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Large Scale Structures and Neutrinos

ISAPP 2023: Neutrino physics, astrophysics and cosmology

July 4, 2023
Villa Monastero, Varenna

Relevant References

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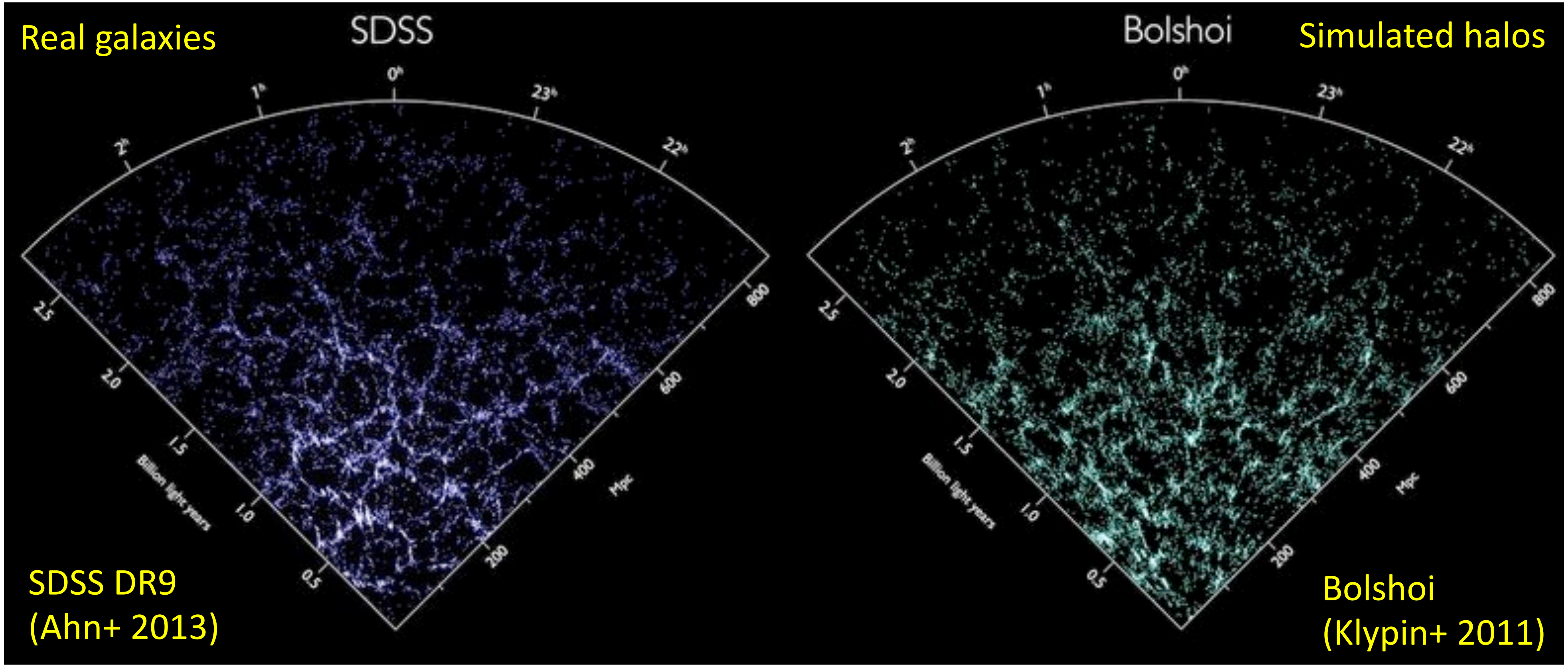
Layout

- Neutrino Cosmology: Recap
- Statistical Properties of the Large Scale Structures: galaxy clustering and (some) gravitational lensing.
- Current and future datasets (and their analysis)
- Neutrinoi constraints from LSS.

A simple upper limit to neutrino masses



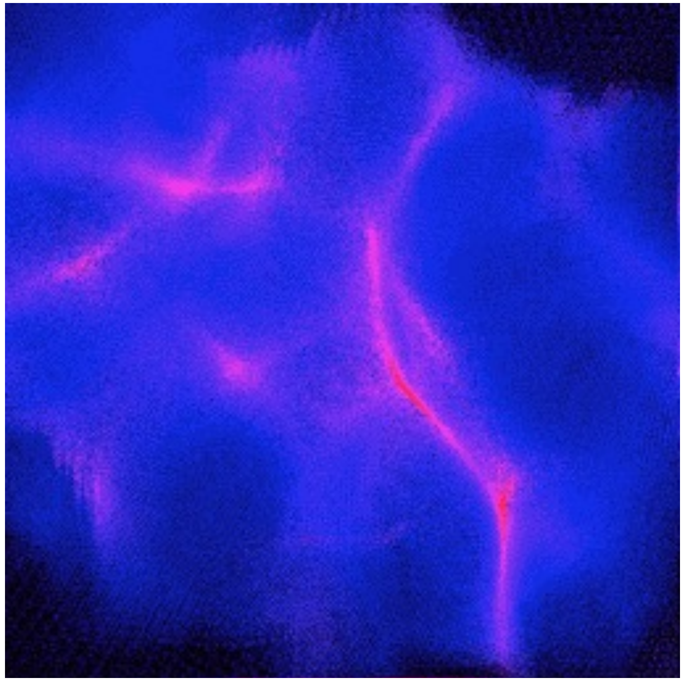
A simple upper limit to neutrino masses from the LSS

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} = \frac{8\pi G}{3H_0^2} n_{\nu,0} \sum m_{\nu,i}$$

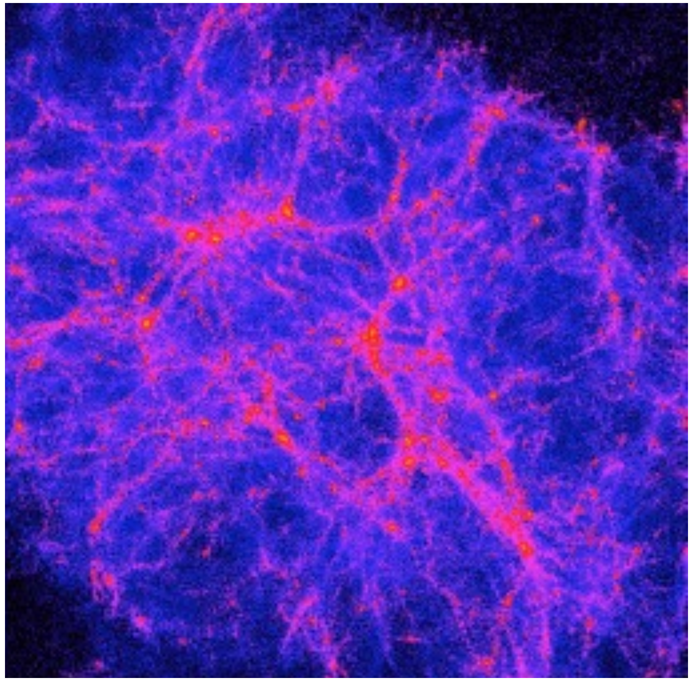
$$n_\nu = \frac{3}{11} n_\gamma = \frac{6\varsigma(3)}{11\pi^2} T_\gamma^3$$

$$\boxed{\Omega_\nu = \frac{\sum m_{\nu,i}}{93.14 h^2\ eV}}$$

Hot Dark Matter



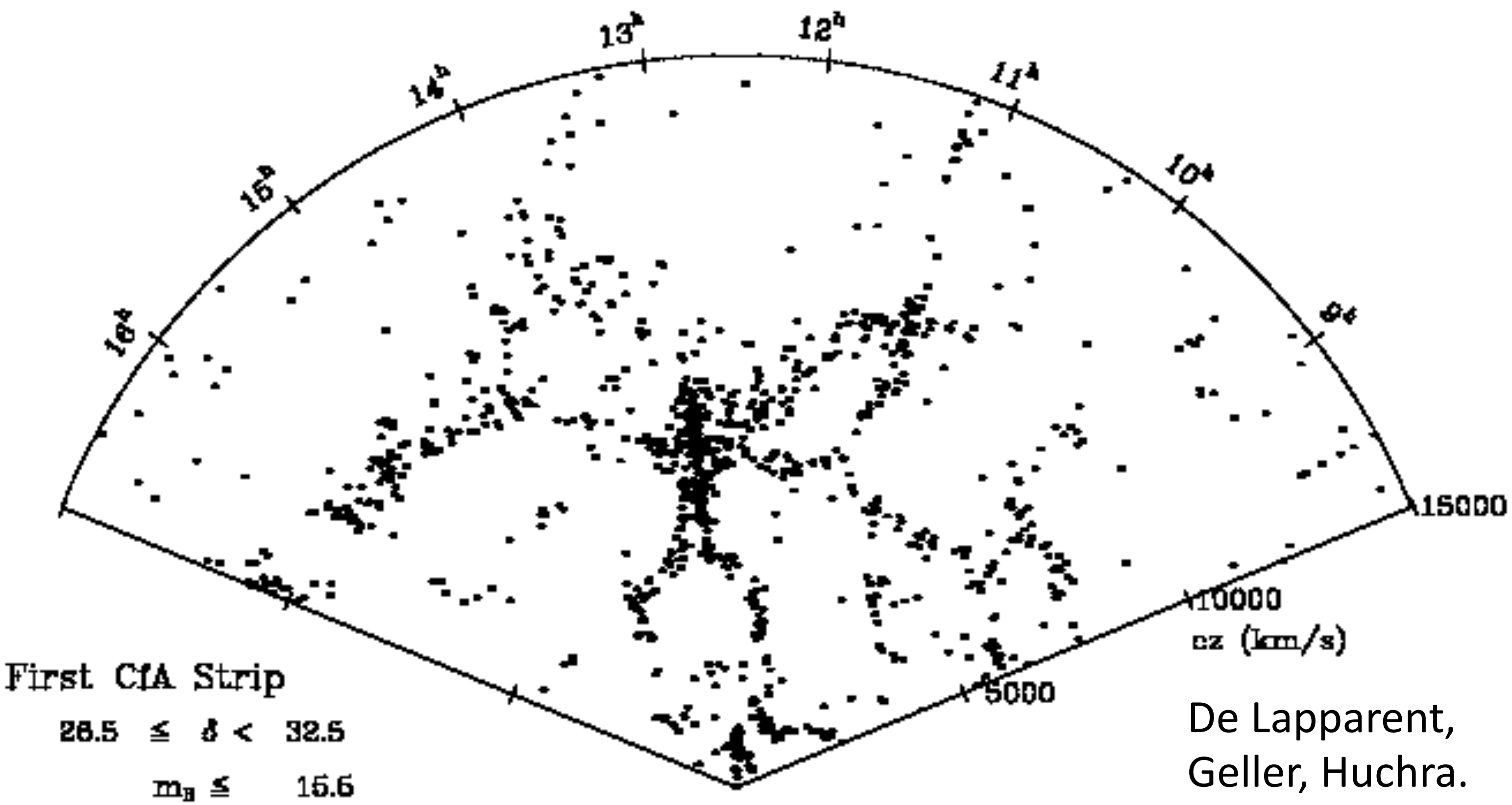
Cold Dark Matter



$$\Omega_\nu \leq \Omega_m \sim 0.3 \ \rightarrow \boldsymbol{m_{\nu,i} \leq 5\ eV}$$

A simple upper limit to neutrino masses from the LSS

Historical remark:
This upper limit could have been set in the '80s when the first galaxy redshift surveys, designed to trace the spatial distribution of matter in the Universe, were performed.



18.000 galaxies
m_B<14.5
cz<15.000 km/s

De Lapparent, Geller, Huchra. 1986

However, we know that active neutrinos have sub-eV masses.

What is the impact of sub-eV neutrinos on the large-scale structure of the Universe? Can observations of the LSS constrain neutrino masses ?

Neutrinos and cosmological perturbations

Recap

Neutrinos impact on cosmological perturbations: recap

Some terminology and frequently used symbols

γ=photons; ν=neutrinos; b=baryons; *CDM*=cold dark matter,
c=CDM+b; *m=CDM+b+*(ν) (for massive neutrinos)

$$\Omega_i = \frac{\bar{\rho}_i}{\rho_c} \;;\; \rho_c = \frac{3H^3}{8\pi G} \;;\; h = \frac{H_0}{100} \;;\; \omega_i = \Omega_i h^2 \;;\; f_\nu = \frac{\rho_\nu}{\rho_c + \rho_\nu} \;;\; \delta_i = \frac{\rho_i}{\bar{\rho}_i} - 1$$

Assumptions:

- I will often assume a flat ΛCDM model.
- I will focus on the neutrino mass rather than on their effective number, which is well constrained by CMB. And set N_{eff}=3.046.
- I will often assume full degeneracy in the neutrino mass hierarchy: $\sum m_{\nu,i} = 3m_\nu$

Neutrinos impact on cosmological perturbations: recap

Relevant Scales: equivalence

At equivalence, the energy density of the radiation (to which relativistic neutrino contributes) equals that of the matter:

$$\rho_r(a_{eq}) = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right]\rho_\gamma(a_{eq}) = \rho_c(a_{eq}) \equiv \rho_{CDM}(a_{eq}) + \rho_b(a_{eq})$$

Since ρ_r scales as a^{-4} and ρ_c scales as a^{-3} then $\left(\frac{a_{eq}}{a_0}\right) = \frac{\rho_{r,0}}{\rho_{c,0}}$

If neutrino are massless: $\rho_{c,0} = \rho_{m,0}$ and $\left(\frac{a_{eq}}{a_0}\right) = \frac{\rho_{r,0}}{\rho_{m,0}}$

If neutrino are massive: $\rho_{c,0} = \rho_{m,0} - \rho_{\nu,0} = \rho_{m,0}(1 - f_\nu)$ and $\left(\frac{a_{eq}}{a_0}\right) = \frac{\rho_{r,0}}{\rho_{m,0}}(1 - f_\nu)^{-1}$

As a result, the epoch of equivalence is delayed: $\boxed{a_{eq}^{f_\nu \neq 0} = a_{eq}^{f_\nu = 0}(1 - f_\nu)^{-1}}$

NEUTRINOS IMPACT ON COSMOLOGICAL PERTURBATIONS: RECAP

Relevant Scales: non-relativistic transition

If neutrinos are massive then they are relativistic early on, when their rest mass equals their thermal energy $m_\nu = T_\nu$

Since $T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma$ the transition occurs at $1 + z_{nr} = 1890 \frac{m_\nu}{1eV}$

Which, for a sub-eV m_ν, is after baryon-radiation decoupling, presumably at $z_{nr} \leq 100.$

The energy density of massive neutrinos in the relativistic and non-relativistic limit is:

$$\rho_\nu = \frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3} T_\gamma^4 \qquad (m_\nu \ll T_\nu)$$

$$\rho_\nu = m_\nu n_\nu \qquad (m_\nu \gg T_\nu)$$

And should be estimated numerically in the intermediate regime.

NEUTRINOS IMPACT ON COSMOLOGICAL PERTURBATIONS: RECAP

Relevant Scales: the free streaming scale 1

Neutrinos behave as a collisionless fluid with a characteristic, average thermal velocity v_{th}. As a result, they travel a typical distance λ_{fs} that defines the free-streaming scale:

For a particle that propagates along radial geodesics with velocity v_{th} in a flat Friedmann Universe $v_{th}dt = a(t)dr$. The distance traveled in the interval $[t_i, t]$ is therefore:

$$\lambda_{fs} = a(t)\int_{t_i}^{t} dr = a(t)\int_{t_i}^{t} \frac{v_{th}dt'}{a(t')}$$

The same definition is used for the (causal) particle horizon (with $v_{th} \rightarrow\ c = 1$) and for the Jeans length of a collisional fluid (with $v_{th} \rightarrow\ c_s$, sound speed).

During either matter domination or radiation domination epochs and for $t \gg t_i$:

$$\lambda_{fs}(t) = 2\pi\sqrt{\frac{2}{3}\frac{v_{th}(t)}{H(t)}} \;\; ; \; k_{fs}(t) = \frac{2\pi}{\lambda_{fs}(t)}$$

NEUTRINOS IMPACT ON COSMOLOGICAL PERTURBATIONS: RECAP

Relevant Scales: the free streaming scale 2

Below the free-streaming scale density fluctuations in the neutrino component are damped. On the contrary, fluctuations larger than this scale the neutrino thermal velocity can be effectively ignored and, after non-relativistic transition, perturbations in the neutrino behave like those in the cold dark matter component.

As long as neutrinos are relativistic $v_{th} = c$, the free-streaming scale coincides with the Hubble radius and increases accordingly ($\lambda_{fs} \propto a^{3/2}$).

After the non-relativistic transition, their thermal velocity decays $v_{th} \propto T_\nu \propto a^{-1}$. As a result, the free-streaming scale increases more slowly, $\lambda_{fs} \propto a^{1/2} = 7.7 \frac{1+z}{\sqrt{\Omega_{\Lambda +} \Omega_m (1+z)^3}} \left(\frac{1\ eV}{m_\nu}\right) h^{-1}\ Mpc$

The comoving free-streaming length, λ_{fs}/a, increase before the non-relativistic transition and then decrease, reaching a maximum at z_{nr} when $\lambda_{fs}(a_{nr}) = 0.32 \left(\frac{m_\nu}{1\ eV}\right)^{-1/2} h^{-1} Mpc$

Neutrinos impact on cosmological perturbations: recap

Perturbation in the linear regime: massless neutrinos 1

Let us describe the qualitative behavior of the fluctuations in all components.

- Cold Dark Matter. Outside the particle horizon ($\approx 1/H(t)$) all density fluctuations, including the CDM ones, are frozen. Inside the horizon, their scale is typically larger than the Jeans' length and can grow by gravitational instability. During radiation domination, they grow slowly ($\delta_{CDM} \propto \ln(a)$) whereas during matter domination they grow faster ($\delta_{CDM} \propto a$).
- Photons. Frozen outside the horizon. Inside horizon and as long as they're coupled to baryons they oscillate. After decoupling they free-
- Baryons Before decoupling they closely follow the photons' evolution. After decoupling they couple gravitationally to CDM and their fluctuations track δ_{CDM}.
- Neutrinos. Frozen outside the horizon. Damped inside the horizon because of free streaming.

Perturbation in the linear regime: massless neutrinos 2

NEUTRINOS IMPACT ON COSMOLOGICAL PERTURBATIONS: RECAP

Instead of describing fluctuations at specific locations, i.e. the fluctuations field in configuration space $\delta(\boldsymbol{x})$, we consider fluctuations in Fourier space.

In a finite volume V_u :

$$\delta(\boldsymbol{x}) = \sum_{\boldsymbol{k}} \delta_{\boldsymbol{k}} \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}) = \sum_{\boldsymbol{k}} \delta^*_{\boldsymbol{k}} \exp(-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x}),$$

Where $\delta_{\boldsymbol{k}}$ are the complex Fourier coefficients with wavenumbers $\boldsymbol{k}$ for which

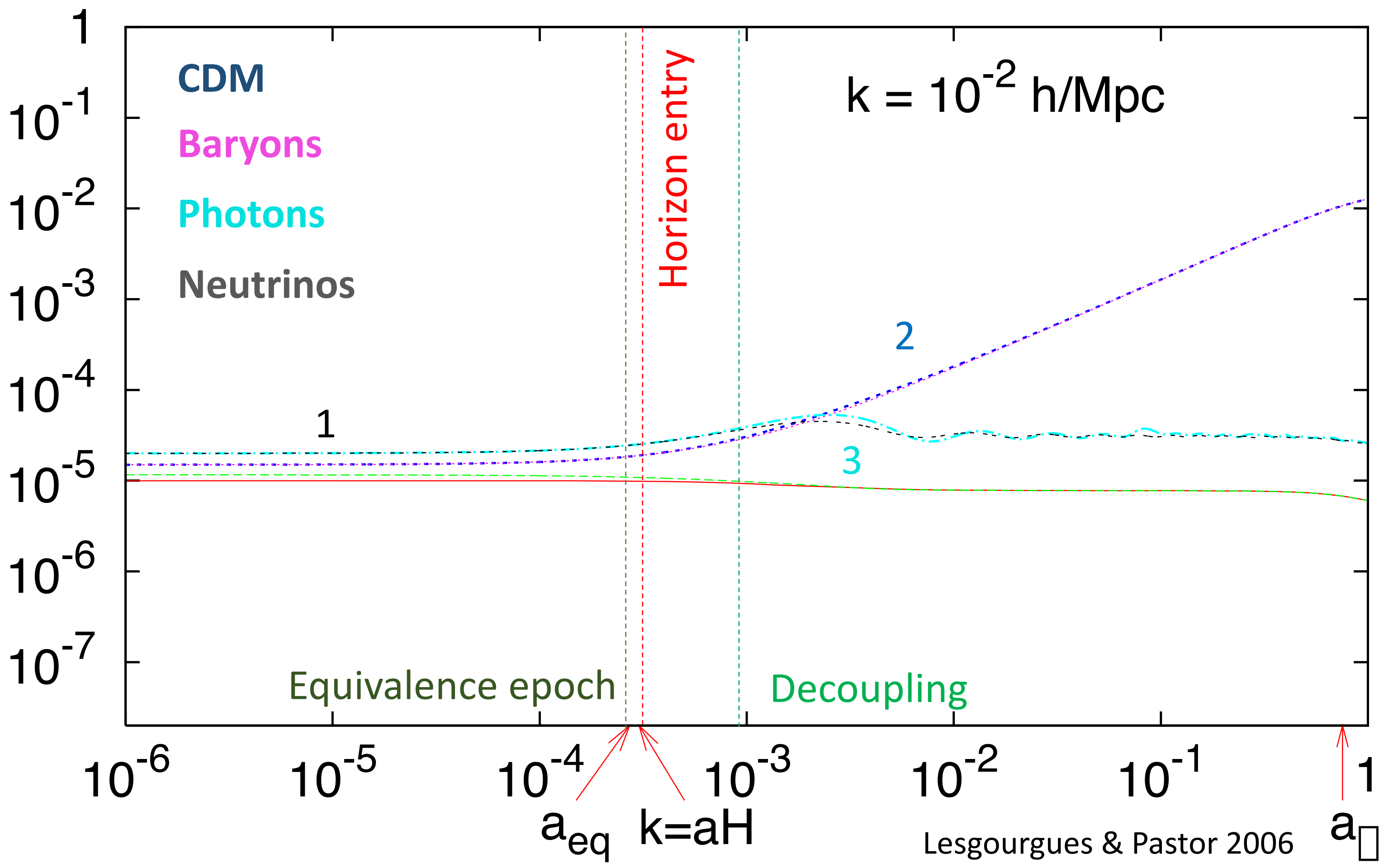
$$\delta_{\boldsymbol{k}} = \frac{1}{V_\mathrm{u}} \int_{V_\mathrm{u}} \delta(\boldsymbol{x}) \exp(-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{x})\, \mathrm{d}\boldsymbol{x}$$

In the linear regime, the Fourier coefficients $\delta_{\boldsymbol{k}}$ evolve independently. It is convenient to use them to characterize the evolution of fluctuations of different scales.

Neutrinos impact on cosmological perturbations: recap

Perturbation in the linear regime: massless neutrinos 3

1. All perturbations are frozen before horizon entry.
2. CDM and baryons fluctuations enter the horizon during matter domination. Their scale is larger than the Jeans scale and therefore they grow via gravitational instability.
3. Photons and neutrinos fluctuations grow less than CDM+baryons and free stream after decoupling.

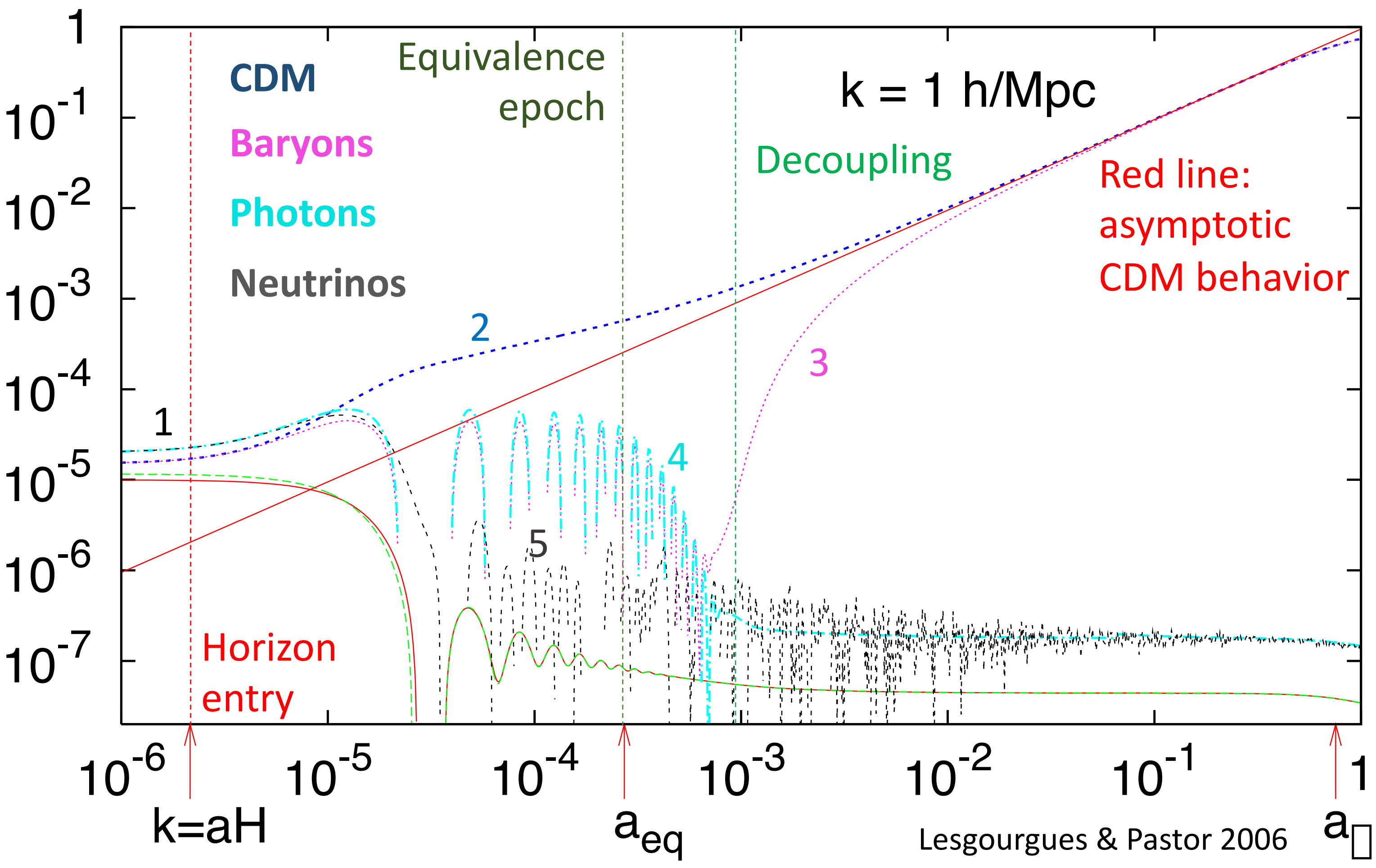


Lesgourgues & Pastor 2006

Neutrinos impact on cosmological perturbations: recap

Perturbation in the linear regime: massless neutrinos 4

1. Super-horizon perturbations are frozen.
2. CDM fluctuation enters the horizon during radiation domination. Slow growth until matter domination. Then it grows $\propto a$.
3. Baryon fluctuation oscillates with photons until decoupling. Then rapid growth.
4. Photon fluctuation oscillates and then free streams.
5. Neutrinos fluctuations are damped by free streaming.



Lesgourgues & Pastor 2006

Perturbation in the linear regime: massive neutrinos 1

Differences between massive and massless cases.

- Cold Dark Matter. Outside the particle horizon ($\approx 1/H(t)$) all density fluctuations, including the CDM ones, are frozen. Inside the horizon, the growth of perturbations is delayed because equality occurs later by a fraction $(1-f_\nu)^{-1}$. It is also suppressed since the presence of an almost homogeneous distribution of neutrinos ($\delta\rho_\nu \ll \delta\rho_{CDM}$) that do not contribute to the gravitational potential, slows down the growth ($\delta_{CDM} \propto a^{(1-3/5 f_\nu)}$).
- Photons. *If the total mass density is kept constant* and neutrinos are added to the balance, then the baryon-to-photo ratio is reduced and oscillations are less pronounced. However, since equality is delayed the damping of fluctuations is reduced. These two effects partly compensate each other. After decoupling photons free-stream.
- Baryons. As long as they are coupled to photons, they closely follow their evolution. After decoupling they couple gravitationally to CDM and track δ_{CDM}.

Neutrinos impact on cosmological perturbations: recap

Perturbation in the linear regime: massive neutrinos 2

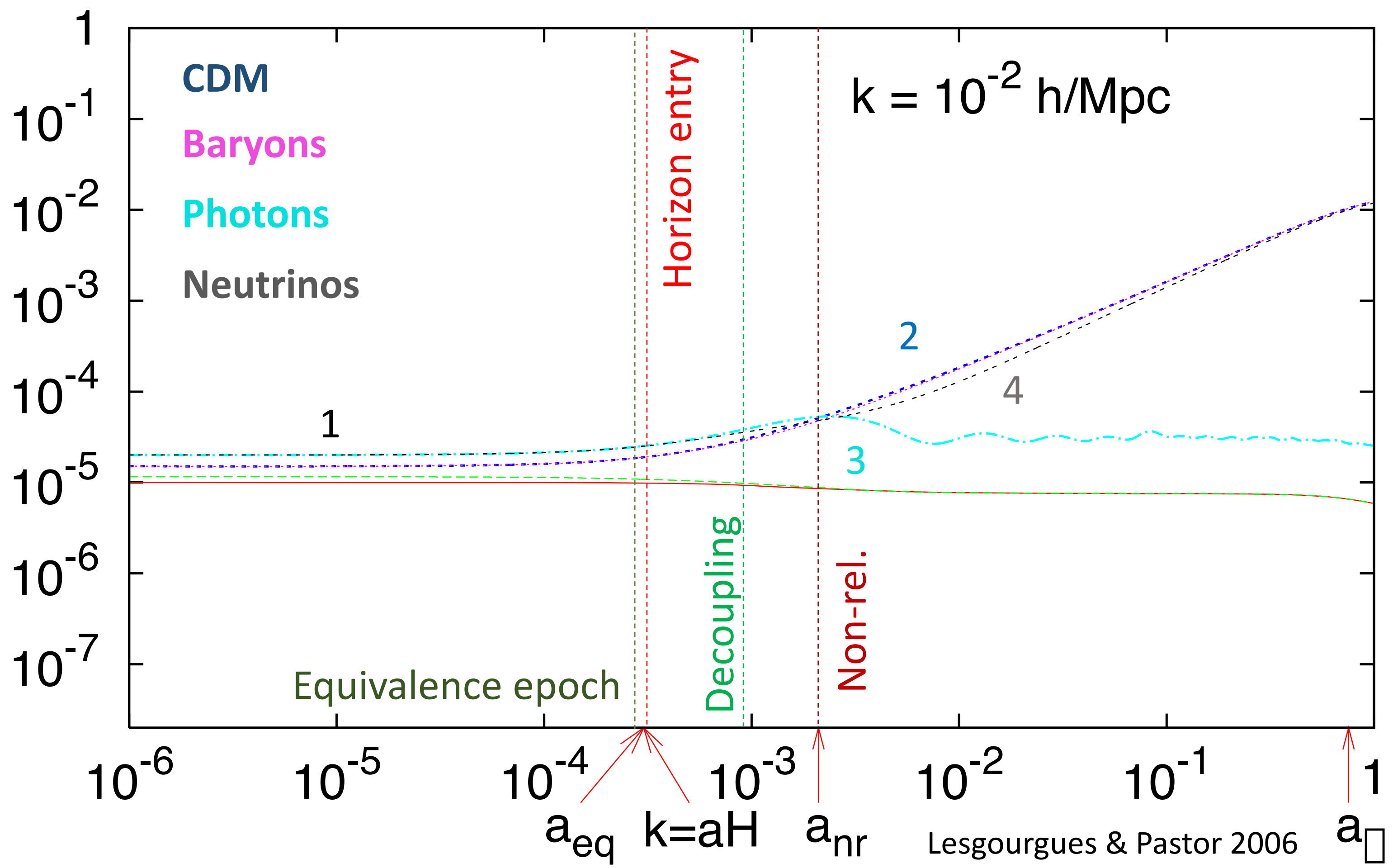
Differences between massive and massless cases.

- Neutrinos. Frozen outside the horizon. I
  - f the perturbation enters the horizon after z_{nr} then: $\delta_\nu \sim \delta_{CDM}$ before horizon entry and then δ_ν tracks δ_{CDM}.
  - If the perturbation enters the horizon before z_{nr} then it is damped by free streaming and then, after z_{nr}, it starts growing to asymptotically match δ_{CDM}.

Neutrinos impact on cosmological perturbations: recap

Perturbation in the linear regime: massive neutrinos 3

1. All perturbations are frozen before horizon entry.
2. CDM and baryons fluctuations enter the horizon during matter domination and grow via gravitational instability.
3. Photons' fluctuation grows less than CDM+baryons and quickly free stream after decoupling.
4. Neutrinos' fluctuation enters the horizon after they are non-relativistic. Then tracks CDM+b.



Neutrinos impact on cosmological perturbations: recap

Perturbation in the linear regime: massive neutrinos 4

1. Super-horizon perturbations are frozen.
2. CDM fluctuation enters the horizon during radiation domination. Slow growth until matter domination. Then it grows $\propto a^{(1-3/5 f_\nu)}$
3. Baryon fluctuation oscillates with photons until decoupling. Then rapid growth.
4. Photon fluctuation oscillates and then free streams
5. Neutrinos fluctuations damped until a_{nr} then growth.