Psi}_2(\boldsymbol{r}_2)\hat{\Psi}_1(\boldsymbol{r}_1)\right\rangle \left\langle \hat{\Psi}_i^\dagger(\boldsymbol{r}_3)\hat{\Psi}_i(\boldsymbol{r}_3)\right\rangle}_{g_{\uparrow\downarrow}(r_2,r_1)\times n_i}$$

With :

$$g_{\uparrow\downarrow}(r_2,r_1) \underset{|r_2-r_1|\to 0}{\sim} \frac{1}{\Omega}\frac{C_2}{4\pi^2|r_2-r_1|^2}$$

S. Tan, 2008

Therefore the impurity decay rate Γ_{if}
should be proportional to Tan’s two-body contact C_2

Bose/Fermi Decay and Contact

$$\dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b$$

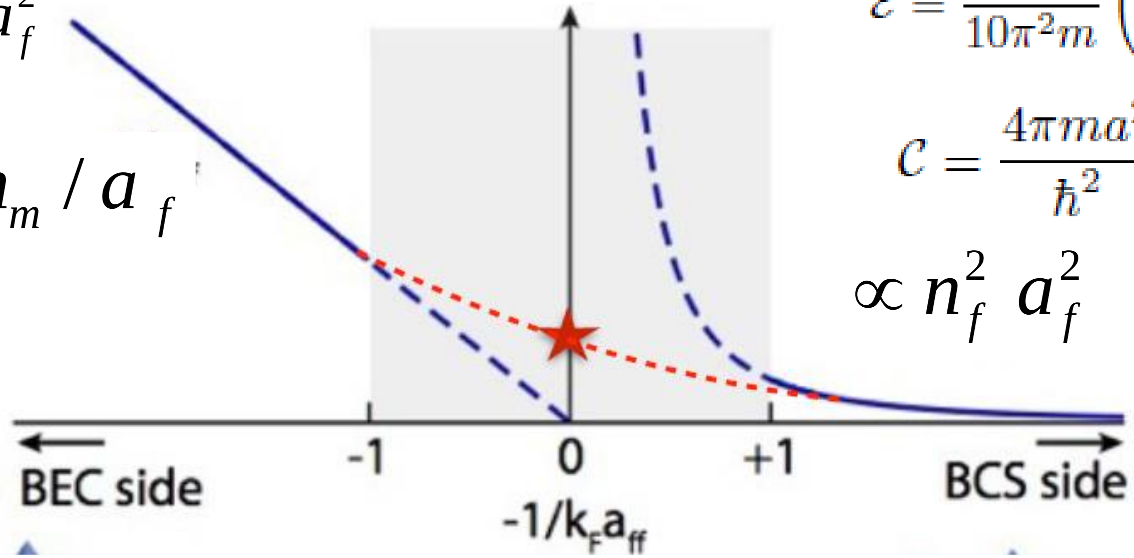
$$\gamma \sim a_{bf}^2$$

is the only parameter that contains short range physics

$$E_{\mathrm{dim}er} = -\frac{\hbar^2}{m a_f^2}$$

$$C_2 \propto n_m / a_f$$

Γ_{bf}



$$\mathcal{E} = \frac{\hbar^2 k_F^5}{10\pi^2 m}\left(1 + \frac{10}{9\pi} k_F a + \ldots\right)$$

$$\mathcal{C} = \frac{4\pi m a^2}{\hbar^2} \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}a}$$

$$\propto n_f^2 \; a_f^2$$

Bose/Fermi decay and Tan's Contact

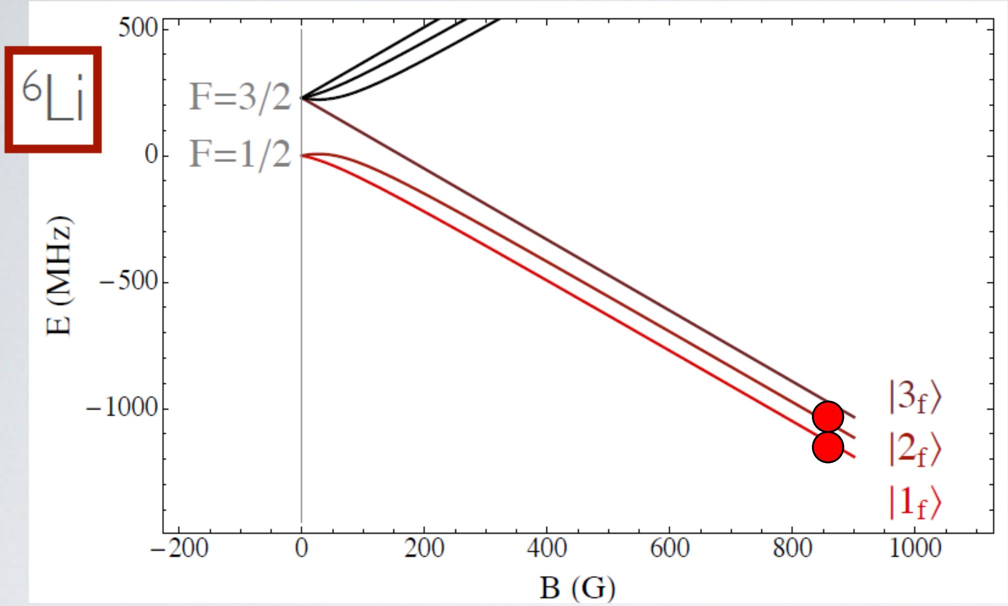
$$\dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b$$

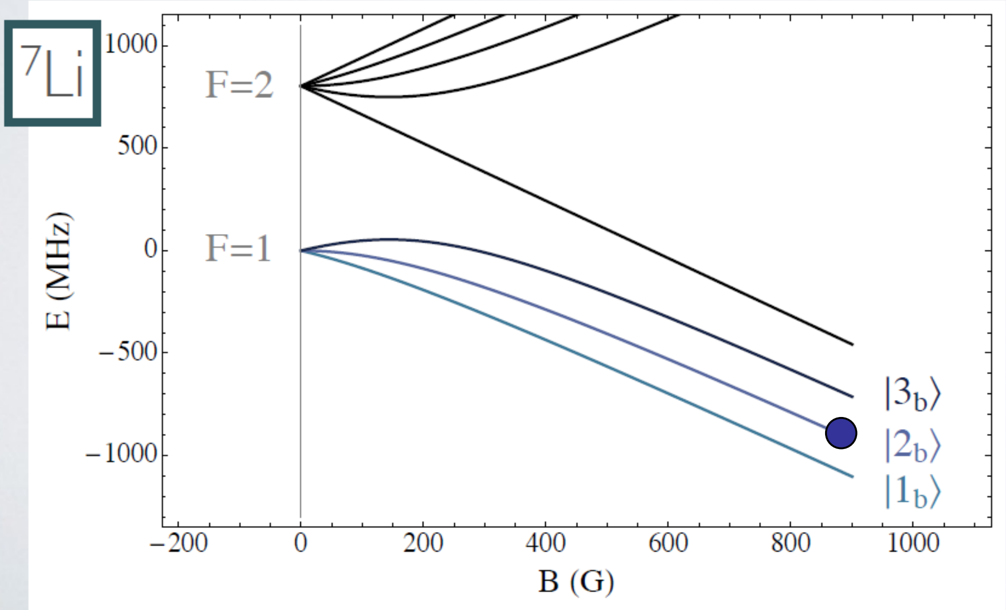
$\gamma \propto a_{bf}^2$ is the only parameter that contains short range physics

	BEC	Unitary	BCS
$(\dot{n}_b/n_b)$	$\propto (n_m/a_{ff})$ [20]	$\propto n_f^{4/3}$	$\propto a_{ff}^2 n_f^2$ [20]
C_2	$8\pi(n_m/a_{ff})$	$(2\zeta/5\pi)k_F^4$	$4\pi^2 a_{ff}^2 n_f^2$

$$\varsigma = 0.87(3)$$

^{7}Li and ^{6}Li isotopes





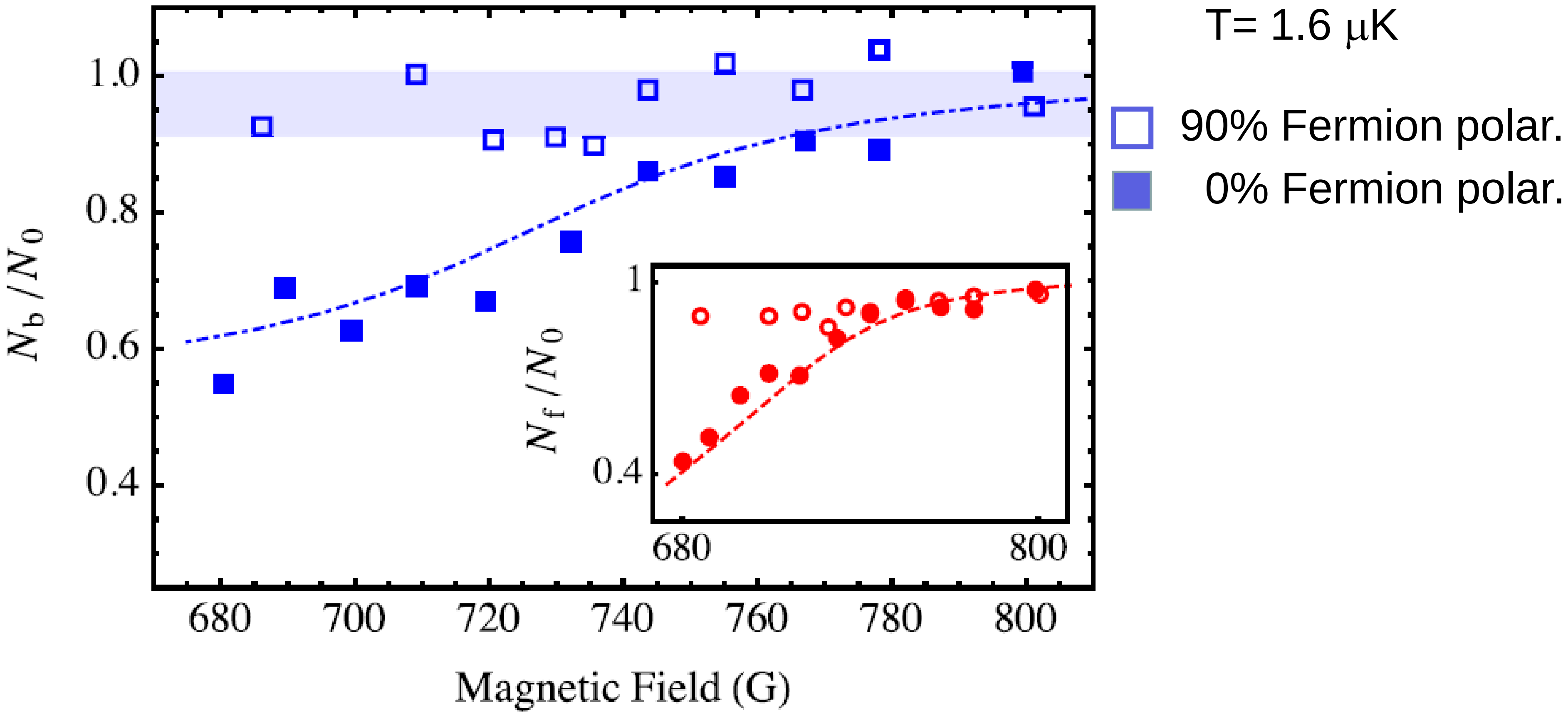
Experiments $^6Li/^7Li$ BEC side

Remaining fraction of Bosons after wait time of 1 s

$N_{0,b} = 1.5\ 10^5$

$N_{0,f} = 3\ 10^5$

T= 1.6 μK

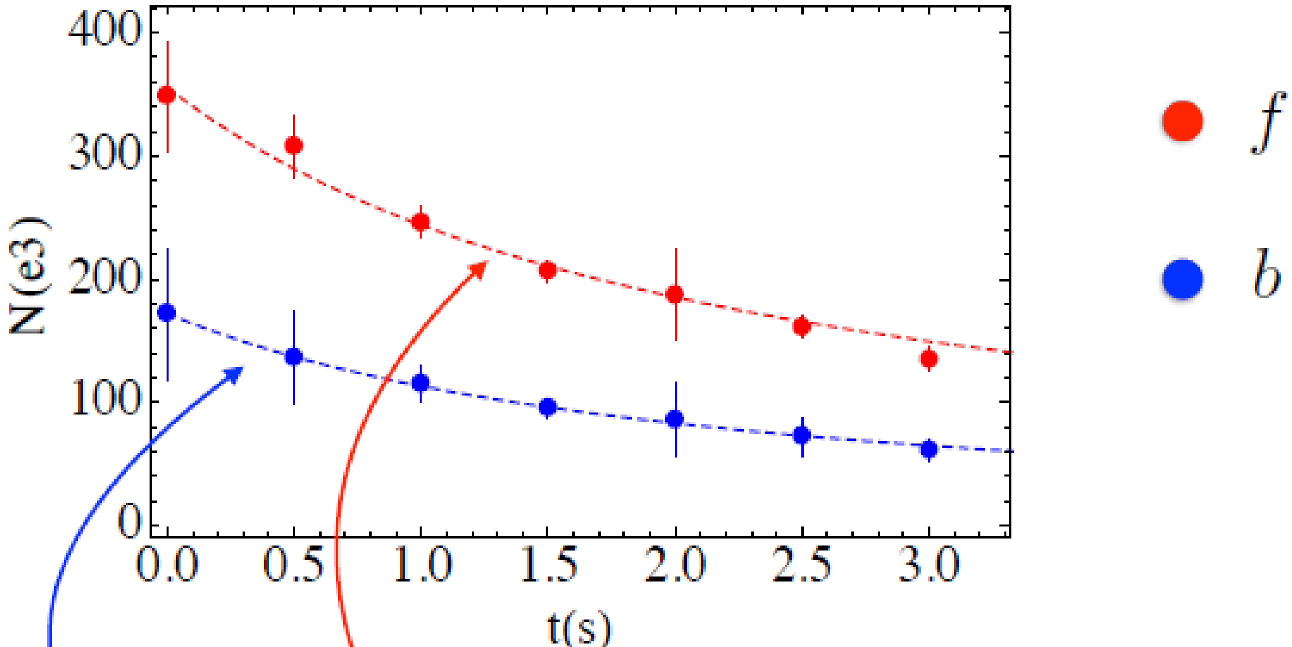


Far on BEC side, Fermions can also decay: inelastic dimer-dimer collisions

rate prop. to $a_f^{-2.55}$

D. Petrov, C.S., G. Shlyapnikov, PRL 2004

Extracting the loss rate



$$N_f(t) = \frac{N_0}{1 + \alpha t}$$

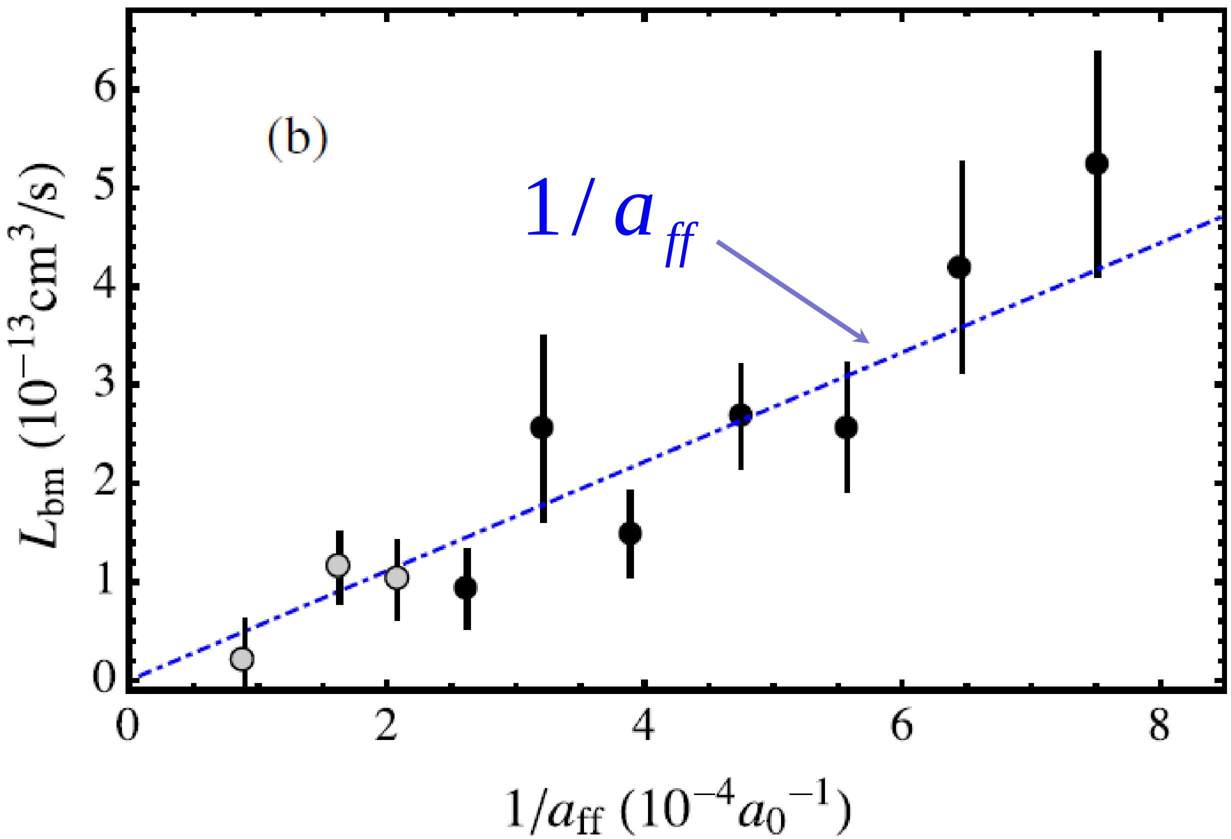
$$\dot{N}_b = -L_{bf} < n_f > N_b - \Gamma_v N_b$$

BEC side: test of *1/a_f law*

$$\dot{N}_b = L_{bm} < n_m > N_b - \Gamma_v N_b$$

$$L_{bm} < n_m > = \gamma C_2 = \gamma\, 8\pi < n_m > / a_{ff} = \alpha < n_m > / a_{ff}$$

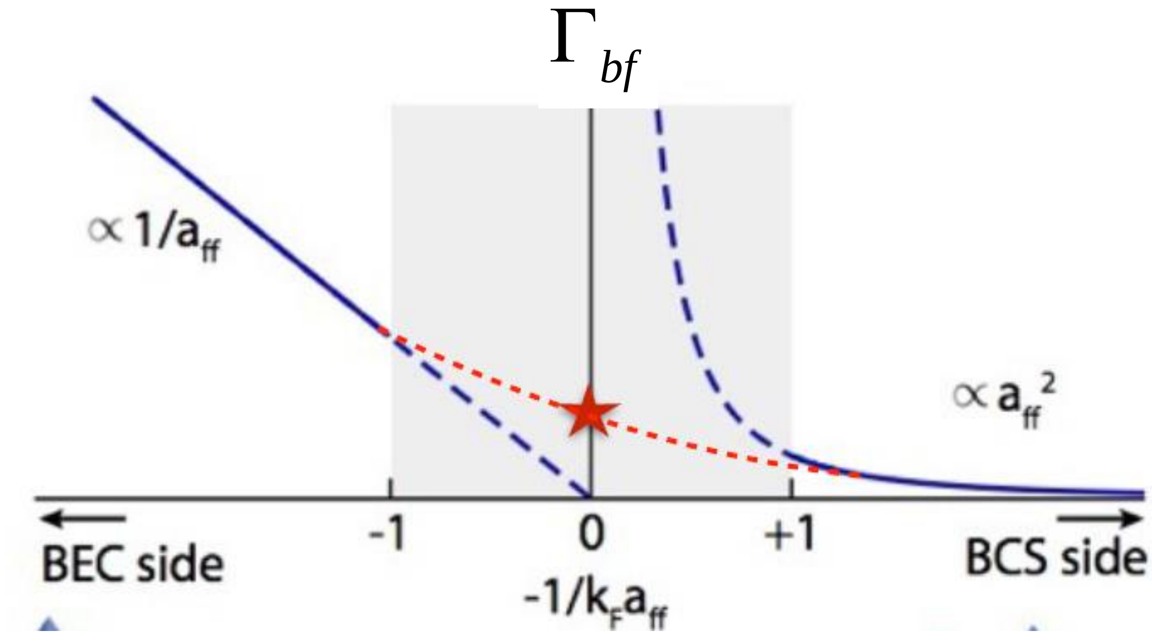
$$\gamma = \alpha / 8\pi = 1.17(11) 10^{-27} m^4 s^{-1}$$



B: 690-800 Gauss

Bose/Fermi decay in strongly interacting regime

$$\dot{n}_b = -\gamma C_2 n_b = -\Gamma_{bf} n_b$$



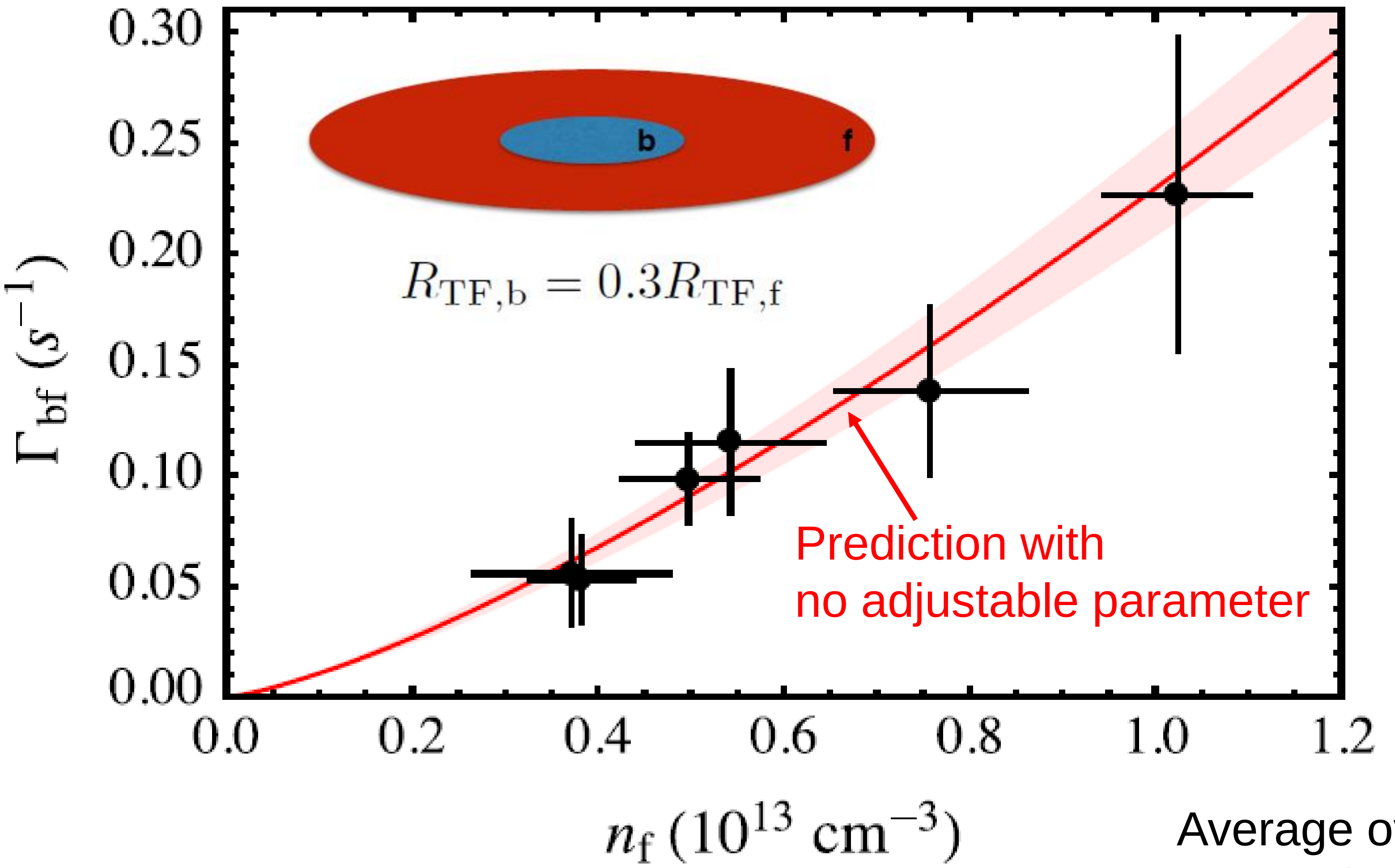
BEC + Fermi Superfluid

$$\Gamma_{bf} \propto n_f^{4/3}$$

Small complication:

three-body losses of the ^{7}Li BEC at 832 Gauss

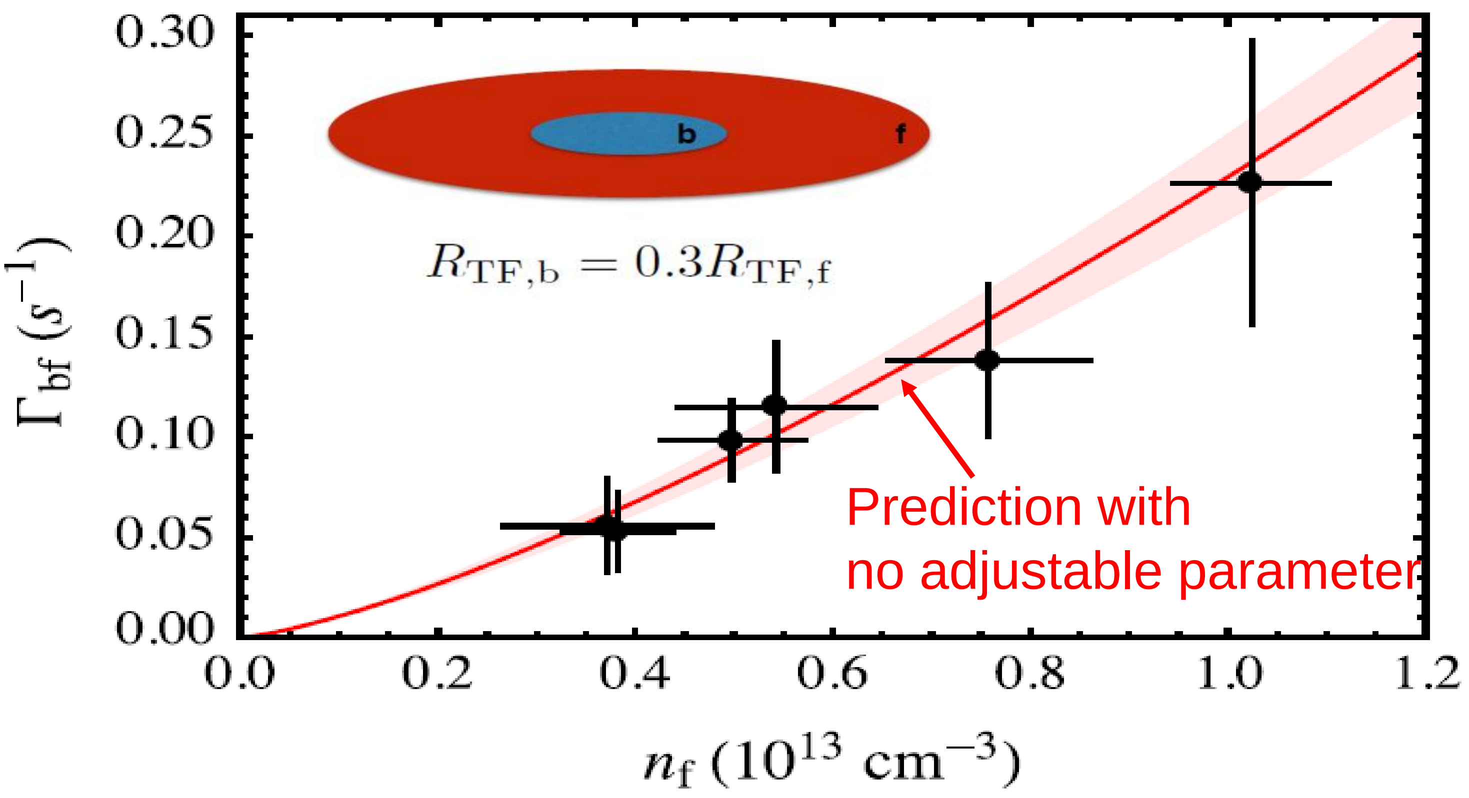
Probing the local unitary Contact



$$\Gamma_{bf} = \gamma C_2 = \frac{2\varsigma}{5\pi}\left(3\pi^2 n_f^{4/3}\right)\times 0.9$$

Average over BEC TF profile

Probing the local unitary Contact

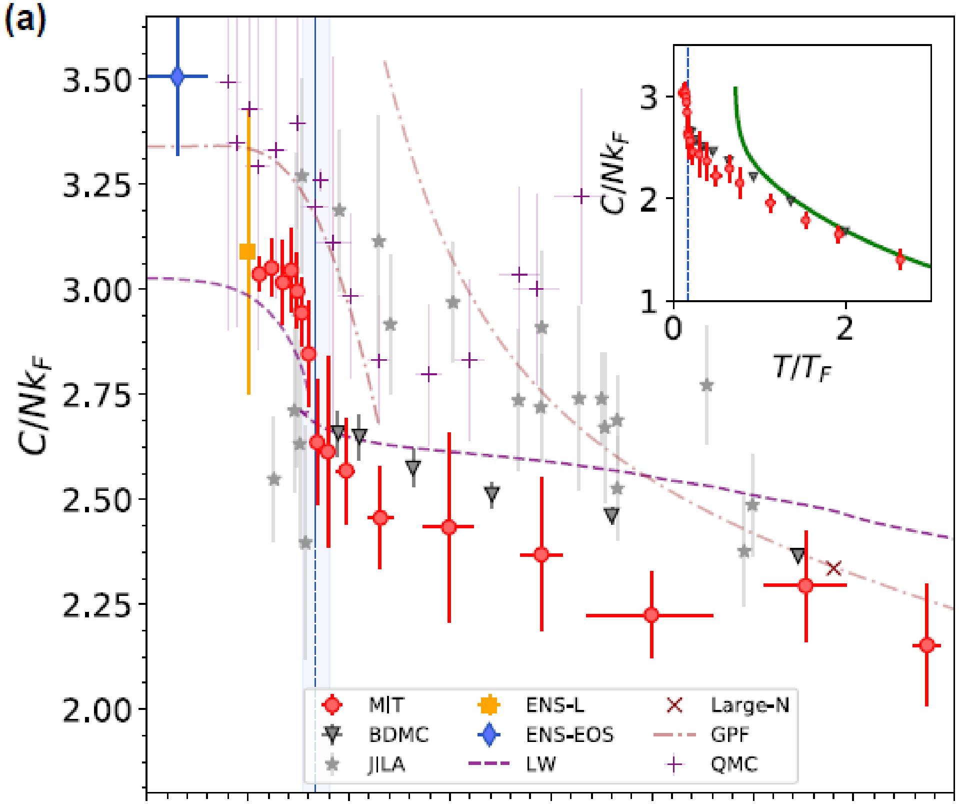


Power law fit: A n^p gives p = 1.36 (15) close to 4/3

Fit: A $n^{4/3}$ gives A and local contact $C_2(0)$

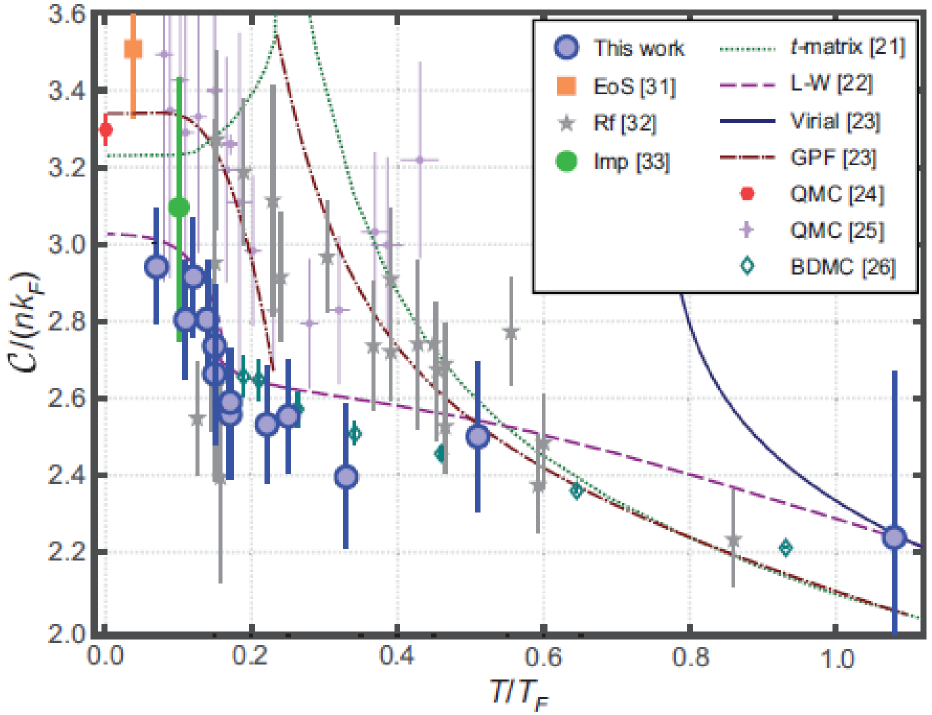
Impurity decay is a local probe of quantum correlations in a many-body system

Temperature dependence of Tan's Contact



T/T$_F$

MIT: B. Mukherjee et al., PRL 122, 203402 (2019)



Swinburne, C. Carcy et al. PRL 122, 203401 (2019)

Summary

- Lifetime of Bose-Fermi mixture is governed by Tan's contact
- Theory applies to $^{174}Yb/^{6}Li$, $^{40}K/^{6}Li$, $^{87}Rb/^{40}K$, assumes small a_{bf}
- What happens when a_{bf} increases ?

  Beyond mean field corrections: M. Pierce, X. Leyronas, F. Chevy, PRL 123, 080403, 2019, ArXiv 1903.01110, and Efimov effect
- Two-body and three-body contact in unitary Bose gas

  R. Fletcher et al., Science 2017, Hadzibabic group, Cambridge
- Bose – Fermi mixtures have a lot of open problems !