Aalto University
School of Science

Material for the Varenna Summer School lectures

Päivi Törmä

Aalto University

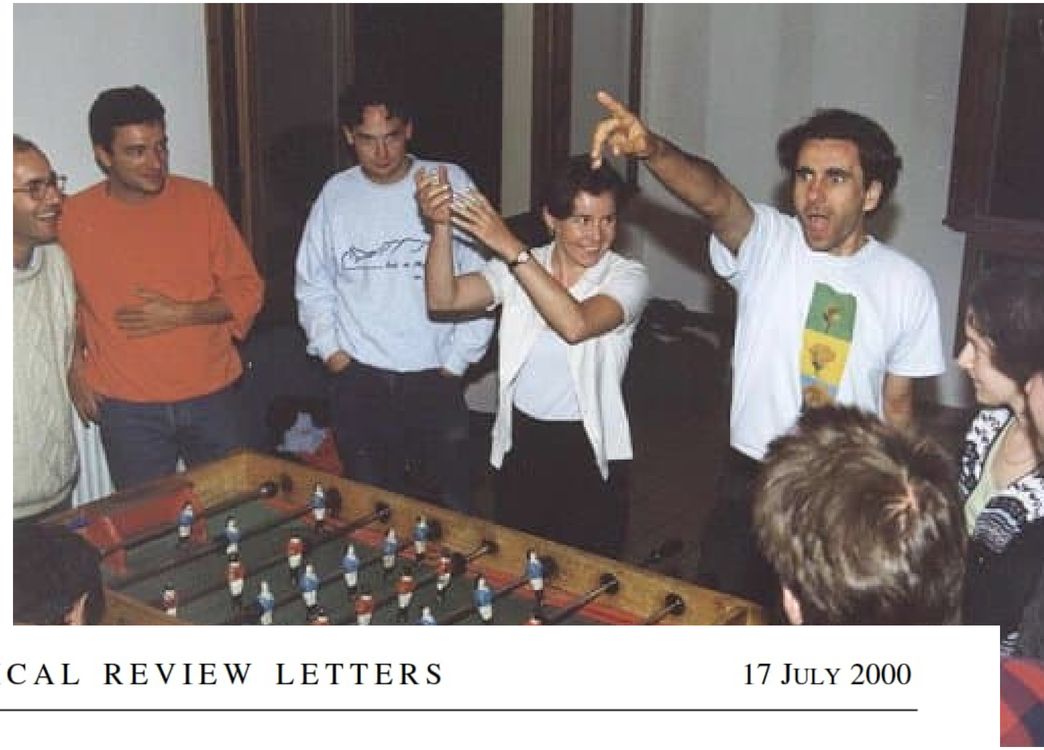
International School of Physics "Enrico Fermi" Course 211 - Quantum Mixtures with Ultra-cold Atoms

23.7.2022

PREIN Centre for Quantum Engineering QUANTERA erc HORIZON2020 Marie Skłodowska-Curie actions ACADEMY OF FINLAND

My summer school memories

Les Houches 1999



VOLUME 85, NUMBER 3 PHYSICAL REVIEW LETTERS 17 JULY 2000

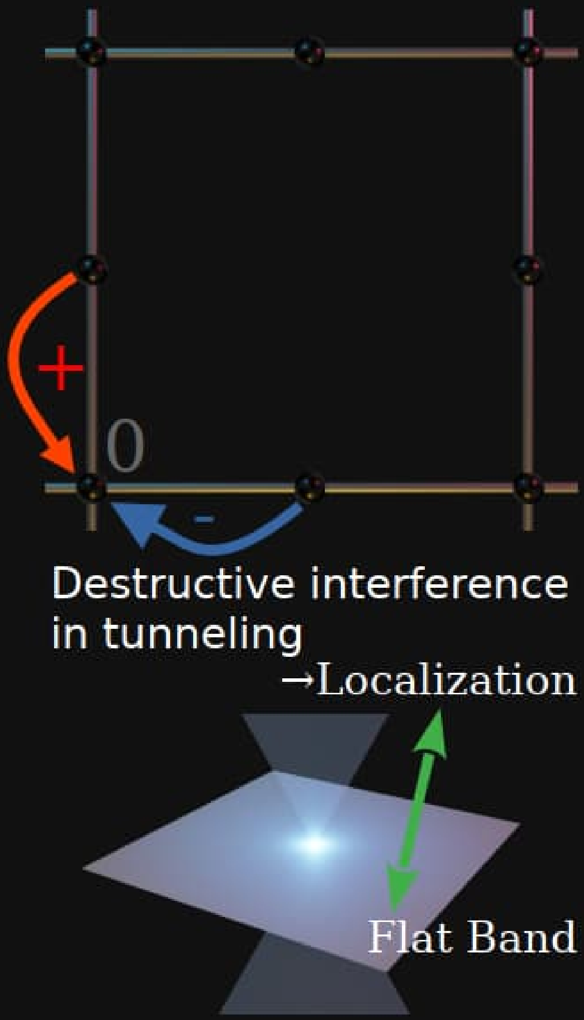
Laser Probing of Atomic Cooper Pairs

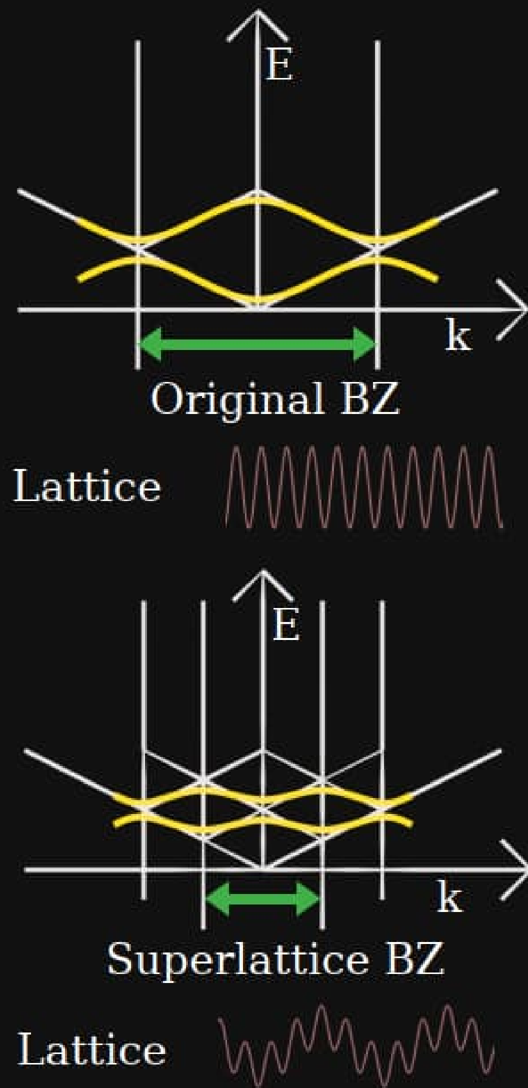
P. Törmä[1,2,3] and P. Zoller[1]

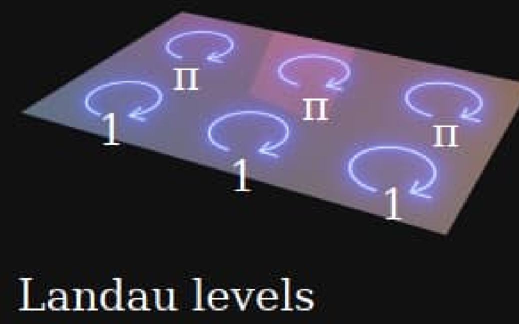
[1]*Institute for Theoretical Physics, University of Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria*
[2]*Helsinki Institute of Physics, P.O. Box 9, FIN-00014 University of Helsinki, Finland*
[3]*Laboratory of Computational Engineering, P.O. Box 9400, FIN-02015 Helsinki University of Technology, Finland*
(Received 24 January 2000)

We consider a gas of attractively interacting cold fermionic atoms which are manipulated by laser light. The laser induces a transition from an internal state with large negative scattering length to one with almost no interactions. The process can be viewed as a tunneling of atomic population between the superconducting and the normal states of the gas. It can be used to detect the BCS ground state and to measure the superconducting order parameter.

Formation of flat bands



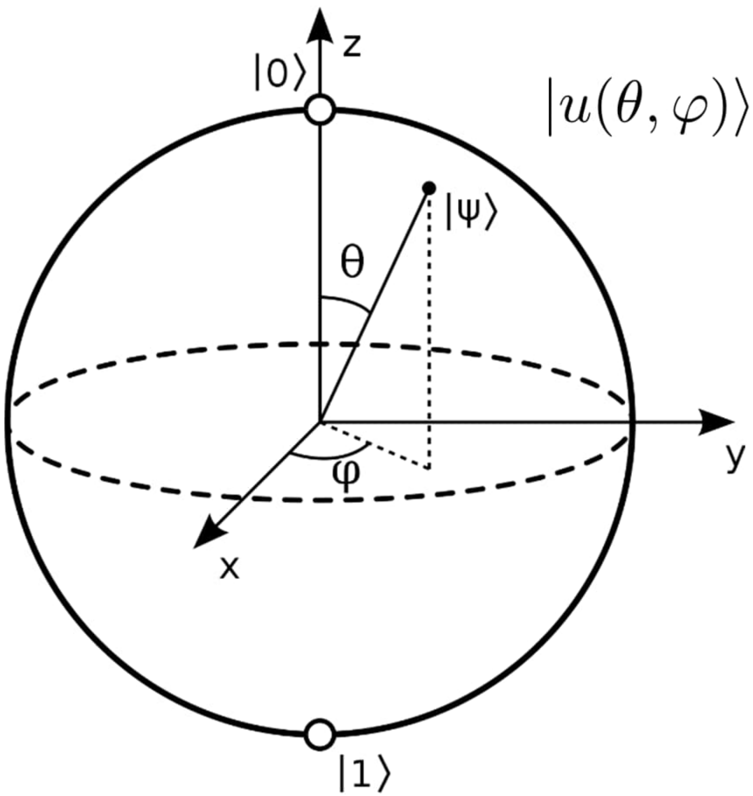




Review: Leykam, Andreanov, Flach, Advances in Physics 2018

$$\mathcal{B}_{ij}(\mathbf{k}) = 2\langle \partial_{k_i} u | (1 - |u\rangle\langle u|) | \partial_{k_j} u \rangle$$

$$\mathrm{Re}\,\mathcal{B}_{ij} = g_{ij}$$ quantum metric $$d\ell^2 = \sum_{ij} g_{ij} dk_i dk_j$$



$$|u(\theta,\varphi)\rangle = e^{-i\varphi/2}\cos(\theta/2)\,|0\rangle + e^{i\varphi/2}\sin(\theta/2)\,|1\rangle$$

$$g_{\varphi\varphi} = \sin^2\theta$$

The Cooper problem: two particles

PHYSICAL REVIEW VOLUME 104, NUMBER 4 NOVEMBER 15, 1956

Letters to the Editor

PUBLICATION of brief reports of important discoveries in physics may be secured by addressing them to this department. The closing date for this department is five weeks prior to the date of issue. No proof will be sent to the authors. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents. Communications should not exceed 600 words in length and should be submitted in duplicate.

Bound Electron Pairs in a Degenerate Fermi Gas*

LEON N. COOPER
Physics Department, University of Illinois, Urbana, Illinois
(Received September 21, 1956)

IT has been proposed that a metal would display superconducting properties at low temperatures if the one-electron energy spectrum had a volume-independent energy gap of order $\Delta \simeq kT_c$, between the ground state and the first excited state.[1,2] We should like to point out how, primarily as a result of the exclusion principle, such a situation could arise.

Consider a pair of electrons which interact above a

$=(1/V)\exp[i(\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_2\cdot\mathbf{r}_2)]$ which satisfy periodic boundary conditions in a box of volume V, and where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the coordinates of electron one and electron two. (One can use antisymmetric functions and obtain essentially the same results, but alternatively we can choose the electrons of opposite spin.) Defining relative and center-of-mass coordinates, $\mathbf{R}=\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$, $\mathbf{r}=(\mathbf{r}_2-\mathbf{r}_1)$, $\mathbf{K}=(\mathbf{k}_1+\mathbf{k}_2)$ and $\mathbf{k}=\frac{1}{2}(\mathbf{k}_2-\mathbf{k}_1)$, and letting $\mathcal{E}_K+\epsilon_k=(\hbar^2/m)(\frac{1}{4}K^2+k^2)$, the Schrödinger equation can be written

$$(\mathcal{E}_K+\epsilon_k-E)a_{\mathbf{k}}+\sum_{\mathbf{k}'}a_{\mathbf{k}'}(\mathbf{k}|H_1|\mathbf{k}')\times\delta(\mathbf{K}-\mathbf{K}')/\delta(0)=0 \quad (1)$$

where

$$\Psi(\mathbf{R},\mathbf{r})=(1/\sqrt{V})e^{i\mathbf{K}\cdot\mathbf{R}}\chi(\mathbf{r},K),$$
$$\chi(\mathbf{r},K)=\sum_{\mathbf{k}}(a_{\mathbf{k}}/\sqrt{V})e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (2)$$

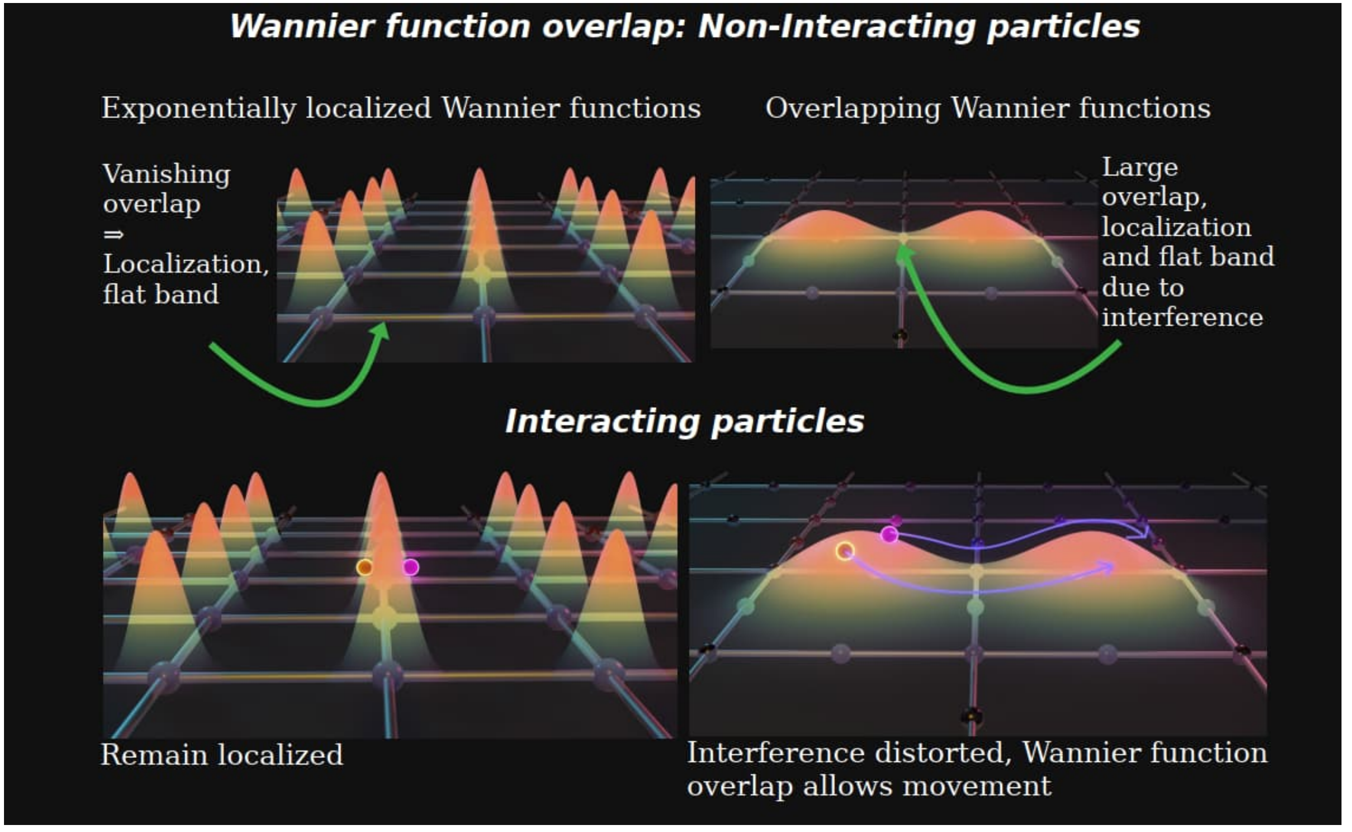
and

$$(\mathbf{k}|H_1|\mathbf{k}')=\left(\frac{1}{V}\int d\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}H_1e^{i\mathbf{k}'\cdot\mathbf{r}}\right)_{0\text{ phonons}}.$$

We have assumed translational invariance in the metal. The summation over $\mathbf{k}'$ is limited by the exclusion principle to values of k_1 and k_2 larger than q_0, and by the delta function, which guarantees the conservation of the total momentum of the pair in a single scattering.

$$T_c \propto e^{-1/(Un_0(E_f))}$$

Why can there be transport in a flat band?

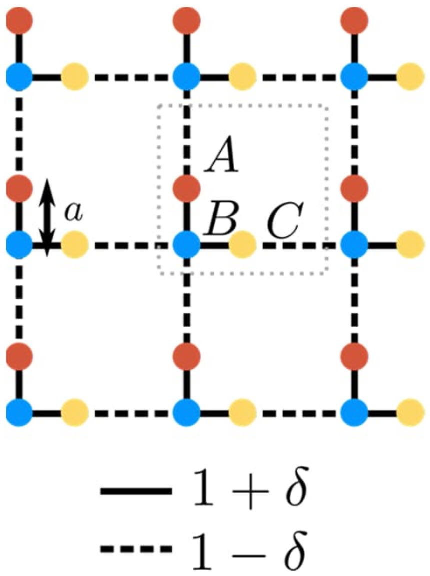


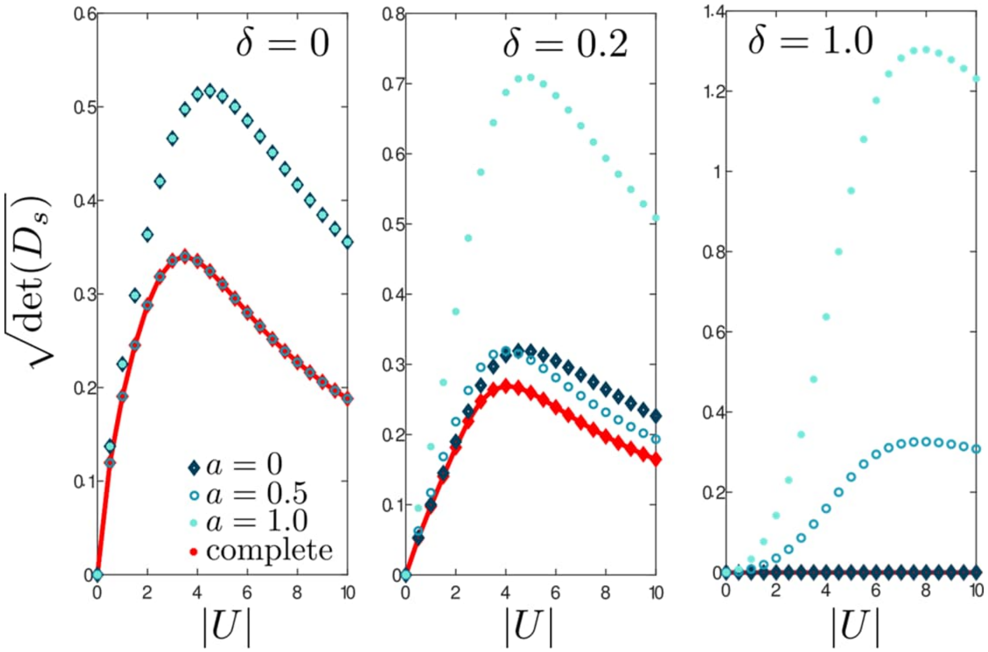
$C \neq 0 \Leftrightarrow$ non-localized $w(\mathbf{r}) = \mathcal{F}[u(\mathbf{k})]$

Brouder, Panati, Calandra, Marzari, PRL 2007

$$D_s \propto g_{ij} \geqslant C$$

Example: the Lieb lattice



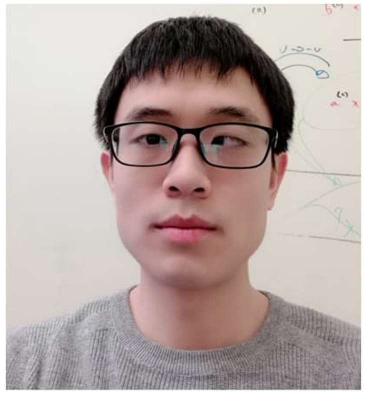


At worst, $\left.\frac{1}{V}\frac{\partial^2 \Omega}{\partial q_i \partial q_j}\right|_{\boldsymbol{q}=0}$ can give an incorrectly nonzero superfluid weight.

When the orbitals are at high-symmetry positions, the quantum metric is guaranteed to be minimal

Superfluidity and quantum geometry

Sebastiano Peotta



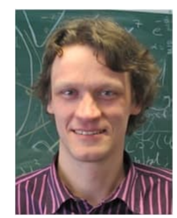
Long Liang

Sebastian Huber

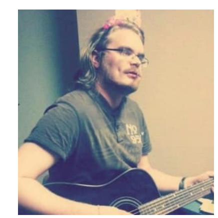
Murad Tovmasyan

Aleksi Julku

Tuomas Vanhala



Ari Harju

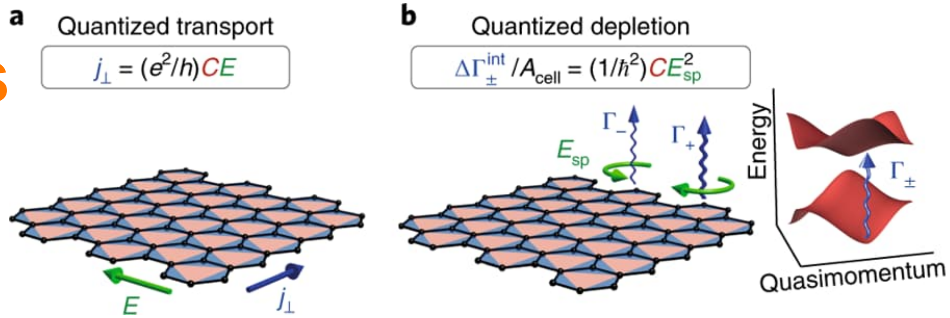


Topi Siro

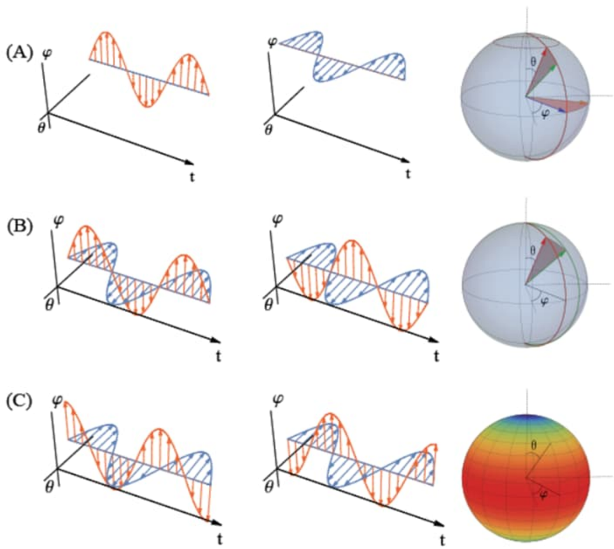
Dong-Hee Kim

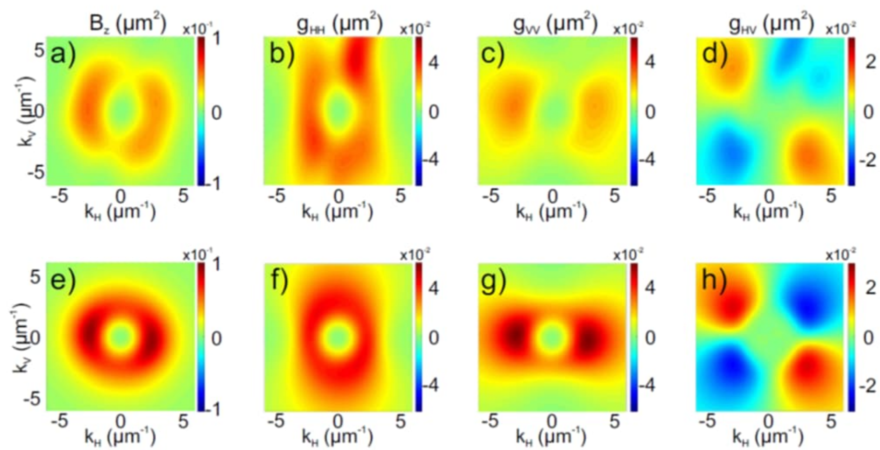
Peotta, PT, Nat Comm 2015
Julku, Peotta, Vanhala, Kim, PT, PRL 2016
Tovmasyan, Peotta, PT, Huber, PRB 2016
Liang, Vanhala, Peotta, Siro, Harju, PT, PRB 2017
Liang, Peotta, Harju, PT, PRB 2017
Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018
PT, Liang, Peotta, PRB(R) 2018

Experimental observations of the quantum metric



Observation of BZ-integrated quantum metric with ultracold gases
Asteria, Tran, Ozawa, Tarnowski, Rem, Fläschner, Sengstock, Goldman, Weitenberg, Nat. Phys. 2019





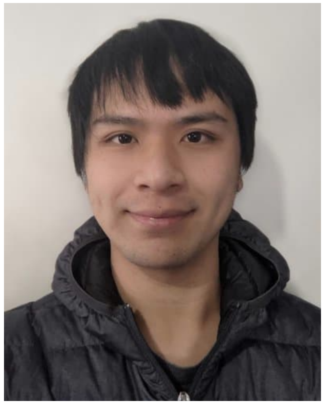
Observation of the quantum metric in a solid state spin system
Yu, Yang, Gong, Cao, Lu, Liu, Plenio, Jelezko, Ozawa, Goldman, Zhang, Cai, National Science Review 2020

Observation of the quantum metric in a continuum polariton system
Gianfrate, Bleu, Dominici, Ardizzone, De Giorgi, Ballarini, West, Pfeiffer, Solnyshkov, Sanvitto, Malpuech, Nature 2019

Revisiting flat band superconductivity: dependence on minimal quantum metric and band touchings

Kukka-Emilia Huhtinen

Jonah Herzog-Arbeitman

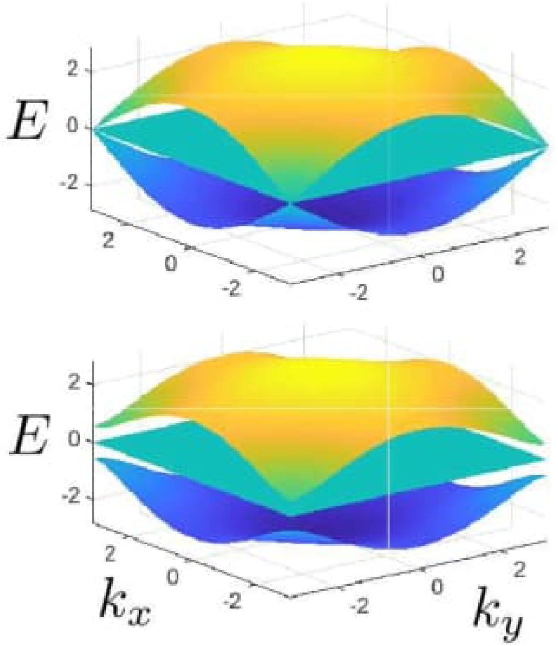


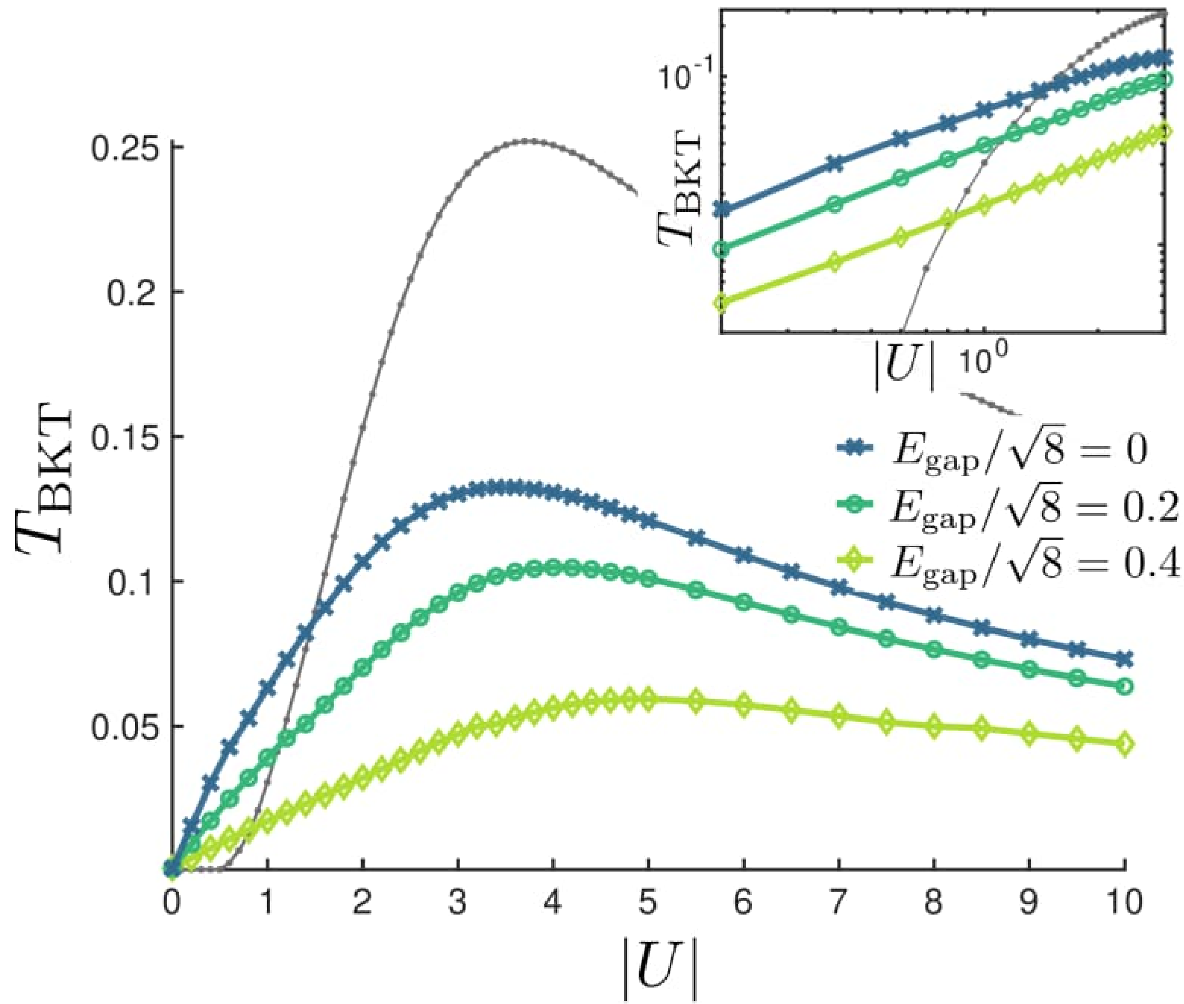
Aaron Chew

Andrei Bernevig

Huhtinen, Herzog-Arbeitsman, Chew, Bernevig, PT, PRB Editor’s Suggestion
arXiv:2203:11133 (2022)

Band touchings increase the critical temperature



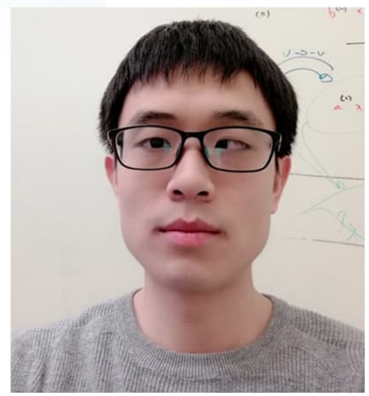


For correct formulas on superconductivity and quantum geometry, use Huhtinen, Herzog-Arbeitsman, Chew, Bernevig, PT, arXiv:2203:11133 (2022)

Twisted bilayer graphene (TBG) superconductivity and quantum metric

Aleksi Julku

Teemu Peltonen



Long Liang

Tero Heikkilä

Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors’ Suggestion

For APS Physics news, google Geometry resques superconductivity

Twisted **B**ilayer **G**raphene (**TBG**) superconductivity since 2018

Reviews: L. Balents, C. Dean, D. Efetov, A. Young, Nat Phys 2020

E. Andrei, D. Efetov, P. Jarillo-Herrero, A. MacDonald, K. Mak, T. Senthil, E.Tutuc, A. Yazdani, A. Young, Nat Rev Mater 2021

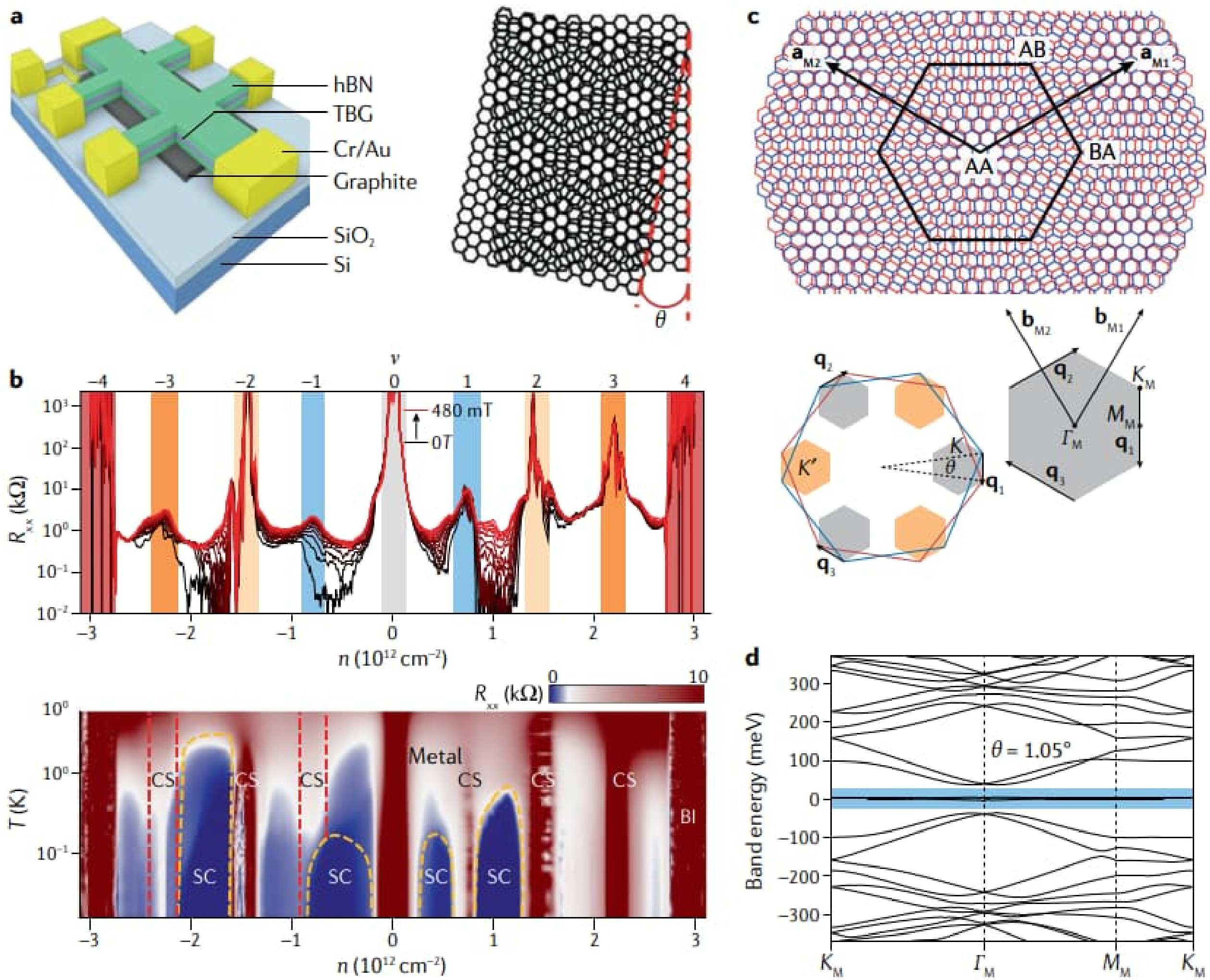


Figure credits see Fig.1 in PT, Peotta, Bernevig, Nat Rev Phys 2022

VIEWPOINT

PDF Version

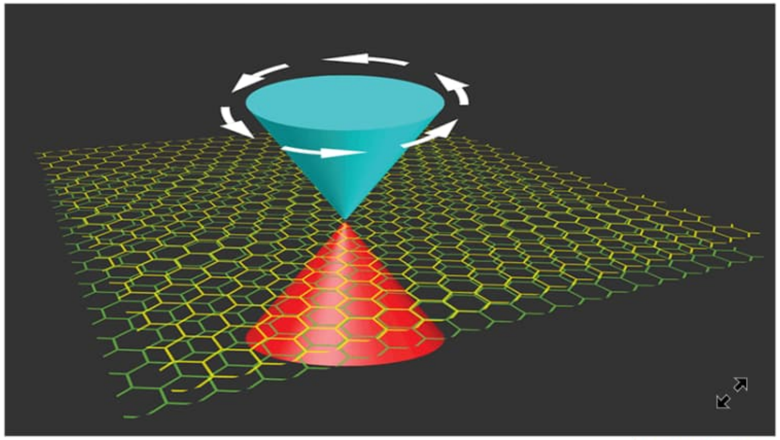
Geometry Rescues Superconductivity in Twisted Graphene

Laura Classen

School of Physics and Astronomy, University of Minnesota, Minneapolis, MN, USA

February 24, 2020 • *Physics* 13, 23

Three papers connect the superconducting transition temperature of a graphene-based material to the geometry of its electronic wave functions.



APS/Alan Stonebraker

Figure 1: Electrons moving through the sheets of twisted bilayer graphene (TBG) have special points in their band structure where two cone-shaped bands meet. The inherent "curvature" of the states in these bands turns out to contribute to the magnitude of TBG'... Show more

On its own, a sheet of graphene is a semimetal—its electrons interact only weakly with each other. But as experimentalists discovered in 2018 [1, 2], the situation changes when two sheets of graphene are stacked together, with a slight ($\sim 1°$) rotation between them (Fig. 1). At this so-called magic twist angle [3] and at low temperatures [1], the electrons become correlated, forming insulating or superconducting phases depending on the carrier density [2–7]. These phases appear to come from a twist-induced flattening of the electronic energy bands, which

Geometric and Conventional Contribution to the Superfluid Weight in Twisted Bilayer Graphene

Xiang Hu, Timo Hyart, Dmitry I. Pikulin, and Enrico Rossi

Phys. Rev. Lett. 123, 237002 (2019)

Published December 5, 2019

Read PDF

Superfluid weight and Berezinskii-Kosterlitz-Thouless transition temperature of twisted bilayer graphene

A. Julku, T. J. Peltonen, L. Liang, T. T. Heikkilä, and P. Törmä

Phys. Rev. B 101, 060505 (2020)

Published February 24, 2020

Read PDF

Topology-Bounded Superfluid Weight in Twisted Bilayer Graphene

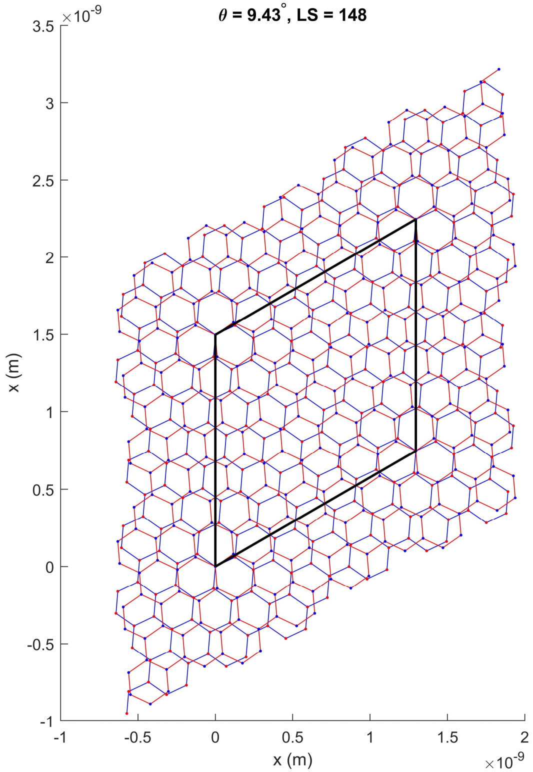
Fang Xie, Zhida Song, Biao Lian, and B. Andrei Bernevig

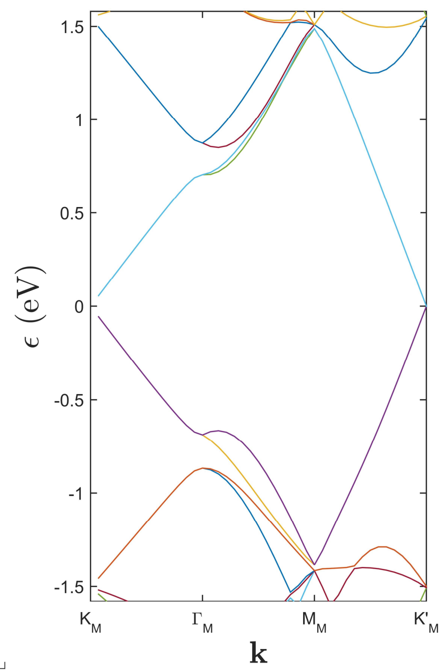
Phys. Rev. Lett. 124, 167002 (2020)

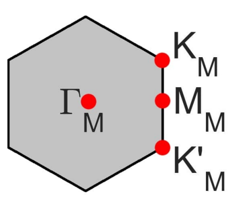
Published April 24, 2020

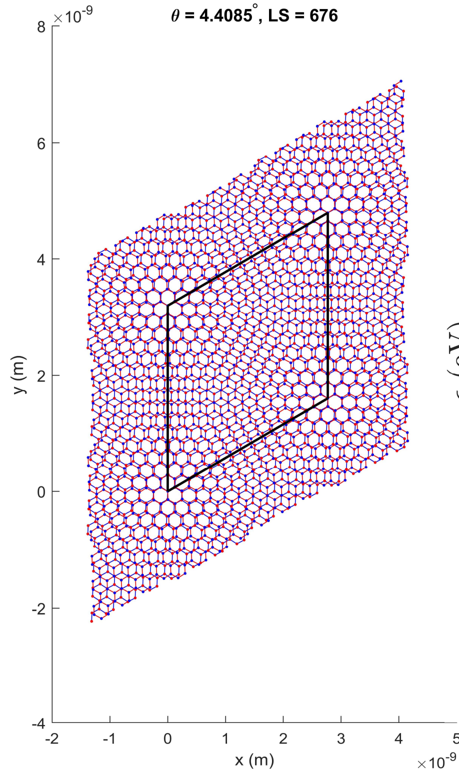
Read PDF

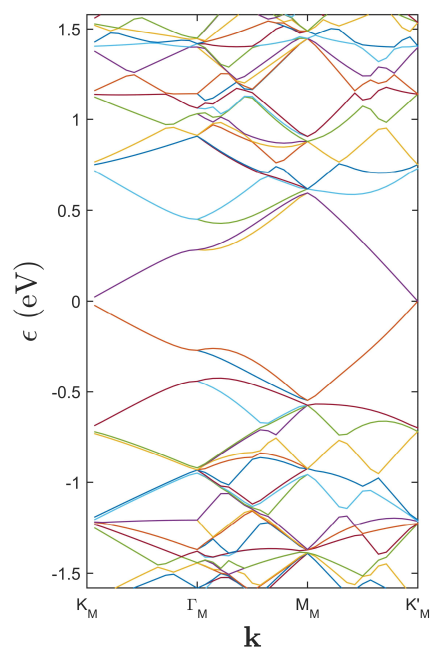
Non-interacting bands



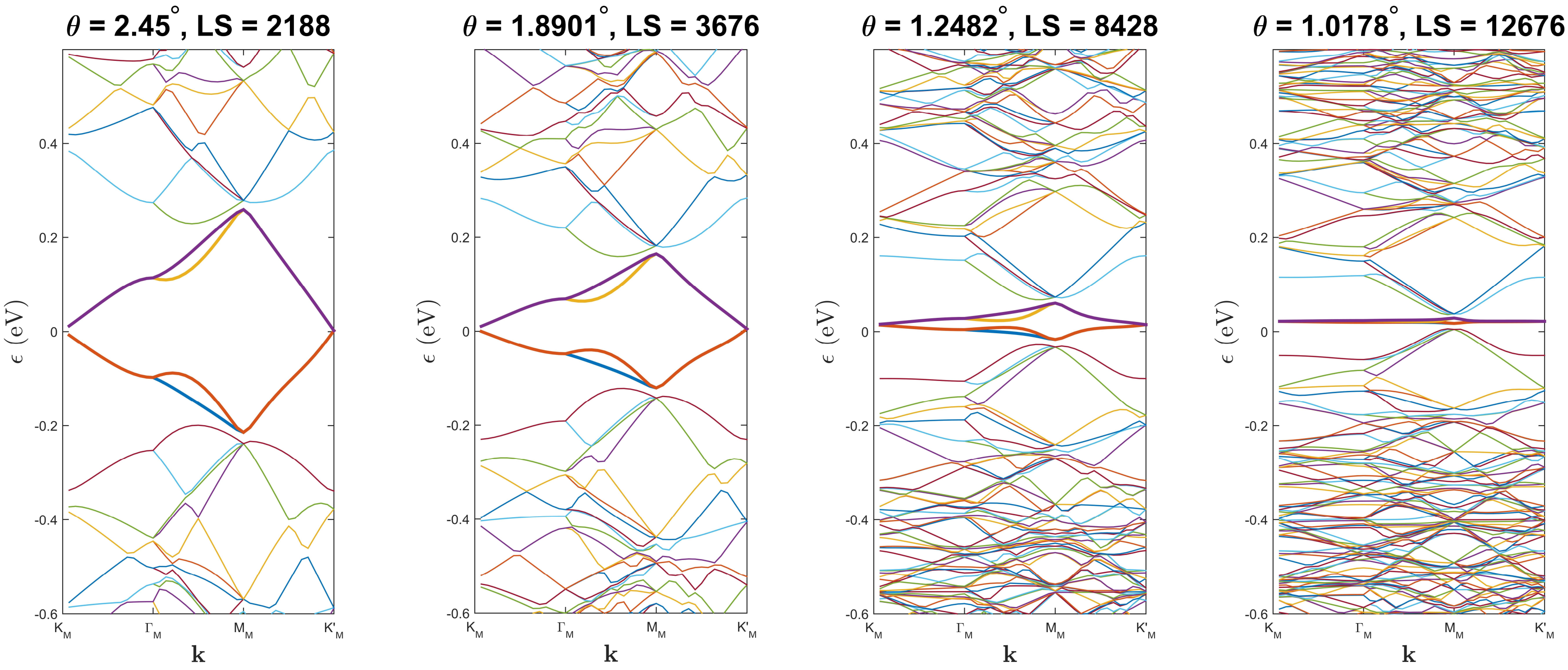








Non-interacting bands

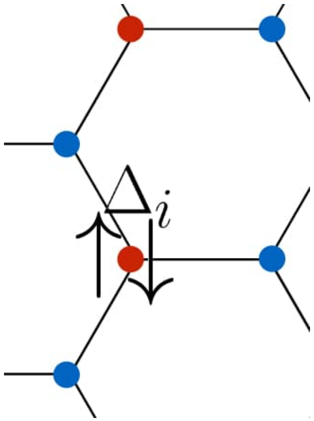


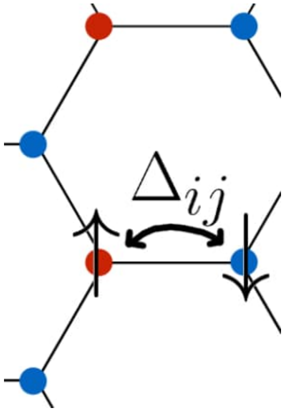
Fermi-Hubbard lattice model with TBG geometry:

$$H = \sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + H_{\text{int}}$$

Two distinct pairing schemes:

$$H_{\text{int}} = J \sum_{i} c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow}$$





$$H_{\text{int}} = \frac{J}{2} \sum_{\langle ij \rangle} h^{\dagger}_{ij} h_{ij}$$

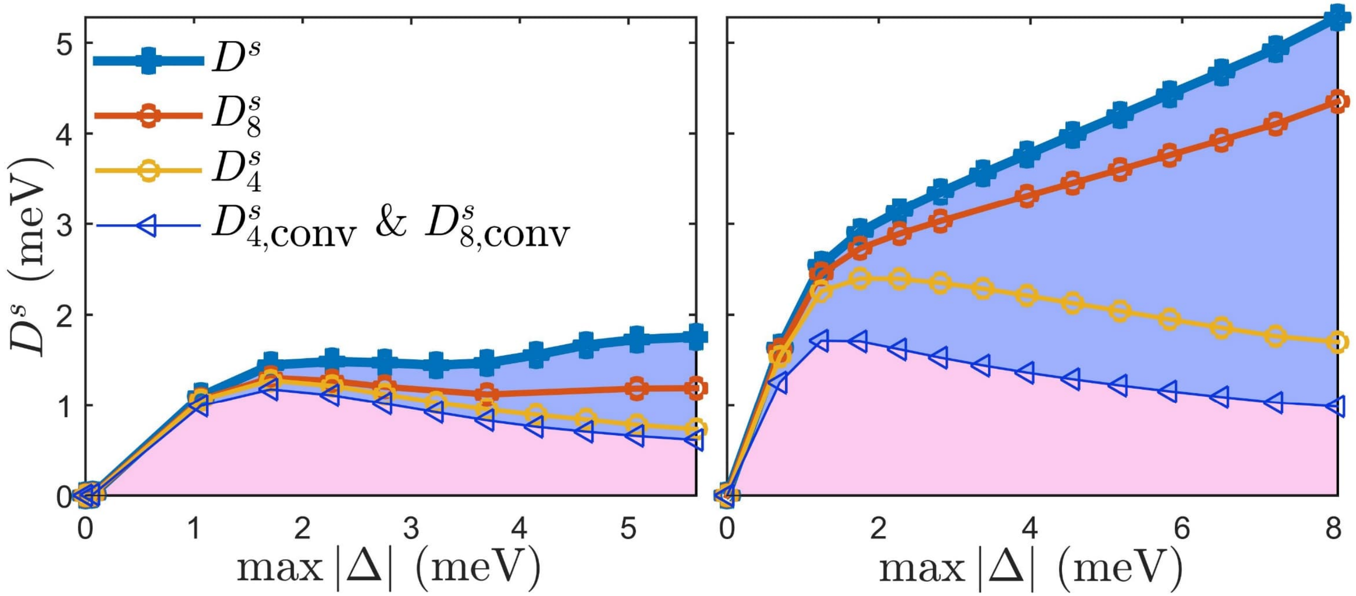
$$h_{ij} = c_{i\downarrow} c_{j\uparrow} - c_{i\uparrow} c_{j\downarrow}$$

J< 0 is attractive interaction strength

Geometric contribution in TBG

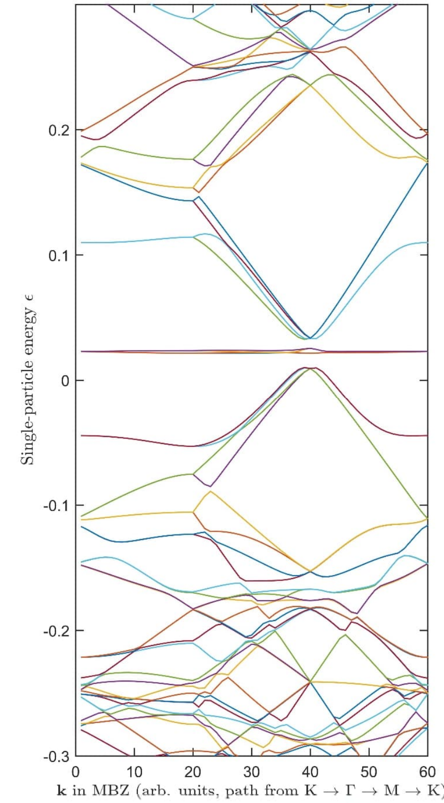
$$D^s = D^s_{\mathrm{conv}} + D^s_{\mathrm{geom}}$$

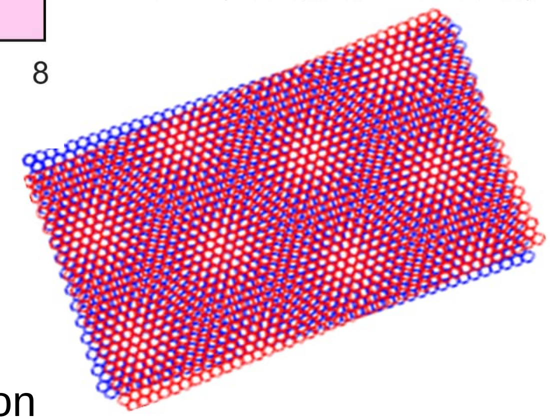
$$T_{\mathrm{BKT}} = \frac{\pi}{8}\sqrt{\det D^s(T_{\mathrm{BKT}})}$$



Non-local (RVB) interaction

Local (s-wave) interaction





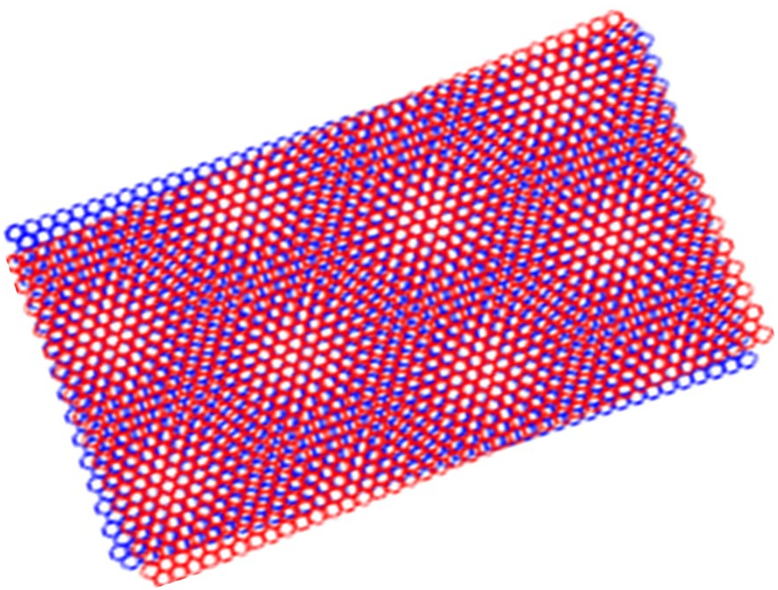
Julku, Peltonen, Liang, Heikkilä, PT, PRB(R) (2020); Editors’ Suggestion
Confirmed by (only s-wave): Hu, Hyart, Pikulin, Rossi, PRL (2019)

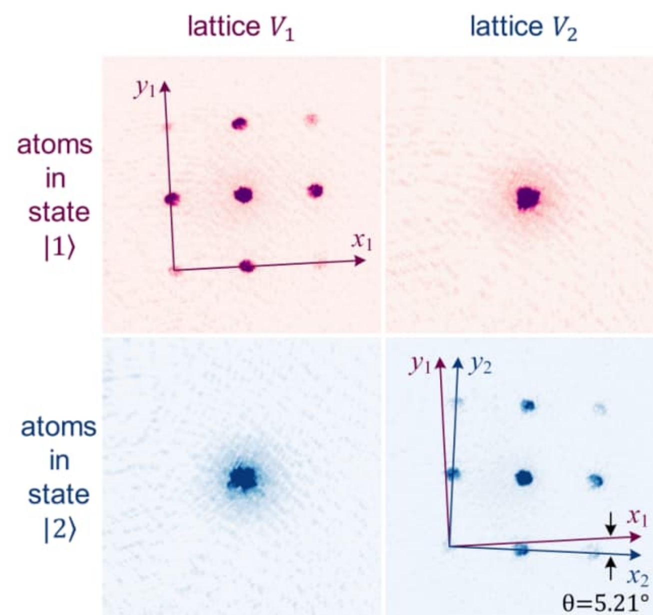
Review

Superconductivity, superfluidity and quantum geometry in twisted multilayer systems
PT, S. Peotta, B.A. Bernevig,
Nat. Rev. Phys. (2022)

$$\frac{1}{4\pi}\int_{\mathrm{BZ}} d^2k\,\mathrm{tr}\,g(\mathbf{k}) \geq \frac{1}{2\pi}\int_{\mathrm{BZ}'} d^2k\,|f_{xy}| \geq \left|\frac{1}{2\pi}\int_{\mathrm{BZ}'} d^2k\, f_{xy}\right| = |e_2|\,.$$

$$\det \mathcal{M}^{\mathrm{R}} \geq \mathcal{C}^2\,,\ \text{with}\ \mathcal{M}^{\mathrm{R}}_{ij} = \frac{1}{2\pi}\int d^2\mathbf{k}\, g_{ij}(\mathbf{k})$$





J. Zhang group

How to experimentally confirm geometric origin of flat band superconductivity?

Linear dependence on U
of critical temperature, pairing gap, superfluid density, etc.

Tunability of interactions (U) crucial: ultracold gases!

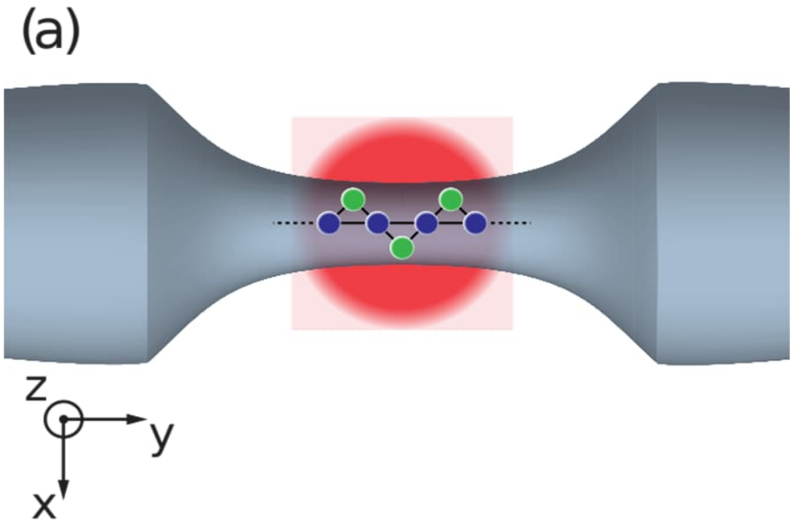
Flat band transport and Josephson effect through a saw-tooth lattice

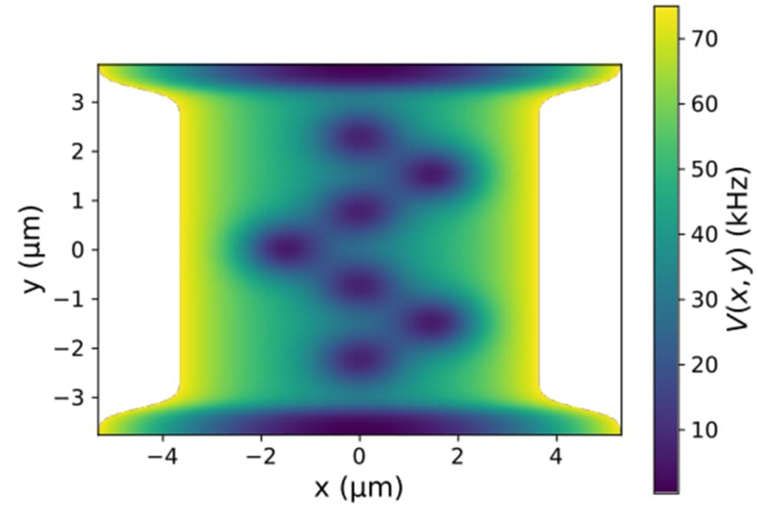
    

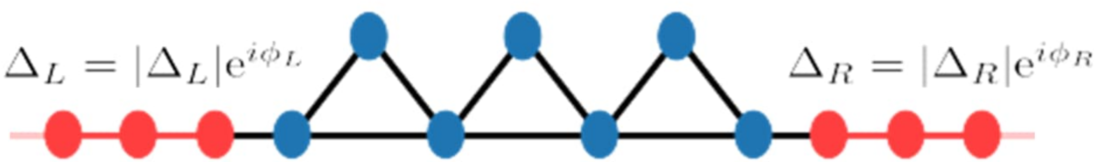
Ville Pyykkönen   Sebastiano Peotta   Philipp Fabritius   Jeffrey Mohan   Tilman Esslinger

Pyykkönen, Peotta, Fabritius, Mohan, Esslinger, PT, PRB (2021)

Possible experiment: Lithium lab, ETH Zürich



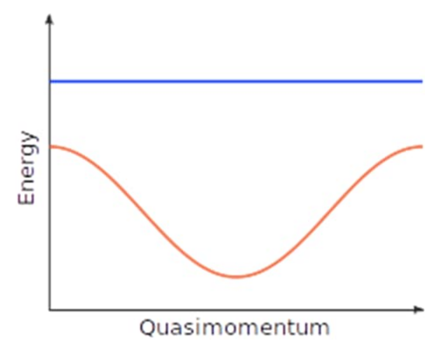




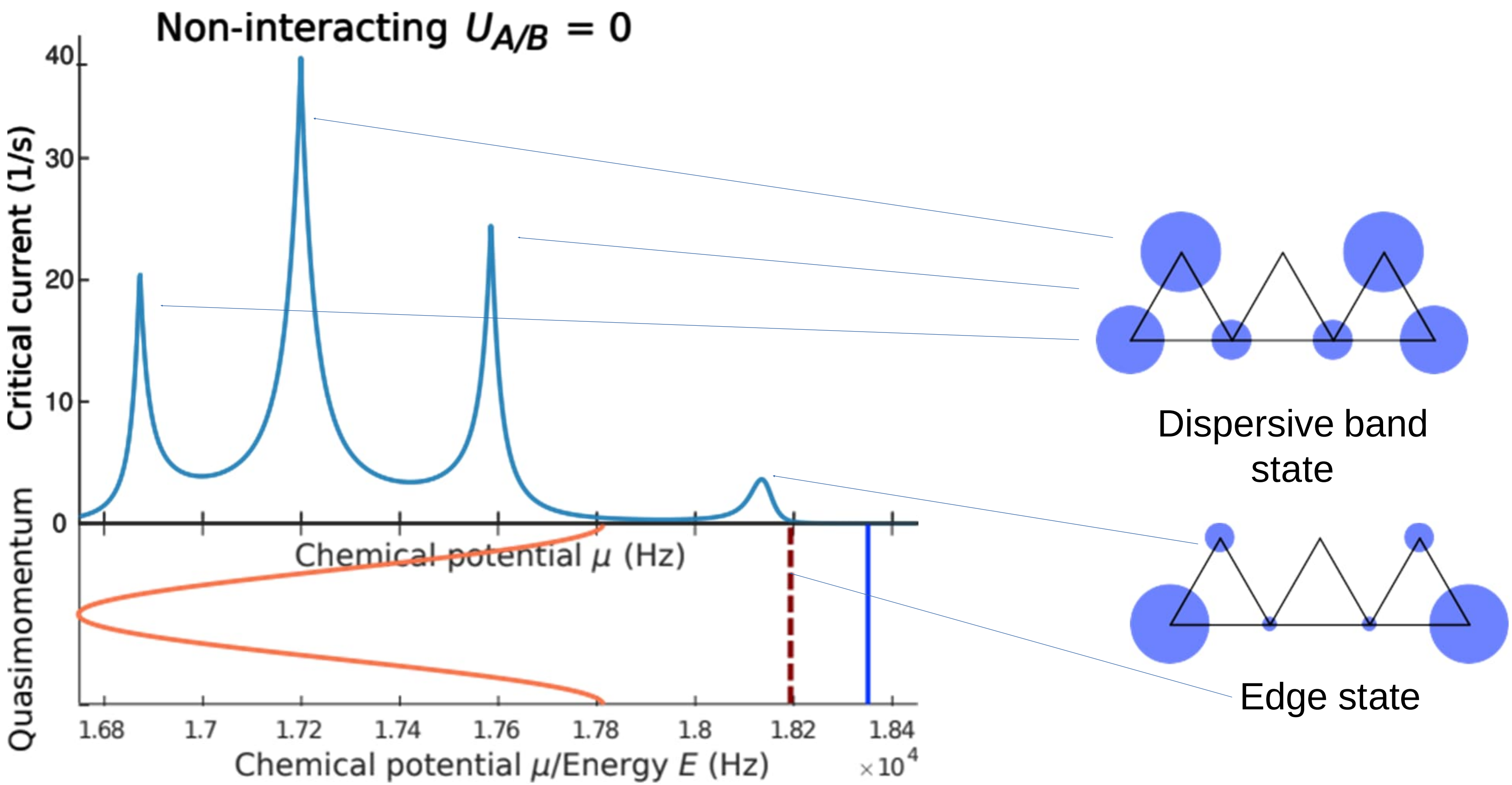
Mean-field fixed

Mean-field self-consistent

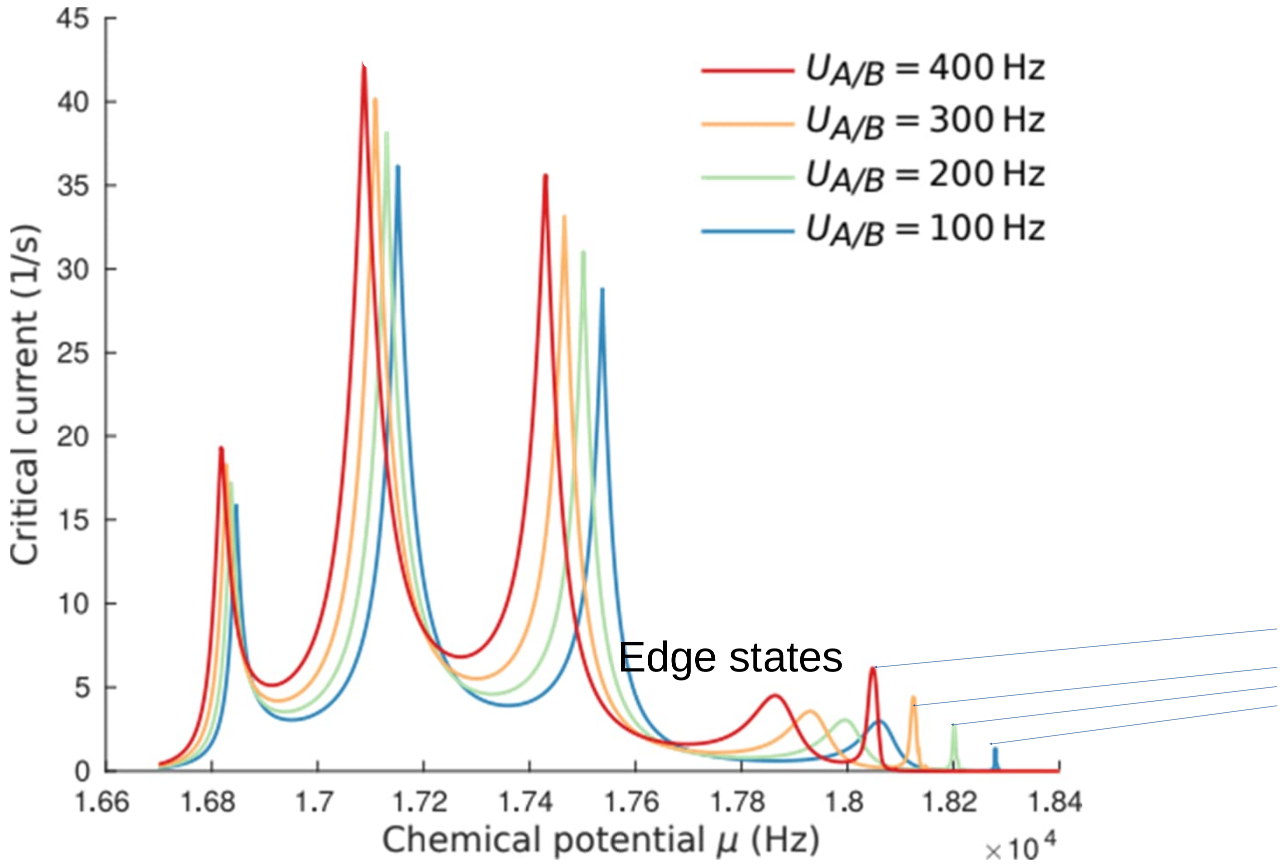
Mean-field fixed

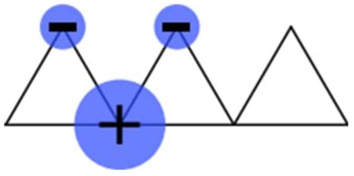


Results: Non-interacting transport



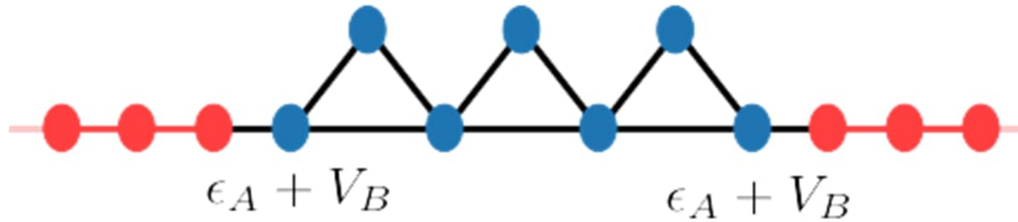
Results: Interacting transport

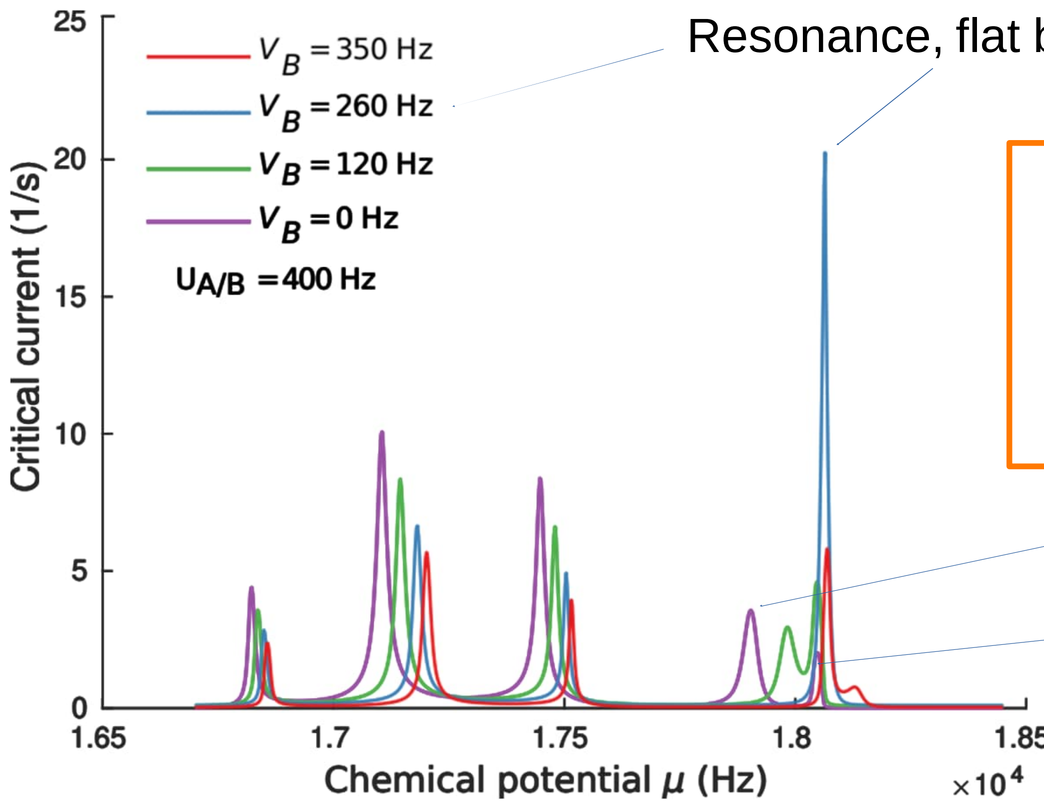




Flat band current finite and linear in U but small

Effect of boundary potential





Resonance, flat band and edge states

Degeneracy between flat band and edge states increases the flat band current!

Edge states

Flat band states

How to experimentally confirm geometric origin of flat band superconductivity?

Linear dependence on U
of critical temperature, pairing gap, superfluid density, etc.

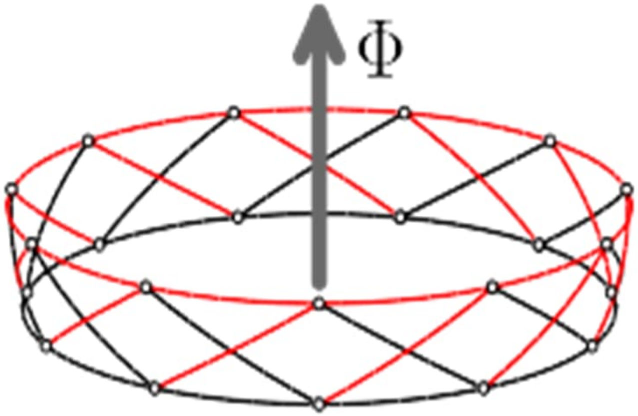
Tunability of interactions (U) crucial: ultracold gases!

But what about temperature…
Interesting effects visible already in the normal state!

Preformed pairs in a flat band

Tovmasyan, Peotta, Liang, PT, Huber, PRB 2018

What are the charge carriers in the ***normal state*** of a flat band superconductor?
We find: only pairs move (Pi-periodic ground state); non Landau-Fermi liquid.



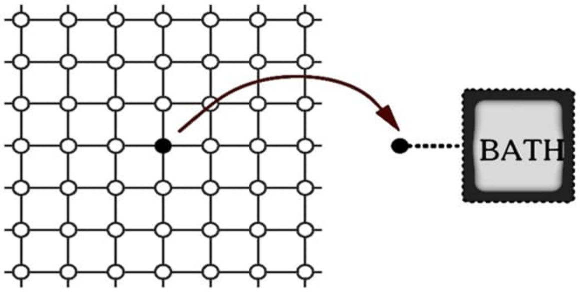
Aharonov-Bohm effect in a ring geometry

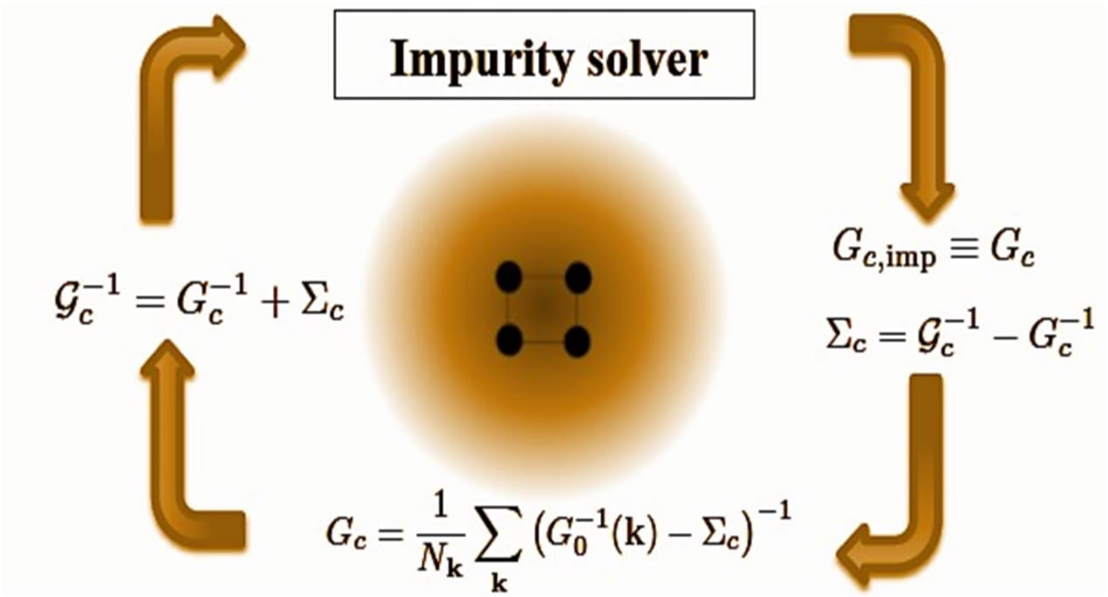
Insulator – pseudogap crossover in the Lieb lattice normal state

Kukka-Emilia Huhtinen

KE Huhtinen, PT, PRB(L) (2021)

Dynamical Mean Field Theory (DMFT) to capture quantum effects *beyond mean-field*

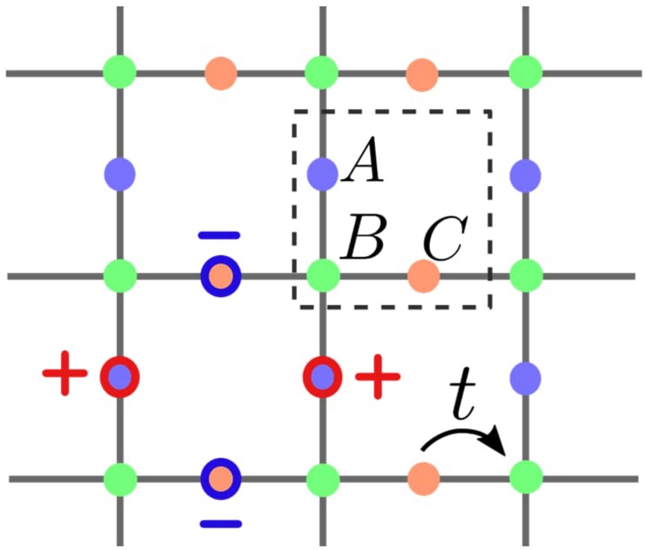




Single site DMFT

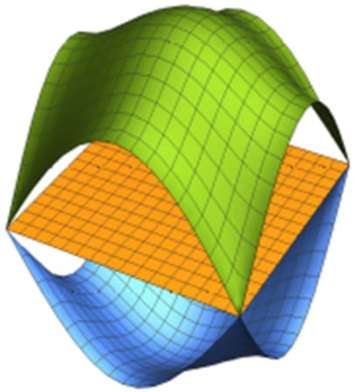
Cellular/cluster DMFT; Non-local correlations

Hubbard model on the Lieb lattice



Attractive Hubbard model

$$H = \sum_{\sigma} \sum_{i\alpha, j\beta} t_{ij} c^{\dagger}_{\sigma,i\alpha} c_{\sigma,j\beta} - \sum_{\sigma} \sum_{\mathrm{i}\alpha} \mu_{\sigma} n_{\sigma,i\alpha}$$
$$+U \sum_{i\alpha} (n_{\uparrow,i\alpha} - 1/2)(n_{\downarrow,i\alpha} - 1/2)$$

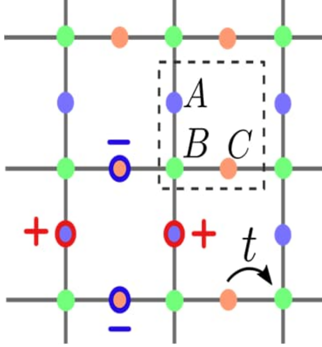


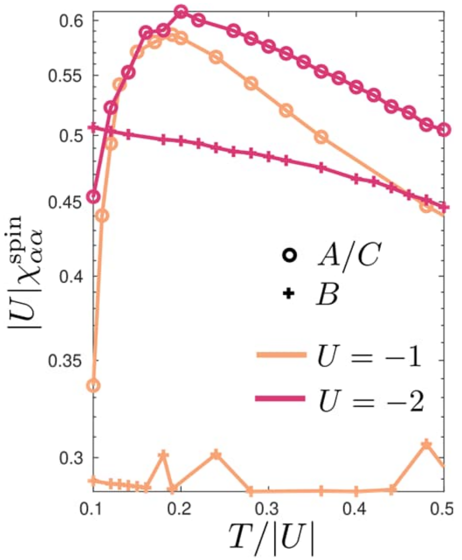
Flat band states reside at A and C sites

DMFT cluster: A, B and C

FOCUS ON THE NORMAL STATE ABOVE SUPERCONDUCTIVITY

Large (U>t) interactions: pseudogap





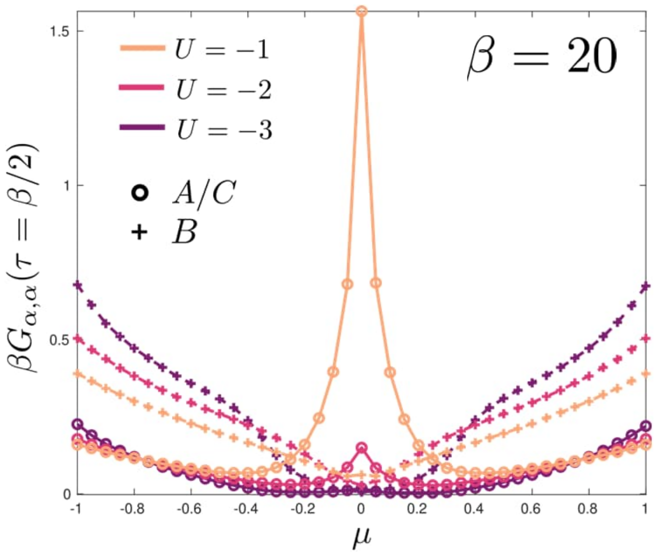
Generalized spin susceptibility:

$$\chi^{\rm spin}_{\alpha\alpha} = \frac{2}{\beta^2}\sum_{\omega,\omega'}\left(\chi^{{\rm ph},\omega,\omega',\nu=0}_{\uparrow\alpha,\uparrow\alpha,\uparrow\alpha,\uparrow\alpha} - \chi^{{\rm ph},\omega,\omega',\nu=0}_{\uparrow\alpha,\uparrow\alpha,\downarrow\alpha,\downarrow\alpha}\right)$$

$$\chi_{ijkl}(\tau_1,\tau_2,\tau_3) = G^{(4)}_{ijkl}(\tau_1,\tau_2,\tau_3) - G_{ij}(\tau_1,\tau_2)G_{kl}(\tau_3,0)$$

$$G^{(4)}_{ijkl}(\tau_1,\tau_2,\tau_3) = \langle T_\tau[c_i^\dagger(\tau_1)c_j(\tau_2)c_k^\dagger(\tau_3)c_l(0)]\rangle$$

$$G_{ij}(\tau_1,\tau_2) = \langle T_\tau[c_i^\dagger(\tau_1)c_j(\tau_2)]\rangle$$

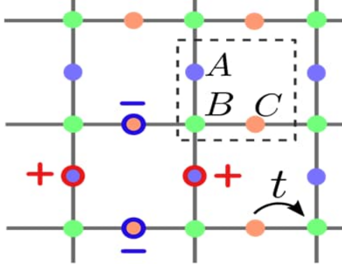


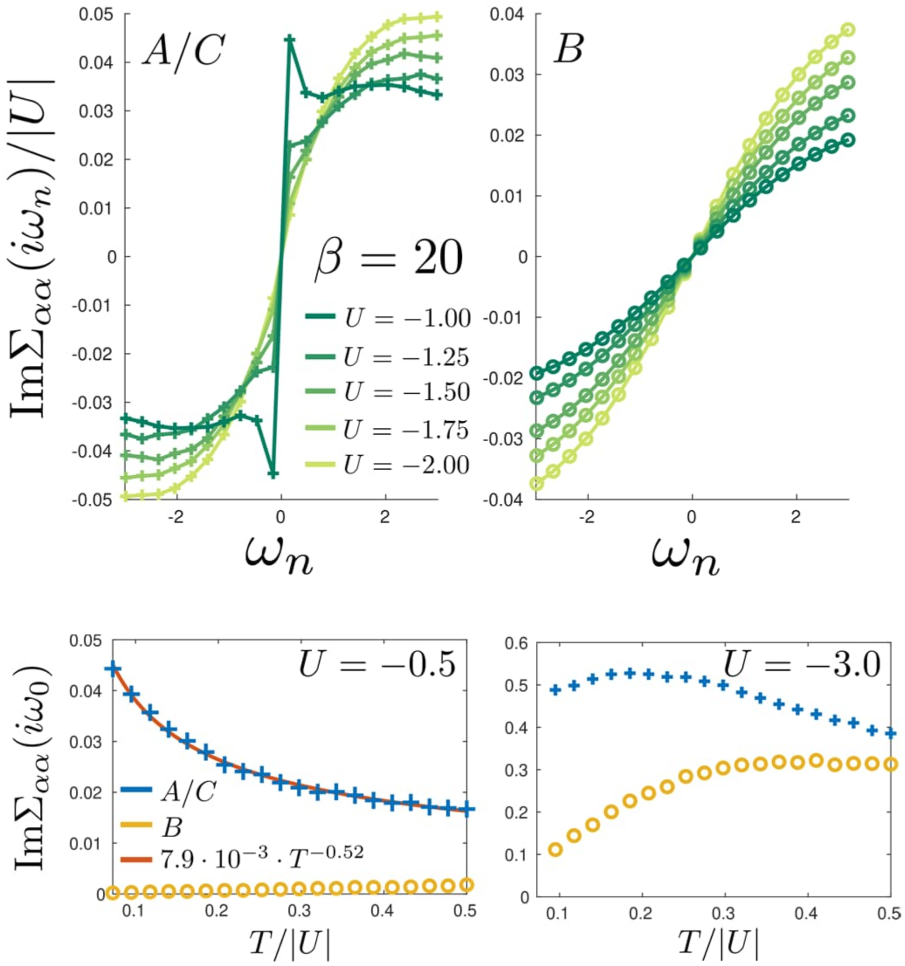
Local contribution to spin susceptibility decreases sharply with temperature at A/C sites.

At low temperatures, $\beta G_{\alpha\alpha}(\beta/2) \approx \mathcal{A}_\alpha(\omega=0)$, where $\mathcal{A}_\alpha$ is the orbital-resolved spectral function.

As interaction is increased, the spectral function becomes depleted around half-filling.

Low interaction (U<t): insulator





$$Z = \left(1 - \frac{\mathrm{Im}\Sigma(i\omega_n)}{\omega_n}\bigg|_{\omega_n \to 0}\right)^{-1}$$

In DMFT, $Z = m/m^*$, where m is the bare mass and m^* is the effective mass.

The self-energy diverges at low frequencies when the interaction strength is decreased.

The temperature dependence is $T^{-1/2}$ rather than T^{-1} found for Mott insulator.

Flat band interacting normal state; Lieb lattice

- **Non-Fermi liquid features in double occupancy and entropy**
- **SU(N) scaling relation**

Pramod Kumar

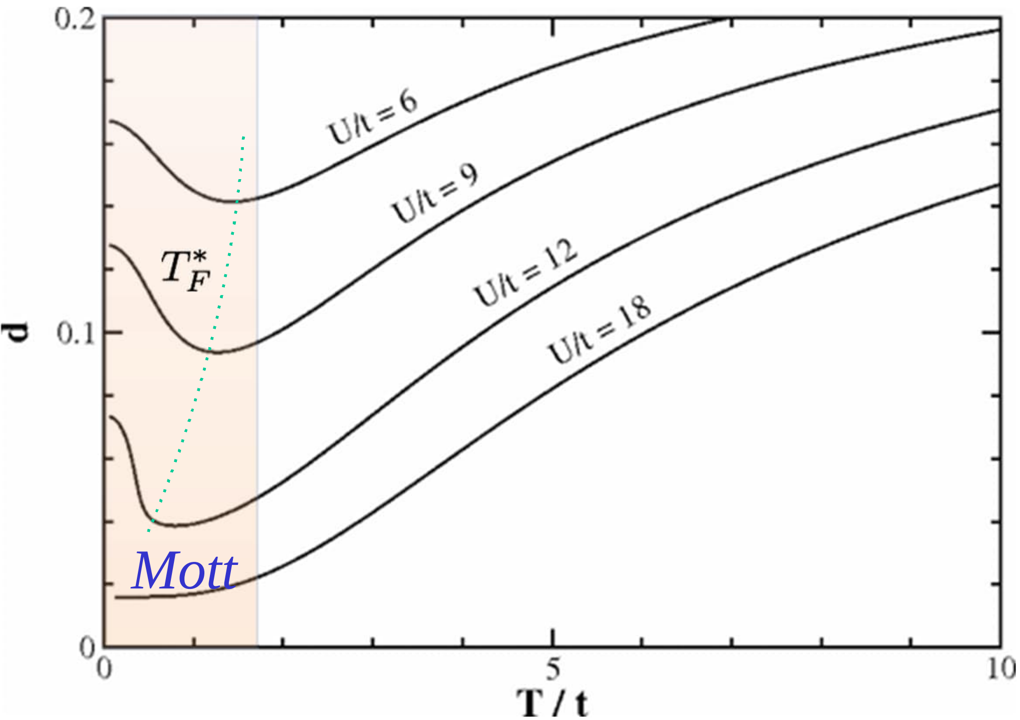
Sebastiano Peotta

Yosuke Takasu

Yoshiro Takahashi

P Kumar, S Peotta, Y Takasu, Y Takahashi, PT, PRB(L) 2021

Double occupancy and Fermi-liquid



Hubbard model on Bethe lattice

Werner, Parcollet, Georges, Hassan
PRL 95, 056401 (2005)

Entropy of Fermi-liquid

$$s \propto m_{eff} T$$

Maxwell's relation

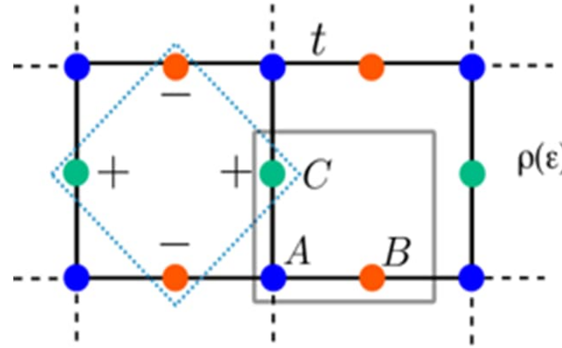
$$\frac{\partial s}{\partial U} = -\frac{\partial d}{\partial T}$$

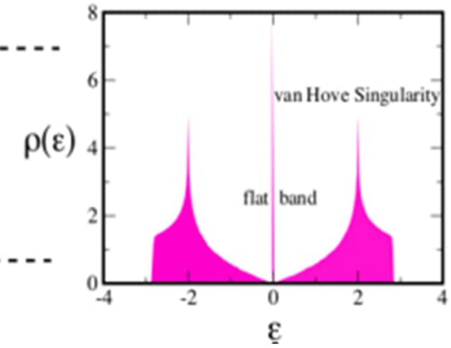
$T < T_F*$

$$\partial_T d < 0$$

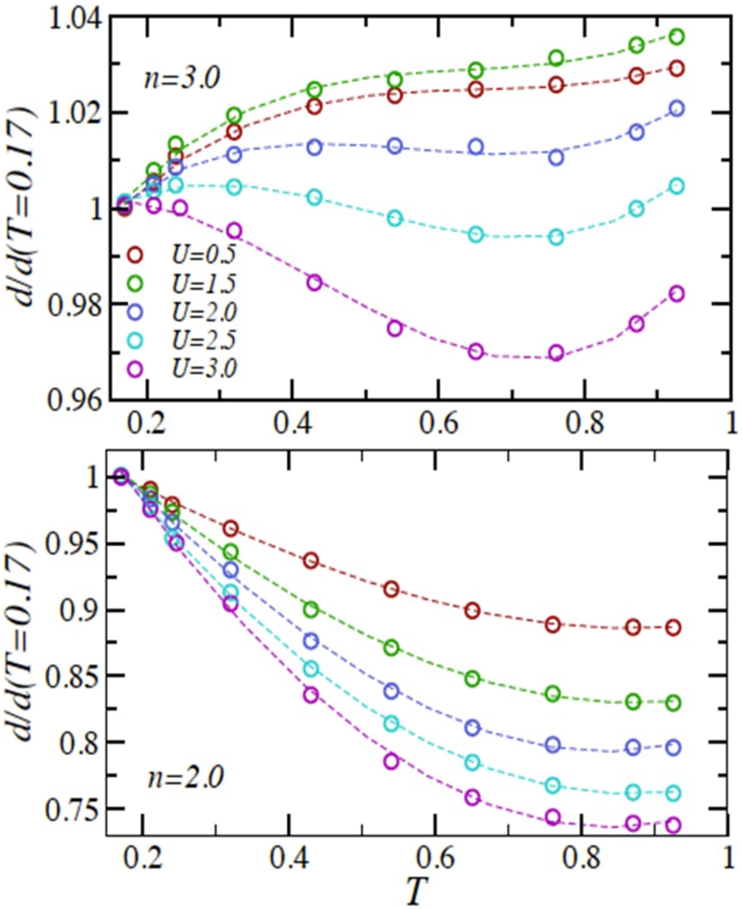
Lieb lattice: repulsive Hubbard model

Normal state properties





average double occupancy



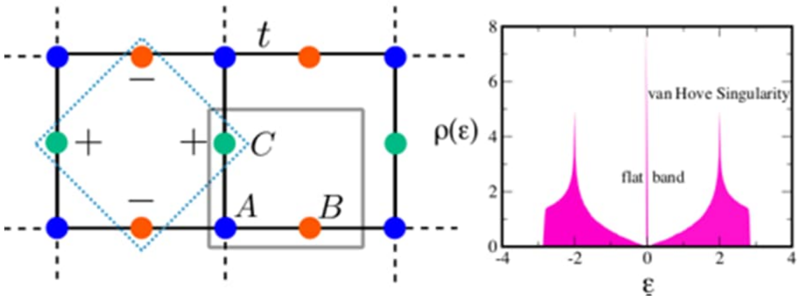
half-filling: flat band significant

Non-Fermi liquid behavior
for small interactions
at the flat band

lowest band filled

Multi-component SU(N) fermions in a Lieb lattice

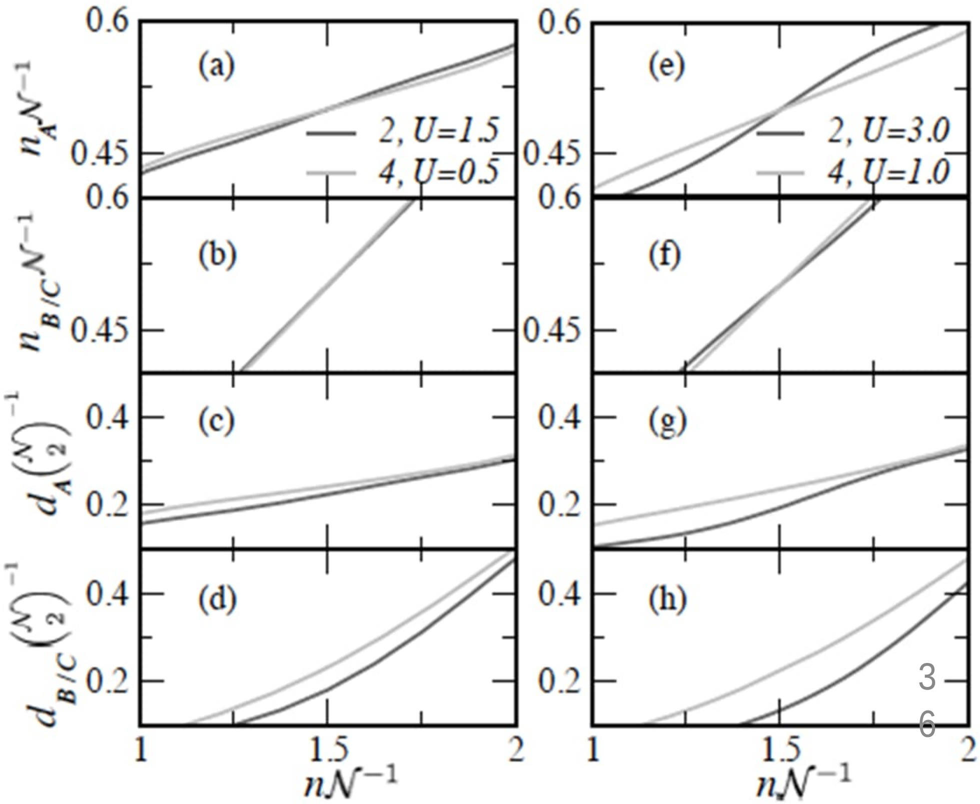
Yb^{174}



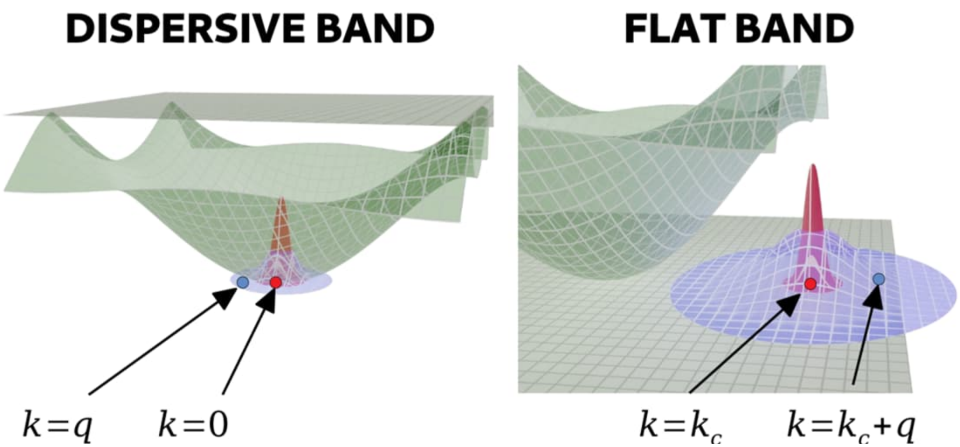
Scaling relation
(Mean-Field theory)

$$U(\mathcal{N}-1) = U'(\mathcal{N}'-1)$$

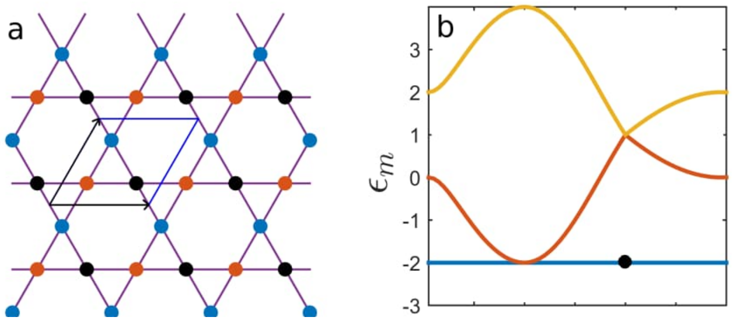
Scaling relation is consistent with DMFT for moderate interaction strength



Flat band BEC & quantum geometry



Kagome lattice:

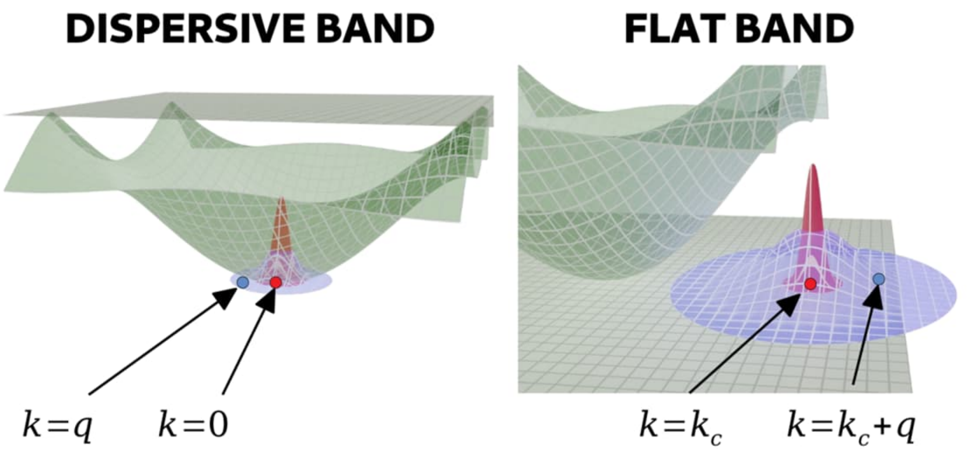


Aleksi Julku

Georg Bruun

Julku, Bruun, PT, PRL 2021, PRB 2021

Flat band BEC & quantum geometry



n_0 Condensate density n_e Excitation density U Interaction $u(k)$ Bloch function

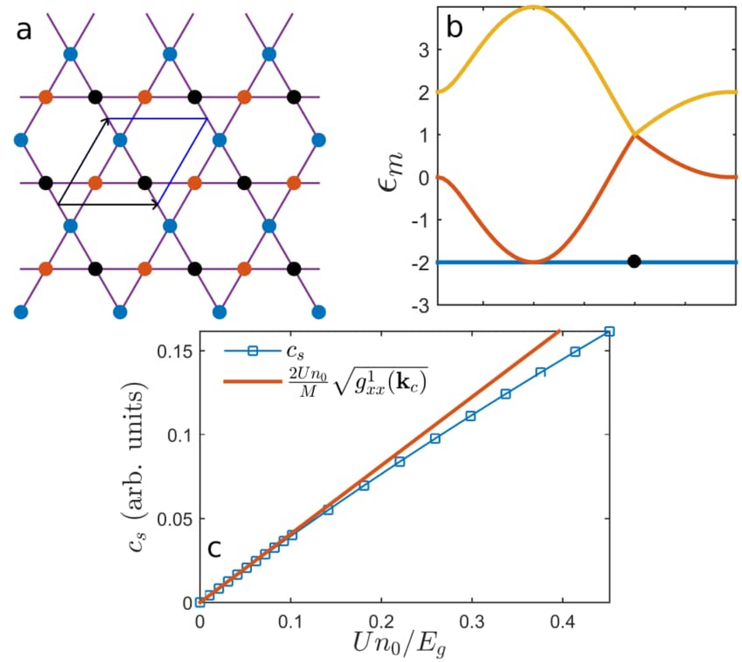
SPEED OF SOUND

$c_s \propto \sqrt{U n_0}$ $\quad$ $c_s \propto U n_0 \sqrt{g_{\alpha\beta}(k_c)}$

Quantum metric

$$g_{\alpha\beta} = \Re[\langle \partial_\alpha u | \partial_\beta u \rangle - \langle \partial_\alpha u | u \rangle \langle u | \partial_\beta u \rangle]$$

Kagome lattice:



Quantum metric dictates the speed of sound

Aleksi Julku

Georg Bruun

Julku, Bruun, PT, PRL 2021, PRB 2021

Flat band BEC & quantum geometry

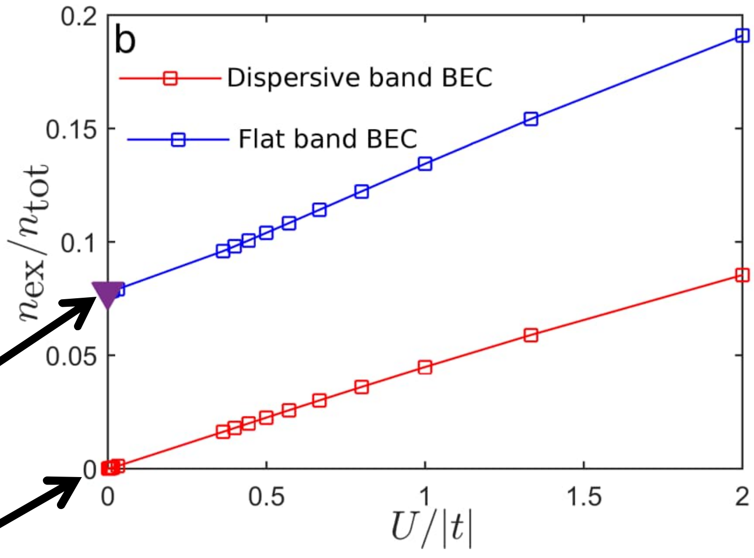
- Excitations do not cost energy? Can BEC stable?

Answer: Yes it can, finite **quantum distance** between Bloch states sets the limit for excitation density -> stable BEC

$$n_e(k) \xrightarrow[U\to 0]{} \frac{1-D}{2D}$$

Quantum distance

$$D=\sqrt{1-\left|\langle u(k_c+q)|u(k_c-q)\rangle\right|^2}$$

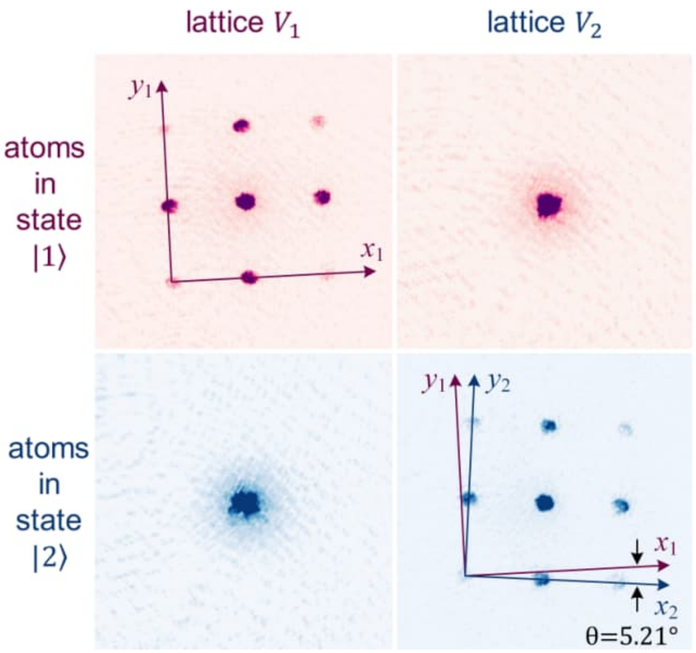


Excitation density can be finite in the non-interacting limit...

...in contrast to dispersive band BEC

Interaction effects prominent even in the limit of vanishing interactions

Summary



Quantum geometry governs
- flat band superfluidity
- normal state properties
- BEC excitations

Outlook

Towards room temperature superconductivity

Flat bands in ultracold gases; tunable interactions, quantum fluctuations