Varenna Summer School on Quantum Mixtures of Ultracold Atomic Gases 18-23 July 2022

SPIN ORBIT COUPLED BEC GASES



Sandro Stringari



CNR-INO

Università di Trento



Why Spin-Orbit Coupled BEC Gases?

- Give rise to artificial gauge fields opening perspectives for novel quantum effects in neutral systems
- Spin orbit coupling breaks Galilean invariance with crucial consequence on superfluid behavior
- Emergence of a supersolid phase breaking spontaneously both phase and translational invariance and giving rise to novel Goldstone modes

LECTURE 1

- The quantum phases of a spin-orbit coupled mixture of Bose-Einstein condensates
- Order parameter and nature of the phase transitions
- Sound and Dynamic properties of SOC BEC's

LECTURE 2

- Superfluidity and rotation of SOC BEC's
- Supersolidity and the novel Goldstone modes

Simplest realization of (1D) spin-orbit coupling in s=1/2 Bose-Einstein condensates (Spielman, Nist, 2009)

BEC

Two detuned and polarized laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new s.p. Hamitonian in the Laboratory frame.

$$h_0^{lab} = \frac{\vec{p}^2}{2} + \frac{\Omega}{2}\sigma_x \cos(2k_0 x - \Delta\omega_L t) + \frac{\Omega}{2}\sigma_y \sin(2k_0 x - \Delta\omega_L t) - \frac{\omega_Z}{2}\sigma_z$$



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The Hamiltonian is invariant with respect to **helicoidal** (skew) translations (continuous symmetry)

$$e^{id(p_x-k_0\sigma_z)}$$

Corresponding to rigid translation plus rotation in spin space Since the Hamiltonian is time dependent in the laboratory frame, it is convenient to consider the unitary transformation

$$e^{i\Theta\sigma_z/2}$$

(corresponding to position and time dependent spin rotation with $\Theta = 2k_0x - \Delta\omega_L t$. The lab Hamiltonian h_0^{lab} is transformed into a translationally invariant and time independent Hamiltonian

$$h_{0} = \frac{1}{2} [(p_{x} - k_{0}\sigma_{z})^{2} + p_{\perp}^{2}] + \frac{1}{2}\Omega\sigma_{x} + \frac{1}{2}\delta\sigma_{z}$$

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characterized by 1D spin-orbit coupling

 $p_x = -i\hbar\partial_x$ is **canonical** momentum k_0 is laser wave vector difference Ω is strength of Raman coupling $\delta = \Delta\omega_L - \omega_Z$ is effective Zeeman field

The Spin Orbit coupled Hamiltonian

$$h_{0} = \frac{1}{2} [(p_{x} - k_{0}\sigma_{z})^{2} + p_{\perp}^{2}] + \frac{1}{2}\Omega\sigma_{x} + \frac{1}{2}\delta\sigma_{z}$$

is translationally invariant

However it breaks

- Galilean invariance since the physical momentum operator $P_x = mv_x = (p_x - k_0\sigma_z)$ does not commute with the Hamiltonian The Spin Orbit coupled Hamiltonian

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- The SOC Hamiltonian violates also parity and time reversal symmetries

Symmetry properties of spin-orbit Hamiltonian $h_0 = \frac{1}{2} [(p_x - k_0 \sigma_z)^2 + p_\perp^2] + \frac{1}{2} \Omega \sigma_x + \frac{1}{2} \delta \sigma_z$

- **Translational** invariance: uniform ground state configuration, unless crystalline order is spontaneously formed (**stripes, supersolidity**)
- Violation of parity and time reversal symmetry \square breaking of symmetry $\omega(q) = \omega(-q)$ in excitation spectrum
- Violation of Galilean invariance: breakdown of Landau criterion for superfluid velocity, emergence of dynamical instabilities and suppression of superfluidity

Different strategies to realize novel quantum phases with SOC

First strategy (Lin et al., Nature 2009).
 Spatially dependent detuning (δ(y)) in strong Raman coupling (Ω >> k₀²) regime yields position dependent vector potential

$$h_0 = \frac{1}{2m^*} (p_x - A_x(y))^2$$

and effective Lorentz force in neutral atoms.

Application of detuning $\delta(y)$ effectively corresponds to bringing the system into a rotating non inertial frame and causes the appearence of quantized vortices.



- Second strategy (Lin et al. Nature 2011) – smaller values of $\,\Omega\,$

Solution of Schrodinger equation with single particle Hamiltonian $h_0 = \frac{1}{2m} [(p_x - \hbar k_0 \sigma_z)^2 + p_{\perp}^2] + \frac{1}{2} \hbar \Omega \sigma_x$

causes, for small values of Ω_{i} the appearence of **two single-particle** states in the lowest band, which can host a Bose-Einstein condensate with different canonical momentum

$$p_x = \pm \hbar k_1 = \pm \hbar k_0 \sqrt{1 - \Omega^2 (m/2\hbar k_0^2)^2}$$



If all the atoms occupy the plane wave state e^{ik_1x} or e^{-ik_1x} the corresponding quantum phase is called **plane wave** phase.

At large Ω the two minima reduce to a single minimum with $p_x = 0$

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A quantum phase transition takes place at $\Omega_C = 2\hbar k_0^2 / m = 4E_R / \hbar$

Plane wave-single minimum phase transition



Phase transition is driven by single-particle Hamiltonian.
 (weakly affected by two-body interactions in 87Rb)

- Spin polarizability diverges at the transition (see later) (G. Martone, Yun Li, S.S. EPL 2012) Are two body interactions relevant?

Crucial effects show up in

Novel dynamic and superfluid features

- Emergence of the new supersolid quantum phase

Role of two-body interactions

Interactions in 1D SO coupled s=1/2 BECs (T=0) discussed by Ho and Zhang (PRL 2011), Yun Li, Pitaevskii, Stringari (PRL 2012),

$$H = \sum_{i} h_{0}(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r} g_{\alpha\beta} n_{\alpha} n_{\beta} \quad h_{0} = \frac{1}{2} [(p_{x} - \hbar k_{0} \sigma_{z})^{2} + p_{\perp}^{2}] + \frac{1}{2} \hbar \Omega \sigma_{x}$$

- One assumes $g_{\uparrow\uparrow}g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^2$ which ensures phase mixing in the absence of Raman coupling

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- One assumes $g_{\uparrow\uparrow}g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^2$ which ensures **phase mixing** in the absence of Raman coupling
- Interactions are treated within mean field approximation (s=1/2 coupled Gross-Pitaevskii equations)
- Setting $k_0 = 0$ (no momentum transfer) yields Rabi coupled spin mixtures (see lectures by Oberthaler and Lamporesi)

Gross Pitaevskii equations in presence of SO coupling

$$i\hbar\frac{\partial}{\partial t}\Psi_{\uparrow} = \left(-\frac{\hbar^{2}}{2m}\left(\nabla_{x}+k_{0}\right)^{2}+\nabla_{\perp}^{2}\right)+V_{ext}+g_{\uparrow\uparrow}|\Psi_{\uparrow}|^{2}+g_{\uparrow\downarrow}|\Psi_{\downarrow}|^{2}\right)\Psi_{\uparrow}-\frac{\hbar\Omega}{2}\Psi_{\downarrow}$$

$$i\hbar\frac{\partial}{\partial t}\Psi_{\uparrow} = \left(-\frac{\hbar^{2}}{2m}\left(\nabla_{x}-k_{0}\right)^{2}+\nabla_{\perp}^{2}\right)+V_{ext}+g_{\downarrow}|\Psi_{\downarrow}|^{2}+g_{\uparrow\downarrow}|\Psi_{\uparrow}|^{2}\right)\Psi_{\downarrow}-\frac{\hbar\Omega}{2}\Psi_{\uparrow}$$

Interplay between modified single particle Hamiltonian and two-body interactions give rise to

- Novel dynamic and superfluid properties
- Emergence of a novel supersolid phase.





Order parameter in the SOC quantum phases

- 1) Supersolid phase $\Psi = \sqrt{\frac{N}{2V}} \begin{bmatrix} \cos \theta \\ -\sin \theta \end{bmatrix} e^{ik_1 x} + \begin{pmatrix} \sin \theta \\ -\cos \theta \end{bmatrix} e^{-ik_1 x} + higher harmonics$ $\cos \frac{\theta}{2} = \frac{k_1}{k_0} \qquad <\sigma_z >= 0 \qquad n(x) = n(1 + \frac{\Omega}{2k_0^2} \cos 2k_1 x) \quad \leftarrow \text{ density fringes fixed by } k_1$
- I) Plane wave phase (magnetized phase)

$$\Psi = \sqrt{\frac{N}{2V}} \begin{pmatrix} \sin \theta \\ -\cos \theta \end{pmatrix} e^{-ik_1 x}$$



I) Zero momentum phase

 $\Psi = \sqrt{\frac{N}{N}}$

Supersolid- Plane Wave phase transition

Transition is **first order**. Critical frequency is (Ho and Zhang PRL 2011, Yun Li et al. PRL 2012)

$$\Omega_{cr} = 2k_0^2 \sqrt{\frac{2\gamma}{1+2\gamma}} \quad \text{where} \quad \gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow}}$$

Value of Ω_{cr} crucially depends on interactions. Vanishes if $g_{\alpha\beta} = g$

The phase transition between a spin mixed and a spin separated phase was observed at the predicted value of Ω





First experimental evidence for density fringes in the spin mixed state has become available with Bragg scattering (MIT 2017)

Bragg + Rayleigh

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ible to immiscible transition The properties of the supersolid phase Raman Coupling The phase transition between a spin $\Omega I E_{l}$ $= 0.6 E_L$ mixed and a spin separated phase w will be discussed in Lecture 2 observed at the predicted



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Nature 2011

Hydrodynamic equations of SOC BEC's

In usual BEC's the hydrodynamic eqs consist of two equations

- Equation of Continuity (eq. for the density)
- Euler like equation (eq. for the phase)
- In a two component quantum mixture one would expect four equations (two for each component)

$$\Psi_{\uparrow} = \sqrt{n_{\uparrow}} e^{i\varphi_{\uparrow}} \ \Psi_{\downarrow} = \sqrt{n_{\downarrow}} e^{i\varphi_{\downarrow}}$$

 $\Psi = \sqrt{ne^{i\varphi}}$

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 $\Psi = \sqrt{ne^{i\varphi}}$

- In SOC BEC's the emergence of the Raman coupling results in the locking of the relative phase of the spin-up and spindown condensates in both the Plane wave and in the Single Minimum phases. The hydrodynamic equations then reduce to three equations (see Martone et al. PRA 86, 063621 2012)
 - Equation of continuity
 - Euler like equation
 - Equation for the spin density

How to derive the Hydrodynamic Equations

Assuming:

- $g_{\alpha\beta} = g$ (good approximation for PW and SM phases of 87Rb)
- locking of relative phase (imposed by Raman coupling if $\omega << \Omega$)
- Small magnetization ($s_z = n_{\uparrow} n_{\downarrow} \ll n$) in Single Minimum Phase

Action associated with GP energy functional reads ($\varphi = \varphi_{\uparrow} = \varphi_{\downarrow}$)

$$A(\varphi, n, s_z) = \cos t + \int dt d\vec{r} \left(\frac{(\vec{\nabla}\varphi)^2}{2m} n - \frac{\hbar^2 k_0}{m} \nabla_x \varphi s_z + \frac{\hbar\Omega}{4} \frac{s_z^2}{n} + \frac{g}{2} n^2 + i\hbar \frac{\partial\varphi}{\partial t} n \right)$$

Imposing vanishing variation $\delta A(\varphi, n, s_z) = 0$ of the action yields

- With respect to spin density \longrightarrow coupling between s_z and $\nabla_x \varphi$

The novel hydrodynamic equations

$$\partial_t n + \vec{\nabla} \cdot \vec{j} = 0$$

$$j_x \equiv nv_x = \frac{\hbar}{m} n \nabla_x \varphi - \frac{\hbar}{m} k_0 s_z$$

Equation of **Continuity** (current is modified by SO coupling)

$$n\partial_t \vec{\nabla} \varphi + \vec{\nabla} \left(\frac{(\hbar \vec{\nabla} \varphi)^2}{2m} + gn + V_{ext} \right) = 0$$
 Equation for gradient of the phase (Euler like equation)

$$-\hbar \frac{k_0}{m} \nabla_x \varphi + \frac{1}{2} \Omega \frac{s_z}{n} = 0$$

Equation for the **spin density** (coupling with gradient of the phase)

(Equations hold in Single Minimum phase – easy generalization to Plane Wave phase)

After merging the equations for density and spin density the HD equations reduce to:

 $m * \frac{\partial v_x}{\partial t} + \nabla_x \left(\frac{m^*}{2}v_x^2 + gn + V_{ext}\right) = 0$ $m\frac{\partial v_{y,z}}{\partial t} + \nabla_{y,z}\left(\frac{m}{2}v_{y,z}^2 + gn + V_{ext}\right) = 0$ for the velocity field

 $\left| \frac{\partial n}{\partial t} + \vec{\nabla}(\vec{v}n) = 0 \right|$ for the density and

with the effective mass $\frac{m}{m^*} = 1 - \left(\frac{\Omega}{\Omega_C}\right)^2$ for $\Omega < \Omega_C = 4E_R/\hbar$ (Plane Wave phase) with the given by $\frac{m}{m^*} = 1 - \frac{\Omega_C}{\Omega} \quad \text{for } \Omega > \Omega_C = 4E_R/\hbar \text{ (Single Minimum phase)}$

Crucial Role of the Effective Mass $(g_{\alpha\beta} = g)$

Propagation of sound

In uniform matter sound propagates at the velocity:

$$c_x^2 = \frac{m}{m^*} gn$$
 along x - direction
 $c_{y,z}^2 = gn$ along y, z directions

Softening of sound velocity along x directions is particulary important near the transition between Single Minimum and Plane Wave phases, where effective mass becomes large.

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Bragg spectroscopy exps. confirm softening of sound velocity. At the transition the phonon dispersion is replaced by parabolic behavior (Si-Cong Ji et al PRL 2015)



Anisotropic Expansion after release of the trap

After release of the trap ($V_{ext} = 0$ at t = 0) the expansion exhibits strong asymmetry in the x-y plane. (a) $\Omega = \Omega_c$ (b) $\Omega = 1.5\Omega_c$

In the absence of SOC, expansion would be symmetic ($\omega_x = \omega_y$)

The large value of the effective mass near the transition between Single Minimum and Plane Wave phases, makes the expansion along the x-direction much slower than along y.

Red lines: HD prediction **Circles**: Time Dep. GP simulation



Qu,Pitaevskii and SS New J.Phys.2017



Magnetic Susceptibility

By adding a small effective magnetic field $-\hbar \delta \sigma_z$, the HD eq. for spin density takes the form (Single Minimum Phase with $g_{\alpha\beta} = g$)

$$-\frac{\hbar k_0}{m}\nabla_x \varphi + \frac{1}{2}\Omega \frac{s_z}{n} = \delta$$

Equilibrium condition of vanishing current,

 $j_x = \frac{\hbar}{m} n \nabla_x \varphi - \frac{\hbar}{m} k_0 s_z = 0$ then yields result for the susceptibility



 $\Omega > \Omega_c$

In the Plane Wave Phase ($\Omega < \Omega_c$) one instead finds $\chi = \frac{2\Omega^2}{\Omega_c (\Omega^2 - \Omega^2)}$

In both cases χ exhibits **divergent** behavior at **the transition**, (weakly affected by interactions)

[AFM vs FM like transition (similar to Rabi coupled BECs see Lamporesi lectures)]

theory : Martone, Yun Li, Stringari, EPL 2012 exp : Zhang et al, PRL 2012



Softening of Center of Mass frequency

The dipole (center of mass) oscillation in harmonic trap Is particularly sensitive to the value of the effective mass.

Assuming $g_{\alpha\beta} = g$, the HD equations applied to a harmonically trapped BEC, actually provide the expression $\omega_D = \omega_x \sqrt{\frac{m}{m^*}}$ for the dipole frequency



Experiments (Zhang et al. PRL 2012) confirm the softening of the frequency, near the transition between the PW and SM phases.

Nonlinear effects crucial in the region of the phase transition.

Other collective modes (scissors, quandrupole and compression modes **so far unexplored experimentally** in SOC BEC gases)

Emergence of Rotonic structure

New futures emerge at large wave vectors in the Plane Wave phase:



 $\omega(q) \neq \omega(-q)$ consequence of violation of **parity** and **time reversal** symmetry

Exp: Si-Cong Ji et al., PRL 114, 105301 (2015) Theory: Martone et al., PRA 86, 063621 (2012)

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