Varenna Summer School on
Quantum Mixtures of Ultracold Atomic Gases
18-23 July 2022

SPIN ORBIT COUPLED BEC GASES

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Why Spin-Orbit Coupled BEC Gases?

- Give rise to artificial gauge fields opening perspectives for novel quantum effects in neutral systems

- Spin orbit coupling breaks Galilean invariance with crucial consequence on superfluid behavior

- Emergence of a supersolid phase breaking spontaneously both phase and translational invariance and giving rise to novel Goldstone modes

LECTURE 1

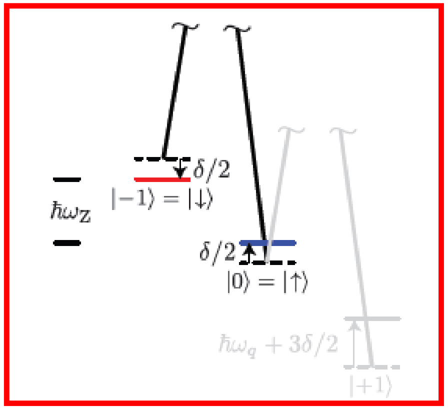
- The quantum phases of a spin-orbit coupled mixture of Bose-Einstein condensates
- Order parameter and nature of the phase transitions
- Sound and Dynamic properties of SOC BEC's

LECTURE 2

- Superfluidity and rotation of SOC BEC's
- Supersolidity and the novel Goldstone modes

Simplest realization of (1D) spin-orbit coupling in **s=1/2** Bose-Einstein condensates (Spielman, Nist, 2009)

Two detuned and polarized laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new s.p. Hamitonian in the Laboratory frame.



$$h_0^{lab} = \frac{\vec{p}^{\,2}}{2} + \frac{\Omega}{2}\sigma_x \cos(2k_0 x - \Delta\omega_L t) + \frac{\Omega}{2}\sigma_y \sin(2k_0 x - \Delta\omega_L t) - \frac{\omega_Z}{2}\sigma_z$$

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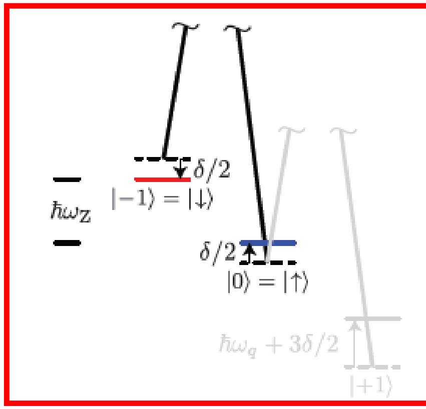

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The Hamiltonian is invariant with respect to **helicoidal (skew) translations** (**continuous** symmetry)

$$e^{id(p_x - k_0\sigma_z)}$$

Corresponding to rigid **translation** plus **rotation** in **spin space**

Since the Hamiltonian is time dependent in the laboratory frame, it is convenient to consider the unitary transformation

$$e^{i\Theta\sigma_z/2}$$

(corresponding to position and time dependent spin rotation with $\Theta = 2k_0 x - \Delta\omega_L t$. The lab Hamiltonian h_0^{lab} is transformed into a **translationally invariant** and **time independent** Hamiltonian

$$h_0 = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

characterized by 1D spin-orbit coupling

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characterized by 1D spin-orbit coupling

$p_x = -i\hbar\partial_x$ is **canonical** momentum
k_0 is laser wave vector difference
Ω is strength of Raman coupling
$\delta = \Delta\omega_L - \omega_Z$ is effective Zeeman field

The Spin Orbit coupled Hamiltonian

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

is **translationally invariant**

However it breaks

- **Galilean invariance** since the physical momentum operator $P_x = mv_x = (p_x - k_0\sigma_z)$ does not commute with the Hamiltonian

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- The SOC Hamiltonian **violates** also **parity** and **time reversal** symmetries

Symmetry properties of spin-orbit Hamiltonian

$$h_0 = \frac{1}{2}[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- **Translational** invariance: ⇨ uniform ground state configuration, unless crystalline order is spontaneously formed (**stripes, supersolidity**)

- **Violation** of **parity** and **time** reversal symmetry ⇨ breaking of symmetry $\omega(q) = \omega(-q)$ in excitation spectrum

- **Violation** of **Galilean** invariance: ⇨ breakdown of Landau criterion for superfluid velocity, emergence of dynamical instabilities and **suppression** of **superfluidity**

Different strategies to realize novel quantum phases with SOC

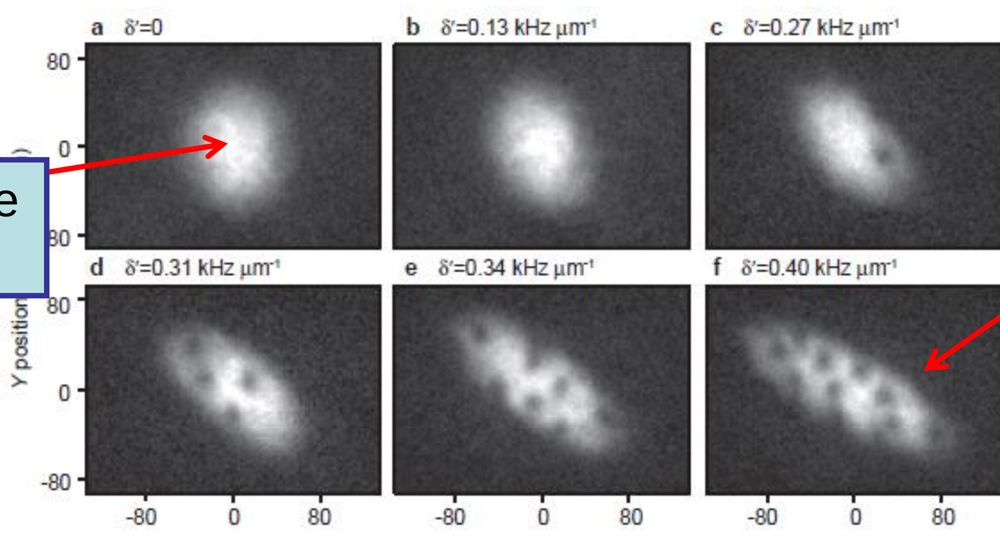
- **First strategy** (Lin et al., Nature 2009).
  Spatially dependent detuning ($\delta(y)$) in strong Raman coupling ($\Omega >> k_0^2$) regime yields position dependent vector potential

$$h_0 = \frac{1}{2m^*}\left(p_x - A_x(y)\right)^2$$

and **effective Lorentz force in neutral atoms.**

Application of detuning $\delta(y)$ effectively corresponds to bringing the system into a rotating non inertial frame and causes the appearence of quantized vortices.

No y-dependence In detuning



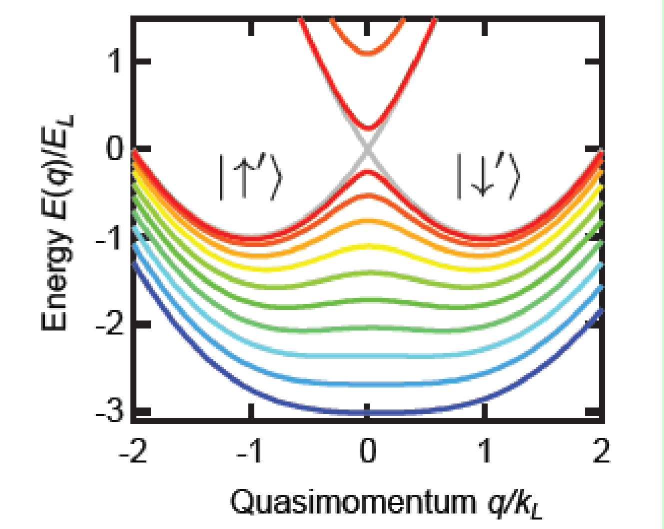
strong y-dependence In detuning

- Second strategy (Lin et al. Nature 2011) – smaller values of Ω

Solution of Schrodinger equation with single particle Hamiltonian

$$h_0 = \frac{1}{2m}[(p_x - \hbar k_0 \sigma_z)^2 + p_\perp^2] + \frac{1}{2}\hbar\Omega\sigma_x$$

causes, for small values of Ω,
the appearence of **two single-particle** states in the lowest band, which can host a Bose-Einstein condensate with different canonical momentum



$$p_x = \pm\hbar k_1 = \pm\hbar k_0\sqrt{1-\Omega^2(m/2\hbar k_0^2)^2}$$

If all the atoms occupy the plane wave state $e^{ik_1 x}$ or $e^{-ik_1 x}$
the corresponding quantum phase is called **plane wave** phase.

At large Ω the two minima reduce to a single minimum with $p_x = 0$

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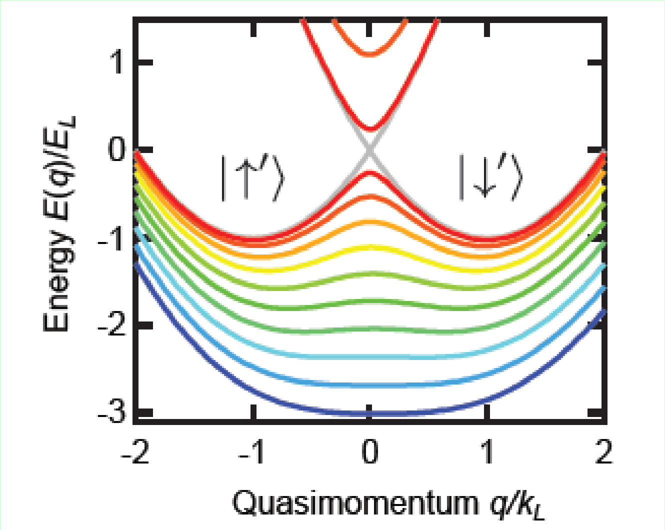

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At large Ω the two minima reduce to a single minimum with $p_x = 0$

A **quantum phase transition** takes place at $\Omega_C = 2\hbar k_0^2/m = 4E_R/\hbar$

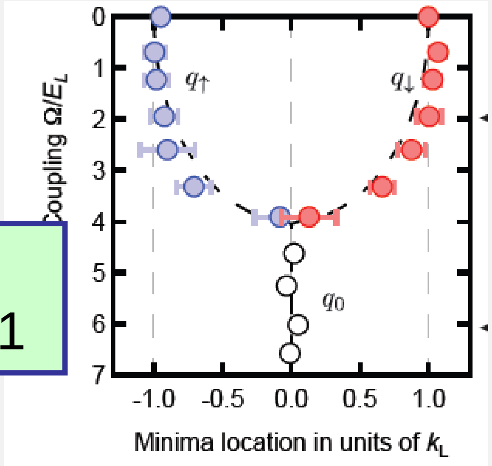
Plane wave-single minimum phase transition

Transition has **second order** nature
It has been observed
at the predicted value

$$\Omega_C \approx 2\hbar k_0^2 / m \equiv 4E_R / \hbar$$

of Raman coupling

Lin et al.,
Nature 2011



- Phase transition is driven by single-particle Hamiltonian.
(weakly affected by two-body interactions in 87Rb)

- **Spin polarizability diverges** at the transition (see later)
(G. Martone, Yun Li, S.S. EPL 2012)

Are two body **interactions** relevant ?

Crucial effects show up in

- Novel **dynamic and superfluid** features
- Emergence of the new **supersolid** quantum phase

Role of two-body interactions

Interactions in 1D SO coupled s=1/2 BECs (T=0) discussed by
Ho and Zhang (PRL 2011), Yun Li, Pitaevskii, Stringari (PRL 2012), …..

$$H = \sum_i h_0(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r}\, g_{\alpha\beta} n_\alpha n_\beta$$

$$h_0 = \frac{1}{2}[\left(p_x - \hbar k_0 \sigma_z\right)^2 + p_\perp^2] + \frac{1}{2}\hbar\Omega\sigma_x$$

- One assumes $g_{\uparrow\uparrow} g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^2$ which ensures **phase mixing** in the absence of Raman coupling

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- Interactions are treated within **mean field** approximation (s=1/2 coupled Gross-Pitaevskii equations)

- Setting $k_0 = 0$ (**no momentum transfer**) yields **Rabi coupled spin mixtures** (see lectures by Oberthaler and Lamporesi)

Gross Pitaevskii equations in presence of SO coupling

$$i\hbar\frac{\partial}{\partial t}\Psi_\uparrow = \left(-\frac{\hbar^2}{2m}(\nabla_x + k_0)^2 + \nabla_\perp^2] + V_{ext} + g_{\uparrow\uparrow}|\Psi_\uparrow|^2 + g_{\uparrow\downarrow}|\Psi_\downarrow|^2 \right)\Psi_\uparrow - \frac{\hbar\Omega}{2}\Psi_\downarrow$$

$$i\hbar\frac{\partial}{\partial t}\Psi_\uparrow = \left(-\frac{\hbar^2}{2m}(\nabla_x - k_0)^2 + \nabla_\perp^2] + V_{ext} + g_{\downarrow}|\Psi_\downarrow|^2 + g_{\uparrow\downarrow}|\Psi_\uparrow|^2 \right)\Psi_\downarrow - \frac{\hbar\Omega}{2}\Psi_\uparrow$$

Interplay between modified single particle Hamiltonian and two-body interactions give rise to

- Novel dynamic and superfluid properties
- Emergence of a novel supersolid phase.

Quantum phase diagram predicted by SOC Hamiltonian at T=0

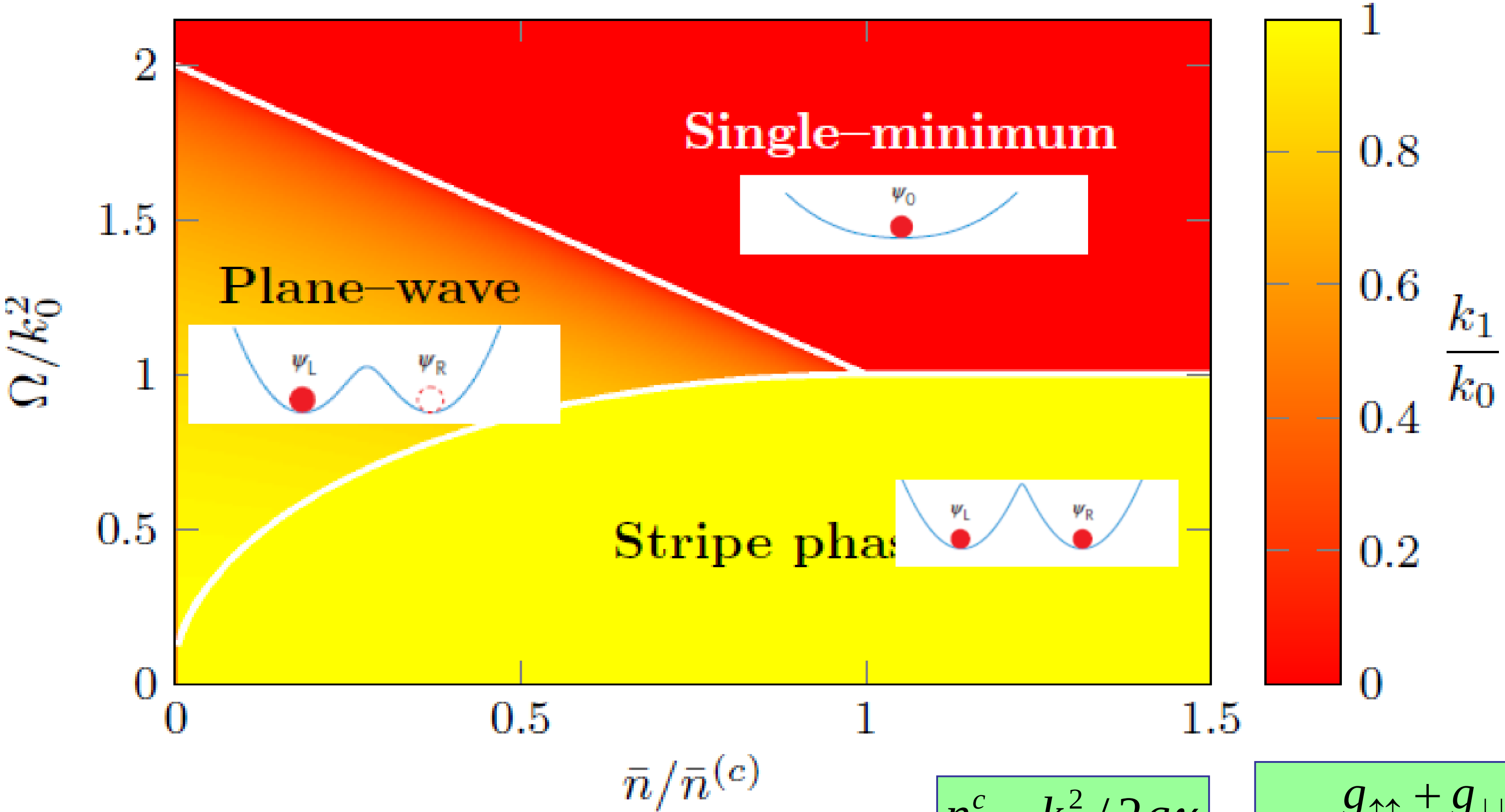
^{87}Rb

$|\uparrow\rangle = |F=1, m_F=0\rangle$

$|\uparrow\rangle = |F=1, m_F=-1\rangle$

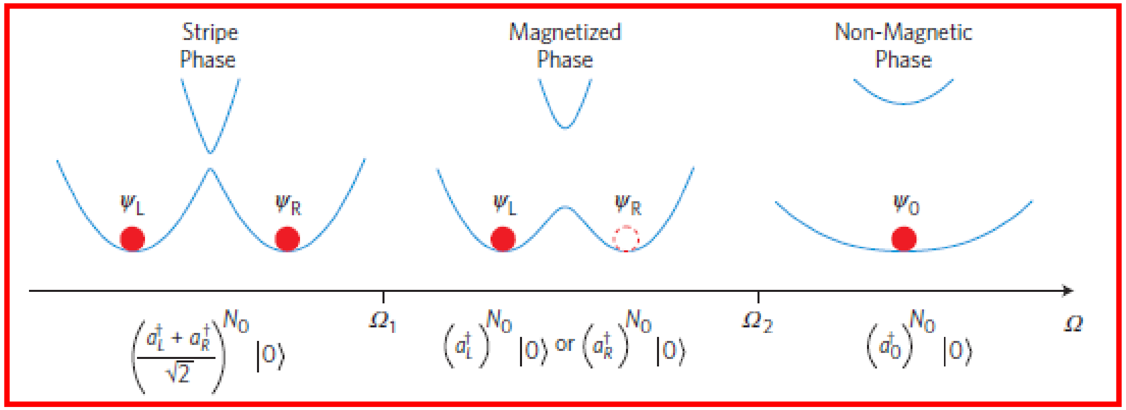
$a_{\uparrow\uparrow} = 101.41 a_{\mathbf{B}}$

$a_{\downarrow\downarrow} = a_{\uparrow\updownarrow} = 100.94 a_B$



$n^c = k_0^2 / 2g\gamma$

$$\gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}}$$



Order parameter in the SOC quantum phases

I) Supersolid phase

$$\Psi = \sqrt{\frac{N}{2V}}\left[\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix}e^{ik_1x} + \begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}e^{-ik_1x}\right]$$

+ higher harmonics

$$\cos\frac{\theta}{2} = \frac{k_1}{k_0}$$

$$<\sigma_z>=0$$

$$n(x) = n\left(1+\frac{\Omega}{2k_0^2}\cos 2k_1x\right)$$

⟵ density fringes fixed by k_1

I) Plane wave phase (magnetized phase)

$$\Psi = \sqrt{\frac{N}{2V}}\begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}e^{-ik_1x}$$

$$<\sigma_z>=\frac{k_1}{k_0}$$

I) Zero momentum phase

$$\Psi = \sqrt{\frac{N}{V}}\begin{pmatrix}1 \\ -1\end{pmatrix}$$

Supersolid- Plane Wave phase transition

Transition is **first order**.
Critical frequency is

$$\Omega_{cr} = 2k_0^2\sqrt{\frac{2\gamma}{1+2\gamma}}$$

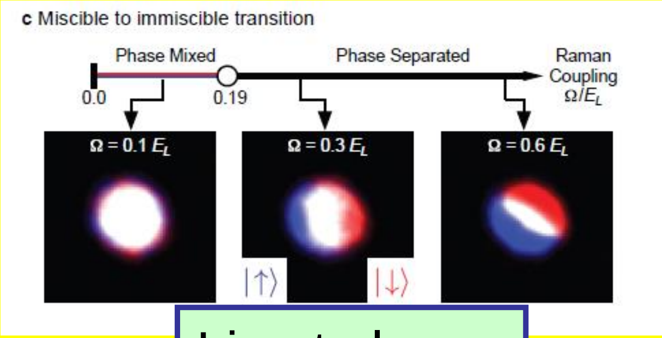
where

$$\gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}}$$

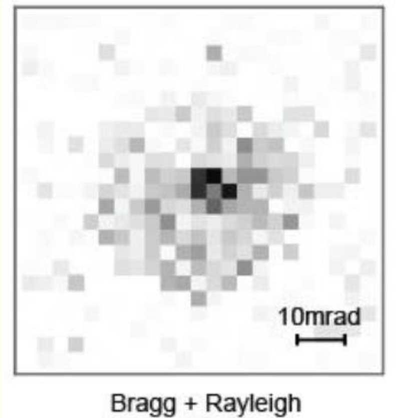
(Ho and Zhang PRL 2011, Yun Li et al. PRL 2012)

Value of Ω_{cr} **crucially depends** on interactions. Vanishes if $g_{\alpha\beta} = g$

The phase transition between a spin mixed and a spin separated phase was observed at the predicted value of Ω



Lin et al., Nature 2011



First experimental evidence for density fringes in the spin mixed state has become available with Bragg scattering (MIT 2017)

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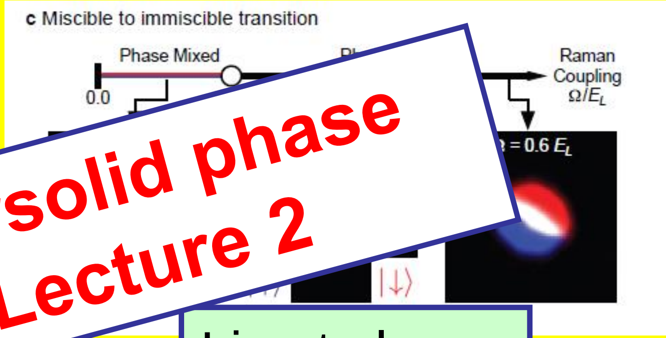
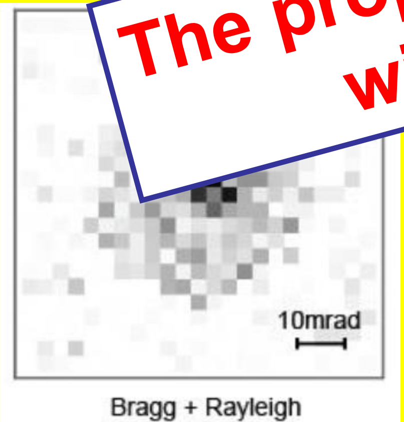


Lin et al., Nature 2011

The properties of the supersolid phase will be discussed in Lecture 2



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Hydrodynamic equations of SOC BEC's

In usual BEC's the hydrodynamic eqs consist of **two equations**

- Equation of Continuity (eq. for the density)
- Euler like equation (eq. for the phase)

$$\Psi = \sqrt{n} e^{i\varphi}$$

- In a two component quantum mixture one would expect **four equations** (two for each component)

$$\Psi_\uparrow = \sqrt{n_\uparrow} e^{i\varphi_\uparrow}$$

$$\Psi_\downarrow = \sqrt{n_\downarrow} e^{i\varphi_\downarrow}$$

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- In SOC BEC's the emergence of the Raman coupling results in the **locking of the relative phase** of the spin-up and spin-down condensates in both the Plane wave and in the Single Minimum phases. The hydrodynamic equations then reduce to **three equations** (see Martone et al. PRA **86**, 063621 2012)

  - Equation of continuity
  - Euler like equation
  - Equation for the spin density

How to derive the Hydrodynamic Equations

Assuming:

- $g_{\alpha\beta} = g$ (good approximation for PW and SM phases of 87Rb)
- locking of relative phase (imposed by Raman coupling if $\omega << \Omega$)
- Small magnetization ($s_z = n_\uparrow - n_\downarrow << n$) in Single Minimum Phase

Action associated with GP energy functional reads ($\varphi = \varphi_\uparrow = \varphi_\downarrow$)

$$A(\varphi, n, s_z) = \cos t + \int dt d\vec{r} \left(\frac{(\vec{\nabla}\varphi)^2}{2m} n - \frac{\hbar^2 k_0}{m} \nabla_x \varphi s_z + \frac{\hbar\Omega}{4} \frac{s_z^2}{n} + \frac{g}{2} n^2 + i\hbar \frac{\partial \varphi}{\partial t} n \right)$$

Imposing vanishing variation $\delta A(\varphi, n, s_z) = 0$ of the action yields

- With respect to phase ⇨ Equation of Continuity
- With repect to density ⇨ Euler-like equation
- With respect to spin density ⇨ coupling between s_z and $\nabla_x \varphi$

The novel hydrodynamic equations

$$\partial_t n + \vec{\nabla} \cdot \vec{j} = 0$$

$$j_x \equiv n v_x = \frac{\hbar}{m} n \nabla_x \varphi - \frac{\hbar}{m} k_0 s_z$$

Equation of **Continuity**
(current is modified by SO coupling)

$$m \partial_t \vec{\nabla} \varphi + \vec{\nabla} \left(\frac{(\hbar \vec{\nabla} \varphi)^2}{2m} + gn + V_{ext} \right) = 0$$

Equation for gradient of the **phase**
(Euler like equation)

$$-\hbar \frac{k_0}{m} \nabla_x \varphi + \frac{1}{2} \Omega \frac{s_z}{n} = 0$$

Equation for the **spin density**
(coupling with gradient of the phase)

(Equations hold in Single Minimum phase – easy generalization to Plane Wave phase)

Crucial Role of the Effective Mass ($g_{\alpha\beta} = g$)

After merging the equations for density and spin density the HD equations reduce to:

$$\frac{\partial n}{\partial t} + \vec{\nabla}(\vec{v}n) = 0$$

for the density and

$$m^* \frac{\partial v_x}{\partial t} + \nabla_x \left(\frac{m^*}{2} v_x^2 + gn + V_{ext} \right) = 0$$

$$m \frac{\partial v_{y,z}}{\partial t} + \nabla_{y,z} \left(\frac{m}{2} v_{y,z}^2 + gn + V_{ext} \right) = 0$$

for the velocity field

with the effective mass given by

$$\frac{m}{m^*} = 1 - \left(\frac{\Omega}{\Omega_C} \right)^2 \quad \text{for } \Omega < \Omega_C = 4E_R/\hbar \text{ (Plane Wave phase)}$$

$$\frac{m}{m^*} = 1 - \frac{\Omega_C}{\Omega} \quad \text{for } \Omega > \Omega_C = 4E_R/\hbar \text{ (Single Minimum phase)}$$

Propagation of sound

In uniform matter sound propagates at the velocity:

$$c_x^2 = \frac{m}{m^*} gn \quad \text{along x - direction}$$

$$c_{y,z}^2 = gn \quad \text{along y, z directions}$$

Softening of sound velocity along x directions is particulary important near the transition between Single Minimum and Plane Wave phases, where effective mass becomes large.

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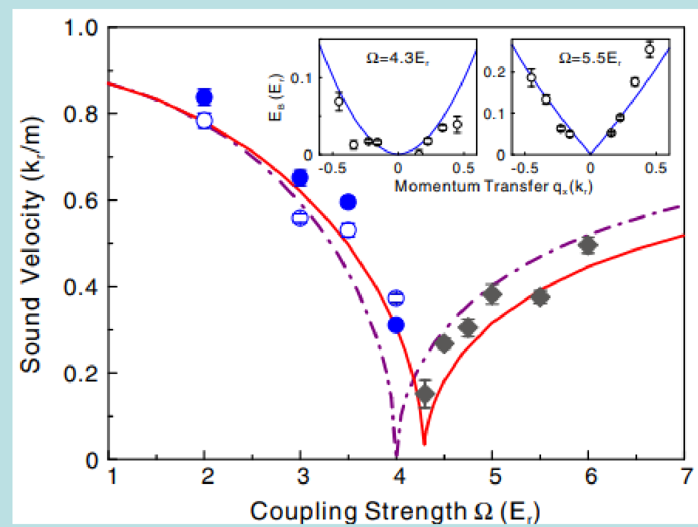
Bragg spectroscopy exps. confirm softening of sound velocity.
At the transition the phonon dispersion is replaced by parabolic behavior
(Si-Cong Ji et al PRL 2015)

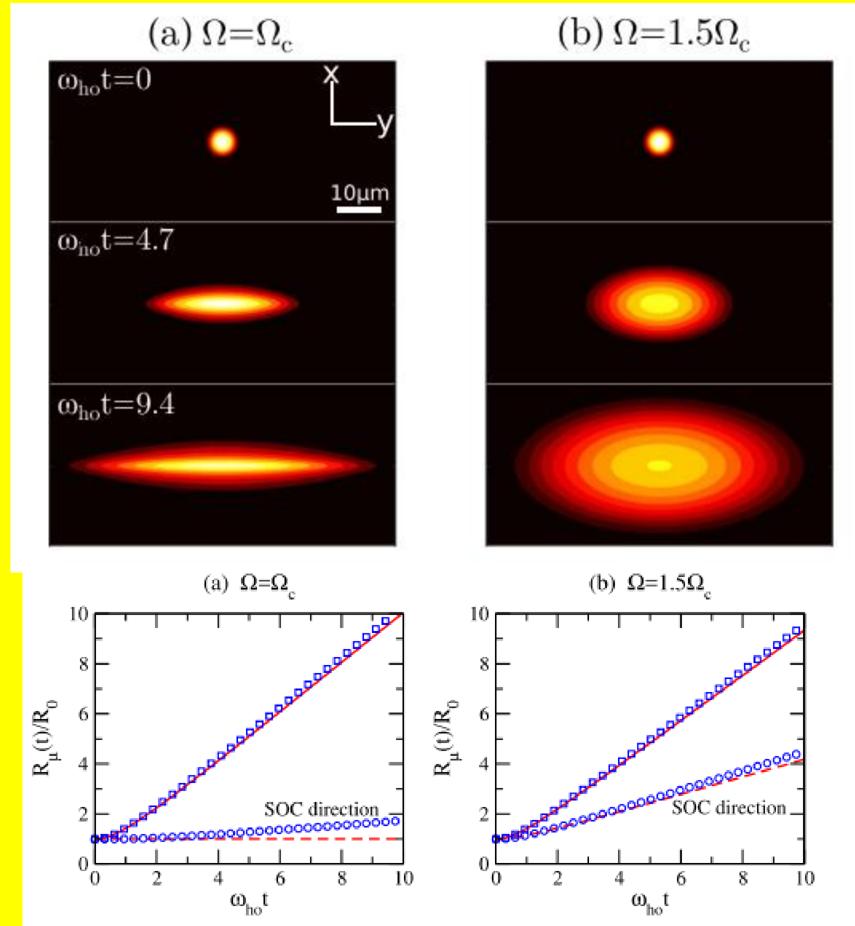


Anisotropic Expansion after release of the trap

After release of the trap ($V_{ext} = 0$ at $t = 0$) the expansion exhibits strong asymmetry in the x-y plane.

In the absence of SOC, expansion would be symmetic ($\omega_x = \omega_y$)

The large value of the effective mass near the transition between Single Minimum and Plane Wave phases, makes the expansion along the x-direction much slower than along y.



Red lines: HD prediction

Circles: Time Dep. GP simulation

Qu,Pitaevskii and SS
New J.Phys.2017

Quantum phase diagram predicted by SOC Hamiltonian at T=0

^{87}Rb

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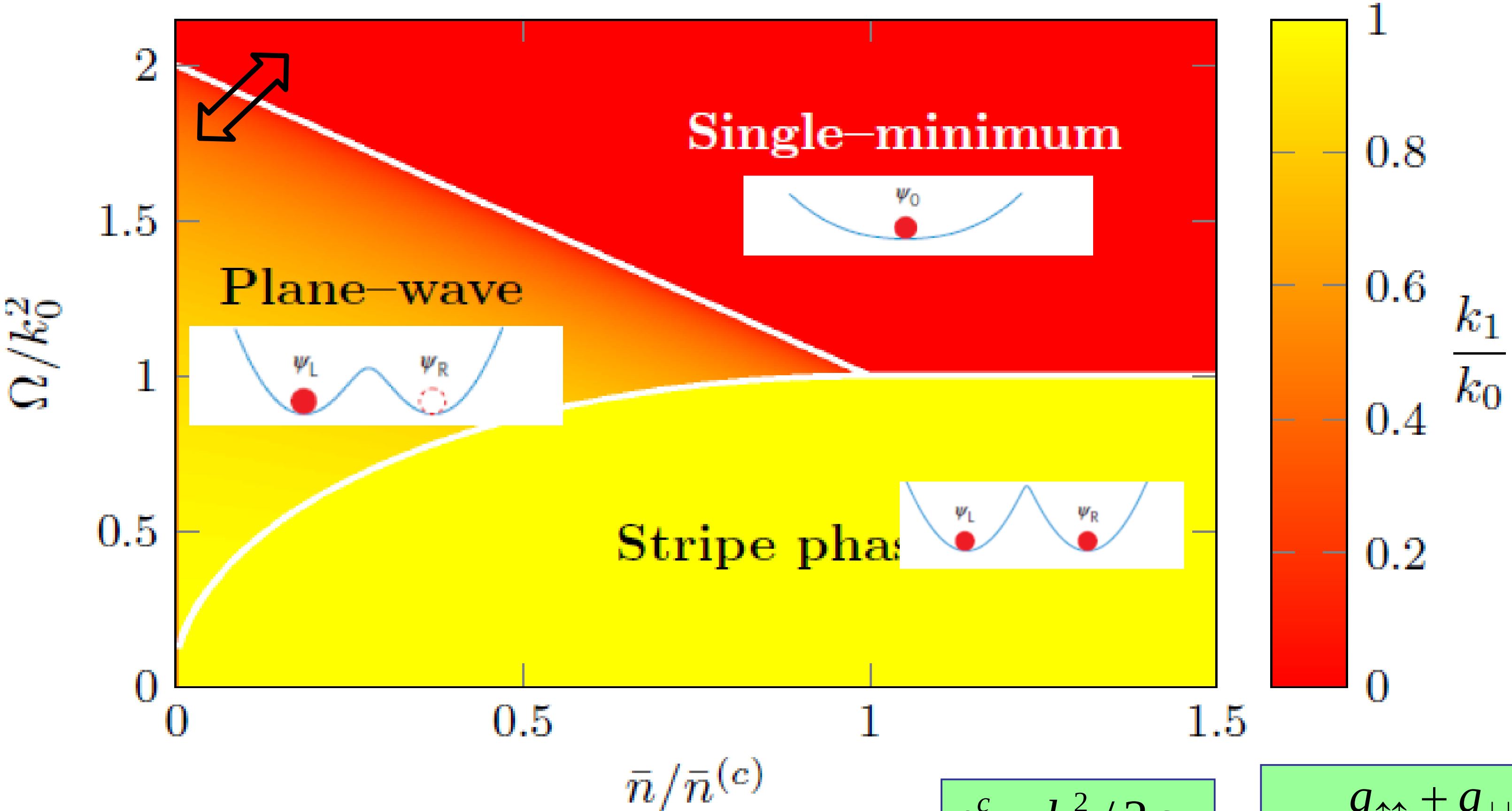


$n^c = k_0^2 / 2g\gamma$

$$\gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}}$$

Magnetic Susceptibility

By adding a small effective magnetic field $-\hbar\delta\sigma_z$, the HD eq. for spin density takes the form (Single Minimum Phase with $g_{\alpha\beta} = g$)

$$-\frac{\hbar k_0}{m}\nabla_x\varphi + \frac{1}{2}\Omega\frac{s_z}{n} = \delta$$

Equilibrium condition of vanishing current,

$$j_x = \frac{\hbar}{m}n\nabla_x\varphi - \frac{\hbar}{m}k_0 s_z = 0$$

then yields result for the susceptibility

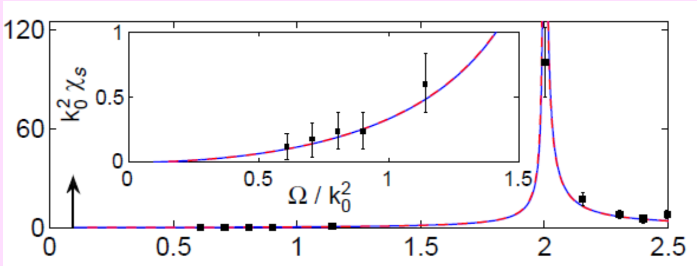
$\Omega > \Omega_c$

$$\chi = \frac{< s_z >}{n\lambda} = \frac{2}{\Omega - \Omega_c}$$

In the Plane Wave Phase ($\Omega < \Omega_c$) one instead finds

$$\chi = \frac{2\Omega^2}{\Omega_C(\Omega_C^2 - \Omega^2)}$$

In both cases χ exhibits **divergent** behavior at **the transition**, (weakly affected by interactions)



[AFM vs FM like transition (similar to Rabi coupled BECs see Lamporesi lectures)]

theory : Martone, Yun Li, Stringari, EPL 2012

exp : Zhang et al, PRL 2012

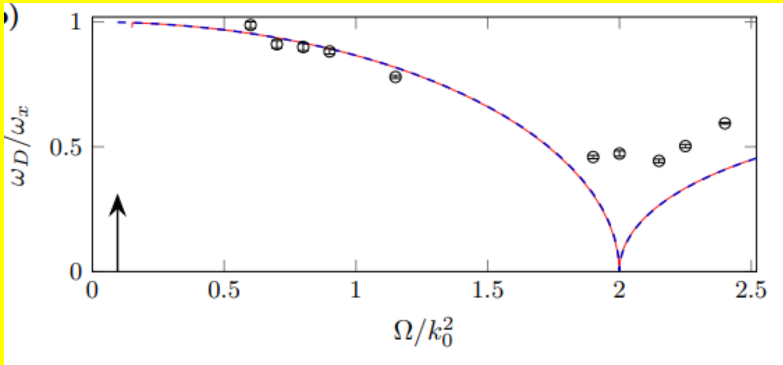
Softening of Center of Mass frequency

The dipole (center of mass) oscillation in harmonic trap
Is particularly sensitive to the value of the effective mass.

Assuming $g_{\alpha\beta} = g$, the HD equations applied to a harmonically trapped BEC, actually provide the expression

$$\omega_D = \omega_x \sqrt{\frac{m}{m^*}}$$

for the dipole frequency



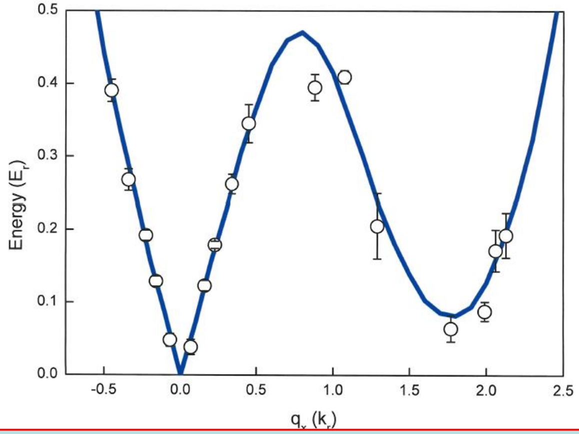
Experiments (Zhang et al. PRL 2012) confirm the softening of the frequency, near the transition between the PW and SM phases.

Nonlinear effects crucial in the region of the phase transition.

Other collective modes (scissors, quandrupole and compression modes **so far unexplored experimentally** in SOC BEC gases)

Emergence of Rotonic structure

New futures emerge at large wave vectors in the Plane Wave phase:



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Exp: Si-Cong Ji et al., PRL 114, 105301 (2015)
Theory: Martone et al., PRA 86, 063621 (2012)

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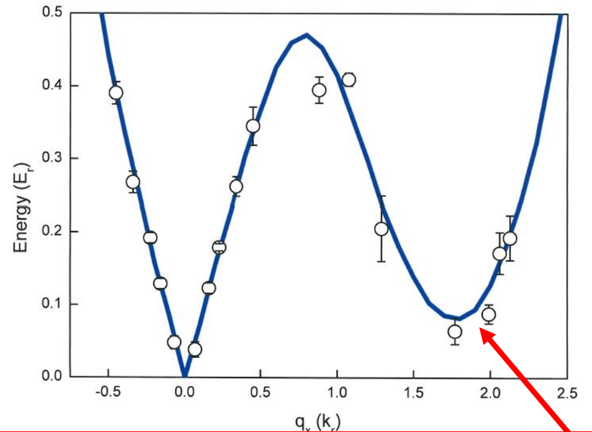

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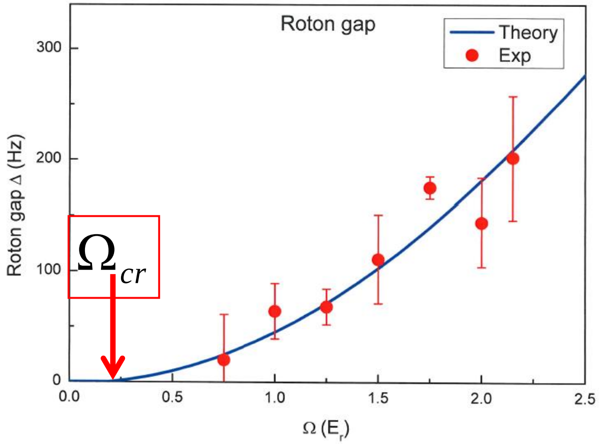

Roton gap decreases as Raman coupling is lowered:
Onset of crystallization
(transition to **stripe** phase)

LECTURE 1

- The quantum phases of a spin-orbit coupled mixture of Bose-Einstein condensates

- Order parameter and nature of the phase transitions

- Sound and Dynamic properties of SOC BEC’s

LECTURE 2

- Superfluidity and rotation of SOC BEC’s

- Supersolidity and the novel Goldstone modes